

Dynamic Mode Decomposition for Subcriticality Measurement Using Measured Data at Single Location

Toshihiro Yamamoto^{a*}, Hiroki Sakamoto^b

^a*Institute for Integrated Radiation and Nuclear Science, Kyoto University, 2 Asashiro Nishi, Kumatori-cho, Sennan-gun, Osaka, 590-0494, Japan*

^b*Independent researcher, Radiation Dose Analysis and Evaluation Network, 4-13-14, Kokubunji-shi, Tokyo, 185-0001, Japan*

Abstract

Dynamic mode decomposition (DMD) is an effective approach for extracting a fundamental mode eigenvalue in subcriticality measurements. According to several previous studies, data for DMD must be obtained at a sufficient number of locations. However, the number of measurement locations is typically limited owing to experimental constraints. In this study, a time-shifting augmentation technique for DMD is employed, where an augmented data matrix is constructed by stacking multiple time-shifted copies of time-series data at a single location. The proposed DMD technique is applied to numerical simulations of subcriticality measurements, such as those involving the pulsed neutron method and Rossi- α method. The fundamental mode eigenvalues of these measurements are accurately extracted via DMD using the augmented data matrix for a single location without introducing the masking time technique.

Keywords: Rossi- α ; pulsed neutron; subcriticality measurement; dynamic mode decomposition

1. Introduction

Subcritical measurement techniques such as the pulsed neutron method, neutron noise method (e.g., Rossi- α (Orndoff, 1957)), and exponential experiment method cannot readily determine the fundamental mode eigenvalue owing to the significant number of higher harmonics involved. Some techniques have been proposed to determine the fundamental mode time-decay constant α using measured data affected by higher harmonics (e.g., Berglöf et al., 2011; Szieberth et al., 2015;

* Corresponding author

E-mail address: yamamoto.toshihiro.8u@kyoto-u.ac.jp (T. Yamamoto)

Katano 2019; Talamo et al., 2013; Talamo et al., 2020). Researchers have recently proposed an approach for extracting the fundamental mode eigenvalue by applying dynamic mode decomposition (DMD) (Schmid, 2010; Kutz et al., 2016) to subcriticality measurement techniques. DMD, which has been originally used in the fluid dynamics community, is an equation-free, data-driven method for precise decomposition of a complex system into spatiotemporal coherent structures. Recently, applications of DMD have extended into the nuclear engineering field (Abdo et al., 2019; Di Ronco et al., 2020; McClarren and Haut, 2022). A unique application of DMD is the acceleration of the power iteration method for criticality calculations (Roberts, et al., 2019). McClarren (2019) and Hardy et al. (2019) applied DMD to analyze the time-decay constant α in the pulsed neutron method. The authors of the present study applied DMD to determine the spatial decay constant in an exponential experiment (Yamamoto and Sakamoto, 2021) and the time decay constant for the Rossi- α method (Yamamoto and Sakamoto, 2022). These DMD applications were performed in numerical experiments pertaining to subcriticality measurement methods. Nishioka et al. (2022) and Endo et al. (2022) applied DMD to an actual subcriticality experiment for the Kyoto University Critical Assembly, where the fundamental mode α -eigenvalue was estimated by the pulsed neutron method and Rossi- α method.

In these DMD applications, a series of measured data obtained at multiple measurement locations was used to construct the snapshot matrix for DMD analysis. Yamamoto and Sakamoto (2021, 2022) discussed the optimal number of measurement locations and their suitable spatial arrangement in an experimental setup. The measurement locations should be as many as possible to accurately estimate the fundamental mode eigenvalue via DMD. However, the desirable number of measurement locations cannot be assured owing to experimental constraints. The objective of this study is to develop a method that enables an accurate fundamental mode eigenvalue to be estimated via DMD using data measured at a single location. This study is inspired by a method that introduces a shift-stacked data matrix to eliminate the inapplicability of DMD to single sine or cosine wave analysis (Tu et al., 2014; Kutz et al., 2016). The time-series data of a single sine or cosine wave constitute a row vector. The DMD applied to this row vector returns only a single real

eigenvalue and no periodic oscillations. To eliminate this problem, a two-row matrix was constructed by stacking a time-shifted copy of the row vector in the second row. Using the two-row matrix, DMD successfully returns an imaginary eigenvalue, and a single wave can be reconstructed via DMD. By extending this method to subcriticality measurement analysis, DMD is expected to be applicable to datasets measured at a single location.

The remainder of this paper is structured as follows: In Section 2, a DMD method that is applicable to subcriticality measurements at a single location is discussed. In Sections 3 and 4, the application of the newly proposed method to pulsed neutron and Rossi- α , respectively, is described. Finally, Section 5 presents the conclusions of the study.

2. DMD for single data location

2.1 DMD for multiple data locations

Because the fundamentals of DMD have been presented extensively in previous publications, only an overview of DMD is presented in this section. For more details, readers can refer to previous publications (e.g., Tu et al., 2014; Kutz et al., 2016).

In the conventional DMD approach, time-series datasets are prepared at n locations in a subcritical system. For the k th time point, a column vector composed of the measured values $x_{i,k}$ ($i = 1, 2, \dots, n$) is constructed, as follows:

$$\mathbf{x}_k = [x_{1,k} \ x_{2,k} \ \dots \ x_{i,k} \ \dots x_{n,k}]^T, \quad (1)$$

where $x_{i,k}$ are the values measured at the i th position. This time-series data is a sequence of data points at regularly spaced time intervals. The measured value $x_{i,k}$ is the neutron count for the pulsed neutron method and is an autocorrelation for the Rossi- α method. When data measured at n locations and J time points are available, two $n \times (J - 1)$ matrices can be constructed as follows:

$$\mathbf{X} = [\mathbf{x}_1 \ \mathbf{x}_2 \ \dots \ \mathbf{x}_j \ \dots \mathbf{x}_{J-1}], \quad (2)$$

$$\mathbf{X}' = [\mathbf{x}_2 \ \mathbf{x}_3 \ \dots \ \mathbf{x}_j \ \dots \mathbf{x}_J]. \quad (3)$$

$\mathbf{X}$ is advanced by approximately one time step to $\mathbf{X}'$ by assuming a linear operator matrix $\mathbf{A}$, as

follows:

$$\mathbf{X}' \approx \mathbf{A}\mathbf{X}. \quad (4)$$

The best-fit (in the least-squares manner) linear operator matrix $\mathbf{A}$ is expressed as

$$\mathbf{A} = \mathbf{X}'\mathbf{X}^\dagger, \quad (5)$$

where the dimensions of $\mathbf{A}$ are $n \times n$, and $\dagger$ denotes the Moore–Penrose pseudo-inverse of $\mathbf{X}$.

Performing a singular value decomposition for $\mathbf{X}$ yields

$$\mathbf{X} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^*, \quad (6)$$

where the asterisk $*$ indicates the conjugate transpose, and the dimensions of $\mathbf{U}$, $\mathbf{\Sigma}$, and $\mathbf{V}$ are

$n \times n$, $n \times n$, and $(J - 1) \times n$, respectively, when $J - 1 > n$. $\mathbf{X}^\dagger$ and $\mathbf{A}$ are expressed as

$$\mathbf{X}^\dagger = \mathbf{V}\mathbf{\Sigma}^{-1}\mathbf{U}^*, \quad (7)$$

$$\mathbf{A} = \mathbf{X}'\mathbf{V}\mathbf{\Sigma}^{-1}\mathbf{U}^*. \quad (8)$$

DMD is often performed using a rank-reduced matrix instead of a full matrix $\mathbf{A}$. The rank-reduced representation of the $n \times n$ full matrix $\mathbf{A}$ is expressed as an $r \times r$ matrix $\tilde{\mathbf{A}}$ as follows:

$$\tilde{\mathbf{A}} = \mathbf{U}_r^* \mathbf{A} \mathbf{U}_r, \quad (9)$$

where $r \leq \min(n, J - 1)$ and $\mathbf{U}_r$ is composed of the first r columns of the left singular matrix $\mathbf{U}$. Eq. (9) expresses the similarity transformation from $\mathbf{A}$ to $\tilde{\mathbf{A}}$, where the two matrices have the same eigenvalues.

The eigendecomposition of $\tilde{\mathbf{A}}$ is performed as follows:

$$\tilde{\mathbf{A}}\mathbf{W} = \mathbf{W}\mathbf{\Lambda}, \quad (10)$$

where the columns of $\mathbf{W}$ are eigenvectors, and $\mathbf{\Lambda}$ is a diagonal matrix comprising the corresponding eigenvalues λ_m ($1 \leq m \leq r$). According to the “exact DMD modes” (Tu et al., 2014), the eigenvectors of $\mathbf{A}$ are specified in the columns of $\mathbf{\Phi}$, as follows:

$$\mathbf{\Phi} = \mathbf{X}'\mathbf{V}_r\mathbf{\Sigma}_r^{-1}\mathbf{W}, \quad (11)$$

where $\mathbf{V}_r$ is composed of the first r columns of $\mathbf{V}$, and $\mathbf{\Sigma}_r$ is composed of the first r rows and r columns of $\mathbf{\Sigma}$. The α -eigenvalue is associated with the discrete eigenvalue λ_m of $\mathbf{A}$ as follows:

$$\lambda_m = \exp(\alpha_m \Delta), \quad (12)$$

where Δ is the time interval between two consecutive data points $x_{i,k}$ and $x_{i,k+1}$. The measured values at any time t can be approximated as follows:

$$\mathbf{x}(t) \approx \mathbf{\Phi} \exp(\mathbf{\Omega}t) \mathbf{b}, \quad (13)$$

where $\mathbf{\Omega}$ is a diagonal matrix whose entries are time eigenvalues, α_m . Substituting $t = 0$ to Eq.

(13) yields $\mathbf{x}_1 = \mathbf{x}(0) = \Phi \mathbf{b}$. Amplitude vector $\mathbf{b}$ is expressed as follows:

$$\mathbf{b} = \Phi^\dagger \mathbf{x}_1, \quad (14)$$

where $\Phi^\dagger$ is the Moore–Penrose pseudo-inverse of Φ .

2.2 DMD for single data location

This section presents a new method to achieve the objective of this study. The proposed method uses a time-series dataset $\mathbf{x}_i$ at a single location, as follows:

$$\mathbf{x}_i = [x_{i,1} \ x_{i,2} \ \dots \ x_{i,k} \ \dots x_{i,M}], \quad (15)$$

where $x_{i,k}$ is the data point at the i th location and k th time point, and M is the total number of time points used for the DMD analysis. An augmented data matrix is constructed by stacking multiple time-shifted copies of row vector $\mathbf{x}_i$. If the stacking is N times, then two $N \times (M - N - 1)$

1) augmented matrices are constructed as follows:

$$\mathbf{X}_{\text{aug},i} = \begin{bmatrix} x_{i,1} & x_{i,2} & \cdots & x_{i,M-N-1} \\ x_{i,2} & x_{i,3} & \cdots & x_{i,M-N} \\ \vdots & \vdots & \ddots & \vdots \\ x_{i,N} & x_{i,N+1} & \cdots & x_{i,M-2} \end{bmatrix}, \quad (16)$$

$$\mathbf{X}'_{\text{aug},i} = \begin{bmatrix} x_{i,2} & x_{i,3} & \cdots & x_{i,M-N} \\ x_{i,3} & x_{i,4} & \cdots & x_{i,M-N+1} \\ \vdots & \vdots & \ddots & \vdots \\ x_{i,N+1} & x_{i,N+2} & \cdots & x_{i,M-1} \end{bmatrix}. \quad (17)$$

Although the augmented matrices are composed of time-series data at a single location, the matrices can be regarded as composed of data obtained at N locations. Once the two augmented matrices are prepared, then the remaining DMD analysis can be performed in the same manner as in Section 2.1, by replacing $\mathbf{X}$ and $\mathbf{X}'$ with $\mathbf{X}_{\text{aug},i}$ and $\mathbf{X}'_{\text{aug},i}$, respectively. If datasets are available at another location j , the augmented matrix is further augmented by stacking two augmented matrices as follows:

$$\mathbf{X}_{\text{aug},ij} = \begin{bmatrix} \mathbf{X}_{\text{aug},i} \\ \mathbf{X}_{\text{aug},j} \end{bmatrix}. \quad (18)$$

3. Pulsed neutron method

3.1 DMD for multiple locations

The DMD method proposed in Section 2.2 was applied to a numerical simulation of the pulsed neutron method in a one-dimensional one-energy group subcritical system. A one-energy group-diffusion approximation was employed for the numerical test. The test setup comprised a 70-cm-thick one-dimensional slab, as illustrated in Fig. 1. Pulsed neutrons were repeatedly generated at a constant time interval. The slab thickness included the extrapolated length, and the neutron flux was set to zero at the external boundaries, i.e., $x = -35$ and 35 cm. The pulsed neutron source was positioned 13 cm from the slab center, which was in an off-center position, to excite the asymmetric and symmetric higher harmonics. The time-dependent neutron flux distribution is calculated as follows (Duderstadt and Hamilton, 1976):

$$\phi(x, t) = \sum_{n=0,2,\dots}^{\infty} c_n e^{-\alpha_n t} \cos B_{n+1} x + \sum_{n=1,3,\dots}^{\infty} d_n e^{-\alpha_n t} \sin B_{n+1} x + C(x), \quad (19)$$

where

$$c_n = \frac{2S}{H} \cos B_{n+1} x_s, \quad (20)$$

$$d_n = \frac{2S}{H} \sin B_{n+1} x_s, \quad (21)$$

$$B_n = \frac{n\pi}{H}, \quad (22)$$

$$\alpha_n = v(DB_{n+1}^2 + \Sigma_a - (1 - \beta)v\Sigma_f). \quad (23)$$

Here, x_s = source position (13 cm), H = slab thickness (70 cm), S = number of neutrons per pulse (2×10^6). The constants used in Eqs. (19)-(23) are listed in Table 1. The summation in Eq. (19) was performed until the 2000th mode. The last term $C(x)$ in Eq. (19) is due to delayed neutrons that reach an equilibrium state after many repetitions of pulse generation. This term is dependent on the frequency of pulse generation. $C(x)$ was determined using the method of Yamamoto and Sakamoto (2018) as follows:

$$C(x)T = C_t(x) - C_p(x), \quad (24)$$

where T is the period of pulse generation,

$$C_t(x) = \sum_{n=0,2,\dots}^{\infty} \frac{2S}{H\alpha_{t,n}} \cos B_{n+1}x_s \cos B_{n+1}x + \sum_{n=1,3,\dots}^{\infty} \frac{2S}{H\alpha_{t,n}} \sin B_{n+1}x_s \sin B_{n+1}x, \quad (25)$$

$$C_p(x) = \sum_{n=0,2,\dots}^{\infty} \frac{2S}{H\alpha_n} \cos B_{n+1}x_s \cos B_{n+1}x + \sum_{n=1,3,\dots}^{\infty} \frac{2S}{H\alpha_n} \sin B_{n+1}x_s \sin B_{n+1}x, \quad (26)$$

$$\alpha_{t,n} = v(DB_{n+1}^2 + \Sigma_a - v\Sigma_f). \quad (27)$$

Table 1 Constants used for Sections 3, 4.1, and 4.2

Diffusion coefficient D	1 cm
Absorption cross section Σ_a	0.2 cm^{-1}
Production cross section $v\Sigma_f$	0.199 cm^{-1}
Neutron speed v	$2 \times 10^5 \text{ cm/s}$
Delayed neutron fraction β	0.007

The exact α -eigenvalues until the third higher harmonic are presented in Table 1.

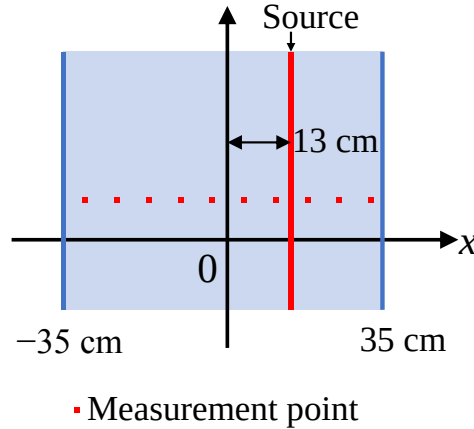


Fig. 1 One-dimensional slab for pulsed neutron and Rossi- α methods

The neutron fluxes after pulse generation at 10 equally spaced locations are shown in Fig. 2, and the 10 locations are shown in Fig. 1. The frequency of pulse generation was 1 Hz. The data matrix $\mathbf{X}$ defined in Eq. (2) was formed by the neutron fluxes at the 10 locations with a sampling time interval of $\Delta = 0.01$ ms. The dimension of $\mathbf{X}$ was $10 \times (1600 - 1)$, which implies that the time-series data until 16 ms after the pulse generation were used for the DMD analysis. However, the DMD failed to decompose the dynamics of data matrix $\mathbf{X}$. As shown in Fig. 2, the

neutron fluxes were affected significantly by the higher harmonics that were dominant during the early period after pulse generation. Therefore, the early time-series data were excluded (masked) from data matrix $\mathbf{X}$ to reduce the effect of the higher harmonics, as is typically performed in least-square fitting for the pulsed neutron method. DMD analyses were performed on the data matrix, in which several initial columns were truncated to mask the data during the early period. The α -eigenvalues of the fundamental and first-harmonic modes obtained from the DMD as a function of the masking time are shown in Fig. 3. This figure shows that the fundamental mode α -eigenvalue depends on the masking time until 1.6 ms, beyond which the eigenvalue becomes almost stable regardless of the masking time. The eigenvalue that corresponds to the constant term $C(x)$ in Eq. (19) is supposed to be zero. However, this eigenvalue was not able to be successfully captured for the masking time that was less than 2 ms. Beyond this masking time, the absolute value of the eigenvalue was less than 1 s^{-1} . For this numerical simulation, the DMD that uses the data at multiple locations does not perform favorably because of its dependence on the masking time.

The least square fitting method is typically adopted for the pulsed neutron method and the Rossi- α method. The fitting method significantly depends on the masking time determination, and the effect of the higher harmonic modes cannot be completely excluded in some situations. On the other hand, DMD largely eases these difficulties of obtaining the fundamental mode eigenvalue as already demonstrated in (Yamamoto and Sakamoto, 2022).

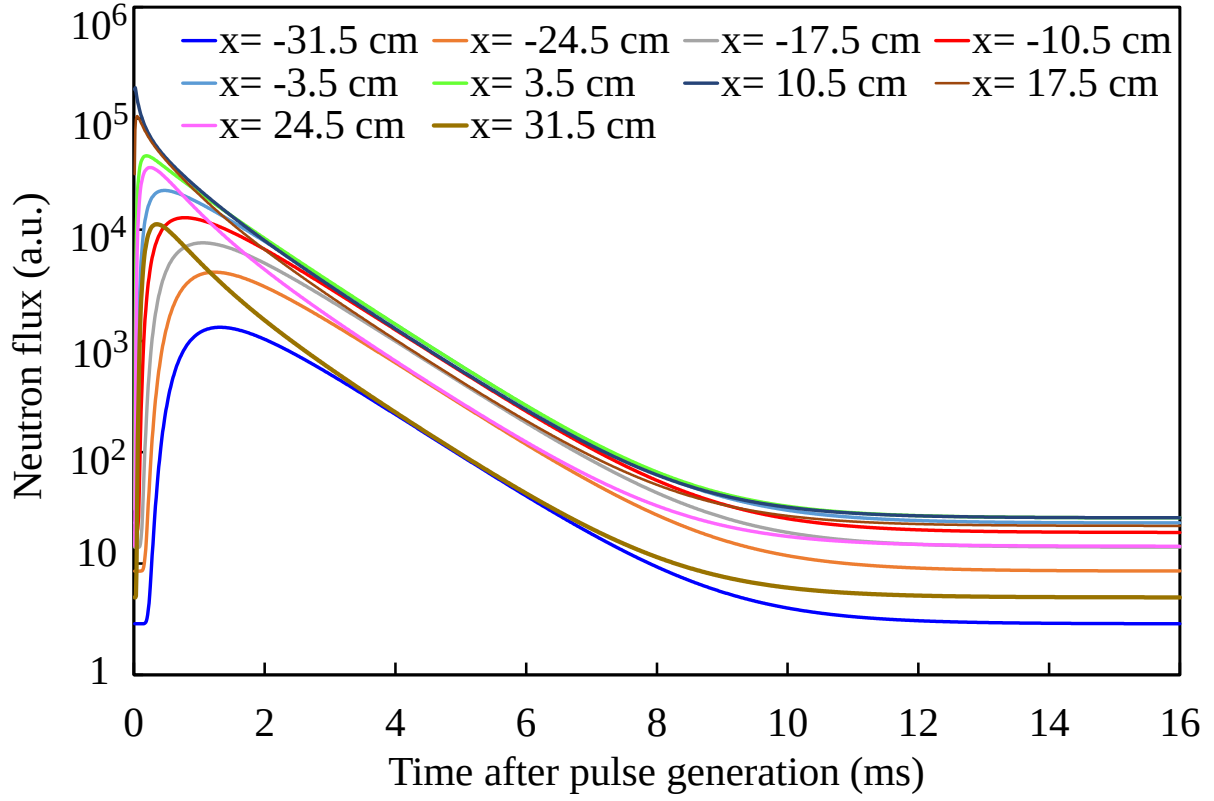


Fig. 2 Neutron flux responses after pulsed neutron generation at 10 locations

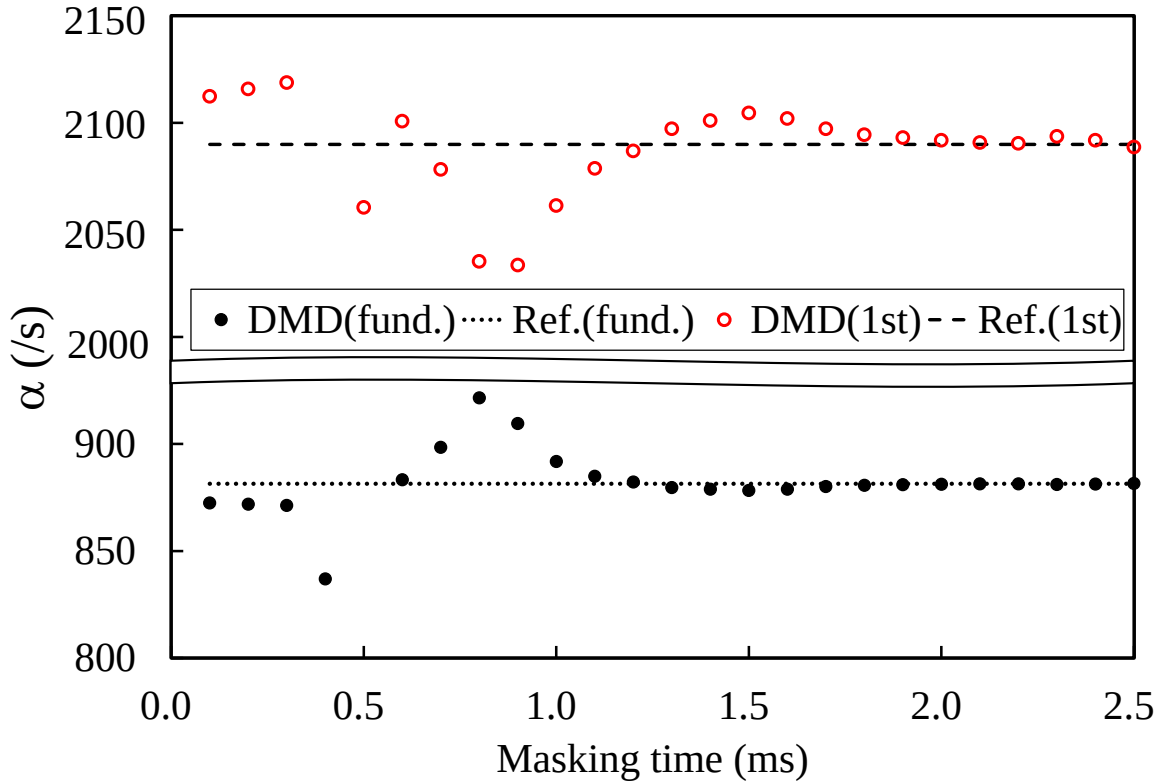


Fig. 3 DMD results of α_0 (fund.) and α_1 (1st) as a function of masking time in pulsed neutron method

3.2 DMD for single location

Using the data matrix $\mathbf{X}$ in Section 3.1, the time-shifted augmented data matrices $\mathbf{X}_{\text{aug},i}$ and $\mathbf{X}'_{\text{aug},i}$, defined in Eqs. (16) and (17), respectively, were obtained for each location. The number of stackings (rows) N was 600; hence, the dimensions of the augmented data matrix were $600 \times (1600 - 600 - 1)$. DMD analyses were performed on the augmented data matrices without employing a masking time. A full-rank ($r = 600$) was used for the DMD analyses to exclude ambiguity in determining the optimal rank. The α -eigenvalues at each location are listed in Table 1 until the third-harmonic mode. The fundamental mode α -eigenvalue obtained via DMD precisely reproduced the reference value to five digits at all locations. The reproduction of the first-harmonic eigenvalues was almost exact at all locations. The second- and third-harmonic modes were captured at some locations, but they were not as accurate as those of the fundamental and first-harmonic modes. The absolute value of the eigenvalue that corresponds to the constant term $C(x)$ in Eq. (19) was almost zero, and it was less than 0.001 s^{-1} . Fig. 4 shows a comparison of the time-dependent neutron flux reproduced using Eq. (13) with the exact value obtained using Eq. (19) at $x = -31.5$ and 10.5 cm. The neutron fluxes reproduced by the DMD agreed precisely with the exact ones.

Table 2 α -eigenvalues with time-shifted data matrix for pulsed neutron method

x (cm)	α_0 (s^{-1})	α_1 (s^{-1})	α_2 (s^{-1})	α_3 (s^{-1})
-31.5	881.44	2089.8	— ^a	6215.0
-24.5	881.44	2089.9	4173.9	— ^a
-17.5	881.44	2089.4	4152.2	7856.5
-10.5	881.44	2090.0	3814.0	— ^a
-3.5	881.44	2089.6	4030.9	6516.9
3.5	881.44	2089.2	— ^a	7541.2
10.5	881.44	2089.9	4059.9	7558.0
17.5	881.44	2090.0	4120.7	— ^a
24.5	881.44	2090.1	— ^a	— ^a
31.5	881.44	2090.3	4063.5	7038.3
Reference ^b	881.44	2090.0	4104.2	6924.1

^aThis mode was not captured by DMD. ^bBased on Eq. (23).

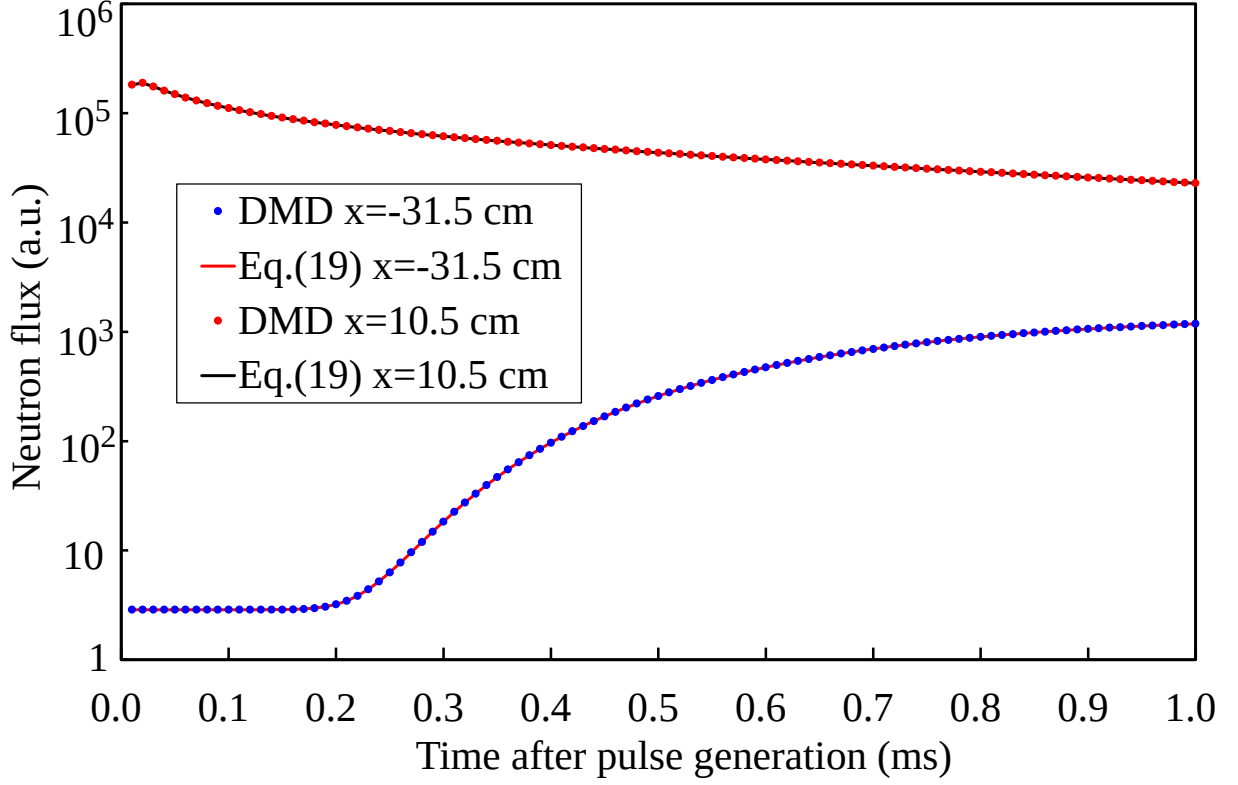


Fig. 4 Time-dependent neutron flux reproduced by DMD using augmented matrix

4. Rossi- α method

4.1 DMD for multiple locations (one region, one-energy group)

The same subcritical system as in the previous section was used for the Rossi- α method. The constants listed in Table 1 were used again in this section. A Poisson neutron source with an intensity of 8000 neutrons/s was positioned 13 cm from the slab center. The autocorrelation function (ACF), which is proportional to the joint probability of a pair neutron detection (Uhrig, 1970), was calculated using the following equation (Yamamoto and Sakamoto, 2022; Endo et al., 2006; Muñoz-Cobo et al., 2011):

$$ACF(\tau, x) = \sum_{n=0}^{\infty} R_n(\tau, x) + C_R(x)^2, \quad (28)$$

$$R_n(\tau, x) = e^{-\alpha_n \tau} \sum_{m=0}^{\infty} \sum_{k=0}^{\infty} \frac{S_k F_{kmn} C_m(x) C_n(x)}{\alpha_k (\alpha_m + \alpha_n)}, \quad (29)$$

where

$$S_k = S \phi_k(x_s), \quad (30)$$

$$C_m(x) = \Sigma_d \varphi_m(x), \quad (31)$$

$$C_R(x) = \sum_{k=0}^{\infty} \frac{S_k C_k(x)}{\alpha_k}, \quad (32)$$

$$F_{kmn} = \int_{x_{min}}^{x_{max}} \overline{\nu(\nu-1)(1-\beta)} \Sigma_f \varphi_k(x) \varphi_m(x) \varphi_n(x) dx, \quad (33)$$

$$\varphi_k(x) = \begin{cases} 2\sqrt{\frac{\nu}{H}} \cos B_{k+1}x, & k = 0, 2, \dots, \\ 2\sqrt{\frac{\nu}{H}} \sin B_{k+1}x. & k = 1, 3, \dots, \end{cases} \quad (34)$$

$$(35)$$

Here, $\Sigma_d = \Sigma_a - \Sigma_f = 0.1171 \text{ cm}^{-1}$, $\Sigma_f = 0.08292 \text{ cm}^{-1}$, $x_{max} = 35 \text{ cm}$, $x_{min} = -35 \text{ cm}$, $x_s =$ source position (13 cm), and $S =$ source intensity (8000 s^{-1}), $\overline{\nu(\nu-1)} = 3.6$. Delayed neutrons were disregarded in the Rossi- α method in this study. These formulations were verified based on a comparison with a Monte Carlo simulation by Yamamoto and Sakamoto (2022).

If a subcritical system is driven by an inherent neutron source that distributes throughout the fuel region, the effect of the higher harmonic modes is expected to be reduced in comparison with a localized neutron source that activates some particular modes. The formulation of the correlation function that accounts for multiples neutrons emitted from the inherent source, which is presented in (Yamamoto, 2015), needs to be used for the analysis.

The ACFs as a function of time lag τ at 10 equally spaced locations are shown in Fig. 5, where the time interval Δ was 0.01 ms. In contrast to the pulsed neutron method, the ACFs decreased monotonically with the time lag at all locations. Similar to Section 3.1, a $10 \times (1600 - 1)$ data matrix $\mathbf{X}$ was constructed using the ACFs at 10 locations for the DMD analysis. The α -eigenvalues of the fundamental and first-harmonic modes obtained from the DMD as a function of the masking time are shown in Fig. 6. The α -eigenvalues of the fundamental and first-harmonic modes were estimated precisely by masking only the first-time lag. The absolute value of the eigenvalue that corresponds to the constant term $C_R(x)$ in Eq. (28) was almost zero, and it was less than 0.01 s^{-1} . Owing to the monotonic decrease in the ACFs, the performance of the DMD-estimated eigenvalues for the Rossi- α method was far superior than that for the pulsed neutron

method.

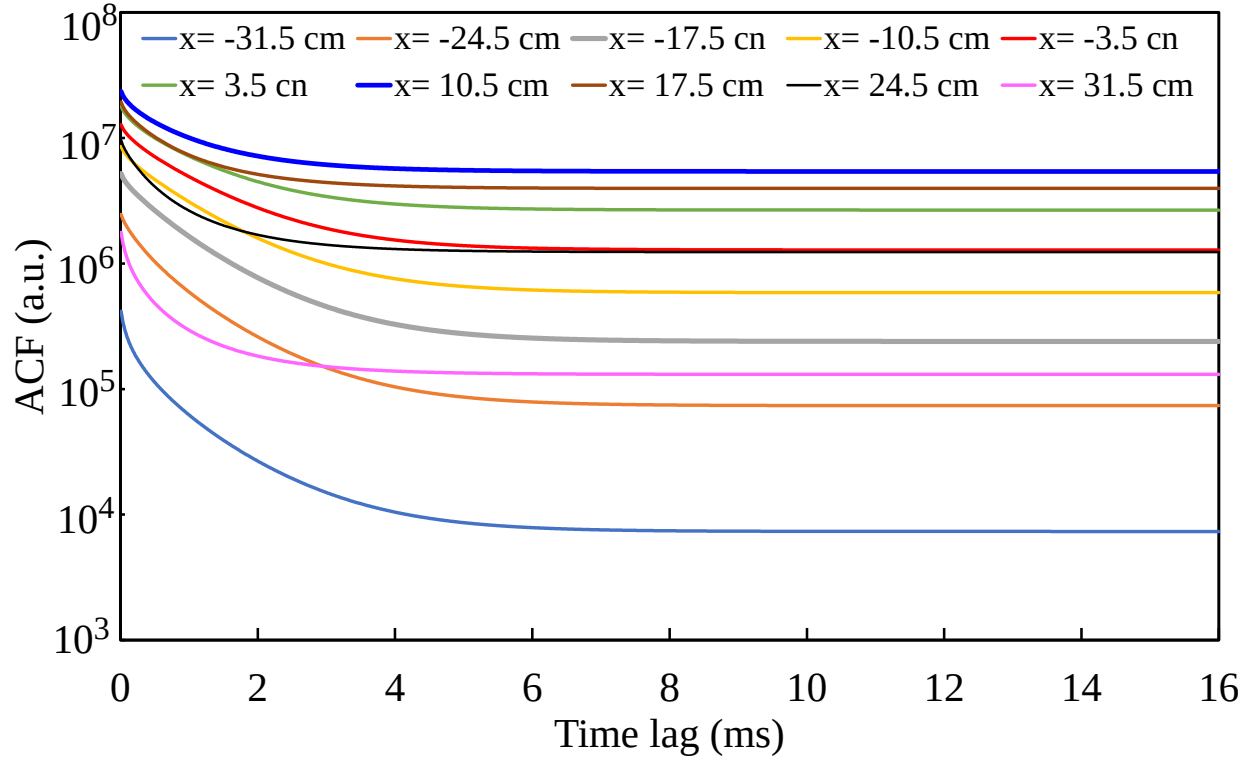


Fig. 5 Autocorrelation functions as a function of time lag in one-region slab

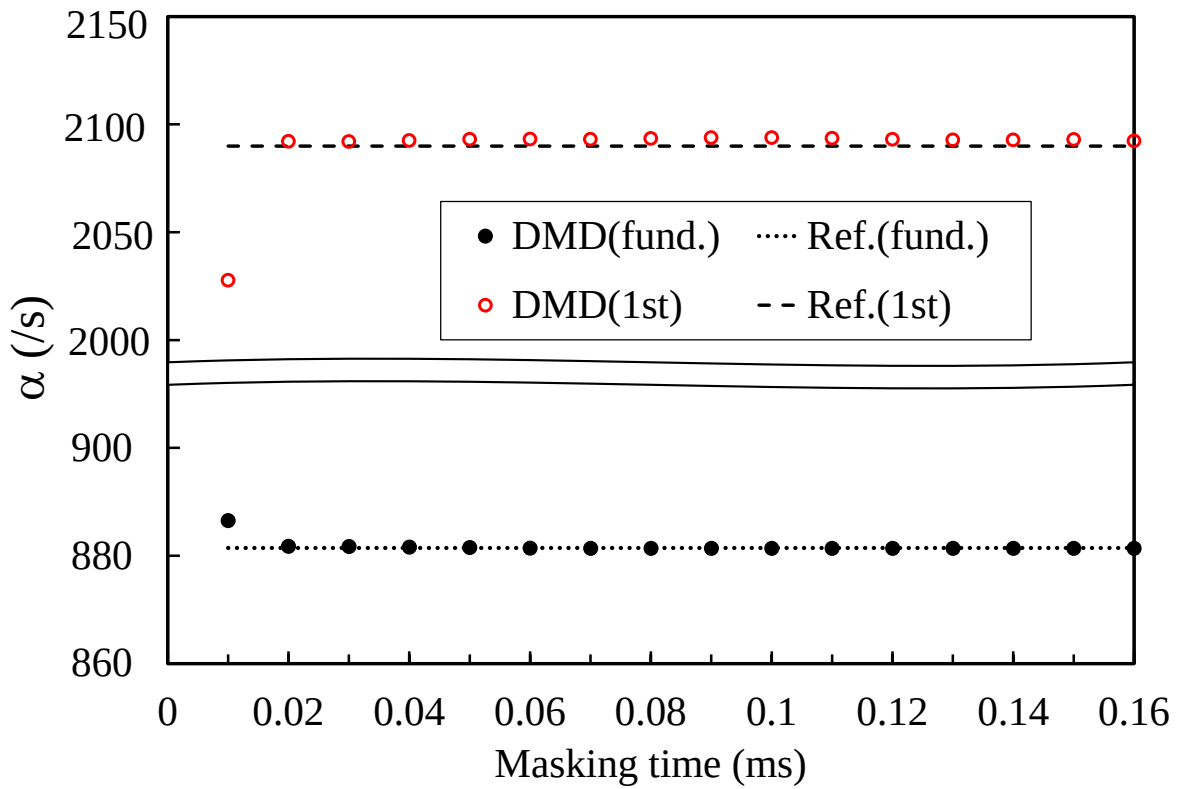


Fig. 6 DMD results of α_0 (fund.) and α_1 (1st) as a function of masking time in Rossi- α method

4.2 DMD for single location (one region, one-energy group)

Similar to Section 3.2, augmented data matrices were formed by stacking 600 time-shifted copies for each data location in Section 4.1. Using $600 \times (1600 - 600 - 1)$ augmented data matrices, DMD analyses were performed without employing the masking time. The α -eigenvalues at each location until the second-harmonic mode are listed in Table 2. Unlike the pulsed neutron method, the third-harmonic mode was not successfully captured. Compared with Table 1, the eigenvalues of the Rossi- α method were slightly less accurate than those of the pulsed neutron method. However, the fundamental mode eigenvalue was reproduced to four digits at all the locations. The absolute value of the eigenvalue that corresponds to the constant term $C_R(x)$ in Eq. (28) was almost zero, and it was less than 0.0001 s^{-1} .

Table 3 α -eigenvalues with time-shifted data matrix for Rossi- α method

x (cm)	α_0 (s^{-1})	α_1 (s^{-1})	α_2 (s^{-1})
-31.5	881.44	2084.5	4278.7
-24.5	881.44	2089.8	4192.5
-17.5	881.45	2090.3	4127.3
-10.5	881.45	2093.2	4836.9 ^a
-3.5	881.45	2061.8	4108.9
3.5	881.44	2105.9	4202.6
10.5	881.45	2091.0	— ^b
17.5	881.43	2089.8	4119.2
24.5	881.48	2091.8	4157.8
31.5	881.44	2088.3	4100.0
Reference ^c	881.44	2090.0	4104.2

^aThis poor agreement with the reference may be caused by the smallness of the second harmonic mode at this position that is close to the node of the mode (-11.7 cm).

^bThis mode is not captured by DMD. In addition to the proximity to the node (11.7 cm), the second harmonic mode is masked by the enhanced uncorrelated constant term ($C_R(x)^2$ in Eq. (24)).

^cBased on Eq. (23).

4.3 DMD for multiple locations (three regions, two-energy group)

DMD analysis was performed for a slightly more complicated two-energy group system used by Yamamoto and Sakamoto (2022). As shown in Fig. 7, the geometry of the numerical test was composed of three regions. For more details regarding the calculation method of the ACFs, readers can refer to (Yamamoto and Sakamoto, 2022). The ACFs as a function of time lag are shown in Fig. 8, where the time interval Δ was 0.01 ms, and the time lag was until 7 ms. Using the ACFs at the 10 locations shown in Fig. 7, a $10 \times (700 - 1)$ data matrix $\mathbf{X}$ was formed for the DMD analysis. In Table 3, the α -eigenvalues of the fundamental and first-harmonic modes obtained from the DMD are shown for several masking times. Whereas the increase in the masking time caused the eigenvalues to approach the reference values, the dependence of the eigenvalues on the masking time was slight, and the DMD yielded sufficiently accurate eigenvalues without employing the masking time. The absolute value of the eigenvalue that corresponds to the constant term $C_R(x)$ in Eq. (28) was almost zero, and it was less than 0.02 s^{-1} .

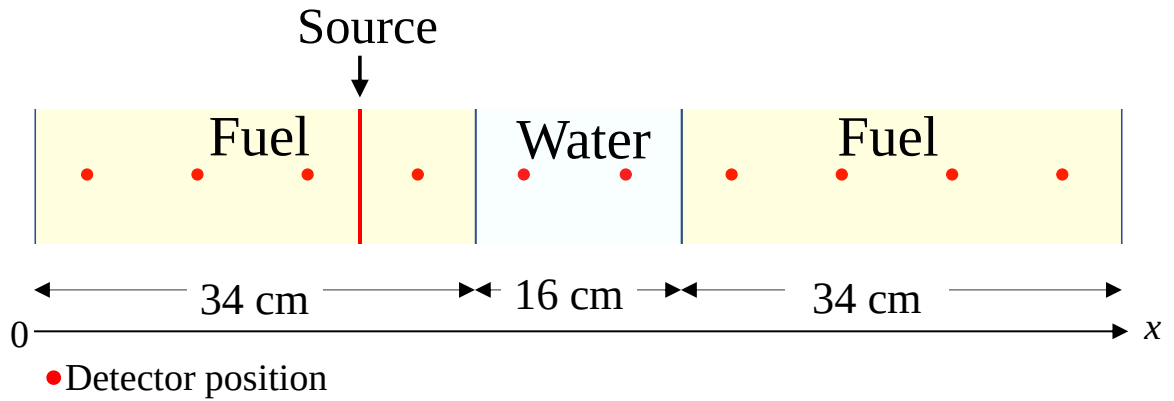


Fig. 7 One-dimensional three-region slab for Rossi- α method

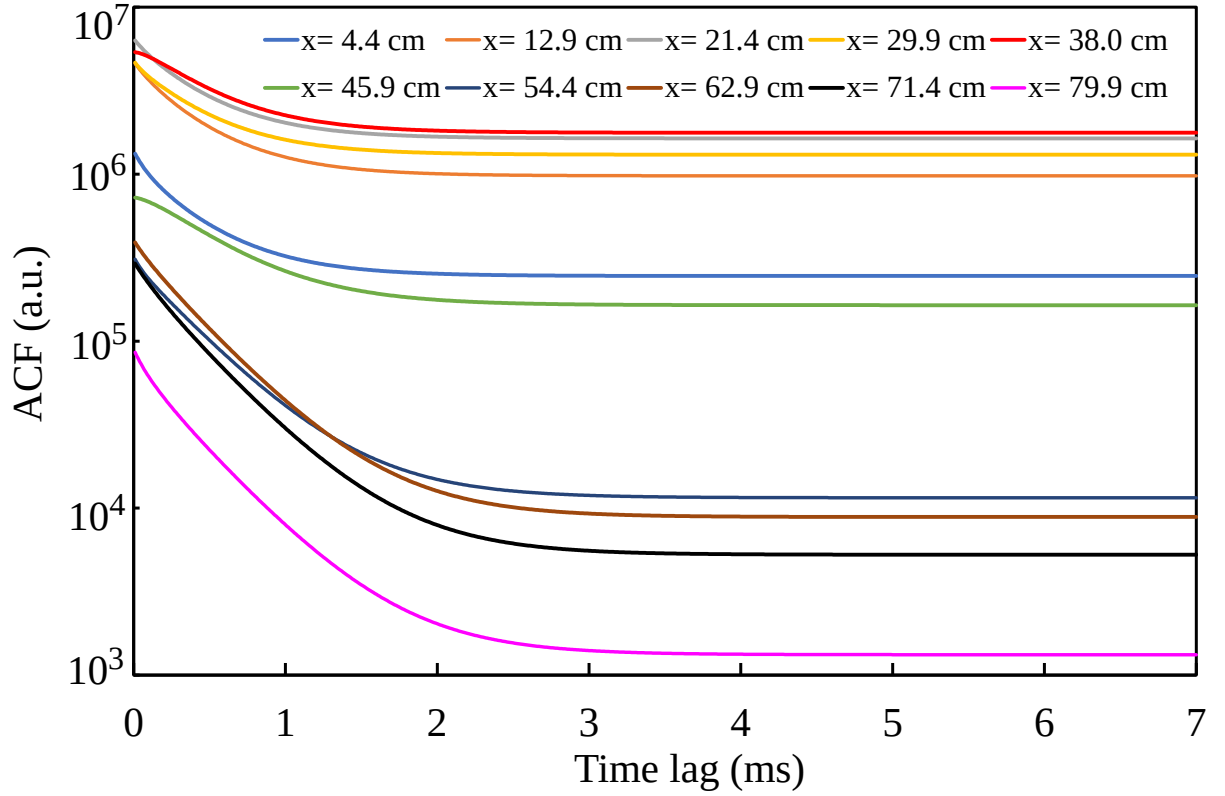


Fig. 8 Autocorrelation functions as a function of time lag in three-region slab

Table 4 Eigenvalues obtained via DMD and reference values for Rossi- α method

	Masking time (ms)	α_0 (s^{-1})	α_1 (s^{-1})
DMD (10 locations)	0	2136.2	2693.8
	0.1	2131.4	2674.1
	0.2	2130.2	2671.3
	0.3	2129.9	2671.8
	0.4	2129.3	2668.7
	0.5	2129.6	2672.7
Reference ^a		2128.6	2661.4

^a From Yamamoto and Sakamoto (2022).

4.4 DMD for single location (three regions, two-energy group)

For the same numerical test described in Section 4.3, an augmented data matrix was formed by stacking 200 time-shifted copies for each location. DMD analyses were performed for $200 \times (700 - 200 - 1)$ augmented data matrices without employing a masking time. Fig. 9 shows the α -eigenvalues of the fundamental and first-harmonic modes at 75 locations. Although the

fundamental mode eigenvalue exhibited space dependence, unlike the one-region system presented in Section 4.2, the eigenvalues were estimated within the narrow range of 2121–2143 s^{-1} for the reference value of 2128.6 s^{-1} . The absolute value of the eigenvalue that corresponds to the constant term $C_R(x)$ in Eq. (28) was almost zero, and it was less than 0.01 s^{-1} .

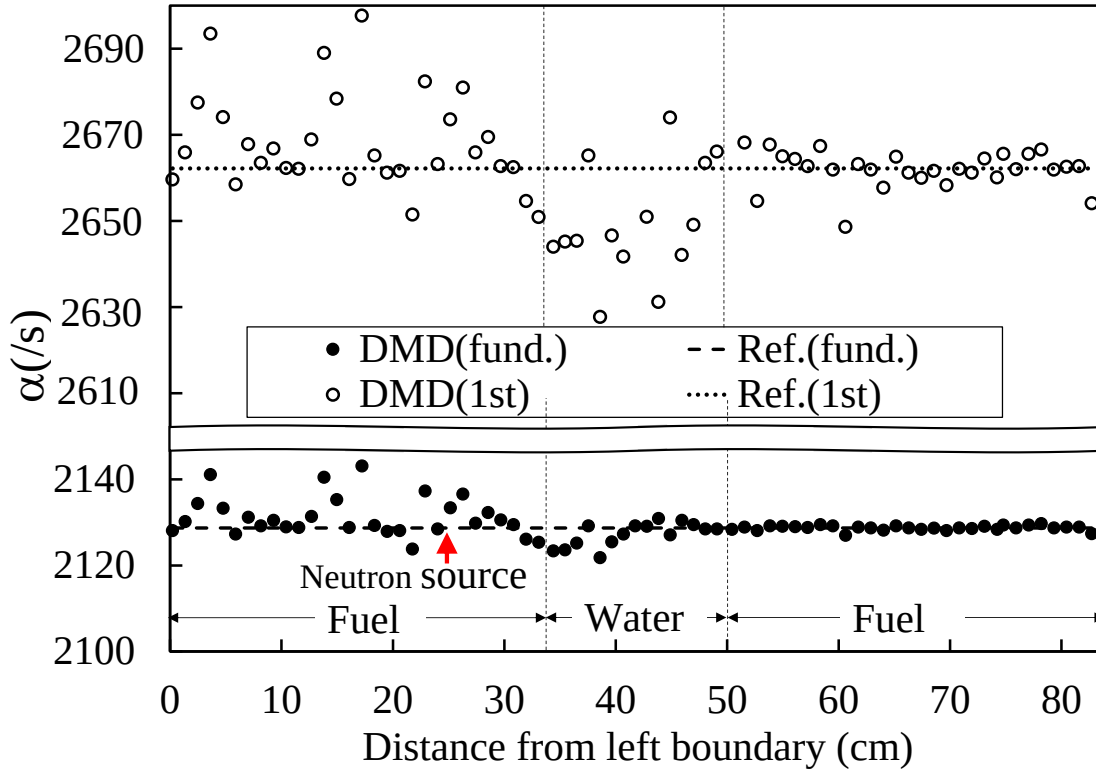


Fig. 9 α_0 (fund.) and α_1 (1st) estimated via DMD for each single location

5. Conclusions

The applications of DMD to fundamental mode extraction in subcriticality measurements hitherto involve the use of measurement data obtained at multiple locations. In some cases, a measurement can only be performed at a few locations. A DMD method that can be performed at a single location using a shift-stacked data matrix was proposed herein. An augmented matrix is constructed by stacking time-shifted (or position-shifted) copies of the data at a single location. The proposed method was applied in numerical studies involving the pulsed neutron method and Rossi- α method. In this study, for the DMD analysis of the pulsed neutron method that uses the

1 data at multiple locations, the data had to be excluded (masked) in the early times, whereas the
2 Rossi- α method was primarily independent of the masking time.

3 Using a shift-stacked data matrix composed of data at a single location for DMD, the
4 fundamental mode eigenvalues were successfully extracted for the numerical simulations of
5 subcriticality measurements. The newly proposed method did not require data masking in the early
6 times, which might have eliminated the necessity of masking time determination.

7 When DMD analysis is performed for a single location, the prior knowledge of the spatial
8 distribution of the fundamental and higher harmonic neutron fluxes is desirable compared with
9 DMD for the multiple locations. Although the requirement of the prior knowledge is a drawback
10 of DMD for a single location, this method is a promising alternative when a measurement can be
11 performed only at a single location.

12 The numerical data used in this study did not contain any type of noise or bias, which is
13 inevitable in actual subcriticality measurements. In the future, the applicability of the proposed
14 method to data with measurement noise will be investigated; furthermore, the proposed method
15 will be compared with the conventional DMD method, which uses noisy data at multiple locations.

17 **References**

- 18 Abdo, M., Elzohery, R., Roberts, J.A., 2019. Modeling isotopic evolution with surrogates based
19 on dynamic mode decomposition. Ann. Nucl. Energy 129, 280–288.
20 <https://doi.org/10.1016/j.anucene.2019.01.048>.
- 21 Berglöf, C., Fernández-Ordóñez, M., Villamarín, D., Bécares, V., González-Romero, E.M.,
22 Bournos, V., Muñoz-Cobo, J.L., 2011. Auto-correlation and variance-to-mean measurements
23 in a subcritical core obeying multiple alpha-modes. Ann. Nucl. Energy 38, 194–202.
24 <https://dx.doi.org/10.1016/j.anucene.2010.11.009>.
- 25 Di Ronco, A., Introini, C., Cervi, E., Lorenzi, S., Jeong, Y.S., Seo, S.B., Bang, I.C., Giacobbo, F.,
26 Cammi, A., 2020. Dynamic mode decomposition for the stability analysis of the Molten Salt
27 Fast Reactor core. Nucl. Eng. Des. 362, 110529.

- 1 <https://doi.org/10.1016/j.nucengdes.2020.110529>.
- 2 Duderstadt, J.J., Hamilton, L.J., 1976. Nuclear Reactor Analysis. John Wiley & Sons, Inc., New
- 3 York.
- 4 Endo, T., Yamane, Y., Yamamoto, A., 2006. Space and energy dependent theoretical formula for
- 5 the third order neutron correlation technique. *Ann. Nucl. Energy* 33, 521–537.
- 6 <https://dx.doi.org/10.1016/j.anucene.2006.02.002>.
- 7 Endo, T., Nishioka, F., Yamamoto, A., Watanabe, K., Pyeon, C.H., 2022. Application of dynamic
- 8 mode decomposition to Rossi- α method in a critical state using file-by-file moving block
- 9 bootstrap method. *J. Nucl. Sci. Technol.* (in press).
- 10 <https://dx.doi.org/10.1080/00223131.2022.2030260>
- 11 Hardy, Z.K., Morel, J.E., Ahrens, C., 2019. Dynamic mode decomposition for subcritical metal
- 12 systems. *Nucl. Sci. Eng.* 193 1173–1185. <https://doi.org/10.1080/00295639.2019.1609317>.
- 13 Katano, R., 2019. Estimation method of prompt neutron decay constant reducing higher order
- 14 mode effect by linear combination. *Nucl. Sci. Eng.* 193, 431–439.
- 15 <https://doi.org/10.1080/00295639.2018.1528803>.
- 16 Kutz, J.N., Brunton, S.L., Proctor, J.L., 2016. Dynamic mode decomposition: Data-driven
- 17 modeling for complex systems, Society for Industrial and Applied Mathematics, Philadelphia,
- 18 Pennsylvania. <https://doi.org/10.1137/1.9781611974508>.
- 19 McClarren, R.G., 2019. Calculating time eigenvalues of the neutron transport equation with
- 20 dynamic mode decomposition. *Nucl. Sci. Eng.* 193, 854–867.
- 21 <https://doi.org/10.1080/00295639.2018.1565014>.
- 22 McClarren, R.G., Haut, T.S., 2022. Data-driven acceleration of thermal radiation transfer
- 23 calculations with the dynamic mode decomposition and a sequential singular value
- 24 decomposition. *J. Comput. Phys.* 448, 110756. <https://doi.org/10.1016/j.jcp.2021.110756>.
- 25 Muñoz-Cobo, J.L., Berglöf, C., Peña, J., Villamarín, D., Bournos, V., 2011. Feynman- α and Rossi-
- 26 α formulas with spatial and modal effects. *Ann. Nucl. Energy* 38, 590–600.
- 27 <https://doi.org/10.1016/j.anucene.2010.09.027>.

- 1 Nishioka, F., Endo, T., Yamamoto, A., Yamanaka, M., Pyeon, C.H., 2022. Applicability of
2 dynamic mode decomposition to estimate fundamental mode component of prompt neutron
3 decay constant from experimental data. Nucl. Sci. Eng. 196, 133–143.
4 <https://doi.org/10.1080/00295639.2021.1968225>.
- 5 Orndoff, J.D., 1957. Prompt neutron periods of metal critical assemblies. Nucl. Sci. Eng. 2, 450–
6 460. <https://dx.doi.org/10.13182/NSE57-A25409>.
- 7 Roberts, J.A., Xu, L., Elzohery, R., Abdo, M., 2019. Acceleration of the power method with
8 dynamic mode decomposition. Nucl. Sci. Eng. 193, 1371–1378.
9 <https://doi.org/10.1080/00295639.2019.1634928>.
- 10 Schmid, P.J., 2010. Dynamic mode decomposition of numerical and experimental data. J. Fluid
11 Mech. 656, 5–28. <https://doi.org/10.1017/S0022112010001217>.
- 12 Szieberth, M., Klujber, G., Kloosterman, J.L., de Haas, D., 2015. Measurement of multiple α -
13 modes at the Delphi subcritical assembly by neutron noise techniques. Ann. Nucl. Energy 75,
14 146–157. <https://dx.doi.org/10.1016/j.anucene.2014.07.018>.
- 15 Talamo, A., Gohar, Y., Sadovich, S., Kiyavitskaya, H., Bournos, V., Fokov, Y. Routkovskaya, C.,
16 2013. Correction factor for the experimental prompt neutron decay constant. Ann. Nucl. Energy
17 62, 421–428. <https://dx.doi.org/10.1016/j.anucene.2013.06.035>.
- 18 Talamo, A., Gohar, Y., Yamanaka, M., Pyeon, C.H., 2020. Calculation of the prompt neutron decay
19 constant for the KUCA facility driven by a stationary or pulsed external neutron source. J. Nucl.
20 Sci. Technol. 57, 145–156. <https://doi.org/10.1080/00223131.2019.1651232>.
- 21 Tu, J.H., Rowley, C.W., Luchtenburg, D.M., 2014. On dynamic mode decomposition: theory and
22 applications. J. Comput. Dyn. 1, 391–421. <https://doi:10.3934/jcd.2014.1.319>.
- 23 Uhrig, R.E., 1970. Random noise techniques in nuclear reactor systems. Ronald Press, New York.
- 24 Yamamoto, T., Miyoshi, Y., Tonoike, K., Okamoto, H., Ida, T., Aoki, S., 2003. Effect of higher-
25 harmonics flux in exponential experiment for subcriticality measurement. J. Nucl. Sci. Tehcnol.
26 40, 77–83. <https://doi.org/10.1080/18811248.2003.9715336>.
- 27 Yamamoto, T., 2015. Higher harmonic analyses of the ^{252}Cf source driven noise analysis method.

- 1 Ann. Nucl. Energy 76, 521–529. [https:// dx.doi.org/10.1016/j.anucene.2014.10.034](https://dx.doi.org/10.1016/j.anucene.2014.10.034).
- 2 Yamamoto, T., Sakamoto, H., 2018. Effect of higher harmonics in the area-ratio pulsed neutron
- 3 source technique. Prog. Nucl. Energy 108, 286–294.
- 4 <https://doi.org/10.1016/j.pnucene.2018.05.022>.
- 5 Yamamoto, T., Sakamoto, H., 2021. Application of dynamic mode decomposition to exponential
- 6 experiment for spatial decay constant determination. Ann. Nucl. Energy 162, 108506.
- 7 <https://dx.doi.org/10.1016/j.anucene.2021.108506>.
- 8 Yamamoto, T., Sakamoto, H., 2022. Higher harmonic analyses of the Rossi- α method and
- 9 application of dynamic mode decomposition for time decay constant determination in a 1D
- 10 subcritical system. Ann. Nucl. Energy 168, 108886.
- 11 <https://dx.doi.org/10.1016/j.anucene.2021.108886>.
- 12
- 13 List of figures
- 14 Fig. 1 One-dimensional slab for pulsed neutron and Rossi- α methods
- 15 Fig. 2 Neutron flux responses after pulsed neutron generation at 10 locations
- 16 Fig. 3 DMD results of α_0 (fund.) and α_1 (1st) as a function of masking time in pulsed neutron
- 17 method
- 18 Fig. 4 Time-dependent neutron flux reproduced by DMD using augmented matrix
- 19 Fig. 5 Autocorrelation functions as a function of time lag in one-region slab
- 20 Fig. 6 DMD results of α_0 (fund.) and α_1 (1st) as a function of masking time in Rossi- α method
- 21 Fig. 7 One-dimensional three-region slab for Rossi- α method
- 22 Fig. 8 Autocorrelation functions as a function of time lag in three-region slab
- 23 Fig. 9 α_0 (fund.) and α_1 (1st) estimated via DMD for each single location