

Evaluation Method for Electromagnetic Induction Testing of Dielectrics using Impedance Plane Diagrams Drawn using Ampère-Maxwell Equation and Simple Electrical Circuit Model

Wataru Matsunaga ^{a,*} and Yoshihiro Mizutani ^b

^a School of Engineering, Department of Mechanical Engineering, Tokyo Institute of Technology, I1-457, 2-12-1, Ookayama, Meguro-ku, Tokyo 1528552, Japan

Email: matsunaga.wataru.5w@kyoto-u.ac.jp

Current affiliation: Department of Mechanical Engineering and Science, Kyoto University, c2N03, Kyoto daigaku-katsura, Nishikyo-ku, Kyoto 615-8540, Japan

^b School of Engineering, Department of Mechanical Engineering, Tokyo Institute of Technology, I1-415, 2-12-1, Ookayama, Meguro-ku, Tokyo 1528552, Japan

Email: mizutani.y.aa@m.titech.ac.jp

*Corresponding author: Wataru Matsunaga. Email: matsunaga.wataru.5w@kyoto-u.ac.jp

Acknowledgments

This work was supported by JSPS KAKENHI under Grant Number JP20J21500. The authors thank Senior Assistant Professor Koichi Mizukami in the Graduate School of Science and Engineering, Civil and Environmental Engineering, Ehime University, for providing advice on how to judge the results of the numerical analyses.

Abstract

In this study, methods to draw the impedance plane diagram that is applied to electromagnetic induction testing (EIT) of conductors were investigated using the Ampère-Maxwell equation and an electric circuit model to enable application of this diagram to EIT of dielectrics. EIT has been applied only to conductors as eddy current testing (ECT), but recently it has been found that EIT can also be applied to dielectrics by applying a high-frequency AC voltage. On the other hand, the evaluation method of EIT for dielectrics has not been sufficiently established, and in particular, there are no examples of studies on the drawing of impedance plane diagrams, which is widely applied as an evaluation method when ECT is applied to real structures. Therefore, in this study, we attempt to draw an impedance plane diagram from the Ampère-Maxwell equation and a simple electrical circuit, as in ECT. First, based on the Ampère-Maxwell equation that considers eddy and displacement currents, a theoretical solution for the impedance was derived by considering both conductivity and permittivity, and an

impedance plane diagram was then drawn. From the impedance diagrams obtained, it was shown that the same trends can be drawn for impedance diagrams for both conductors and dielectrics. Next, an electrical circuit model for EIT was proposed that takes both the conductivity and the permittivity of the material into account. Using the electric circuit equations, impedance plane diagrams for conductors and dielectrics were drawn, and for dielectrics in particular, it was shown that the impedance plane diagrams can be drawn by considering the conductivity of the dielectric material. In addition, similar to the impedance plane diagrams drawn from the electric circuit model and derived from the Ampère-Maxwell equation, the change behavior in the impedance plane diagrams clearly differs between the cases where the conductivity and permittivity of the dielectric change and the case where the lift-off changes. This demonstrates the effectiveness of the electric circuit model in providing a qualitative understanding of the effects of the dielectric conditions and the measurement conditions on EIT of dielectrics.

Keywords: nondestructive evaluation, electromagnetic induction testing, dielectrics, impedance plane diagram

1. Introduction

Electromagnetic induction testing (EIT) is a nondestructive inspection method in which an alternating current (AC) voltage is applied to a coil to induce eddy currents or displacement currents in a specimen under test; the changes in these electromagnetic fields are then used to detect defects in materials and evaluate the degree of curing and the moisture absorption of resins. EIT of conductors, which is called eddy current testing (ECT), is performed to detect manufacturing defects and to verify the alloy composition and the hardness of a product when incorporated into a production line. EIT has conventionally been limited to use on conductors, but Heuer et al. reported recently that EIT can be applied to dielectric materials by using higher frequency AC voltages [1]. Through use of high-frequency waves, Mizukami et al. reported that it is possible to detect both local thinning and slit-like defects in glass fiber-reinforced plastics (GFRPs) [2]. Gäbler et al. also reported that it is possible to detect poor epoxy resin curing and thinning of polymethyl methacrylate [3]. Furthermore, the authors of this work reported that the moisture absorption of GFRPs can be measured using EIT [4]. However, when EIT for conductors (i.e., ECT) is compared with EIT for dielectrics, ECT is widely used in industry, as noted above, but EIT for dielectrics still suffers many problems in practice and remains a laboratory-level research task. One challenge that faces EIT for dielectrics is that the relevant theoretical system is not well established when compared with that for ECT. In particular, while established methods are available to evaluate the effects of various parameters, including the frequency, lift-off, material conductivity, magnetic permeability, and coil dimensions, on the impedance in ECT evaluations based on application of an impedance plane diagram to ECT [5, 6]. In addition to the single coil, which is the most basic type of ECT, it has been reported that it is possible to evaluate rectangular planar coils [7] and Double Gradient Winding Coils, in which the coil is wound in a unique way from a planar coil [8]. In addition to conventional metals [9], this evaluation method has recently been reported to be applicable to carbon fiber reinforced plastics, common anisotropic materials [10, 11]. Hence, the application of EIT to dielectrics can be expected to be expanded once the technique for drawing impedance plane diagrams is established. However, no similar methods have been established for use in EIT for dielectrics to date.

The objective of this study is to draw impedance plane diagrams for dielectric materials when EIT is applied to them, and also to evaluate the effects of frequency, lift-off, material conductivity, and permittivity on the impedance plane diagrams. In other words, methods to draw impedance plane diagrams that are applicable to both conductors and dielectrics are investigated. In EIT for conductors (ECT), there are two possible methods to draw impedance plane diagrams: theoretically, based on the Ampère-Maxwell equation [7], and by simply modeling the test conditions in an electrical circuit [5, 6]. Impedance diagrams drawn using the former method are highly accurate, but the equations derived in this manner are complex and the actual physical phenomena can easily become a black box. In contrast, an impedance diagram drawn using the latter method is less accurate than that obtained by the former method, but the electrical circuit model is simpler, which means that it is easier

to understand the physical phenomena that occur in the material. These two methods are used appropriately, depending on the purpose of use of the material. In this study, the use of these two methods to draw impedance plane diagrams that are applicable to both conductors and dielectrics is investigated, and attempts are also made to draw impedance plane diagrams using the two methods described above.

The content of this paper is described as follows. First, based on the Ampère-Maxwell equation, which takes the effects of both the eddy and displacement currents into account, a theoretical solution is derived for the impedance of a coil that is applicable to the EIT of both conductors and dielectrics. Using this theoretical solution, impedance plane diagrams are drawn for conductors and dielectrics to investigate the effects of the conductivity, permittivity, frequency, and lift-off on their impedance behavior. Next, by combining the electric circuit model used to discuss the impedance of dielectrics with the electric circuit model applied to EIT of conductors [12, 13], an electric circuit model that is applicable to both conductor and dielectric EIT is proposed. The circuit equations are derived from the circuit model and are solved to draw impedance plane diagrams for conductors and dielectrics during EIT, and the effects of the resistance, capacitance, frequency, and coupling coefficient on the impedance behavior are investigated. Furthermore, the impedance plane diagrams drawn based on the electric circuit model are compared with those derived from the theoretical solution, and the validity of the proposed electric circuit model is examined.

2. Drawing impedance plane diagrams from the Ampère-Maxwell equation

2. 1. Calculation of impedance when considering conductivity and permittivity

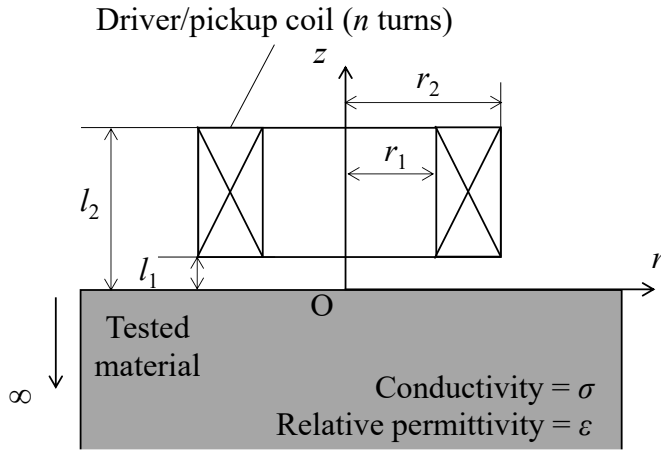


Fig. 1 Analytical model for derivation of analytical solutions for the vector potential when considering the conductivity σ and the relative permittivity ϵ

An analytical method for the electromagnetic field of a coil in ECT has been proposed in works by Dodd and

Deeds and by Luquire et al. [14, 15]. In this study, this analytical method is used to derive a theoretical solution for the coil impedance when the conductivity and permittivity are considered. To derive a theoretical solution for the impedance, it is necessary to calculate the electromagnetic field formed in the material by the coil, and this requires calculation of a vector potential. Mizukami et al. used the model shown in Fig. 1 to derive a theoretical solution for the vector potential formed in the dielectric and in the coil [2]. Figure 1 shows an analytical model in which a coil of height $l_2 - l_1$, inner diameter $2r_1$, and outer diameter $2r_2$, with a number of turns of n , is placed directly above a dielectric material with relative permittivity ϵ , conductivity σ , and a depth of ∞ at the lift-off l_1 , and an alternating current with an angular frequency ω and a current density i_0 is applied to the coil. In Fig. 1, the Ampère-Maxwell equation in cylindrical coordinates when the effects of eddy currents and displacement currents are taken into account is given as (1) below:

$$\frac{\partial^2 A_\theta}{\partial r^2} + \frac{1}{r} \frac{\partial A_\theta}{\partial r} + \frac{\partial^2 A_\theta}{\partial z^2} - \frac{A_\theta}{r^2} = -\mu_0 i_0 + (j\omega\mu_0\sigma - \omega^2\mu_0\epsilon_0\epsilon)A_\theta \quad (1)$$

In equation (1), A_θ is the vector potential in the θ direction, j is the imaginary unit, μ_0 is the magnetic permeability of a vacuum, and ϵ_0 is the permittivity of a vacuum. The first term in parentheses on the right side of equation (1) represents the eddy currents, and the second term in parentheses represents the displacement currents. In solving equation (1), Mizukami et al. derived the vector potential by taking only the displacement currents into account and assuming that the effects of the eddy currents are very small at high frequencies [2]. Because the boundary conditions in the model shown in Fig. 1 do not change when the conductivity is taken into consideration, it is possible to derive a vector potential that takes both the eddy and displacement currents into account by adding the effect of the conductivity to the part of the solution for the vector potential derived by Mizukami et al. [2]. In other words, even if the eddy and displacement currents are considered, the vector potential is almost the same as that derived by Mizukami et al. The vector potential on the coil shown in Fig. 1 can be calculated using equation (2):

$$A_\theta(r, z) = \frac{\mu_0 i_0}{2} \int_0^\infty \frac{1}{\alpha \alpha_0^2} I(r_2, r_1) J_1(\alpha r) (F + G) d\alpha \quad (2)$$

Here, $J_1(\alpha r)$ in equation (2) is a first-order Bessel function, and α_0 , $I(r_1, r_2)$, F , G , and i_0 can be calculated using equations (3)–(7):

$$\alpha_0 = (\alpha - \omega^2\mu_0\epsilon_0)^{1/2} \quad (3)$$

$$I(r_1, r_2) = \int_{\alpha r_1}^{\alpha r_2} x J_1(x) dx \quad (4)$$

$$F = 2 - \exp\{-\alpha_0(l_2 - z)\} - \exp\{-\alpha_0(z - l_1)\} \quad (5)$$

$$G = \frac{\alpha_0 - \alpha_1}{\alpha_0 + \alpha_1} \exp(-\alpha_0 z) \{\exp(-\alpha_0 l_1) - \exp(-\alpha_0 l_2)\} \quad (6)$$

$$i_0 = \frac{nI}{(l_2 - l_1)(r_2 - r_1)} \quad (7)$$

Here, I is the current that passes through the coil. α_1 in equation (6) can be calculated using equation (8):

$$\alpha_1 = (\alpha^2 + j\omega\mu_0\mu\sigma - \omega^2\mu_0\epsilon_0\epsilon)^{1/2} \quad (8)$$

The second term on the right-hand side in equation (8) is the term that represents the eddy currents, which Mizukami et al. neglected when deriving the vector potential because its effect is infinitely smaller than the effect of the third term on the right-hand side, i.e., the displacement current [2]. Using the vector potential from equation (2), the coil voltage V can be calculated using equation (9):

$$V = j \frac{2\pi\omega n}{(l_2 - l_1)(r_2 - r_1)} \int_{l_1}^{l_2} \int_{r_1}^{r_2} r A_\theta(r, z) dr dz \quad (9)$$

From Ohm's law, equation (10) holds when the coil impedance is Z :

$$V = ZI \Leftrightarrow Z = \frac{V}{I} \quad (10)$$

Using equations (9) and (10), the impedance Z is determined as shown in equation (11) below:

$$Z = \frac{V}{I} = j \frac{\mu_0\pi\omega n^2}{(l_2 - l_1)^2(r_2 - r_1)^2} \int_0^\infty \frac{1}{\alpha^3\alpha_0^2} I^2(r_1, r_2) d\alpha \int_{l_1}^{l_2} (F + G) dz \quad (11)$$

In equation (11), integration of the last term $(F + G)$ over the range from l_1 to l_2 yields the coil impedance Z , as shown in equation (12) below:

$$Z = \frac{V}{I} = j \frac{\mu_0\pi\omega n^2}{(l_2 - l_1)^2(r_2 - r_1)^2} \int_0^\infty \frac{H}{\alpha^3\alpha_0^2} I^2(r_1, r_2) d\alpha \quad (12)$$

Here, H in equation (12) is given by equation (13):

$$H = 2(l_2 - l_1) + \frac{\alpha_0 - \alpha_1}{\alpha_0(\alpha_0 + \alpha_1)} \{ \exp(-\alpha_0 l_2) - \exp(-\alpha_0 l_1) \}^2 + \frac{2}{\alpha_0} [\exp \{ -\alpha_0(l_2 - l_1) - 1 \}] \quad (13)$$

In the following sections, we use equations (12) and (13) to draw the impedance plane diagrams when EIT is applied to conductors and dielectrics. The coil was set up such that its inner diameter $2r_1$ was 14 mm, its outer diameter $2r_2$ was 16 mm, the lift-off l_1 was 1.0 mm, the coil height $l_2 - l_1$ was 21 mm, the number of coil turns n was 20 turns, and the applied current I was 0.10 A. In all analyses, the impedance plane diagrams were drawn for a specimen with only the conductivity σ , a specimen with only the relative permittivity ϵ , and a specimen with both σ and ϵ , while the frequency f and the lift-off l_1 were varied.

Numerical analysis software (MATLAB R2019b, MathWorks, Inc., Natick, MA, USA) was used to perform the analysis.

2. 2. Drawing impedance plane diagrams

2. 2. 1. Conductors

When an impedance plane diagram is drawn for ECT, it is common practice not to consider the conductor's

permittivity; therefore, it is assumed that $\varepsilon = 0$ in the discussion here. When drawing impedance plane diagrams for different frequencies f , the lift-off l_1 was fixed at 1.0 mm and the frequency f was varied, with values of 2.0, 4.0, 6.0, 8.0, and 10 kHz being used. Furthermore, when the impedance plane diagrams were drawn for different values of lift-off l_1 , the frequency f was fixed at 10 kHz and the lift-off l_1 was selected from values of 1.0, 2.0, 3.0, 4.0, and 5.0 mm. For all the impedance plane diagrams, the conductivity σ was varied from 1.0 to 1.0×10^7 S/m.

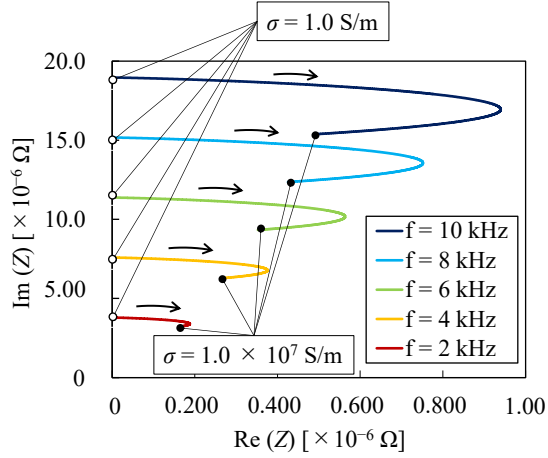


Fig. 2 Impedance plane calculations using the Ampère–Maxwell equation for the conductor when the frequency f and the conductivity σ are varied

Figure 2 shows the results obtained when impedance plane diagrams were drawn for the various values of frequency f . The vertical axis in Fig. 2 represents the imaginary part of the impedance Z denoted by $\text{Im}(Z)$ and the horizontal axis represents the real part of Z denoted by $\text{Re}(Z)$. Figure 2 shows that at any frequency f , the impedance diagrams are elliptical in shape, and that the impedance moves in a clockwise direction when the conductivity σ increases. The figure also shows that the radius of the impedance plane diagram increases when the frequency f increases, and that it moves in the $+$ direction of the y -axis. The results above show that when the frequency f increases, the change in impedance with respect to the change in conductivity σ also increases. However, when the frequency f increases, the eddy current penetration depth δ then decreases because $\delta = 1/(\pi f \mu_0 \mu \sigma)^{1/2}$ [16]. Therefore, the tradeoff between these two parameters must be taken into consideration to increase the sensitivity to changes in conductivity.

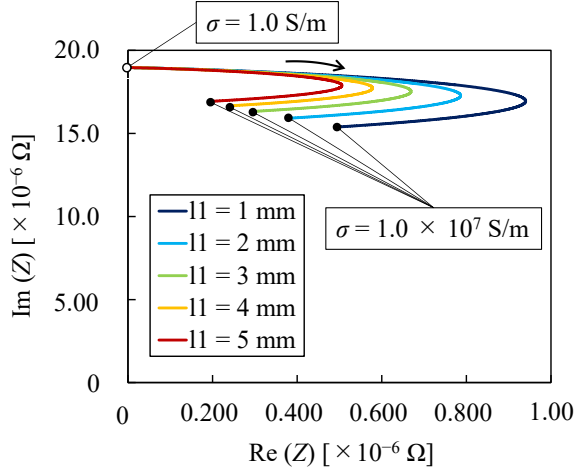


Fig. 3 Impedance plane calculations using the Ampère–Maxwell equation for the conductor when the lift-off l_1 and the conductivity σ are varied

Figure 3 shows the results obtained when the impedance plane diagrams were drawn for different values of the lift-off l_1 . The vertical and horizontal axes in Fig. 3 are the same as in Fig. 2. Figure 3 shows that for any value of the lift-off l_1 , the impedance plane diagrams are elliptical and that the impedance moves in a clockwise direction when the conductivity σ increases. The figure also shows that the impedance plane radius increases as the lift-off l_1 decreases. The results above show that a smaller value of l_1 leads to a greater change in the impedance with respect to any change in the conductivity σ . This indicates that the lift-off value should be as small as possible to increase sensitivity to changes in the conductivity.

The above analysis results are a well-known and are qualitatively consistent with the results of previous studies [14, 17]. These simulations were performed in order to compare with the dielectric simulation results to be performed in the subsequent sections.

2. 2. 2. Dielectrics

Case A. When only the permittivity is considered

First, the properties of ideal dielectric materials are discussed. In the case of an ideal dielectric, the conductivity is infinitely small, as noted by Mizukami et al. [2], and the effect of the eddy currents in equation (1) can be neglected, which means that the conductivity σ in the dielectric is discussed as having a value of 0 S/m. When impedance plane diagrams were drawn for the different values of the frequency f , the lift-off l_1 was fixed at 1.0 mm and the values of frequency f were 2.0, 4.0, 6.0, 8.0, and 10 MHz. To draw the impedance plane diagrams with different values of lift-off l_1 , the frequency f was fixed at 10 MHz, and the values of lift-off l_1 were 1.0, 2.0, 3.0, 4.0, and 5.0 mm. In all impedance plane diagrams, the relative permittivity ϵ was varied from 1.0 to 1.0×10^7 .

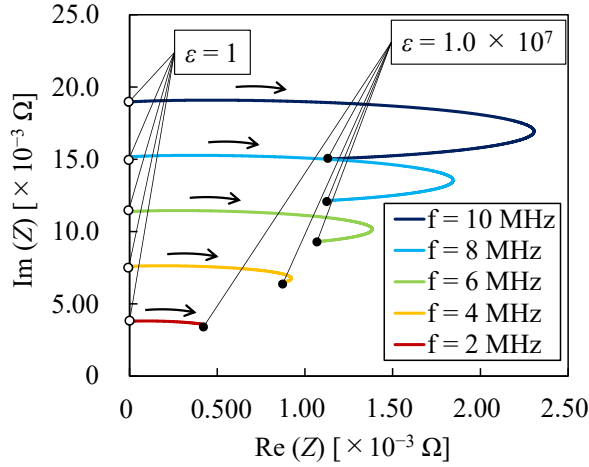


Fig. 4 Impedance plane calculations using the Ampère–Maxwell equation for the dielectric while considering the relative permittivity ϵ only when the frequency f and the relative permittivity ϵ are varied

Figure 4 shows the results obtained when the impedance plane diagrams were drawn for various values of frequency f . The vertical and horizontal axes in Fig. 4 are the same as in Fig. 2. Figure 4 shows that the impedance plane diagrams are elliptical in shape at all frequencies f and that the impedance moves clockwise with increasing relative permittivity ϵ . The impedance diagram radius increases with increasing frequency f and it moves in the + direction of the y -axis. The results above indicate that when the frequency f increases, the change in the impedance also increases with respect to the change in permittivity. It has been reported previously that the penetration depth of the displacement current is independent of the operating frequency up to approximately 10 MHz [2], which means that the highest possible frequency should be applied to increase sensitivity to changes in the permittivity without needing to take the coil's resonance frequency into consideration.

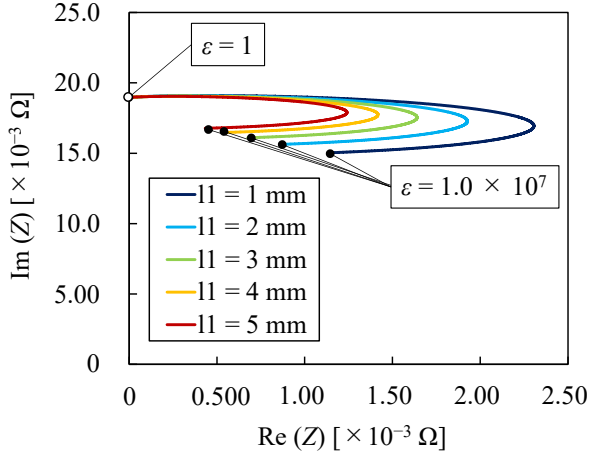


Fig. 5 Impedance plane calculations using the Ampère–Maxwell equation for the dielectric while considering the relative permittivity ε only when the lift-off l_1 and the relative permittivity ε are varied

Figure 5 shows the results obtained when the impedance plane was drawn for different values of lift-off l_1 . The vertical and horizontal axes in Fig. 5 are the same as in Fig. 2. Figure 5 shows that for any value of the lift-off l_1 , the impedance plane diagrams are elliptical and that the impedance moves in a clockwise direction when the relative permittivity ε increases. The figure also shows that the impedance increases in terms of the radius of the impedance plane diagrams when the lift-off l_1 decreases. The results above indicate that a smaller lift-off l_1 leads to a greater change in impedance with respect to any changes in the permittivity, and the lift-off should therefore be as small as possible to increase sensitivity to changes in the permittivity.

These results demonstrate that the conductivity σ in a conductor corresponds to the relative permittivity ε in a dielectric, and that EIT for an ideal dielectric, i.e., testing in which the permittivity alone is considered, can enable an impedance plane diagram to be drawn with the same tendency as that for EIT of a conductor.

Case B. When both the conductivity and permittivity are considered

Next, actual dielectric materials are discussed. In actual dielectrics, it is necessary to consider the conductivity along with the permittivity. This is because it has been reported that the dielectric conductivity σ tends to increase with increasing frequency f . For example, the conductivity of epoxy resin ranges from approximately 1.0×10^{-4} S/m to 1.0×10^{-3} S/m at 10 MHz [18, 19]. This conductivity range is approximately 10^9 to 10^{10} S/m greater than the range estimated by Mizukami et al. when they assumed infinitely small dielectric conductivity [2]. When 1.0×10^{-4} S/m is used as the dielectric conductivity, the magnitudes of the eddy currents J_e and the displacement currents J_d can be compared in the same manner that they were compared by Mizukami et al. by using equation (14).

$$\frac{J_d}{J_e} = \frac{\omega^2 \mu_0 \epsilon_0 \epsilon}{\omega \mu_0 \sigma} = \frac{\omega \epsilon_0 \epsilon}{\sigma} = \frac{2 \times 3.14 \times 1.0 \times 10^7 \times 8.85 \times 10^{-12} \times 3}{1.0 \times 10^{-4}} \cong 16.7 \quad (14)$$

Therefore, it was found to be necessary to account for the effects of the eddy currents along with the effects of the displacement currents in actual dielectrics, and in the analysis presented here, the effects of both conductivity and permittivity were taken into account. When impedance plane diagrams were drawn with different frequencies f , the lift-off l_1 was fixed at 1.0 mm, and the values of the frequency f were 2.0, 4.0, 6.0, 8.0, and 10 MHz. When the impedance plane diagrams were drawn with different values of the lift-off l_1 , the frequency f was fixed at 10 MHz, and the values of l_1 were 1.0, 2.0, 3.0, 4.0, and 5.0 mm. For all the impedance diagrams, the analysis was performed while varying either the conductivity σ or the relative permittivity ϵ . When the conductivity σ was varied, the relative permittivity ϵ was fixed at 3.0, and the conductivity σ was varied from 1.0 to 1.0×10^7 S/m. When the relative permittivity ϵ was varied, the conductivity σ was fixed at 1.0×10^{-4} S/m, and the relative permittivity ϵ was varied from 1.0 to 1.0×10^7 .

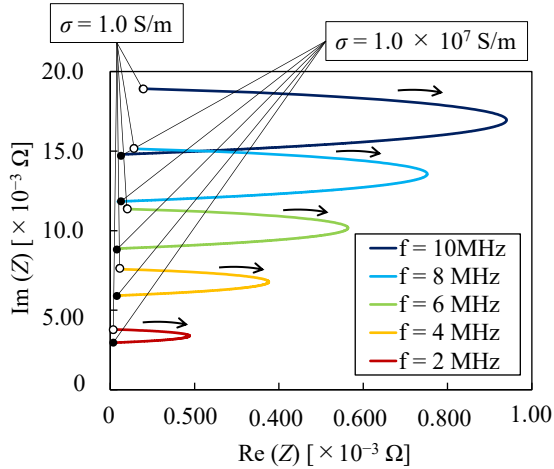


Fig. 6 Impedance plane calculations using the Ampère–Maxwell equation for the dielectric while considering the conductivity σ and the relative permittivity ϵ when the frequency f and the conductivity σ are varied

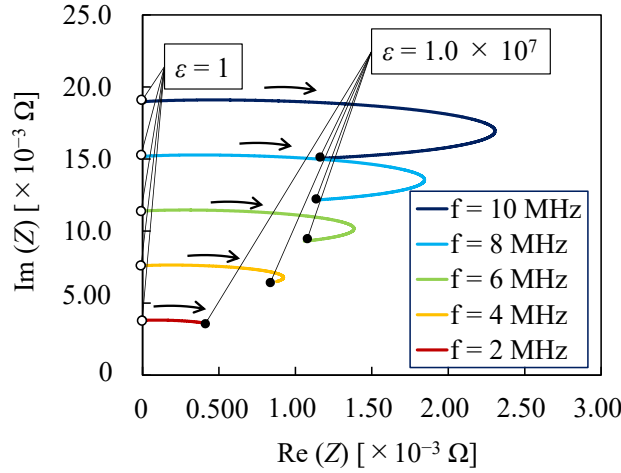


Fig. 7 Impedance plane calculations using the Ampère–Maxwell equation for the dielectric while considering the conductivity σ and the relative permittivity ε when the frequency f and the relative permittivity ε are varied

Figures 6 and 7 show the results obtained when the impedance plane diagrams were drawn at different values of the frequency f . Figure 6 shows the impedance plane diagrams obtained when the relative permittivity ε was fixed at 3.0 and Fig. 7 shows the impedance plane diagrams obtained when the electrical conductivity σ was fixed at 1.0×10^{-4} S/m. The vertical and horizontal axes in Figs. 6 and 7 are the same as in Fig. 2. In the impedance plane diagrams in Figs. 6 and 7, the trend in Fig. 6 matches that of Fig. 2, which takes only conductivity into account, and the trend in Fig. 7 matches that of Fig. 4, which takes only permittivity into account. Figure 6 shows that the impedance plane diagrams do not start on the y -axis when the conductivity σ is varied, but this problem can be resolved by reducing the conductivity σ further and then performing the calculation.

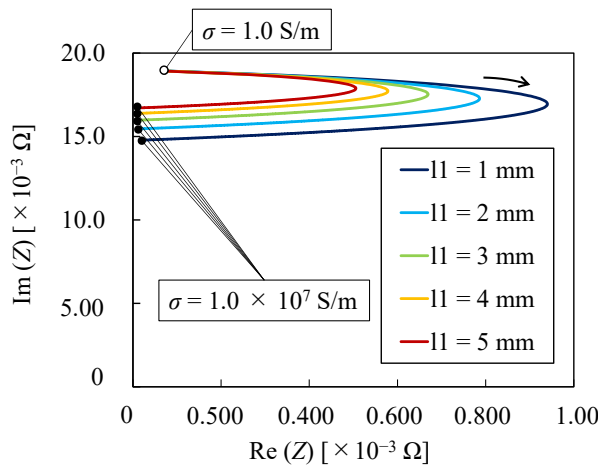


Fig. 8 Impedance plane calculations using the Ampère–Maxwell equation for the dielectric while considering

the conductivity σ and the relative permittivity ε when the lift-off l_1 and the conductivity σ are varied

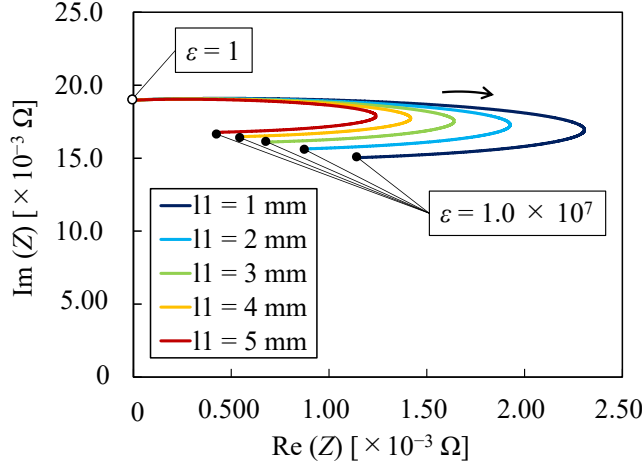


Fig. 9 Impedance plane calculations using the Ampère–Maxwell equation for the dielectric while considering the conductivity σ and the relative permittivity ε when the lift-off l_1 and the relative permittivity ε are varied

Figures 8 and 9 show the results obtained when the impedance plane diagrams were drawn at different values of the lift-off l_1 . The vertical and horizontal axes in Figs. 8 and 9 are the same as in Fig. 2. In the impedance plane diagrams in Figs. 8 and 9, the trend in Fig. 8 matches that of Fig. 3, which takes only conductivity into account, and the trend in Fig. 9 matches that of Fig. 5, which takes only permittivity into account. In Fig. 8, the impedance plane diagrams do not start on the y -axis; however, as shown in Fig. 6, this issue can be resolved by reducing the conductivity σ further and then performing the calculation.

The results above showed that the impedance plane diagrams for actual dielectrics with both conductivity and permittivity can be drawn that show the same tendencies as those for ideal dielectrics, where only the permittivity is considered. Therefore, it is possible to draw impedance plane diagrams for both conductors and dielectrics with similar trends in EIT.

3. Drawing impedance plane diagrams from a simple electrical circuit model

3. 1. Calculation of the impedance when considering a resistor and a capacitor

Based on the electric circuit model that is applied to ECT, i.e., for conductors, an electric circuit model for dielectrics is constructed here and an impedance plane diagram is then drawn. It was reported previously that ECT can simulate the actual test conditions by using an electric circuit model, which then enables the impedance diagram to be drawn [5, 6], and the tendency of this diagram agrees with that of the theoretical impedance plane diagram that was drawn based on the Ampère–Maxwell equation. For this reason, this section also presents an investigation of whether the tendency in EIT for dielectrics matches the tendency above by comparing the

results obtained from the Ampère-Maxwell equation with the impedance plane diagrams drawn based on the electric circuit model.

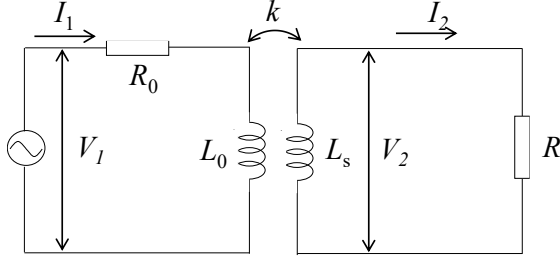


Fig. 10 Electrical circuit model for EIT of electrical conductors

The electric circuit model used in ECT is explained first [5, 6]. Figure 10 shows the electric circuit model when applied to a conductor. The electric circuit shown in Fig. 10 consists of a primary circuit and a secondary circuit, where the primary circuit side represents a coil connected to the power source and the secondary circuit side represents the material under test. Application of a voltage V_1 to the primary circuit side induces a current I_1 , and the flow of I_1 in the coil induces another current I_2 in the secondary circuit by mutual induction. The magnetic coupling between the primary and secondary circuits here can be expressed using the coupling coefficient k (where $0 \leq k \leq 1$). Using k , the inductance L_0 of the primary circuit, the inductance L_s of the secondary circuit, and the mutual inductance M of the primary and secondary circuits in Fig. 10 can be calculated using $M = k(L_0 L_s)^{1/2}$. The current I_2 induced in the secondary circuit by mutual induction flows through the resistance R , which determines the impedance of the entire circuit. Here, R corresponds to the conductivity σ of a conductor, and the relationship between the resistance R and the conductivity σ can be calculated using $R = \rho l / A = l / (\sigma A)$.

In this equation, ρ is the resistivity, A is the cross-sectional area of the test specimen, and l is the length of the specimen. From this equation, l/A must be calculated to link the resistance R to the conductivity σ . In this paper, the value of l/A is assumed to be constant, and the changes in the resistance R are assumed to be changes in the conductivity σ .

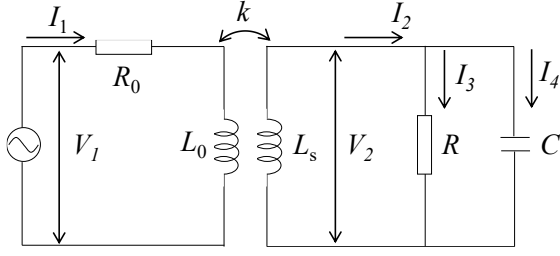


Fig. 11 Electrical circuit model for EIT of dielectrics when considering the resistor and the capacitor

For the electric circuit model for conductors described above, a circuit model is proposed that takes both the conductivity and the permittivity into account by connecting a capacitor in parallel to the resistor on the secondary circuit side. Figure 11 shows the resulting electric circuit model when both the conductivity and the permittivity are considered. In Fig. 10, only an inductor and a resistor were used as circuit elements on the secondary circuit side, whereas in Fig. 11, a capacitor has been added to the secondary circuit of Fig. 10, and this capacitor is connected in parallel with the resistor. The parallel-connected resistor and capacitor in the secondary circuit are the circuit elements that are used to simulate dielectrics when drawing Cole-Cole plots, which are applied in the electrochemical field [12, 13]. The capacitor shown in Fig. 11 corresponds to the permittivity, and the relationship between its capacitance C and its relative permittivity ϵ can be calculated using $C = \epsilon\epsilon_0 S/d$. In this equation, S is the area of the pole plate and d is the distance between the electrodes. As the equation indicates, it is necessary to calculate S/d to link the capacitance C with the relative permittivity ϵ . In this paper, the value of S/d is assumed to be constant and the changes in the capacitance C are assumed to be changes in the relative permittivity ϵ .

The impedance of the secondary circuit relative to that of the primary circuit in the electric circuit shown in Fig. 11 is calculated using Kirchhoff's law, where I_3 is the current passing through the resistor and I_4 is the current passing through the capacitor. Equation (15) is then obtained as follows:

$$\begin{cases} V_1 = j\omega L_0 I_1 + j\omega M I_2 + R_0 I_1 \\ V_2 = j\omega M I_1 + j\omega L_s I_2 \\ V_2 = \left(\frac{j}{\omega C} \right) (-I_3) \\ V_2 = -R I_4 \\ I_2 = I_3 + I_4 \end{cases} \quad (15)$$

To solve equation (15), the impedance of the secondary circuit relative to that of the primary circuit must be calculated, and x and y are defined as shown in equation (16) below:

$$\frac{V_1}{I_1} = R_0 + j\omega L_0 - \omega^2 M^2 \left\{ \frac{j\omega RC + 1}{-R - j\omega L_s(j\omega RC + 1)} \right\} \equiv x + jy \quad (16)$$

Solution of equation (16) then yields equation (17):

$$\begin{cases} x - R_0 = \frac{\omega^2 M^2 R}{(R - \omega^2 RCL_s)^2 + \omega^2 L_s^2} \\ y - \omega L_0 = \frac{-\omega^3 M^2 L_s + \omega^3 M^2 RC(R - \omega^2 RCL_s)}{(R - \omega^2 RCL_s)^2 + \omega^2 L_s^2} \end{cases} \quad (17)$$

In the following sections, equation (17) will be used to draw the impedance plane diagrams when EIT is applied to conductors and dielectrics. Here, the resistance of the primary circuit R_0 was assumed to be 100 Ω . The inductance L_0 on the primary circuit side, i.e., the coil inductance, was calculated using equation (18), which was proposed by Nagaoka [20]:

$$L_0 = K_N \frac{\mu_0 \mu \pi r^2 n^2}{l} \quad (18)$$

Here, μ is the relative permeability, K_N is the coefficient reported by Nagaoka [20], r is the coil radius, l is the coil height, and n is the number of turns in the coil. To calculate (18), the same values that were described above were used, with the exception of the values of r and K_N . The coil radius r was calculated using the average of the inner and outer diameters of the coil, and the resulting value was 15 mm. The ratio of the coil diameter $2r$ to the coil height l , as calculated based on the radius r , gives equation (19) as follows:

$$\frac{2r}{l} \cong 0.714 \quad (19)$$

Using this value, the value of K_N that was proposed in previous studies was calculated to be 0.75 [20]. The inductance L_s on the secondary circuit side was assumed to be 1/10 of the value of L_0 . In all analyses, the impedance plane diagrams were drawn for various values of the frequency f and coupling coefficient k for the specimen while considering only the resistance R of the secondary circuit, only the capacitance C of the secondary circuit, and both the resistance R and the capacitance C of the secondary circuit. The coupling coefficient k represents the degree of magnetic coupling between the primary and secondary circuits, and does not strictly represent the lift-off alone; in this study, however, it was assumed that changes in the coupling coefficient represent the changes in the lift-off.

3. 2. Drawing the impedance plane diagrams

3. 2. 1. Conductors

When impedance diagrams are drawn based on the electric circuit model for a conductor, the permittivity (or the capacitor) is not taken into account, as per the analysis. For this reason, the current flow through the capacitor must be set at zero. The reactance Z_C of the capacitor can be calculated using $Z_C = 1/(j\omega C)$. This equation shows that as the capacitance C becomes infinitely small, the reactance Z_C of the capacitor then approaches infinity.

As a result, no current can flow through the capacitor and the electric circuit model is as shown in Fig. 10. Therefore, to calculate the impedance of a conductor when using the electric circuit model, equation (20) is obtained by using the limit that $C \rightarrow 0$ for equation (17):

$$\begin{cases} x - R_0 = \lim_{C \rightarrow 0} \frac{\omega^2 M^2 R}{(R - \omega^2 R C L_s)^2 + \omega^2 L_s^2} = \frac{\omega^2 M^2 R}{R^2 + \omega^2 L_s^2} \\ y - \omega L_0 = \lim_{C \rightarrow 0} \frac{-\omega^3 M^2 L_s + \omega^3 M^2 R C (R - \omega^2 R C L_s)}{(R - \omega^2 R C L_s)^2 + \omega^2 L_s^2} = -\frac{\omega^3 M^2 L_s}{R^2 + \omega^2 L_s^2} \end{cases} \quad (20)$$

The equations derived above are consistent with the results that were calculated by applying Kirchhoff's law to the electric circuit model shown in Fig. 10. When impedance diagrams were drawn for the different values of frequency f , the coupling coefficient k was fixed at 0.50 and the values of f were 2.0, 4.0, 6.0, 8.0, and 10 kHz. To draw the impedance plane diagrams with different values of the coupling coefficient k , the frequency f was fixed at 10 kHz and the values of the coupling coefficient k were 0.20, 0.40, 0.60, 0.80, and 1.0. For all the impedance diagrams, the secondary circuit resistance R was varied from $1.0 \times 10^{-2} \Omega$ to $1.0 \times 10^{10} \Omega$.

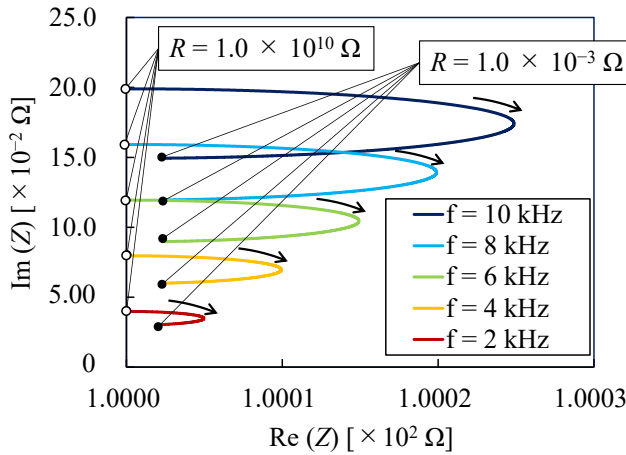


Fig. 12 Impedance plane calculations using the electrical circuit model for the conductor when the frequency f and the resistance R are varied

Figure 12 shows the results obtained when the impedance plane diagrams were drawn at various values of frequency f . The vertical axis of Fig. 12 represents the imaginary part of the impedance Z ($\text{Im}(Z)$), and the horizontal axis represents the real part of Z ($\text{Re}(Z)$). The figure shows that the impedance plane diagrams form circles at any frequency f , and also shows that the impedance plane moves in the clockwise direction as the resistance R decreases, i.e., as the conductivity σ increases. The impedance diagram radius increases as the frequency f increases and the impedance diagrams move in the + direction of the y -axis. The trend of the results presented above are consistent with the impedance diagram drawn based on the Ampère-Maxwell equation (see Fig. 2).

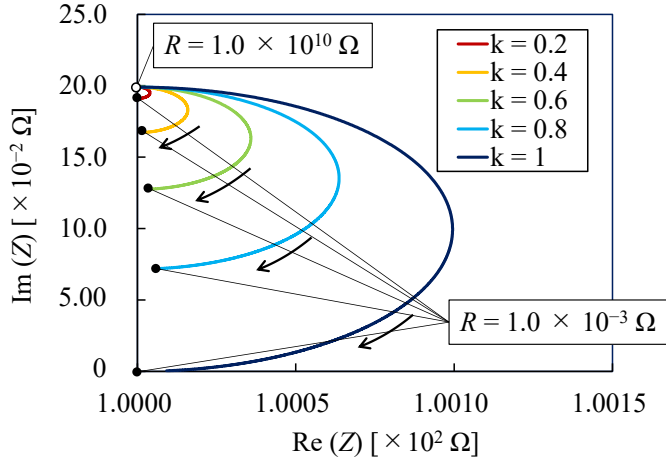


Fig. 13 Impedance plane calculations using the electrical circuit model for the conductor when the coupling coefficient k and the resistance R are varied

Figure 13 shows the results obtained when the impedance plane diagrams were drawn for different values of the coupling coefficient k . The vertical axis in Fig. 13 represents the imaginary part of the impedance Z ($\text{Im}(Z)$) and the horizontal axis represents the real part of Z ($\text{Re}(Z)$). The figure shows that the impedance moves in the clockwise direction as the resistance R decreases, i.e., as the conductivity σ increases. Additionally, it shows that the impedance diagram radius increases as the coupling coefficient k increases, i.e., as the lift-off l_1 decreases. The trend of the results shown above are consistent with the corresponding impedance plane diagrams drawn based on the Ampère-Maxwell equation (see Fig. 3).

3. 2. 2. Dielectrics

Case A. When only the capacitance is considered

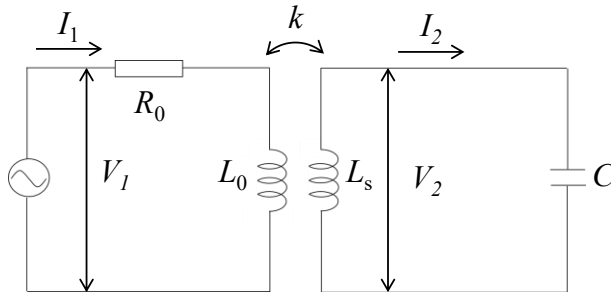


Fig. 14 Electrical circuit model for EIT of dielectrics while considering the capacitor only

When an impedance plane diagram is drawn from an electric circuit model for an ideal dielectric, where only the permittivity is considered and the conductivity is 0 S/m, the conductivity (or resistance) is not taken into account, as per the case of the analysis presented above. For this reason, the current that flows through the resistance must be set at zero. When the resistance R is assumed to be as high as possible, no current can flow through the resistor, and the electric circuit model then becomes as shown in Fig. 14. For this reason, to enable the impedance of the dielectric material to be calculated when considering the permittivity only using the electric circuit model, equation (21) is obtained by assuming the limit that $R \rightarrow \infty$ for equation (17):

$$\begin{cases} x - R_0 = \lim_{R \rightarrow \infty} \frac{\omega^2 M^2 R}{(R - \omega^2 R C L_s)^2 + \omega^2 L_s^2} = 0 \\ y - \omega L_0 = \lim_{R \rightarrow \infty} \frac{-\omega^3 M^2 L_s + \omega^3 M^2 R C (R - \omega^2 R C L_s)}{(R - \omega^2 R C L_s)^2 + \omega^2 L_s^2} = \frac{\omega^3 M^2 C}{(1 - \omega^2 C L_s)} \end{cases} \quad (21)$$

The equation derived above agrees well with the results calculated by applying Kirchhoff's law to the electric circuit model shown in Fig. 14. Equation (21) indicates that x is R_0 for any value of y , which implies a straight line oriented parallel to the y -axis. Therefore, the impedance diagram that is calculated based on the electric circuit model when the permittivity alone is considered clearly differs from the impedance plane diagrams drawn based on the Ampère-Maxwell equation. In other words, the proposed electric circuit model cannot be used to draw an impedance plane diagram for an ideal dielectric.

Case B. When both resistance and capacitance are considered

When an impedance plane diagram is drawn based on the electric circuit model for a dielectric material in which both the permittivity and the conductivity are considered, the resistance and capacitance are then taken into account. When impedance diagrams were drawn with different values of the frequency f , the coupling coefficient k was fixed at 0.50 and the values of f were 2.0, 4.0, 6.0, 8.0, and 10 MHz. Furthermore, when impedance plane diagrams were drawn for different values of the coupling coefficient k , the frequency f was fixed at 10 MHz and the values of the coupling coefficient k were 0.20, 0.40, 0.60, 0.80, and 1.0. In all the impedance diagrams, the analysis was performed while either the resistance R or capacitance C of the secondary circuit was varied. When R was varied, the capacitance C was fixed at 1.0×10^{-12} F and R was varied from $1.0 \times 10^{-2} \Omega$ to $1.0 \times 10^{10} \Omega$. When C was varied, the resistance R was fixed at $1.0 \times 10^2 \Omega$ and C was varied from 1.0×10^{-12} F to 5.5×10^{-8} F.

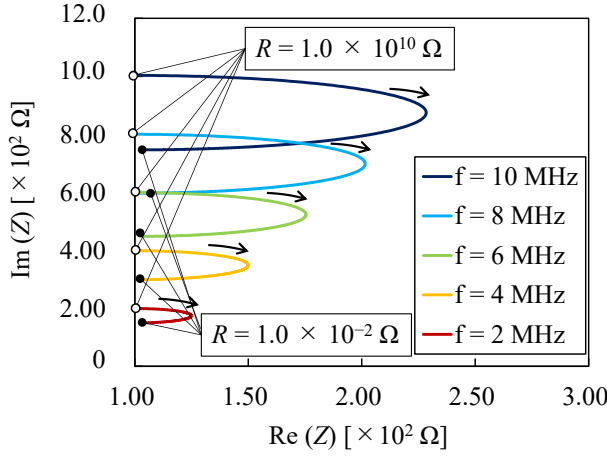


Fig. 15 Impedance plane calculations using the electrical circuit model for the dielectric while considering the resistance R and the capacitance C when the frequency f and the resistance R are varied

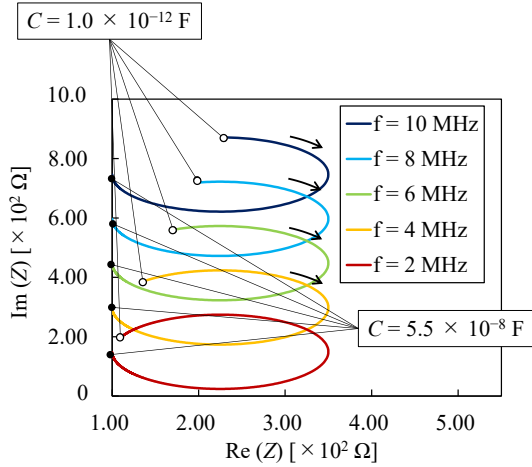


Fig. 16 Impedance plane calculations using the electrical circuit model for the dielectric while considering the resistance R and the capacitance C when the frequency f and the capacitance C are varied

Figures 15 and 16 show the results obtained when the impedance plane diagrams were drawn for different values of the frequency f . Figure 15 shows the impedance plane diagrams obtained when the capacitance C was fixed at 1.0×10^{-12} F and Fig. 16 shows the impedance plane diagrams obtained when R was fixed at $1.0 \times 10^2 \Omega$. The vertical axes in Figs. 15 and 16 represent the imaginary part of the impedance Z ($\text{Im}(Z)$) and the horizontal axes represent the real part of Z ($\text{Re}(Z)$). In Figs. 15 and 16, the impedance plane diagrams are circular in shape at all values of the frequency f , and we can see that the impedance moves clockwise with decreasing resistance R (i.e., with increasing conductivity σ) in Fig. 15 and with increasing capacitance C (i.e., with increasing relative permittivity ϵ) in Fig. 16. Furthermore, these figures also show that the impedance transitions occur in the +

direction of the y -axis when the frequency f increases. In Fig. 15, which shows an impedance diagram with fixed capacitance C and varying resistance R , the impedance diagram radius increases as f increases, and the starting point of this impedance diagram begins on the y -axis at any frequency f . In contrast, in Fig. 16, which shows an impedance diagram with fixed resistance R and varying capacitance C , the impedance plane diagram radius remains constant, even when the frequency f increases, and the starting point of this impedance plane diagram varies with frequency f .

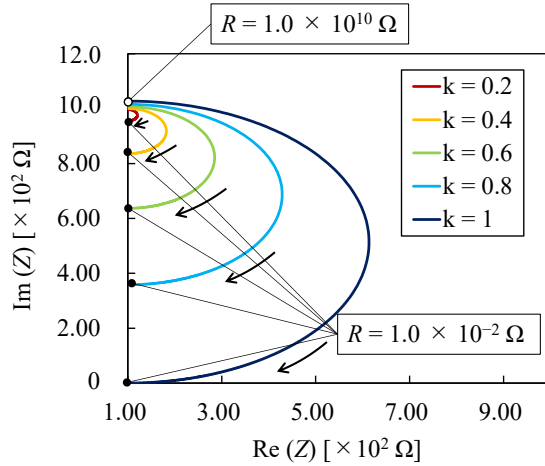


Fig. 17 Impedance plane calculations using the electrical circuit model for the dielectric while considering the resistance R and the capacitance C when the coupling coefficient k and the resistance R are varied

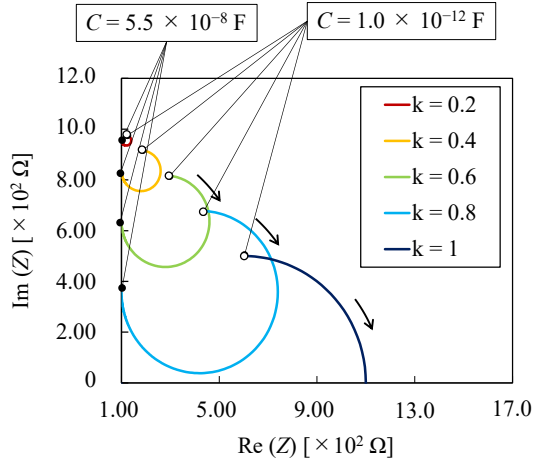


Fig. 18 Impedance plane calculations using the electrical circuit model for the dielectric while considering the resistance R and the capacitance C when the coupling coefficient k and the capacitance C are varied

Figures 17 and 18 show the results obtained when the impedance plane diagrams were drawn for different

values of the coupling coefficient k . Figure 17 shows the impedance plane diagram obtained when the capacitance C was fixed at 1.0×10^{-12} F and Fig. 18 shows the impedance plane diagram obtained when the resistance R was fixed at $1.0 \times 10^2 \Omega$. The vertical axes in Figs. 17 and 18 represent the imaginary part of Z ($\text{Im}(Z)$), and the horizontal axes represent the real part of Z ($\text{Re}(Z)$). In Figs. 17 and 18, the impedance plane diagrams are circular in shape for all values of coupling coefficient k . In Fig. 17, the impedance moves clockwise with decreasing resistance R (i.e., with increasing conductivity σ) and in Fig. 18, the impedance moves clockwise with increasing capacitance C (i.e., with increasing relative permittivity ε). These figures also show that the impedance plane diagram radius increases as the coupling coefficient k increases. In Fig. 17, which shows an impedance diagram drawn with fixed capacitance C and varying resistance R , the impedance diagram is drawn from the same point in each case, regardless of the value of coupling coefficient k that is used. In contrast, in Fig. 18, which shows an impedance diagram drawn with fixed resistance R and varying capacitance C , the starting point of the impedance diagram also varies when the frequency f changes.

4. Discussion

In the work presented above, two methods to draw impedance diagrams were proposed: a rigorous method based on the Ampère-Maxwell equation and an approximate method based on an electric circuit model. In this section, the two proposed methods are compared.

When the impedance plane diagrams were drawn strictly based on the Ampère-Maxwell equation, as illustrated in Figs. 4 and 5, impedance plane diagrams could be drawn even for an ideal dielectric with its conductivity set at 0 S/m. In contrast, when the impedance plane was approximated using the electric circuit model, the impedance plane diagram could not be drawn when the conductivity was 0 S/m, i.e., when the resistance was infinitely large. The above results indicate that if an impedance plane diagram is to be drawn using the electric circuit model proposed in this study, then the dielectric must have a conductivity greater than 0 S/m. However, the actual dielectric does have a low level of conductivity [18, 19]. Therefore, when the measurement object is a dielectric, it is possible to draw an impedance plane diagram using the electric circuit model proposed in this study.

Figures 6 and 8, which show impedance plane diagrams drawn with a fixed relative permittivity ε and varying conductivity σ , and Figs. 15 and 17, which show impedance diagrams drawn with a fixed capacitance C and varying resistance R , show the same tendencies in the rigorous and approximate drawings. Specifically, we found that the changes in radius with changing lift-off and frequency, the initial point of the impedance, and the impedance transition direction were consistent with the changing conductivity (resistance). In contrast, Figs. 7 and 9, which show impedance plane diagrams drawn with a fixed conductivity σ and varying relative permittivity ε , and Figs. 16 and 18, which show impedance plane diagrams drawn with a fixed resistance R and varying capacitance C , show partial differences between the tendencies for the rigorous and approximate

methods. Specifically, it was found that the initial point of the impedance when the radius and the permittivity (capacitance) changed with increasing frequency did not coincide with the initial point of the impedance when the impedance plane was drawn either rigorously or approximately. These results indicate that it is difficult to draw a rigorous impedance plane diagram for dielectrics when using the electric circuit model proposed in this study, unlike the case for conductors, which has been researched in depth in previous studies. The above reason is thought to be due to the circuit configuration of the secondary circuit in the electric circuit model. In this study, for simplicity, we applied the ECT electric circuit model and proposed a circuit model in which equal voltage is applied to the resistor and capacitor due to mutual induction between the coils in the primary and secondary circuits. When an AC voltage is applied to an actual dielectric, a displacement current is generated due to polarization due to electric dipoles inside the dielectric, and eddy currents are also generated due to the slight movement of free electrons. The magnitude of displacement currents and eddy currents, in addition to permittivity and conductivity, depend on the magnitude of frequency [21], and permittivity and conductivity also depend on frequency [22–24]. The proposed electric circuit models do not sufficiently take the latter into account, and to express this in an electric circuit model, it is possible to control the voltage applied to each capacitor and resistor according to the frequency of the applied voltage, for example. To this end, the error between the circuit model and the Ampère-Maxwell equation could be improved by making the entire circuit more complex, such as by changing the coupling method between the primary and secondary circuits or by introducing new electric circuit elements to make the secondary circuit more complex. Nevertheless, it was still possible to simulate the direction of the transition in the impedance diagram when the capacitance increased (increase in permittivity), the transition in the impedance diagram in the + direction along the y -axis as the frequency increased, and the increase in the impedance diagram radius as the coupling coefficient increased (reduction in lift-off) when using an impedance diagram that was approximated based on an electric circuit model. Therefore, it is assumed that the electric circuit model proposed in this study is applicable to qualitative evaluation, although it is difficult to calculate the quantitative impedance changes. In fact, when performing inspections via ECT using an impedance planar diagram, it is important that the qualitative impedance planar diagram changes with any changes in the defects and the physical properties. As a result, it is important that the impedance plane that is derived from the electric circuit model matches the impedance plane derived from the theoretical equation in terms of operation during actual testing. It is thus assumed that the impedance plane diagram derived using the electric circuit model proposed in this study is fully applicable to actual impedance plane inspection operations and that it provides useful knowledge to aid in understanding of the physical phenomenon.

When the electric circuit model is used to approximate impedance plane diagrams for different frequencies and different lift-offs, the direction of the transition in the impedance plane diagrams with decreasing resistance (increasing conductivity) and increasing capacitance (increasing permittivity) can be seen to differ clearly from

that in the impedance plane diagrams with the increasing coupling coefficient (decreasing lift-off). Therefore, even when using an approximate drawing method based on an electric circuit model, the behavior observed in the impedance plane diagram when the conductivity or the dielectric constant changes because of the appearance of a crack, for example, is different to the behavior observed when the lift-off changes because of wall thinning or other phenomena. This helps to provide a qualitative understanding of the effects of the dielectric state and measurement conditions on EIT.

The discussion above indicates that the electric circuit model proposed in this study enables an impedance plane diagram to be drawn when EIT is applied to a dielectric material and also enables qualitative evaluation of the effects of the frequency, lift-off, material conductivity, and dielectric constant on the impedance plane diagram. However, the current electrical circuit models are insufficient for use in quantitative impedance evaluation. This problem could be resolved by increasing the complexity of the circuit model proposed in this study, but that will form part of our future work.

5. Conclusions

In this study, to expand the application scope of the impedance plane diagram, which has been widely applied as an evaluation method in ECT, to evaluation of EIT results for dielectrics, the method of drawing this diagram rigorously based on the Ampère-Maxwell equation and a method of drawing the diagram approximately from an electric circuit model were investigated and compared.

In the drawing realized using the Ampère-Maxwell equation, by considering displacement currents in addition to eddy currents, an impedance plane diagram was drawn that took both the conductivity and dielectric constant of the specimen into account. It was shown that the impedance planar diagram obtained for the dielectric can be used to check its impedance behavior when the measurement frequency and lift-off are varied, as per the case for a conductor.

In the drawing that was developed using a simplified electric circuit model, an electric circuit model that is applicable to dielectrics was proposed by newly connecting a capacitor in parallel to the secondary circuit side of the circuit model that was conventionally applied to the ECT of conductive materials. The impedance planar diagram that was calculated simply using this electric circuit model shows a qualitative trend that agrees with that of the rigorous planar diagram obtained using the Ampère-Maxwell equation, although it lacks quantitative properties at present. Because qualitative changes in the impedance plane diagram are important for evaluations performed with the impedance plane diagram, the effectiveness of the impedance plane diagram derived using a simplified electric circuit model for use in the inspection of dielectrics by performing actual EIT was demonstrated.

Statements and Declarations

Ethics approval and consent to participate

Not applicable.

Consent for publication

All authors consent to publication.

Availability of data and material

The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

Competing interests

The authors have no competing interests to declare that are relevant to the content of this article..

Funding

This work was supported by JSPS KAKENHI under Grant Number JP20J21500.

Author contributions

Conceptualization, methodology, formal analysis and investigation, writing (original draft preparation and review and editing), and funding acquisition were performed by Wataru Matsunaga. Writing (review and editing), resource acquisition, and supervision were performed by Yoshihiro Mizutani.

References

1. Heuer, H., Schulze, M.H., Meyendorf, N.: Non-destructive evaluation (NDE) of composites: Eddy current techniques. *Non-Destructive Evaluation (NDE) of Polymer Matrix Composites: Techniques and Applications*. 33–55 (2013).
2. Mizukami, K., Mizutani, Y., Todoroki, A., Suzuki, Y.: Design of eddy current-based dielectric constant meter for defect detection in glass fiber reinforced plastics. *NDT and E International*. 74, 24–32 (2015). <https://doi.org/10.1016/j.ndteint.2015.04.005>
3. Gäbler, S., Heuer, H., Heinrich, G.: Measuring and imaging permittivity of insulators using high-frequency eddy-current devices. *IEEE Trans Instrum Meas.* 64, 2227–2238 (2015). <https://doi.org/10.1109/TIM.2015.2390851>
4. Matsunaga, W., Mizukami, K., Mizutani, Y., Todoroki, A.: Estimation of the moisture absorption rate

- of glass fiber reinforced plastic using electromagnetic induction testing. *Composites Part C: Open Access*. 8, 100260 (2022). <https://doi.org/10.1016/j.jcomc.2022.100260>
5. Dominique, P., Isabelle, D.: Eddy current sensors for nondestructive inspection of graphite composite materials. In: *Conference Record - IAS Annual Meeting (IEEE Industry Applications Society)*. pp. 1676–1682 (1992)
 6. Le Bihan, Y.: Study on the transformer equivalent circuit of eddy current nondestructive evaluation. *NDT and E International*. 36, 297–302 (2003). [https://doi.org/10.1016/S0963-8695\(03\)00003-3](https://doi.org/10.1016/S0963-8695(03)00003-3)
 7. Theodoulidis, T.P., Kriezis, E.E.: Impedance evaluation of rectangular coils for eddy current testing of planar media. *NDT & E International*. 35, 407–414 (2002). [https://doi.org/10.1016/S0963-8695\(02\)00008-7](https://doi.org/10.1016/S0963-8695(02)00008-7)
 8. Xu, P., Huang, S., Zhao, W.: Differential eddy current testing sensor composed of double gradient winding coils for crack detection. In: *2010 IEEE Sensors Applications Symposium (SAS)*. pp. 59–63. IEEE (2010)
 9. Egorov, A. V., Polyakov, V. V., Salita, D.S., Kolubaev, E.A., Psakhie, S.G., Chernyavskii, A.G., Vorobei, I. V.: Inspection of aluminum alloys by a multi-frequency eddy current method. *Defence Technology*. 11, 99–103 (2015). <https://doi.org/10.1016/j.dt.2014.12.002>
 10. Mizukami, K., Watanabe, Y., Ogi, K.: Eddy current testing for estimation of anisotropic electrical conductivity of multidirectional carbon fiber reinforced plastic laminates. *Compos Part A Appl Sci Manuf*. 143, (2021). <https://doi.org/10.1016/j.compositesa.2021.106274>
 11. Wu, D., Chen, W., Yang, J., Yang, F.: Self-resonance eddy-current testing method in noncontact detection of carbon fiber reinforced plastics. *Compos Struct*. 259, (2021). <https://doi.org/10.1016/j.compstruct.2020.113247>
 12. Huu Chuc, N., Thuy, D.V., Park, J., Kim, D., Koo, J., Lee, Y., Nam, J.-D., Choi, H.R.: A dielectric elastomer actuator with self-sensing capability. In: *Electroactive Polymer Actuators and Devices (EAPAD) 2008*. p. 69270V. , San Diego (2008)
 13. Bindra, P., Hazra, A.: Capacitive gas and vapor sensors using nanomaterials. *Journal of Materials Science: Materials in Electronics*. 29, 6129–6148 (2018). <https://doi.org/10.1007/s10854-018-8606-2>
 14. Dodd, C. V., Deeds, W.E.: Analytical Solutions to Eddy-Current Probe-Coil Problems. *J Appl Phys*. 39, 2829–2838 (1968). <https://doi.org/10.1063/1.1656680>
 15. Luquire, J.W., Deeds, W.E., Dodd, C. V.: Alternating current distribution between planar conductors. *J Appl Phys*. 41, 3983–3991 (1970). <https://doi.org/10.1063/1.1658399>
 16. Chen, Z., Jomdecha, C., Xie, S.: Eddy Current Testing. In: *Ida, N. and Meyendrof, N. (eds.) Handbook of Advanced Nondestructive Evaluation*. pp. 645–728. Springer International Publishing (2019)
 17. Auld, B.A., Moulder, J.C.: Review of Advances in Quantitative Eddy Current Nondestructive

- Evaluation. *J Nondestr Eval.* 18, (1999). <https://doi.org/10.1023/A:1021898520626>
18. Kim, Y.J., Shin, T.S., Choi, H. Do, Kwon, J.H., Chung, Y.C., Yoon, H.G.: Electrical conductivity of chemically modified multiwalled carbon nanotube/epoxy composites. *Carbon N Y.* 43, 23–30 (2005). <https://doi.org/10.1016/j.carbon.2004.08.015>
 19. Gardea, F., Lagoudas, D.C.: Characterization of electrical and thermal properties of carbon nanotube/epoxy composites. *Compos B Eng.* 56, 611–620 (2014). <https://doi.org/10.1016/j.compositesb.2013.08.032>
 20. Nagaoka H (1909) The inductance coefficients of solenoids. *Journal of the College of Science, Imperial University of Tokyo, Japan* 27:1–33 (in Japanese). <https://doi.org/10.15083/00037799>
 21. Mizukami, K., Mizutani, Y., Todoroki, A., Suzuki, Y.: Design of eddy current-based dielectric constant meter for defect detection in glass fiber reinforced plastics. *NDT and E International.* 74, 24–32 (2015). <https://doi.org/10.1016/j.ndteint.2015.04.005>
 22. Sandler, J., Shaffer, M.S.P., Prasse, T., Bauhofer, W., Schulte, K., Windle, A.H.: Development of a dispersion process for carbon nanotubes in an epoxy matrix and the resulting electrical properties. *Polymer (Guildf).* 40, 5967–5971 (1999). [https://doi.org/10.1016/S0032-3861\(99\)00166-4](https://doi.org/10.1016/S0032-3861(99)00166-4)
 23. Gardea, F., Lagoudas, D.C.: Characterization of electrical and thermal properties of carbon nanotube/epoxy composites. *Compos B Eng.* 56, 611–620 (2014). <https://doi.org/10.1016/j.compositesb.2013.08.032>
 24. Bertasius, P., Meisak, D., Macutkevicius, J., Kuzhir, P., Selskis, A., Volnyanko, E., Banys, J.: Fine tuning of electrical transport and dielectric properties of epoxy/carbon nanotubes composites via magnesium oxide additives. *Polymers (Basel).* 11, (2019). <https://doi.org/10.3390/polym11122044>

Figure Captions:

Fig. 1 Analytical model for derivation of analytical solutions for the vector potential when considering the conductivity σ and the relative permittivity ε

Fig. 2 Impedance plane calculations using the Ampère–Maxwell equation for the conductor when the frequency f and the conductivity σ are varied

Fig. 3 Impedance plane calculations using the Ampère–Maxwell equation for the conductor when the lift-off l_1 and the conductivity σ are varied

Fig. 4 Impedance plane calculations using the Ampère–Maxwell equation for the dielectric while considering the relative permittivity ε only when the frequency f and the relative permittivity ε are varied

Fig. 5 Impedance plane calculations using the Ampère–Maxwell equation for the dielectric while considering the relative permittivity ε only when the lift-off l_1 and the relative permittivity ε are varied

Fig. 6 Impedance plane calculations using the Ampère–Maxwell equation for the dielectric while considering

the conductivity σ and the relative permittivity ε when the frequency f and the conductivity σ are varied

Fig. 7 Impedance plane calculations using the Ampère–Maxwell equation for the dielectric while considering the conductivity σ and the relative permittivity ε when the frequency f and the relative permittivity ε are varied

Fig. 8 Impedance plane calculations using the Ampère–Maxwell equation for the dielectric while considering the conductivity σ and the relative permittivity ε when the lift-off l_1 and the conductivity σ are varied

Fig. 9 Impedance plane calculations using the Ampère–Maxwell equation for the dielectric while considering the conductivity σ and the relative permittivity ε when the lift-off l_1 and the relative permittivity ε are varied

Fig. 10 Electrical circuit model for EIT of electrical conductors

Fig. 11 Electrical circuit model for EIT of dielectrics when considering the resistor and the capacitor

Fig. 12 Impedance plane calculations using the electrical circuit model for the conductor when the frequency f and the resistance R are varied

Fig. 13 Impedance plane calculations using the electrical circuit model for the conductor when the coupling coefficient k and the resistance R are varied

Fig. 14 Electrical circuit model for EIT of dielectrics while considering the capacitor only

Fig. 15 Impedance plane calculations using the electrical circuit model for the dielectric while considering the resistance R and the capacitance C when the frequency f and the resistance R are varied

Fig. 16 Impedance plane calculations using the electrical circuit model for the dielectric while considering the resistance R and the capacitance C when the frequency f and the capacitance C are varied

Fig. 17 Impedance plane calculations using the electrical circuit model for the dielectric while considering the resistance R and the capacitance C when the coupling coefficient k and the resistance R are varied

Fig. 18 Impedance plane calculations using the electrical circuit model for the dielectric while considering the resistance R and the capacitance C when the coupling coefficient k and the capacitance C are varied