

Transport Theory for Quark-Gluon Plasma based on an Operator Field Langevin Equation

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1 Introduction

The quark-gluon plasma (QGP) phase is expected to be produced in high energy nuclear collision, and experiments aiming to confirm QGP are actively performed[1]. In particle physics, the fundamental dynamics governing a world of quarks and gluons is known to be a quantum field theory called quantum chromodynamics (QCD). The whole collision process producing QGP is a very complicated problem of many body with space-time evolution. To analyze such a process theoretically, we must establish a hierarchy of characteristic space-time scales of physics and introduce appropriate *coarse graining*. In the previous works we discussed the following three-level hierarchy in approaches to the problem[2]: (i) The *phenomenological* level described by hydrodynamical equation, (ii) the *semi-phenomenological* level controlled by a Langevin equation and (iii) the *fundamental* level governed by QCD.

For a complex system such as QGP, the phenomenological approach gives us only a little information on detailed properties of dynamics, while it is quite a long way to reproduce the physics of QGP from fundamental QCD. We emphasize that the approach at the *semi-phenomenological level* is the most realistic for QGP to input experimental data into theoretical calculation or to compare experiment with theory.

More concretely we set up a Langevin equation for a dominant mode in QGP in the *semi-phenomenological* approach. Then the phenomenological parameters appearing in the hydrodynamical approach are written down as functions of the parameters in this Langevin equation. On the other hand the parameters in the Langevin equation are to be calculated from the fundamental parameters in QCD. Thus semi-phenomenological approach (the Langevin approach) bridges between phenomenology with fundamental ones. The purpose of this paper is to describe the whole program to relate the three hierarchical approaches with each other, emphasizing a central role of the semi-phenomenological one into the center.

2 Semi-Phenomenological Approach

2.1 Langevin Equation in Classical Mechanics

Let us see the well-known approach of Langevin equation in classical mechanics[3]. There the fundamental dynamics is Newton's equation of motion,

$$m \frac{d^2}{dt^2} x(t) = F(t). \quad (1)$$

After performing a *coarse graining* in appropriate time scale, we obtain the Langevin equation,

$$m \frac{d^2}{dt^2} x(t) = -\gamma \frac{d}{dt} x(t) + F_r(t) + f(t), \quad (2)$$

where the original force $F(t)$ is divided into the parts responsible for systematic motion (the friction term $-\gamma \frac{dx(t)}{dt}$ and the residual force $F_r(t)$) and the part responsible for fluctuating motion around it (the random force $f(t)$). Equation (2) is a basis of the semi-phenomenological approach in classical mechanics.

The classical Langevin equation (2) is also closely related to phenomenology such as transport phenomena, that is entropy production. For example, the Langevin equation describing a classical Brownian particle in a uniform electric field gives the Ohmic law as its averaged motion, and from the Ohmic law we can easily evaluate Joule heat, which means that we can discuss entropy production. This way, in classical mechanical region, the Langevin equation is a very powerful tool for clear understanding of the phenomena.

2.2 Necessity for Quantum Langevin Equation in QGP

A quark-gluon plasma produced in high energy nuclear collision must contain many space-time hierarchies and we should introduce several steps of coarse graining. We then expect that at an appropriate time scale an operator $a(\mathbf{k}, t)$ representing a dominant mode in a quark-gluon plasma could be described by a Langevin type equation, which may be called an *operator-valued Langevin equation* or a *quantum Langevin equation*.

As is well-known, the consistent formulation of quantum Langevin equation is not so easy as that of classical Langevin equation[4]. Furthermore, in QGP physics one needs the quantum Langevin formulation for *field operators*, because the dynamics of interest is not for particle but for field. We will describe our formulation of quantum Langevin equation for a field operator in the next subsection[2].

2.3 Quantum Langevin Formulation

Let us start with a canonical oscillator operator, $a(\mathbf{k}, t)$, of some dominant mode in a quark-gluon plasma. We consider the following Langevin equation for this operator,

$$i \frac{d}{dt} a(\mathbf{k}, t) = \int^t K(\mathbf{k}, t') a(\mathbf{k}, t') dt' + f(\mathbf{k}, t) \quad (3)$$

where $f(\mathbf{k}, t)$ is an operator-valued random force and $K(\mathbf{k}, t)$ is a *c*-number kernel function which is the only input of the above Langevin equation, except for the statistical property of $f(\mathbf{k}, t)$ which will be fixed below.

Consider the system in a thermal equilibrium state. From finite-temperature quantum theoretical point of view, we impose two requirement to the solution of the above equation: [R-1] At any time t , $a(\mathbf{k}, t)$ should obey the equal-time canonical (anti-)commutation relation,

$$[a(\mathbf{k}, t), a^\dagger(\mathbf{k}', t)]_\pm = \delta^3(\mathbf{k} - \mathbf{k}'), \quad (4)$$

where $[A, B]_\pm = AB - \xi BA$ and

$$\xi = \begin{cases} 1 & \text{for Bose-particle} \\ -1 & \text{for Fermi-particle} \end{cases}.$$

[R-II] The Kubo-Martin-Schwinger condition,

$$\langle a^\dagger(\mathbf{k}, t) a(\mathbf{k}', t' + i\beta) \rangle = \langle a(\mathbf{k}', t') a^\dagger(\mathbf{k}, t) \rangle e^{\beta\mu}, \quad (5)$$

should be satisfied, where $\beta = 1/T$ is inverse temperature and μ is an chemical potential. [R-1] is a fundamental requirement of *quantum* theory and the KMS condition in [R-2] guarantees the existence of thermal equilibrium state.

We introduce Fourier transform of the kernel $K(\mathbf{k}, t)$ and random force $f(\mathbf{k}, t)$ as

$$\begin{aligned} E(\mathbf{k}, \omega) &= \lim_{\delta \rightarrow +0} \int_0^\infty dt e^{(i\omega - \delta)t} K(\mathbf{k}, t) \\ \tilde{f}(\mathbf{k}, \omega) &= \lim_{\delta \rightarrow +0} \int_0^\infty dt e^{(i\omega - \delta)t} f(\mathbf{k}, t), \end{aligned} \quad (6)$$

and denote the real and imaginary parts of $E(\mathbf{k}, t)$ by

$$E(\mathbf{k}, \omega) = \varepsilon(\mathbf{k}, \omega) - \frac{i}{2}\gamma(\mathbf{k}, \omega). \quad (7)$$

Solving (3) under the initial condition that $a(\mathbf{k}, t = 0) = a_0(\mathbf{k})$, we obtain

$$a(\mathbf{k}, t) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \left[\frac{e^{-i\omega t}}{\omega - E(\mathbf{k}, \omega)} i a_0(\mathbf{k}) + \frac{e^{-i\omega t}}{\omega - E(\mathbf{k}, \omega)} \tilde{f}(\mathbf{k}, \omega) \right]. \quad (8)$$

If we assume that $(\omega - E(\mathbf{k}, \omega))^{-1}$ has poles only in the lower half plane of complex ω , denoting j -th pole and its residue by

$$\begin{aligned} E_j(\mathbf{k}) &= \varepsilon_j(\mathbf{k}) - \frac{i}{2}\gamma_j(\mathbf{k}) \\ R_j(\mathbf{k}) &= \left[1 - \frac{\partial E(\mathbf{k}, \omega)}{\partial \omega} \Big|_{\omega=E_j(\mathbf{k}, \omega)} \right]^{-1}, \end{aligned} \quad (9)$$

the first term of (8) can be rewritten

$$i a_0(\mathbf{k}) \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \frac{e^{-i\omega t}}{\omega - E(\mathbf{k}, \omega)} = a_0(\mathbf{k}) \sum_i R_i(\mathbf{k}) e^{-iE_i(\mathbf{k})t}. \quad (10)$$

If the random force $f(\mathbf{k}, t)$ or its Fourier transform $\tilde{f}(\mathbf{k}, \omega)$ were *c-number*, [R-1] could not be satisfied, because then the (anti-)commutation relation between $a(\mathbf{k}, t)$ and its hermite conjugate $a^\dagger(\mathbf{k}, t)$ decreases with t due to $\gamma_j(\mathbf{k}) \neq 0$. Thus $f(\mathbf{k}, t)$ must be an *operator-valued* random force.

We now expand $f(\mathbf{k}, t)$ in its Fourier components,

$$f(\mathbf{k}, t) = \int_{-\infty}^{+\infty} d\omega' \sqrt{\frac{\gamma(\mathbf{k}, \omega')}{2\pi}} \rho(\mathbf{k}, \omega') A(\mathbf{k}, \omega') e^{-i\omega' t}, \quad (11)$$

where an operator $A(\mathbf{k}, \omega)$ is assumed to obey a(n) (anti-)commutation relation,

$$[A(\mathbf{k}, \omega), A^\dagger(\mathbf{k}', \omega')]_{\pm} = \delta^3(\mathbf{k} - \mathbf{k}')\delta(\omega - \omega'). \quad (12)$$

Remark that the (anti-)commutation relation in (12) contains a δ -function of ω and that $\rho(\mathbf{k}, \omega)$ is a c -number weight function. With use of (11), we have for (8)

$$a(\mathbf{k}, t) = ia_0(\mathbf{k}) \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \frac{e^{-i\omega t}}{\omega - E(\mathbf{k}, \omega)} + \int_{-\infty}^{+\infty} d\omega \sqrt{\frac{\gamma(\mathbf{k}, \omega)}{2\pi}} \frac{\rho(\mathbf{k}, \omega) A(\mathbf{k}, \omega)}{\omega - E(\mathbf{k}, \omega)} e^{-i\omega t}. \quad (13)$$

The functional form of $\rho(\mathbf{k}, \omega)$ has been analyzed in detail by Streater and Hasegawa *et al*[5]. Limiting our discussion here to the fluctuation after reaching equilibrium, we only use stationary part of (13),

$$a(\mathbf{k}, t) = \int_{-\infty}^{\infty} d\omega \sqrt{\frac{\gamma(\mathbf{k}, \omega)}{2\pi}} \frac{\rho(\mathbf{k}, \omega) A(\mathbf{k}, \omega)}{\omega - E(\mathbf{k}, \omega)} e^{-i\omega t}. \quad (14)$$

Then the condition for the $\rho(\mathbf{k}, \omega)$ is relaxed, i.e.,

$$\int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \rho^2(\mathbf{k}, \omega) \frac{\gamma(\mathbf{k}, \omega)}{|\omega - E(\mathbf{k}, \omega)|^2} = 1. \quad (15)$$

The KMS-condition [R-II] is satisfied by giving statistical average of $A(\mathbf{k}, \omega)$ and $A^\dagger(\mathbf{k}, \omega)$ as follows:

$$\begin{aligned} \langle A(\mathbf{k}, \omega) \rangle &= \langle A^\dagger(\mathbf{k}, \omega) \rangle = 0 \\ \langle A^\dagger(\mathbf{k}, \omega) A(\mathbf{k}', \omega') \rangle &= \delta^3(\mathbf{k} - \mathbf{k}')\delta(\omega - \omega')n(\omega, T) \\ \langle A(\mathbf{k}, \omega) A^\dagger(\mathbf{k}', \omega') \rangle &= \delta^3(\mathbf{k} - \mathbf{k}')\delta(\omega - \omega')[1 + \xi n(\omega, T)] \\ \langle A(\mathbf{k}, \omega) A(\mathbf{k}, \omega) \rangle &= \langle A^\dagger(\mathbf{k}, \omega) A^\dagger(\mathbf{k}, \omega) \rangle = 0 \end{aligned} \quad (16)$$

where the Gaussian property is assumed for the higher moments, the number distribution is given by

$$n(\omega, T) = \frac{1}{\exp(\frac{\omega - \mu}{T}) - \xi}, \quad (17)$$

and see the definition of ξ in (5). Remark that in order to satisfy the KMS condition, $n(\omega, T)$ here must be a function of frequency ω but not of the spectrum $\varepsilon(\mathbf{k})$.

We now reach quantum version of fluctuation-dissipation theorem,

$$\begin{aligned} \langle f(\mathbf{k}, t) \rangle &= \langle f^\dagger(\mathbf{k}, t) \rangle = 0 \\ \langle f^\dagger(\mathbf{k}, t) f(\mathbf{k}', t') \rangle &= \delta^3(\mathbf{k} - \mathbf{k}') \int_0^\infty \frac{d\omega}{2\pi} \rho^2(\mathbf{k}, \omega) \gamma(\mathbf{k}, \omega) n(\omega, T) e^{-i\omega(t-t')} \\ \langle f(\mathbf{k}, t) f^\dagger(\mathbf{k}', t') \rangle &= \delta^3(\mathbf{k} - \mathbf{k}') \int_0^\infty \frac{d\omega}{2\pi} \rho^2(\mathbf{k}, \omega) \gamma(\mathbf{k}, \omega) [1 + \xi n(\omega, T)] e^{+i\omega(t-t')} \\ \langle f(\mathbf{k}, t) f(\mathbf{k}', t') \rangle &= \langle f^\dagger(\mathbf{k}, t) f^\dagger(\mathbf{k}', t') \rangle = 0. \end{aligned} \quad (18)$$

As Kubo pointed out in his paper[4], the KMS condition requires that a quantum random force be a colored noise.

Consider a neutral scalar field $\varphi(x)$ (x stands for $(\mathbf{x}, t)$) to describe a dominant mode in QGP, and construct it from $a(\mathbf{k}, t)$ which is a stationary solution of the above quantum Langevin equation (14), and its hermite conjugate $a^\dagger(\mathbf{k}, t)$:

$$\varphi(x) = \int \frac{d^3\mathbf{k}}{\sqrt{(2\pi)^3 \cdot 2\Omega(\mathbf{k})}} [a(\mathbf{k}, t) e^{i\mathbf{k} \cdot \mathbf{x}} + a^\dagger(\mathbf{k}, t) e^{-i\mathbf{k} \cdot \mathbf{x}}]. \quad (19)$$

Require the equal time commutation relation of the field operator,

$$[\varphi(\mathbf{x}, t), \dot{\varphi}(\mathbf{x}', t)] = \delta^3(\mathbf{k} - \mathbf{k}'), \quad (20)$$

then the normalization of the field $\Omega(\mathbf{k})$ is determined to be

$$\Omega(\mathbf{k}) = \int_0^\infty \frac{d\omega}{2\pi} \rho^2(\mathbf{k}, \omega) \omega \frac{\gamma(\mathbf{k}, \omega)}{|\omega - E(\mathbf{k}, \omega)|^2}. \quad (21)$$

3 Phenomenology Using Semi-phenomenological Parameters

Here we see how phenomenological level approaches using the parameters in the quantum Langevin formalism of the last section are developed.

3.1 Thermodynamical Quantities and Transport Coefficients

In this subsection we express phenomenological parameters (thermodynamical quantities and transport coefficients) in terms of the semi-phenomenological parameters in the quantum Langevin equation of the previous section such as $\epsilon(\mathbf{k}, \omega)$ and $\gamma(\mathbf{k}, \omega)$.

To deal with thermodynamical quantities, we define the energy-momentum tensor operator $T^{\mu\nu}(x)$ and the particle current operator $J^\mu(x)$ from $\varphi(x)$ by

$$\begin{aligned} T^{\mu\nu}(x) &\equiv : \varphi(x) \overset{\leftrightarrow}{p}^\mu \overset{\leftrightarrow}{p}^\nu \varphi(x) : \\ J^\mu(x) &= : \varphi(x) \overset{\leftrightarrow}{p}^\mu \varphi(x) : \end{aligned} \quad (22)$$

where $: ** :$ is a normal product and

$$\overset{\leftrightarrow}{p}^\mu \equiv i(\overset{\rightarrow}{\partial}^\mu - \overset{\leftarrow}{\partial}^\mu)/2. \quad (23)$$

The energy density, pressure, particle density and entropy density are then given as

$$\begin{aligned} E(T) &= \langle T^{00} \rangle \\ P(T) &= \frac{1}{3} \sum_{i=1}^3 \langle T^{ii} \rangle \\ \langle J_0(T) \rangle &= \int_0^\infty \frac{k^2 dk}{2\pi^2} J_0(k, T) \\ S(T) &= \frac{1}{2\pi^2} \int_0^\infty k^2 dk \left\{ (1 + J_0(k, T)) \ln(1 + J_0(k, T)) - J_0(k, T) \ln J_0(k, T) \right\}, \end{aligned} \quad (24)$$

where the isotropic property has been taken account of (from now on we use the notation of $k = |\mathbf{k}|$). Using the stationary solution of the quantum Langevin equation in the last section we can calculate these quantities as

$$\begin{aligned} E(T) &= \frac{1}{2\pi^2} \int_0^\infty k^2 dk \int_0^\infty \frac{d\omega}{2\pi} \frac{\omega}{\Omega(k)} \rho^2(k, \omega) \omega \frac{\gamma(k, \omega)}{|\omega - E(k, \omega)|^2} n(\omega, T) \\ P(T) &= \frac{1}{6\pi^2} \int_0^\infty k^2 dk \int_0^\infty \frac{d\omega}{2\pi} \frac{\omega}{\Omega(k)} \rho^2(k, \omega) \frac{k^2}{\omega} \frac{\gamma(k, \omega)}{|\omega - E(k, \omega)|^2} n(\omega, T) \\ J_0(k, T) &= \int_0^\infty \frac{d\omega}{2\pi} \frac{\omega}{\Omega(k)} \rho^2(k, \omega) \frac{\gamma(k, \omega)}{|\omega - E(k, \omega)|^2} n(\omega, T). \end{aligned} \quad (25)$$

Based on a linear response theory, transport coefficients such as heat conductivity $\kappa(T)$, shear viscosity $\zeta_{(s)}(T)$, and bulk viscosity $\zeta_{(v)}(T)$ are given as

$$\begin{aligned}\kappa(T) &= \frac{V}{T} \int_0^\infty d\tau \langle \{\bar{T}^{01}(t), \bar{T}^{01}(t+\tau)\} \rangle_C, \\ \zeta_{(s)}(T) &= \frac{2V}{T} \int_0^\infty d\tau \langle \{\bar{T}^{12}(t), \bar{T}^{12}(t+\tau)\} \rangle_C, \\ \zeta_{(v)}(T) &= \frac{V}{T} \int_0^\infty d\tau \langle \{\bar{P}(t), \bar{P}(t+\tau)\} \rangle_C,\end{aligned}\quad (26)$$

where the following notations are used,

$$\begin{aligned}\bar{P}(t) &= \frac{1}{3} \sum_{i=1}^3 \bar{T}^{ii}(t) - \frac{\partial P(T)}{\partial E(T)} \bar{T}^{00}(t) \\ \bar{T}^{\mu\nu}(t) &= \frac{1}{V} \int d^3\mathbf{k} T^{\mu\nu}(\mathbf{x}) \\ \{A, B\} &= \frac{1}{2}(AB + BA) \\ \langle AB \rangle_C &= \langle AB \rangle - \langle A \rangle \langle B \rangle.\end{aligned}\quad (27)$$

The stationary solution of the quantum Langevin equation yields

$$\begin{aligned}\kappa(T) &= \frac{1}{6\pi^2 T} \int_0^\infty k^2 dk \int_0^\infty d\omega \frac{\omega^2 k^2}{\Omega^2(k)} \rho^4(k, \omega) \frac{\gamma^2(k, \omega)/4}{\pi |\omega - E(k, \omega)|^4} n(\omega, T) [1 + n(\omega, T)] \\ \zeta_{(s)}(T) &= \frac{1}{15\pi^2 T} \int_0^\infty k^2 dk \int_0^\infty d\omega \frac{k^4}{\Omega^2(k)} \rho^4(k, \omega) \frac{\gamma^2(k, \omega)/4}{\pi |\omega - E(k, \omega)|^4} n(\omega, T) [1 + n(\omega, T)] \\ \zeta_{(v)}(T) &= \frac{1}{2\pi^2 T} \int_0^\infty k^2 dk \int_0^\infty d\omega \frac{1}{\Omega^2(k)} \left[\frac{1}{3} k^2 - \frac{\partial P(T)}{\partial E(T)} \omega^2 \right]^2 \\ &\quad \times \rho^4(k, \omega) \frac{\gamma^2(k, \omega)/4}{\pi |\omega - E(k, \omega)|^4} n(\omega, T) [1 + n(\omega, T)].\end{aligned}\quad (28)$$

We recall here that the transport coefficients are proportional to the *inverse* of damping term, $\frac{1}{2}\gamma(k, \omega)$, as is well-known.

As the simplest case, let us suppose that $(\omega - E(k, \omega))^{-1}$ has a single simple pole at $\omega = \varepsilon(k) - i\gamma(k)/2$ and furthermore that $\varepsilon(k) \gg \gamma(k)$ (weak damping limit). In this case the thermodynamical quantities are simplified as

$$\begin{aligned}E(T) &= \frac{1}{2\pi^2} \int_0^\infty k^2 dk \varepsilon(k) n(\varepsilon(k), T) \\ P(T) &= \frac{1}{6\pi^2} \int_0^\infty k^2 dk \frac{k^2}{\varepsilon(k)} n(\varepsilon(k), T) \\ S(T) &= \frac{1}{2\pi^2} \int_0^\infty k^2 dk \{ (1 + J_0(k, T)) \ln(1 + J_0(k, T)) - J_0(k, T) \ln J_0(k, T) \}\end{aligned}\quad (29)$$

where $J_0(k, T) = n(\varepsilon(k), T)$. The transport coefficients can also be rewritten in simple forms as

$$\begin{aligned}\kappa(T) &= \frac{1}{3\pi^2 T} \int_0^\infty k^2 dk \frac{k^2}{\gamma(k)} n(\varepsilon(k), T) [n(\varepsilon(k), T) + 1] \\ \zeta_{(s)}(T) &= \frac{2}{15\pi^2 T} \int_0^\infty k^2 dk \frac{k^4}{\varepsilon^2(k)} \frac{1}{\gamma(k)} n(\varepsilon(k), T) [n(\varepsilon(k), T) + 1] \\ \zeta_{(v)}(T) &= \frac{1}{\pi^2 T} \int_0^\infty k^2 dk \frac{1}{\varepsilon^2(k)} \frac{1}{\gamma(k)} \left[\frac{1}{3} k^2 - \frac{\partial P(T)}{\partial E(T)} \varepsilon^2(k) \right]^2 n(\varepsilon(k), T) [n(\varepsilon(k), T) + 1].\end{aligned}\quad (30)$$

Here we discussed a neutral scalar boson only for simplicity. Extension to general bosonic and fermionic fields is straightforward.

3.2 Simple Model for Phase Transition in QGP

The lattice calculation[6] suggests the transition between confinement and deconfinement phases in QGP. We study this phase transition phenomenologically, assuming under the one-pole approximation the following dependence of ε and γ on (k, T) below and above the critical temperature T_c , including their possible dependence of T :

$$\begin{aligned}\varepsilon(k, T) &= B\sqrt{k^2 + M^2} \frac{1}{2} \left[1 - \tanh \frac{T - T_c}{d} \right] + k \frac{1}{2} \left[1 + \tanh \frac{T - T_c}{d} \right], \\ \gamma(k, T) &= Ck\end{aligned}\tag{31}$$

with positive constants B , M , d , and C . Here we suppose that an approximate phase transition takes place around T_c in a narrow region with very small width d , because

$$\begin{aligned}\varepsilon &\simeq k && \text{for } T \gg T_c, \\ &\simeq B\sqrt{k^2 + M^2} && \text{for } T \ll T_c\end{aligned}\tag{32}$$

meaning that the QGP fluid is composed of free gas of massless particle for $T \gg T_c$ and free gas of massive particle for $T \ll T_c$. Let us assume that we have the QGP fluid of free gluons and (u, d, s) -quarks for $T \gg T_c$, while we have only pions and kaons for $T \ll T_c$. In this case we have a naive counting of degrees of freedom as follows:

$$\begin{aligned}\mathcal{N} &= 2 \times 8 + 2 \times 2 \times 3 \times (7/8) = 47, \\ \mathcal{N}' &\equiv \mathcal{N}/B^3 = 7 \text{ or } B \simeq 1.89.\end{aligned}\tag{33}$$

Here we have introduced the correction factor $(7/8)$ for fermion quarks, because we have used the above boson formula for quarks for simplicity[7]. This value of $\mathcal{N}$ is undoubtedly underestimated because many more degrees of freedom can be excited for $T \gg T_c$.

We can reproduce the shape of $\mathcal{E}(T)$ given by lattice simulation with $T_c = 160\text{MeV}$, $d = 2.0\text{MeV}$ and $M = 200\text{MeV}$. (We can safely use $C = 0.1$ for γ .) And the evaluation of the temperature dependencies of transport coefficients are also very easy. This choice enables us to set a phase transition-like behavior of the QGP fluid in our model equation of state.

In previous papers[7, 8, 9], we numerically solved the hydrodynamical equation accompanied by the equation of state, and then obtained

1. the space-time evolution of (3+1)-dimensional perfect QGP fluid with phase transition[8],
2. the cooling-down curve of temperature in the central region for (1+1)- and (3+1)-dimensional perfect QGP fluid with phase transition[8],
3. the cooling-down curve of temperature for (1+1)-dimensional viscous QGP fluid with phase transition[7],
4. the space-time evolution of (3+1)-dimensional baryon-rich perfect QGP fluid with phase transition[9].

We have observed a remarkable effect of phase transition through these analyses.

4 Semi-Phenomenological Parameters from Fundamental Theory

We turn our attention to set up a program for calculating the semi-phenomenological parameters, $\varepsilon(k)$ and $\gamma(k)$, from the fundamental theory, i.e., QCD.

So far the thermodynamical quantities and transport coefficients are calculated for a system of *microscopically infinite volume and in thermal equilibrium*, and are used for a *macroscopic one point* of the phenomenological hydrodynamical model. But a more realistic view is that the space-time evolution of a fire ball born in nuclear collision gives rise to its *finite size and finite life-time* effects. The information of such a space-time size may be observed through two-body correlation of the produced particles, this effect is called Hanbury-Brown Twiss effect[10]. Although the finite size and finite life-time effects may be crucial for understanding QGP phenomena, we introduce below a common practice from quantum field theory, namely, perturbative QCD at finite-temperature. This is a demonstration of explicit determination $\varepsilon(k)$ and $\gamma(k)$ from a fundamental theory. A possibility of accommodating part of finite size effect in perturbative QCD at finite-temperature will also be considered.

4.1 Perturbative QCD at Finite-Temperature

Before going into the calculation of perturbative QCD at finite-temperature, we list several reasons which make the validity of its naive calculation questionable for QGP: (i) First of all, it is not certain whether thermal equilibrium state is realized in nuclear collisions or not. If not, results from finite-temperature perturbative QCD have little meaning. (ii) Finite-temperature perturbative QCD implies an expansion around the particle picture at zero-temperature (or a particle picture not very different from it) with perturbative corrections. However the QGP is a system of many body, which usually changes the particle picture drastically from the one of the vacuum theory. Indeed many examples exist in solid state physics or condensed matter physics, in which it would be impossible without appropriate choice of particle picture (called quasi-particle picture) to describe the physics. (iii) The running coupling constant of QCD g is not small at the present experimental energy, and a perturbative expansion with respect to g is not a good approximation scheme. (iv) A serious objection to use naive finite-temperature perturbative QCD comes from the fact that there appear infrared divergences for higher-order diagrams. Linde[11] showed that each diagram for the thermodynamical potential $\Omega(T)$ behaves as

$$\begin{aligned}\Omega(T) &= -\frac{1}{\beta} \log [Tr[\exp\{-\beta H\}]] \\ &\sim g^6 T^4 \left(\frac{g^2 T}{m(T)} \right)^{N-3} \quad \text{as } m(T) \rightarrow 0\end{aligned}\tag{34}$$

where N is the number of vertices and $m(T)$ is the infrared cut-off. We will come back to this point later. (v) We have to check that calculated values for physical quantities are independent of choice of gauge. This gauge independence problem is not yet settled completely for finite-temperature perturbative QCD.

In spite of these doubts, we try finite-temperature perturbative QCD to extract $\varepsilon(k)$ and $\gamma(k)$. To do this we remark that the field operator in the quantum Langevin equation of the previous section should be identified as the quasi particle field in the ordinary terminology. Therefore, for our purpose the best way, known to us, is to study the *self-energy* in quantum field theory at finite temperature and to find *its pole structure*. This program requires a self-consistent procedure: One chooses some candidates for the unperturbative state from experimental data or physical insight,

and does some approximate (e.g., perturbative, loop-wise and so on) calculations around them. Then the results pick up one candidate among many, determining such parameters as energy of the quasi-particle, etc, which closes the self-consistent procedure.

Our work at this stage is a compromise. We extract the $\varepsilon(k)$ and $\gamma(k)$ from the self-energy calculations of the naive perturbative QCD at finite temperature, which apparently lack the self-consistent treatment or the proper choice of the quasi-particle picture, as pointed out in (ii). Although our parameters thus obtained will not reproduce experimental data well, we are presenting a way to calculate the $\varepsilon(k)$ and $\gamma(k)$ from a fundamental theory, and this practice may lead to a more reliable method to calculate the $\varepsilon(k)$ and $\gamma(k)$ at fundamental level.

There already exists many calculations of the polarization tensor (the self-energy) for gluons of finite-temperature QCD denoted by Π , see for example Refs.[12, 13, 14, 15]. We have a naive one-loop calculations in Coulomb gauge, though we suppress their explicit expressions here, which becomes in the long wave limit

$$\lim_{\vec{k} \rightarrow 0} \Pi_T(\vec{k}, \omega) = \lim_{\vec{k} \rightarrow 0} \Pi_L(\vec{k}, \omega) = \frac{1}{3}(gT)^2 \equiv m_g^2 \quad (35)$$

where $\Pi_T(\vec{k}, \omega)$ and $\Pi_L(\vec{k}, \omega)$ stand for the transverse mode and the longitudinal mode, respectively. m_g is a thermally induced gluon mass.

One of the simplest way to obtain the parameters of Langevin equation $\varepsilon(k)$ and $\gamma(k)$ from above (35) are as follows: First, let us consider that the real parts of Π_A ($A = T, L$) (provided that the energy counter terms are included in them) provides a self-consistent equation for the $\varepsilon(k)$, that is,

$$\text{Re } \Pi_A[k, \omega = \varepsilon_A(k)] = 0. \quad (36)$$

Then the imaginary parts of Π_A induce $\gamma(k)$ as,

$$\text{Im } \Pi_A[k, \omega = \varepsilon_A(k)] = -i\varepsilon_A(k)\gamma_A(k). \quad (37)$$

From (36) and (37) we derive the mode spectrum and the damping rate.

4.2 Improvement for Finite-Size System

We discussed the simplest method to evaluate $\varepsilon_A(k)$ and $\gamma_A(k)$ based on naive finite-temperature field theory. However, real QGP produced in high energy nuclear reactions has finite size and some important physical features will originate in the finite-size effect. Up to now, a systematic full treatment of finite-size system in quantum field theory has not yet been established.

We limit our consideration to an effect among those brought by the finiteness of the system, namely a suppression of excitation of low momentum modes. This suppression is effectively taken account of by introducing an infrared cut-off $m_L \sim 1/L$ (L :size of the fluid) in the spectrum of modes. For example, a spectrum of relativistic massless mode in infinite-size system, $\varepsilon(k) = k$ is changed into $\varepsilon(k) = \sqrt{k^2 + m_L^2}$ in finite-size system. We can apply this prescription readily to the analysis of phase transition in subsection 3.2. The calculation of polarization tensor in finite-temperature QCD should be done when propagators with the above infrared cut-off are used.

As for the infrared cut-off, we pointed out the infrared divergence (Linde's argument) in (iv). Recently in modern finite temperature field theory, to avoid the infrared catastrophe within finite-temperature field theory, a systematic way to introduce thermally induced mass m_g (a resummation program) was proposed by Braaten-Pisarski [15] for the coupling region $gT \ll T$ and this m_g works as infrared cut off, too. We remark though that the running coupling constant g is not small in the present experimental setup where $gT \sim T$.

In figure 1, we compare these two cut-off scales (finite-size and thermally induced ones) in the present numerical solutions, and it seems that “whole fluid size” are large enough and cut off from the size is smaller than thermally induced mass. But such a comparison is only the first trial and there exist many ambiguities.

5 Concluding Remarks

The three level hierarchical approaches (phenomenological, semi-phenomenological and fundamental) for quark-gluon plasma are described. The three levels are related to each other as the semi-phenomenological level is at the center of the relationship. This is the reason why we emphasize the importance of the semi-phenomenological view which is rather missing in many past studies of QGP problem.

The programs to calculate the semi-phenomenological parameters from fundamental finite-temperature QCD are set up. Particularly we have considered a possible modification in naive calculations due to finite-size effects. Studies along this line are on the way.

Finally we make two remarks on future improvement in the quantum Langevin approach. First we need a proper formulation of quantum Langevin equation for the longitudinal mode: The residue of the longitudinal mode is not 1 but decreases rapidly as $|\mathbf{k}|$ becomes large, implying that the condition [R-1] in quantum Langevin formalism is inadequate for this mode. To accommodate the longitudinal mode into the quantum Langevin formalism is an open problem. Secondly we have considered a finite-size effect which may affect the values of the semi-phenomenological parameters, calculated from fundamental theory. But a finite-size effect may be considered as part of effects arising from spatially inhomogeneity of the QGP system. As was pointed out, the present formulation is based on a naive picture that a *macroscopic one point* of the phenomenological hydrodynamical model is *microscopically of infinite volume and in thermal equilibrium*. Our future attempt is to take account of the spatially inhomogeneity at the level of the quantum Langevin formalism, thus making theoretical arguments more realistic for QGP experiments.

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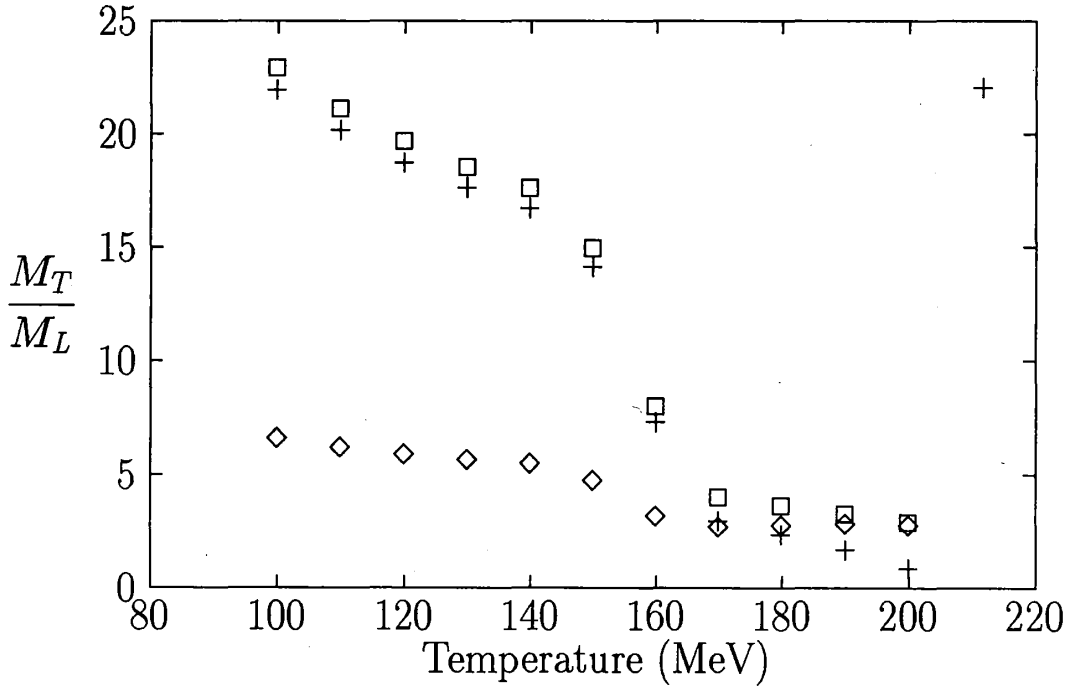


Figure 1

The ratio of thermally induced mass over cut off from size. The square plot stands for the size of fluid $\langle \sqrt{x^2 + y^2 + z^2} \rangle$, the cross for longitudinal-size $\langle \sqrt{z^2} \rangle$, and the diamond for transverse-size $\langle \sqrt{x^2 + y^2} \rangle$, respectively.