

# Convergence of some truncated Riesz transforms on predual of generalized Campanato spaces and its application to a uniqueness theorem for nondecaying solutions of Navier-Stokes equations.

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## 1. INTRODUCTION

This is an announcement of our recent work [8]. In [6] the first author introduced predual of generalized Campanato spaces. In this report, we show convergence of some truncated Riesz transforms on the function spaces and its application to a uniqueness theorem for nondecaying solutions of Navier-Stokes equations. Our uniqueness theorem is an extension of Kato's [3].

## 2. GENERALIZED CAMPANATO SPACE $\mathcal{L}_{p,\phi}(\mathbb{R}^n)$

Let  $1 \leq p < \infty$  and  $\phi : (0, \infty) \rightarrow (0, \infty)$ . For a ball  $B = B(x, r)$ , we shall write  $\phi(B)$  in place of  $\phi(r)$ . The function spaces  $\mathcal{L}_{p,\phi} = \mathcal{L}_{p,\phi}(\mathbb{R}^n)$  is defined to be the sets of all  $f$  such that  $\|f\|_{\mathcal{L}_{p,\phi}} < \infty$ , where

$$\|f\|_{\mathcal{L}_{p,\phi}} = \sup_B \frac{1}{\phi(B)} \left( \frac{1}{|B|} \int_B |f(x) - f_B|^p dx \right)^{1/p},$$
$$f_B = \frac{1}{|B|} \int_B f(x) dx.$$

Then  $\mathcal{L}_{p,\phi}$  is a Banach space modulo constants with the norm  $\|f\|_{\mathcal{L}_{p,\phi}}$ . If  $p = 1$  and  $\phi \equiv 1$ , then  $\mathcal{L}_{1,\phi} = \text{BMO}$ . It is known that if  $\phi(r) = r^\alpha$ ,  $0 < \alpha \leq 1$ , then  $\mathcal{L}_{p,\phi} = \text{Lip}_\alpha$ , and, if  $\phi(r) = r^{-n/p}$ ,  $1 \leq p < \infty$ , then  $\mathcal{L}_{p,\phi} = L^p$ .

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A function  $\phi : (0, \infty) \rightarrow (0, \infty)$  is said to satisfy the doubling condition if there exists a constant  $C > 0$  such that

$$C^{-1} \leq \frac{\phi(r)}{\phi(s)} \leq C \quad \text{for} \quad \frac{1}{2} \leq \frac{r}{s} \leq 2.$$

A function  $\phi : (0, \infty) \rightarrow (0, \infty)$  is said to be almost increasing (almost decreasing) if there exists a constant  $C > 0$  such that

$$\phi(r) \leq C\phi(s) \quad (\phi(r) \geq C\phi(s)) \quad \text{for} \quad r \leq s.$$

**Lemma 2.1.** *Assume that  $\phi(r)r^{n/p}$  is almost increasing and that  $\phi(r)/r$  is almost decreasing. Then  $\phi$  satisfies the doubling condition and*

$$\|f\|_{\mathcal{L}_{p,\phi}} \leq C(\|(1 + |x|^{n+1})f\|_{\infty} + \|\nabla f\|_{\infty}).$$

That is  $\mathcal{S} \subset \mathcal{L}_{p,\phi}$ .

*Proof.* Let  $B = B(z, r)$ .

**Case 1:**  $r < 1$ : In this case  $r \lesssim \phi(r)$ . Then

$$|f(x) - f(y)| \lesssim r \|\nabla f\|_{\infty} \lesssim \phi(r) \|\nabla f\|_{\infty}, \quad x, y \in B.$$

$$\left( \frac{1}{|B|} \int_B |f(x) - f_B|^p dx \right)^{1/p} \lesssim \sup_{x,y \in B} |f(x) - f(y)| \lesssim \phi(r) \|\nabla f\|_{\infty}.$$

**Case 2:**  $1 \leq r$ : In this case  $1 \lesssim \phi(r)r^{n/p}$  and

$$|f(x)| \leq \frac{\|(1 + |x|^{n+1})f\|_{\infty}}{1 + |x|^{n+1}}, \quad \left( \int |f(x)|^p dx \right)^{1/p} \lesssim \|(1 + |x|^{n+1})f\|_{\infty}.$$

Then

$$\begin{aligned} \left( \frac{1}{|B|} \int_B |f(x) - f_B|^p dx \right)^{1/p} &\leq 2 \left( \frac{1}{|B|} \int_B |f(x)|^p dx \right)^{1/p} \\ &\lesssim \frac{\|(1 + |x|^{n+1})f\|_{\infty}}{|B|^{1/p}} \lesssim \phi(r) \|(1 + |x|^{n+1})f\|_{\infty}. \quad \square \end{aligned}$$

### 3. $H_I^{[\phi,\infty]}(\mathbb{R}^n)$ , PREDUAL OF $\mathcal{L}_{1,\phi}(\mathbb{R}^n)$

The space  $H_U^{[\phi,q]}$  was introduced in [6], which is a generalization of Hardy space. The duality  $(H_U^{[\phi,q]})^* = \mathcal{L}_{q',\phi}$  also proved in [6].

In this talk we recall the definition of  $H_I^{[\phi,\infty]}(\mathbb{R}^n)$ , which is a special case of  $H_U^{[\phi,q]}$ .

In what follows, we always assume that  $\phi(r)r^n$  is almost increasing and that  $\phi(r)/r$  is almost decreasing.

**Definition 3.1** ( $[\phi, \infty]$ -atom). A function  $a$  on  $\mathbb{R}^n$  is called a  $[\phi, \infty]$ -atom if there exists a ball  $B$  such that

- (i)  $\text{supp } a \subset B$ ,
- (ii)  $\|a\|_\infty \leq \frac{1}{|B|\phi(B)}$ ,
- (iii)  $\int_{\mathbb{R}^n} a(x) dx = 0$ .

where  $\|a\|_\infty$  is the  $L^\infty$  norm of  $a$ . We denote by  $A[\phi, \infty]$  the set of all  $[\phi, \infty]$ -atoms.

If  $a$  is a  $[\phi, \infty]$ -atom and a ball  $B$  satisfies (i)–(iii), then, for  $g \in \mathcal{L}_{1,\phi}$ ,

$$\begin{aligned} \left| \int_{\mathbb{R}^n} a(x)g(x) dx \right| &= \left| \int_B a(x)(g(x) - g_B) dx \right| \\ &\leq \|a\|_\infty \int_B |g(x) - g_B| dx \\ &\leq \frac{1}{\phi(B)} \frac{1}{|B|} \int_B |g(x) - g_B| dx \\ &\leq \|g\|_{\mathcal{L}_{1,\phi}}. \end{aligned}$$

That is, the mapping  $g \mapsto \int_{\mathbb{R}^n} ag dx$  is a bounded linear functional on  $\mathcal{L}_{1,\phi}$  with norm not exceeding 1. Hence  $a$  is also in  $\mathcal{S}'$ , since  $\mathcal{S} \subset \mathcal{L}_{1,\phi}$ .

**Definition 3.2** ( $H_I^{[\phi, \infty]}$ ). The space  $H_I^{[\phi, \infty]} \subset (\mathcal{L}_{1,\phi})^*$  is defined as follows:

$f \in H_I^{[\phi, \infty]}$  if and only if there exist sequences  $\{a_j\} \subset A[\phi, \infty]$  and positive numbers  $\{\lambda_j\}$  such that

$$(3.1) \quad f = \sum_j \lambda_j a_j \text{ in } (\mathcal{L}_{1,\phi})^* \quad \text{and} \quad \sum_j \lambda_j < \infty.$$

In general, the expression (3.1) is not unique. Let

$$\|f\|_{H_I^{[\phi, \infty]}} = \inf \left\{ \sum_j \lambda_j \right\},$$

where the infimum is taken over all expressions as in (3.1). Then  $H_I^{[\phi, \infty]}$  is a Banach space equipped with the norm  $\|f\|_{H_I^{[\phi, \infty]}}$  and  $(H_I^{[\phi, \infty]})^* = \mathcal{L}_{1,\phi}$ .

#### 4. TRUNCATED RIESZ TRANSFORMS ON $H_I^{[\phi, \infty]}(\mathbb{R}^n)$ AND MAIN RESULT

The Riesz transforms of  $f$  are defined by

$$R_j f(x) = c_n \text{ p.v. } \int \frac{y_j}{|y|^{n+1}} f(x-y) dy, \quad j = 1, \dots, n,$$

where

$$c_n = \Gamma((n+1)/2)\pi^{-(n+1)/2}.$$

Let

$$k(x) = \begin{cases} C_n \frac{1}{|x|^{n-2}} & n \geq 3, \\ C_2 \log \frac{1}{|x|}, & n = 2, \end{cases}$$

where

$$C_n = \Gamma(n/2)(2(n-2)\pi^{n/2})^{-1}, \quad C_2 = (2\pi)^{-1}.$$

Then  $-\Delta k = \delta$ .

It is known that

$$R_j R_k f(x) = \text{p.v.} \int (\partial_j \partial_k k)(y) f(x-y) dy - \delta_{j,k} \frac{1}{n} f(x),$$

for  $j, k = 1, \dots, n$ , and

$$\sum_j R_j^2 f = -f.$$

Let  $\psi \in C^\infty(\mathbb{R}^n)$  be a radial function with  $0 \leq \psi \leq 1$ ,  $\psi(x) = 0$  for  $|x| \leq 1$ , and  $\psi(x) = 1$  for  $|x| \geq 2$ . We set  $\lambda = 1 - \psi$ . For  $0 < \epsilon < 1/2$  we define  $\psi_\epsilon(x) = \psi(x/\epsilon)$ ,  $\lambda_\epsilon(x) = \lambda(\epsilon x)$ , and  $k_\epsilon = \psi_\epsilon \lambda_\epsilon k$  so that  $\text{supp } k_\epsilon \subset \{x : \epsilon \leq |x| \leq 2/\epsilon\}$ .

**Definition 4.1** ( $R_{i,j}^\epsilon$ ). Let  $1 \leq i, j \leq n$ . For  $0 < \epsilon < 1/4$ , the operators  $R_{i,j}^\epsilon$  are defined by  $R_{i,j}^\epsilon f = \partial_i \partial_j k_\epsilon * f$  for  $f \in \mathcal{S}'$ .

We consider the following condition.

$$(4.1) \quad \begin{cases} \int_1^\infty \frac{\phi(t)}{t^2} dt < \infty, & \text{if } n \geq 3, \\ \int_1^\infty \frac{\phi(t) \log(1+t)}{t^2} dt < \infty, & \text{if } n = 2. \end{cases}$$

**Theorem 4.1.** Assume that  $\phi$  satisfies (4.1). If  $\varphi \in \mathcal{S}$  and  $\int \varphi = 0$ , then

$$\lim_{\epsilon \rightarrow 0} R_{i,j}^\epsilon \varphi = R_i R_j \varphi \quad \text{in } H_I^{[\phi, \infty]}.$$

In particular,  $\lim_{\epsilon \rightarrow 0} (-\Delta) k_\epsilon * \varphi = \varphi$  in  $H_I^{[\phi, \infty]}$ .

Using the duality  $\left(H_I^{[\phi, \infty]}\right)^* = \mathcal{L}_{1, \phi}$  and the equality

$$\lim_{\epsilon \rightarrow 0} \left\langle \sum_{j=1}^n R_{i,j}^\epsilon \partial_j f, \varphi \right\rangle = \lim_{\epsilon \rightarrow 0} \langle f, (-\Delta) k_\epsilon * \partial_i \varphi \rangle = \langle f, \partial_i \varphi \rangle$$

for all  $\varphi \in \mathcal{S}$ , we have the following.

**Corollary 4.2.** Assume that  $\phi$  satisfies (4.1). For  $f \in \mathcal{L}_{1,\phi}$ ,

$$\lim_{\epsilon \rightarrow 0} \sum_{j=1}^n R_{i,j}^\epsilon \partial_j f = -\partial_i f \quad \text{in } S'.$$

## 5. PROOF OF THE MAIN RESULT

To prove Theorem 4.1 we state two lemmas.

**Lemma 5.1.** Let  $\ell$  be a continuous decreasing function from  $[0, \infty)$  to  $(0, \infty)$  such that  $\ell(r)r^\theta$  is almost increasing for some  $\theta < 1$  and that

$$\int_1^\infty \frac{\phi(t)}{t^2 \ell(t)} dt < \infty.$$

Define

$$w(x) = (1 + |x|)^{n+1} \ell(|x|) \quad \text{for } x \in \mathbb{R}^n.$$

If a function  $f$  satisfies

$$(5.1) \quad wf \in L^\infty \quad \text{and} \quad \int f = 0,$$

then  $f \in H_I^{[\phi, \infty]}$ . Moreover, there exist a constant  $C > 0$  such that

$$(5.2) \quad \|f\|_{H_I^{[\phi, \infty]}} \leq C \|wf\|_\infty,$$

where  $C$  is independent of  $f$ .

**Lemma 5.2.** Let  $\ell$  be a continuous decreasing function from  $[0, \infty)$  to  $(0, \infty)$  such that  $\ell(r) \geq (1 + r)^{-n-1}$  and that

$$\lim_{r \rightarrow \infty} \ell(r) = 0 \text{ if } n \geq 3, \quad \lim_{r \rightarrow \infty} \ell(r) \log r = 0 \text{ if } n = 2.$$

Define

$$w(x) = (1 + |x|)^{n+1} \ell(|x|) \quad \text{for } x \in \mathbb{R}^n.$$

If  $\varphi \in \mathcal{S}$  and  $\int \varphi = 0$ , then

$$\lim_{\epsilon \rightarrow 0} \|(R_{i,j}^\epsilon \varphi - R_i R_j \varphi)w\|_\infty = 0.$$

*Proof of Theorem 4.1.* If (4.1) holds, then there exists a continuous decreasing function  $m$  such that  $\lim_{r \rightarrow \infty} m(r) = 0$  and that

$$\begin{cases} \int_1^\infty \frac{\phi(t)}{t^2 m(t)} dt < \infty, & \text{if } n \geq 3, \\ \int_1^\infty \frac{\phi(t) \log(1+t)}{t^2 m(t)} dt < \infty, & \text{if } n = 2. \end{cases}$$

Actually, if  $\int_1^\infty F(t) dt < \infty$ ,  $F(t) = \phi(t)/t^2$  or  $\phi(t) \log(1+r)/t^2$ , then we can take a positive increasing sequence  $\{r_j\}$  and a continuous decreasing function  $m$  such that

$$\int_{r_j}^\infty F(t) dt \leq \frac{1}{j^3}, \quad \text{for } j = 1, 2, \dots,$$

and

$$m(t) \geq \frac{1}{j} \quad \text{for } r_j \leq t \leq r_{j+1}.$$

Then

$$\int_{r_1}^\infty \frac{F(t)}{m(t)} dt = \sum_{j=1}^\infty \int_{r_j}^{r_{j+1}} \frac{F(t)}{m(t)} dt \leq \sum_{j=1}^\infty \frac{1}{j^2} < \infty.$$

We may assume that  $m(r)r^\nu$  is almost increasing for some small  $\nu > 0$ . Let  $\ell$  be a continuous decreasing function from  $[0, \infty)$  to  $(0, \infty)$  such that, for  $r \geq 1$ ,

$$\ell(r) = \begin{cases} m(r), & \text{if } n \geq 3, \\ m(r)/\log(1+r), & \text{if } n = 2. \end{cases}$$

Then  $\ell$  satisfies the assumption of both Lemmas 5.1 and 5.2.

Using the following relations,

$$wf \in L^\infty \quad \text{and} \quad \int f = 0, \quad \xRightarrow{\text{Lemma 4.1}} \quad \|f\|_{H_I^{[\phi, \infty]}} \leq C\|wf\|_\infty;$$

$$\varphi \in \mathcal{S} \quad \text{and} \quad \int \varphi = 0 \quad \xRightarrow{\text{Lemma 4.2}} \quad \lim_{\epsilon \rightarrow 0} \|(R_{i,j}^\epsilon \varphi - R_i R_j \varphi)w\|_\infty = 0;$$

we have that, if  $\varphi \in \mathcal{S}$  and  $\int \varphi = 0$ , then

$$\|R_{i,j}^\epsilon \varphi - R_i R_j \varphi\|_{H_I^{[\phi, \infty]}} \leq C\|(R_{i,j}^\epsilon \varphi - R_i R_j \varphi)w\|_\infty \rightarrow 0,$$

as  $\epsilon \rightarrow 0$ . □

## 6. APPLICATION

Let  $n \geq 2$ . We are concerned with the uniqueness of solutions for the Navier-Stokes equation,

$$(6.1) \quad u_t - \Delta u + (u, \nabla)u + \nabla p = 0 \quad \text{in } (0, T) \times \mathbb{R}^n,$$

$$(6.2) \quad \operatorname{div} u = 0 \quad \text{in } (0, T) \times \mathbb{R}^n,$$

with initial data  $u|_{t=0} = u_0$ , where  $u = u(t, x) = (u_1(t, x), \dots, u_n(t, x))$  and  $p = p(t, x)$  stand for the unknown velocity vector field of the fluid and its pressure field respectively, while  $u_0 = u_0(x) = (u_0^1(x), \dots, u_0^n(x))$  is the given initial velocity vector field.

It is well known (see [2]) that for initial data  $u_0 \in L^\infty(\mathbb{R}^n)$  the equations (6.1), (6.2) admit a unique time-local (regular) solution  $u$  with

$$p = \sum_{i,j=1}^n R_i R_j u_i u_j.$$

In this report, following J. Kato [3], by "a solution in the distribution sense" we mean a weak solution in the following sense.

**Definition 6.1.** We call  $(u, p)$  the solution of the Navier-Stokes equations (6.1), (6.2) on  $(0, T) \times \mathbb{R}^n$  with initial data  $u_0$  in the distribution sense if  $(u, p)$  satisfy  $\operatorname{div} u = 0$  in  $\mathcal{S}'$  for a.e.  $t$  and

$$(6.3) \quad \int_0^T \left\{ \langle u(s), \partial_s \Phi(s) \rangle + \langle u(s), \Delta \Phi(s) \rangle + \langle (u \times u)(s), \nabla \Phi(s) \rangle + \langle p(s), \operatorname{div} \Phi(s) \rangle \right\} ds = -\langle u_0, \Phi(0) \rangle$$

for  $\Phi \in C^1([0, T] \times \mathbb{R}^n)$  satisfying  $\Phi(s, \cdot) \in \mathcal{S}(\mathbb{R}^n)$  for  $0 \leq s \leq T$ , and  $\Phi(T, \cdot) \equiv 0$ , where  $\langle (u \times u), \nabla \Phi \rangle = \sum_{i,j=1}^n \langle u_i u_j, \partial_i \Phi_j \rangle$ . Here  $\mathcal{S}$  denotes the space of rapidly decreasing functions in  $\mathbb{R}^n$  and  $\mathcal{S}'$  denotes the space of tempered distributions in the sense of Schwartz. The space  $\mathcal{S}'$  is the topological dual of  $\mathcal{S}$  and its canonical pairing is denoted by  $\langle, \rangle$ .

J. Kato [3] proved the following uniqueness theorem.

**Theorem 6.1** (J. Kato [3]). *Let  $u_0 \in L^\infty$  with  $\operatorname{div} u_0 = 0$ . Suppose that  $(u, p)$  is the solution in the distribution sense satisfying*

$$(6.4) \quad u \in L^\infty((0, T) \times \mathbb{R}^n), \quad p \in L_{\text{loc}}^1((0, T); \text{BMO}).$$

*Then  $(u, \nabla p)$  is uniquely determined by the initial data  $u_0$ . Moreover,  $\nabla p = \sum_{i,j=1}^n \nabla R_i R_j u^i u^j$  in  $\mathcal{S}'$  for a.e.  $t$ .*

On the other hand, Galdi and Maremonti [1] showed that if  $u$  and  $\nabla u$  are bounded in  $(0, T) \times \mathbb{R}^3$ , then the uniqueness of classical solutions holds provided that for some  $C > 0$  and some  $\epsilon > 0$  the inequality

$$(6.5) \quad |p(t, x)| \leq C(1 + |x|)^{1-\epsilon}$$

holds. See also [9] and [4]. The assumption (6.4) does not imply (6.5).

To prove Theorem 6.1, Kato [3] used the duality  $(H^1)^* = \text{BMO}$  and the following fact: If  $\varphi \in \mathcal{S}$  and  $\int \varphi = 0$ , then

$$\lim_{\epsilon \rightarrow 0} R_{i,j}^\epsilon \varphi = R_i R_j \varphi \quad \text{in } H^1.$$

The duality  $\left(H_I^{[\phi, \infty]}\right)^* = \mathcal{L}_{1, \phi}$  is known and we have proved in Theorem 4.1 that if  $\varphi \in \mathcal{S}$  and  $\int \varphi = 0$ , then

$$\lim_{\epsilon \rightarrow 0} R_{i,j}^\epsilon \varphi = R_i R_j \varphi \quad \text{in } H_I^{[\phi, \infty]}.$$

Then we have the following.

**Theorem 6.2.** *Assume that  $\phi \in \mathcal{G}$  satisfies (4.1). Let  $u_0 \in L^\infty$  with  $\text{div } u_0 = 0$ . Suppose that  $(u, p)$  is the solution of (6.1), (6.2) in the distribution sense satisfying*

$$(6.6) \quad u \in L^\infty((0, T) \times \mathbb{R}^n), \quad p \in L_{\text{loc}}^1((0, T); \mathcal{L}_{1, \phi}).$$

*Then  $(u, \nabla p)$  is uniquely determined by the initial data  $u_0$ . Moreover,  $\nabla p = \sum_{i,j=1}^n \nabla R_i R_j u^i u^j$  in  $\mathcal{S}'$  for a.e.  $t$ .*

For example, let

$$(6.7) \quad \phi(r) = \begin{cases} r^{-n} & \text{for } 0 < r < 1, \\ r(\log(1+r))^{-\beta} & \text{for } r \geq 1, \end{cases}$$

where  $\beta > 1$  if  $n \geq 3$  and  $\beta > 2$  if  $n = 2$ . In this case

$$\mathcal{L}_{1, \phi} \supset L^1 \cup \text{BMO}$$

and  $\mathcal{L}_{1, \phi}$  contains functions  $f$  such that

$$|f(x)| \leq C\phi(1+|x|) = C(1+|x|)(\log(2+|x|))^{-\beta} \quad \text{for } x \in \mathbb{R}^n.$$

Therefore, our result is an extension of both Kato's theorem and the result of Galdi and Maremonti. Note that, if  $\beta = 0$ , then the uniqueness fails (see [2]).

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