

Twisted Fourier-Mukai number of a $K3$ surface

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In my poster, I exhibited a counting formula for the twisted Fourier-Mukai (FM) partners of a projective $K3$ surface. Let S be a projective $K3$ surface over $\mathbb{C}$. A twisted $K3$ surface (S', α') is called a *twisted FM partner* of S if there is an exact equivalence $D^b(S) \simeq D^b(S', \alpha')$ between their derived categories. Let $\text{FM}^d(S)$ be the set of isomorphism classes of twisted FM partners (S', α') of S with $\text{ord}(\alpha') = d$. I calculated the number $\#\text{FM}^d(S)$ from several lattice-theoretic informations about the lattice $H^2(S, \mathbb{Z})$ equipped with a natural Hodge structure. The number $\sum_d \#\text{FM}^d(S)$ has the following meanings.

- The number of certain geometric origins of the category $D^b(S)$.
- The number of isomorphism classes of 2-dimensional compact moduli spaces of stable sheaves on S , considered with natural obstruction classes.
- The number of the 0-dimensional cusps of the Kahler moduli of S .

Now the formula is stated as follows.

Theorem 0.1. *Let $\varepsilon(d) = 1$ or 2 according to $d \leq 2$ or ≥ 3 . For a projective $K3$ surface S the following formula holds.*

$$\begin{aligned} \#\text{FM}^d(S) = & \sum_x \left\{ \sum_M \# \left(O_{\text{Hodge}}(T_x, \alpha_x) \backslash O(D_M) / O(M) \right) \right. \\ & \left. + \varepsilon(d) \sum_{M'} \# \left(O_{\text{Hodge}}(T_x, \alpha_x) \backslash O(D_{M'}) / O(M') \right) \right\}. \end{aligned}$$

Here x runs over the set $O_{\text{Hodge}}(T(S)) \backslash I^d(D_{NS(S)})$ and the lattices M, M' run over the sets $\mathcal{G}_1(M_x), \mathcal{G}_2(M_x)$ respectively.

This formula is simplified if S satisfies either of the following conditions : (1) The Neron-Severi lattice $NS(S)$ contains the hyperbolic plane U . (2) $NS(S)$ is 2-elementary. (3) The rank of $NS(S)$ equals to 1.

As an application of the formula, I gave a set of explicit Mukai vectors for a projective $K3$ surface of Picard number 1 such that the set of the corresponding moduli spaces of stable sheaves, considered with natural obstruction classes, coincides with the set $\text{FM}^d(S)$.

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