

Inverse optimal damper placement via shear model for elastic-plastic moment-resisting frames under large-amplitude ground motions

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Abstract

A new method of optimal damper design is presented for elastic-plastic planar frame structures under large-amplitude earthquake ground motions. The transformation of a moment frame into the 1st-mode equivalent shear mass system leads to an efficient search of the optimal damper placement **along the building height** for the elastic-plastic moment frame. In the optimization procedure, a linear elastic bare frame is transformed into the 1st-mode equivalent shear mass system first. Then the optimization of the damper placement is conducted for the elastic shear mass system. After that, the damped shear mass system is inversely transformed into the elastic-plastic moment frame with the added dampers. This inversely transformed damper system in the moment frame is employed as an initial design and

the local search-based optimization is conducted for the efficient distribution of dampers **not only along the building height, but also among the different bay**. The proposed method greatly reduces the computational task in the whole optimization procedure because only a limited number of time-history response analyses is required for elastic-plastic moment frames. It is demonstrated through numerical examples that the proposed method efficiently provides the damper designs with high accuracy. Finally, the importance of using the large-amplitude ground motions for damper optimization is demonstrated through the incremental dynamic analysis (IDA) for the optimally designed models.

Keywords: Earthquake response, Viscous damper, Optimization, Elastic-plastic moment-resisting frame, Large-amplitude ground motion, Damper inverse transformation.

1. Introduction

The structural control via passive dampers has been investigated extensively so far from theoretical and practical viewpoints (for example, see [1-4]). Originally, Zhang and Soong [5] and Garcia [6] proposed simple methods of damper placement using the original and revised sequential allocation algorithms. Takewaki [7] developed an optimality criterion-based approach incorporating an incremental inverse problem for viscous damper placement. Uetani et al. [8] applied the mathematical programming techniques to an optimal damper placement problem for large-scale building frames in which viscous and hysteretic dampers were used simultaneously. Trombetti and Silvestri [9] used finite element models to demonstrate the effectiveness of the mass-proportional damping systems. Kanno [10] treated a mixed-integer programming problem for optimal damper placement. De Domenico and Ricciardi [11] introduced a non-Gaussian stochastic linearization formulation for the expression of the nonlinearity of fluid viscous dampers **and later used it for the energy-based optimal design of nonlinear**

viscous dampers in building structures [12]. De Domenico and Hajirasouliha [13] tackled a multi-level performance-based design problem for optimal nonlinear viscous damper placement. Yang et al. [14] presented two optimal design methodologies for passive dampers based on active control theories, including H_∞ and H_2 performances. Zhou et al. [15] proposed a practical design method for reinforced concrete structures with viscous dampers. Alotta et al. [16] presented a simplified procedure for the design of fluid viscous dampers. Furthermore, approaches using metaheuristics have also been widely developed for the damper optimization [17-21].

Some researchers compared several optimization techniques in view of optimization performances, usability, robustness, computational load and so on [22-25]. It is apparent that almost all of the above-mentioned researches treated elastic responses of structural frames. On the other hand, a small number of researches took into account the elastic-plastic responses of moment-resisting frames in the damper optimization problems [26-33]. In relation to this, some researches incorporated the process of redesign of structural frames into the damper optimization problems [34-36]. Idels and Lavan [37] tackled a realistic simultaneous optimization problem of discrete member sections of a structural frame and nonlinear viscous dampers. Gomez et al. [38] simultaneously treated the topology optimization of main frames and the damper optimization in building structures. Saitua et al. [39] incorporated the procedure of strengthening of structural members into the damper design to cope with the increase of the member stress by adding the dampers. Idels and Lavan [40] dealt with an optimization problem of simultaneous use of viscous dampers and passive negative stiffness devices. In addition, the limit states and the failure mechanism of the dampers could also be worth being studied in the optimal design problem [41].

As stated above, in most researches on design of dampers for moment-resting frames, elastic frames are treated under the constraints on member stress. Especially in the cases of the design of viscous

dampers, velocity responses should be estimated precisely so as to prevent the excessive forces on the structural members and grasp the effectiveness of the damper placement [42, 43]. It should also be pointed out that, although code-specified ground motions are commonly used for damper optimization, recently observed ground motions greatly exceed the level of the code-specified ground motions. Such motions are called large-amplitude ground motions in this paper. Elastic-plastic frames and large-amplitude ground motions should be treated in the damper optimization because the damper design for elastic frames may not be safe for large-amplitude ground motions. It is also noted that even damped structures may not remain elastic under large-amplitude ground motions. This is because the total amount of dampers is limited in view of cost and the overdamping causes undesirable events like excessive member force responses and decrease of the performance under code-specified ground motions. Therefore, regarding to large-amplitude ground motions, it is most important to prevent the deformation concentration to specific stories and the structural collapse. Since the time-history response analysis for elastic-plastic frames requires much computational effort, efficient optimization methods are desired to be developed.

In this paper, a new method of optimal damper design is presented for elastic-plastic planar moment-resisting frames under large-amplitude ground motions. The transformation of a moment frame into the 1st-mode equivalent shear mass system leads to an efficient search of the optimal damper placement along the building height for the elastic-plastic moment frame. In the optimization procedure, the linear elastic bare frame is transformed into the 1st-mode equivalent shear mass system first. Then, the optimization of the damper placement is conducted for the elastic shear mass system. After that, the damped shear mass system is inversely transformed into the elastic-plastic frame with the added dampers. This inversely transformed damper system in the moment frame is employed as an initial design and the local search-based optimization is conducted for the efficient distribution of dampers not only along the building height, but also among the different bay. The

proposed method greatly reduces the computational effort in the whole optimization procedure because only a limited number of time-history response analyses is required for elastic-plastic frames. It is demonstrated through numerical examples that the proposed method efficiently provides the damper designs with high accuracy. Finally, the importance of using the large-amplitude ground motions for the damper optimization is demonstrated through the incremental dynamic analysis (IDA) [44] for the optimally designed models.

2. Problem of optimal damper placement for elastic-plastic frame structures and concept of damper design under large-amplitude ground motions

A problem of optimal damper placement is treated for ‘elastic-plastic’ frame structures under large-amplitude ground motions. The overview of the method proposed in this paper is shown in Fig. 1. In the optimization procedure, the linear elastic bare frame is transformed into the 1st-mode equivalent shear mass system first. Then, the optimization of the damper placement is conducted for the elastic shear mass system. After that, the damped shear mass system is inversely transformed into the elastic-plastic frame with the added dampers. The inverse transformation is conducted so as to maximize the approximate value h_1 of the lowest-mode damping ratio by allocating the dampers appropriately. This inversely transformed damper system in the moment frame is employed as an initial design and the local search-based optimization is conducted for the efficient distribution of dampers not only along the building height, but also among the different bays. The details of the proposed method are explained in Section 2.1-2.4.

‘Elastic’ frames are usually treated in the optimization of damper placement so that the member stress remains below the yield stress. In addition, code-specified ground motions are mostly dealt with as the input ground motions for design. Figure 2a shows the velocity response spectrum of the code-specified ground motions, and the amplitudes of the motions are scaled for structural design. On

the other hand, Figure 2b shows the velocity response spectrum of the recently observed ground motions in Japan, and the amplitudes are unscaled. Since recently observed ground motions greatly exceed the level of the code-specified ground motions as shown in Figures 2a, 2b, Table 1, damper designs for elastic frames may not be safe for large-amplitude ground motions. To obtain damper designs effective for large-amplitude ground motions, elastic-plastic frames and large-amplitude ground motions are treated for the design in this paper. The essential feature of this design concept can be found in Figure 2c. Although much computational effort is required for the time-history response analysis for elastic-plastic frames, the proposed method effectively uses the equivalent shear mass system to greatly reduce the computational load for the whole optimization procedure.

In this section, a 1st-mode equivalent shear mass system corresponding to the linear elastic bare frame is defined. After the application of a damper optimization procedure [36, 45] to the 1st-mode equivalent shear mass system, a method of damper inverse transformation is presented from the 1st-mode equivalent shear mass system to the frame model. The optimization problem of damper placement is stated for elastic-plastic frame structures and the local search-based solution algorithm is proposed.

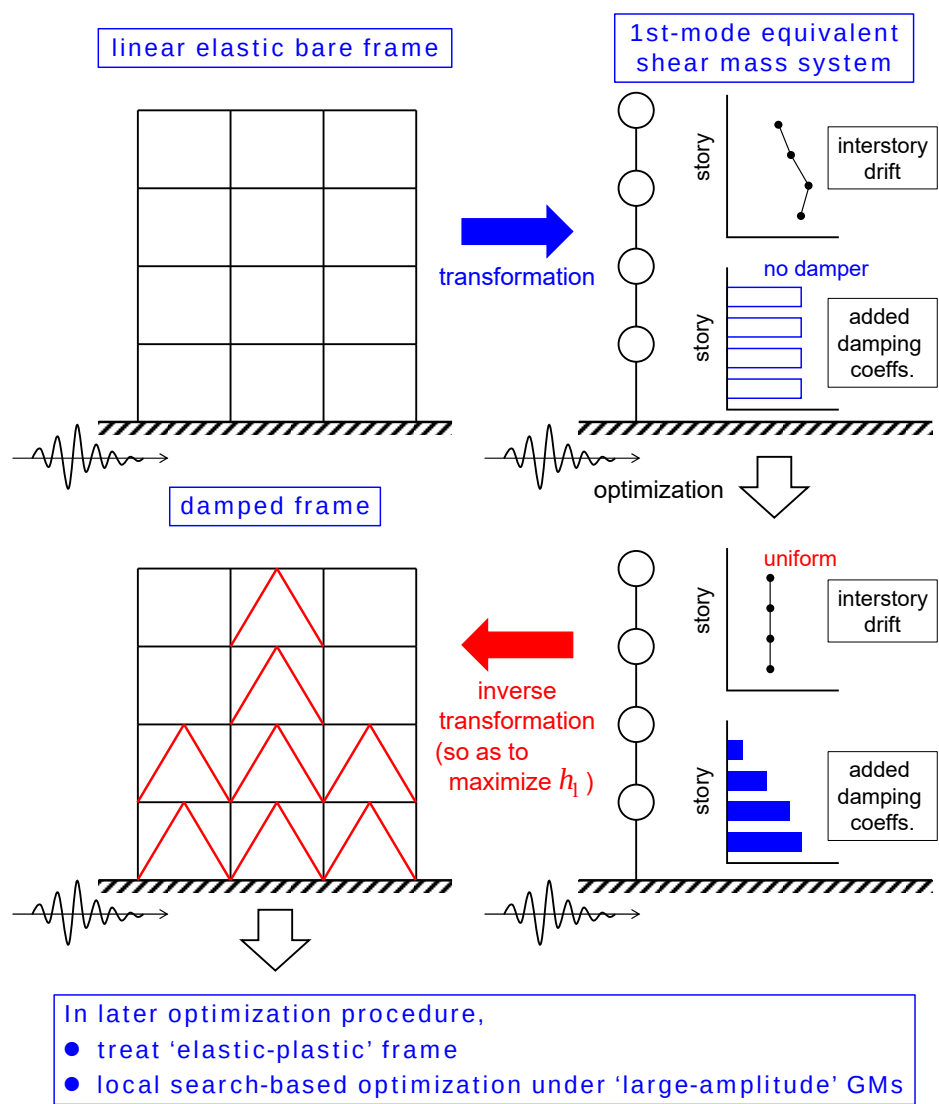


Fig. 1 Overview of proposed method

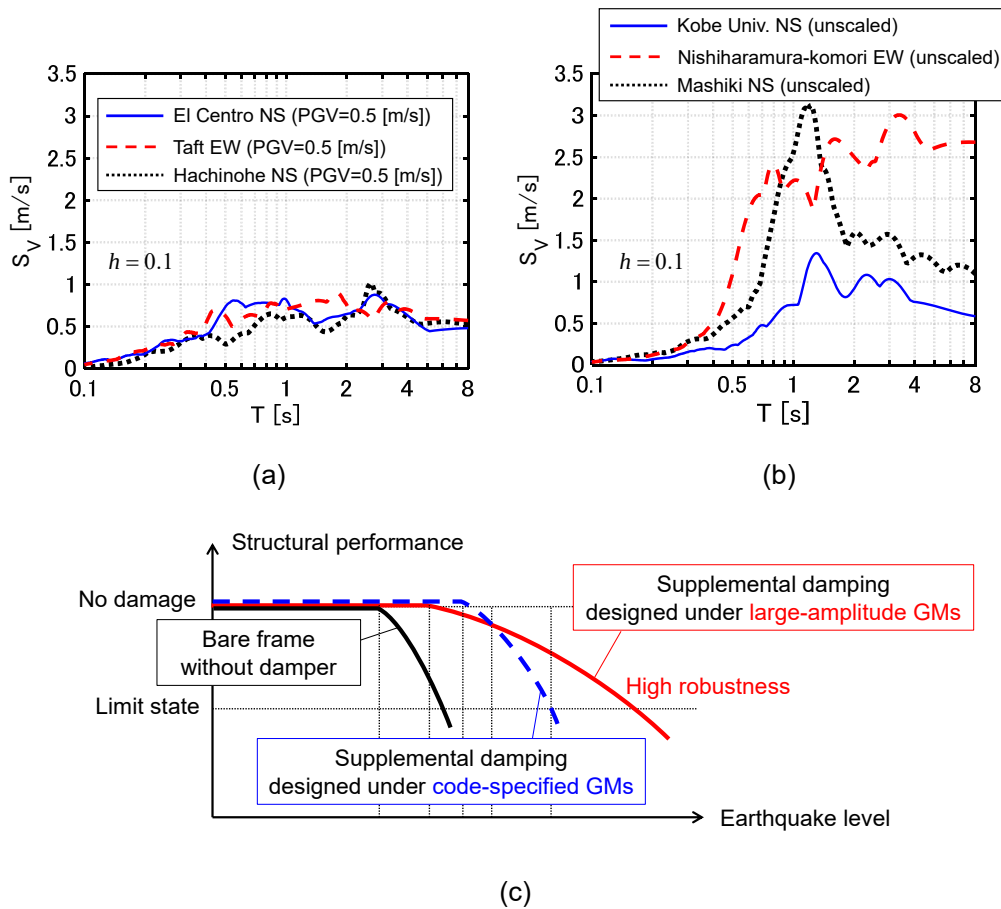


Fig. 2 Velocity response spectrum and concept of damper design under large-amplitude ground motions, (a) velocity response spectrum of code-specified ground motions, (b) velocity response spectrum of recently observed ground motions, (c) concept of damper design under large-amplitude ground motions.

Table 1 Recorded ground motions used in Fig 2

No.	Name of ground motion	No.	Name of ground motion
1.	Imperial Valley 1940 El Centro NS	4.	Hyogoken-Nanbu 1995 Kobe Univ NS
2.	Kern County 1952 Taft EW	5.	Kumamoto 4/16/2016 Nishiharamura-komori EW
3.	Tokachi-Oki 1968 Hachinohe NS	6.	Kumamoto 4/16/2016 Mashikimachi-miyazono NS

2.1 1st-mode equivalent shear mass system

Consider a linear elastic N_F -story N_B -bay planar moment-resisting frame. The horizontal displacements of the nodes in the same floor are assumed to be identical. The equations of motion are expressed as

$$\begin{pmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{J} \end{pmatrix} \begin{pmatrix} \ddot{\mathbf{u}} \\ \ddot{\boldsymbol{\theta}} \end{pmatrix} + \begin{pmatrix} \mathbf{C}_{11} & \mathbf{C}_{12} \\ \mathbf{C}_{21} & \mathbf{C}_{22} \end{pmatrix} \begin{pmatrix} \dot{\mathbf{u}} \\ \dot{\boldsymbol{\theta}} \end{pmatrix} + \begin{pmatrix} \mathbf{K}_{11} & \mathbf{K}_{12} \\ \mathbf{K}_{21} & \mathbf{K}_{22} \end{pmatrix} \begin{pmatrix} \mathbf{u} \\ \boldsymbol{\theta} \end{pmatrix} = - \begin{pmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{J} \end{pmatrix} \begin{pmatrix} \mathbf{1} \\ \mathbf{0} \end{pmatrix} \ddot{u}_g, \quad (1)$$

where $\mathbf{u}, \boldsymbol{\theta}, \mathbf{M} = \text{diag}(m_1, \dots, m_{N_F})$, $\mathbf{J}, \mathbf{K}, \mathbf{C}, \ddot{u}_g$ denote the horizontal displacements of all the floor, the vertical and rotational displacements of all the nodes, the lumped mass matrices with respect to $\mathbf{u}$ and $\boldsymbol{\theta}$, the stiffness matrix, the damping matrix, and the ground acceleration, respectively. By neglecting the inertial force and moment due to $\mathbf{J}$, the equations of motion of the undamped frame are expressed as follows.

$$\begin{pmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \ddot{\mathbf{u}} \\ \ddot{\boldsymbol{\theta}} \end{pmatrix} + \begin{pmatrix} \mathbf{K}_{11} & \mathbf{K}_{12} \\ \mathbf{K}_{21} & \mathbf{K}_{22} \end{pmatrix} \begin{pmatrix} \mathbf{u} \\ \boldsymbol{\theta} \end{pmatrix} = - \begin{pmatrix} \mathbf{M} \mathbf{1} \ddot{u}_g \\ \mathbf{0} \end{pmatrix} \quad (2)$$

Eq. (2) can also be described as

$$\begin{aligned} \mathbf{M} \ddot{\mathbf{u}} + \mathbf{K}_r \mathbf{u} &= -\mathbf{M} \mathbf{1} \ddot{u}_g \\ \boldsymbol{\theta} &= -\mathbf{K}_{22}^{-1} \mathbf{K}_{21} \mathbf{u} \end{aligned}, \quad (3)$$

where

$$\mathbf{K}_r = \mathbf{K}_{11} - \mathbf{K}_{12} \mathbf{K}_{22}^{-1} \mathbf{K}_{21} \quad (4)$$

It is noted that $\mathbf{K}_r$ is not tridiagonal. This procedure is widely known as the static condensation procedure.

Let $\boldsymbol{\varphi}_1 = (\varphi_{1,1} \dots \varphi_{1,N_F})^T$ and ω_1 denote the undamped fundamental natural mode and the undamped fundamental natural circular frequency of the reduced model represented by Eq. (3). By solving the

inverse eigenmode problem, the story stiffnesses $\mathbf{k} = (k_1 \dots k_{N_F})^T$ of the 1st-mode equivalent shear mass system (Figure 3) are obtained by

$$k_i = \omega_1^2 (m_{N_F} \phi_{1,N_F} + \dots + m_i \phi_{1,i}) / (\phi_{1,i} - \phi_{1,i-1}) \quad (i = 2, \dots, N_F), \quad (5)$$

$$k_1 = \omega_1^2 (m_{N_F} \phi_{1,N_F} + \dots + m_1 \phi_{1,1}) / \phi_{1,1} \quad (i = 1) \quad (6)$$

It is noted that only the fundamental natural mode of the frame coincides with that of the shear mass system, and the higher modes of the frame do not coincide with those of the shear mass system.

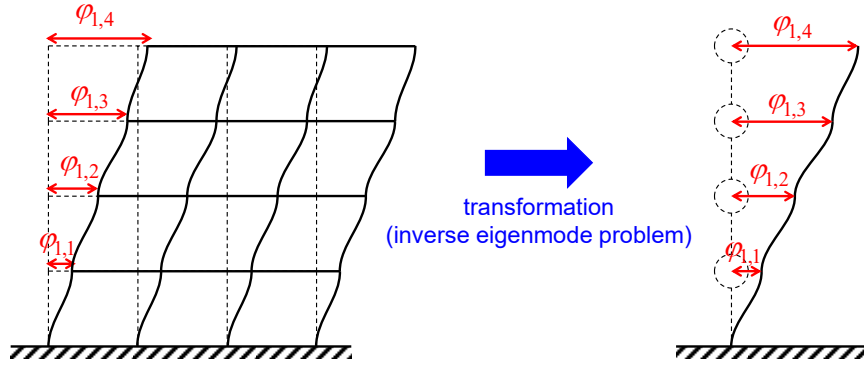


Fig. 3 Transformation of linear elastic bare frame into 1st-mode equivalent shear mass system

2.2 Damper inverse transformation

Consider a linear elastic moment-resisting frame with added dampers as shown in Figure 4. Since the optimal damper placement obtained for a linear elastic moment-resisting frame can be a good initial design for an elastic-plastic frame, the explanation for a linear elastic frame is shown in this section.

Let $c_{i,j}$ denote the damping coefficient added at the j -th bay in the i -th story, and $\phi_{i,j}$ denote the

angle of the damper. Consider a 1st-mode equivalent shear mass system with added viscous damping

coefficients $\mathbf{c} = (c_1 \dots c_{N_F})^T$. The distribution of these added viscous damping coefficients is to be obtained by an optimization algorithm [36, 45].

In this section, an innovative method of the inverse damper transformation is proposed from the 1st-mode equivalent shear mass system into the moment-resisting frame. The transformed added dampers $c_{i,j}$ of the frame structure are to satisfy the following relation.

$$c_i = \sum_{j=1}^{N_B} c_{i,j} \cos^2 \phi_{i,j} \quad (i = 1, \dots, N_F), \quad (7)$$

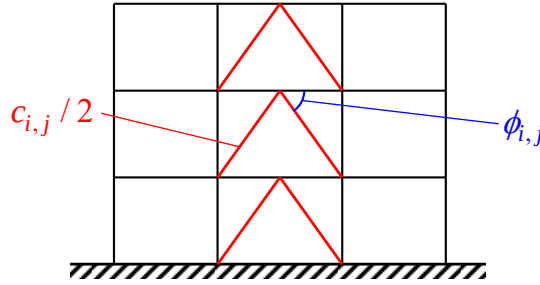


Fig. 4 Angle of damper from horizontal line

When the added viscous dampers are expressed by a Voigt model, the equations of motion of the 1st-mode equivalent shear mass system can be expressed by

$$\mathbf{M}\ddot{\mathbf{u}} + (\mathbf{C} + \mathbf{C}_{add})\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{0}, \quad (8)$$

On the other hand, the equations of motion of the frame structure can be represented by

$$\begin{pmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{J} \end{pmatrix} \begin{pmatrix} \ddot{\mathbf{u}} \\ \ddot{\boldsymbol{\theta}} \end{pmatrix} + \left(\begin{pmatrix} \mathbf{C}_{11} & \mathbf{C}_{12} \\ \mathbf{C}_{21} & \mathbf{C}_{22} \end{pmatrix} + \begin{pmatrix} \mathbf{C}_{11,add} & \mathbf{C}_{12,add} \\ \mathbf{C}_{21,add} & \mathbf{C}_{22,add} \end{pmatrix} \right) \begin{pmatrix} \dot{\mathbf{u}} \\ \dot{\boldsymbol{\theta}} \end{pmatrix} + \begin{pmatrix} \mathbf{K}_{11} & \mathbf{K}_{12} \\ \mathbf{K}_{21} & \mathbf{K}_{22} \end{pmatrix} \begin{pmatrix} \mathbf{u} \\ \boldsymbol{\theta} \end{pmatrix} = - \begin{pmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{J} \end{pmatrix} \begin{pmatrix} \mathbf{1} \\ \mathbf{0} \end{pmatrix} \ddot{u}_g, \quad (9)$$

The damping matrices with the subscript ‘add’ are concerned with the added viscous dampers, and the damping matrices without the subscript ‘add’ are related to the structural damping. As shown in Section 2.1, it is assumed that the influence of the inertial force and moment in the vertical and

rotational directions on the inertial force in the horizontal direction is small. Similarly, this assumption can be applied to the damping terms. The detailed examination will be conducted below.

From Eq.(7), the following relation is obtained

$$\mathbf{C}_{11,add} = \mathbf{C}_{add} \quad (10)$$

The fundamental damping ratio of the shear mass system and that of the frame structure by the added dampers can be evaluated approximately by

$$h_{add,1} = \frac{\boldsymbol{\Phi}_1^T \mathbf{C}_{add} \boldsymbol{\Phi}_1}{2\omega_1 \boldsymbol{\Phi}_1^T \mathbf{M} \boldsymbol{\Phi}_1}, \quad (11)$$

$$\bar{h}_{add,1} = \frac{\bar{\boldsymbol{\Phi}}_1^T \mathbf{C}_{11,add} \bar{\boldsymbol{\Phi}}_1 + \bar{\boldsymbol{\Phi}}_1^T \mathbf{C}_{12,add} \bar{\boldsymbol{\chi}}_1 + \bar{\boldsymbol{\chi}}_1^T \mathbf{C}_{21,add} \bar{\boldsymbol{\Phi}}_1 + \bar{\boldsymbol{\chi}}_1^T \mathbf{C}_{22,add} \bar{\boldsymbol{\chi}}_1}{2\bar{\omega}_1 (\bar{\boldsymbol{\Phi}}_1^T \mathbf{M} \bar{\boldsymbol{\Phi}}_1 + \bar{\boldsymbol{\chi}}_1^T \mathbf{J} \bar{\boldsymbol{\chi}}_1)}, \quad (12)$$

where, $\bar{\boldsymbol{\Phi}}_1, \bar{\boldsymbol{\chi}}_1$ denote the elements corresponding to the horizontal displacements and the vertical and the rotational displacements of the undamped fundamental natural mode of the frame structure, and $\bar{\omega}_1$ denotes the undamped fundamental natural frequency of the frame structure. It is noted that $\mathbf{J}$ is taken into account in the evaluation of $\bar{\boldsymbol{\Phi}}_1, \bar{\boldsymbol{\chi}}_1, \bar{\omega}_1$. Eq. (11) is for the shear mass system and Eq. (12) is for the moment-resisting frame. Most of the difference between the lowest-mode damping ratio of the shear mass system and that of the moment frame comes from $(\bar{\boldsymbol{\Phi}}_1^T \mathbf{C}_{12,add} \bar{\boldsymbol{\chi}}_1 + \bar{\boldsymbol{\chi}}_1^T \mathbf{C}_{21,add} \bar{\boldsymbol{\Phi}}_1)$ in Eq. (12). It is also noted that $(\partial \bar{h}_{add,1} / \partial c_{i,j})$ is constant and $\bar{h}_{add,1}$ is expressed by the linear combination of $c_{i,j}$ as follows.

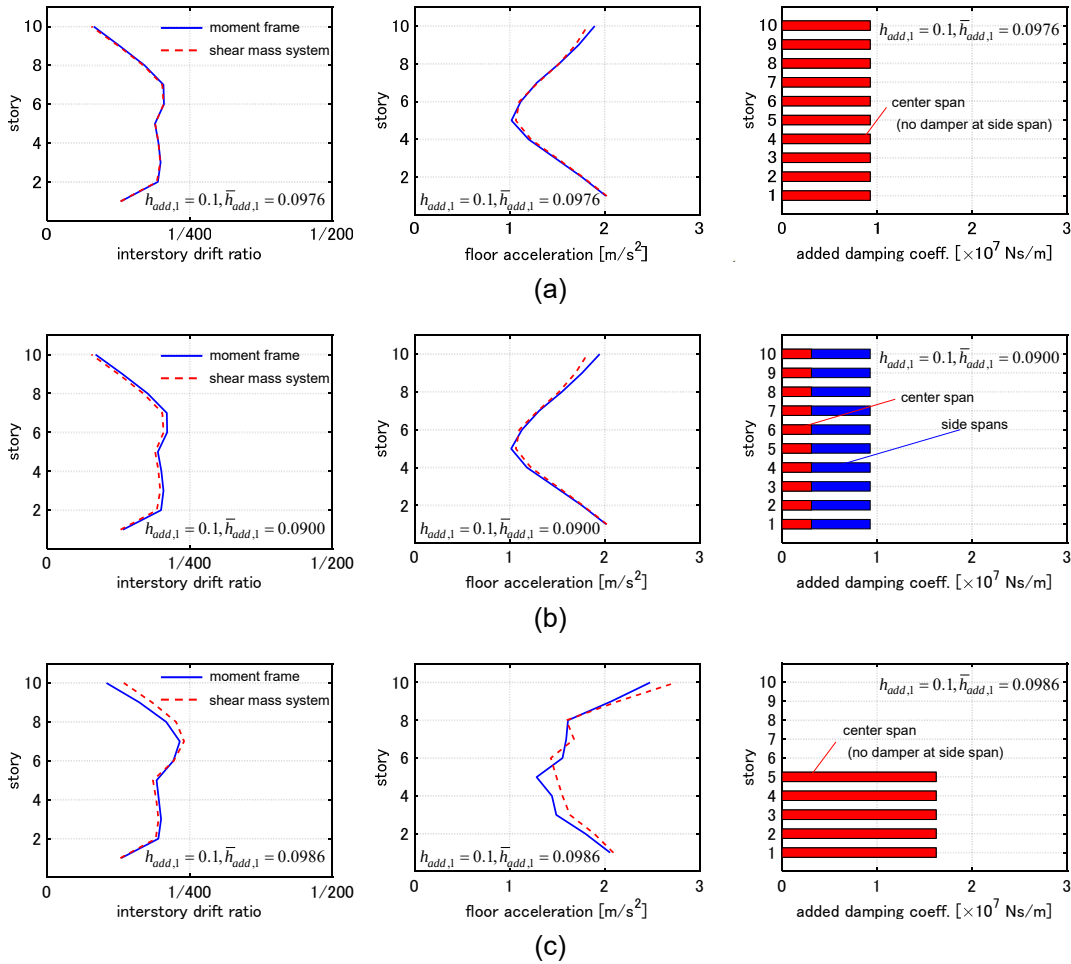
$$\bar{h}_{add,1} = \sum_{i=1}^{N_F} \sum_{j=1}^{N_B} (c_{i,j} \frac{\partial \bar{h}_{add,1}}{\partial c_{i,j}}), \quad (13)$$

When $\bar{\omega}_1 \approx \omega_1, \bar{\boldsymbol{\Phi}}_1 \approx \boldsymbol{\Phi}_1$, and the absolute values of $\bar{\boldsymbol{\chi}}_1$ are much smaller than those of $\bar{\boldsymbol{\Phi}}_1$, the value of $\bar{h}_{add,1}$ gets close to that of $h_{add,1}$. In addition, when all the dampers added to the i -th story are given

to the bay with the maximum value of $(\partial \bar{h}_{add,1} / \partial c_{i,j})$, the value of $\bar{h}_{add,1}$ will be nearly equal to that of $h_{add,1}$. It should be pointed out that the value of $(\partial \bar{h}_{add,1} / \partial c_{i,j})$ in the center bay is larger than those of the side bays. Moreover, the value of $(\partial \bar{h}_{add,1} / \partial c_{i,j})$ in the bay with a small value of $\phi_{i,j}$ is larger than that in the bay with a large value of $\phi_{i,j}$. This is because the elements of the vertical displacements in the fundamental natural mode influence $\bar{h}_{add,1}$.

Figure 5 shows the distributions of the maximum interstory drifts, the distributions of the maximum floor accelerations, and the distributions of added damping coefficients in the 10-story 3-bay building frames with four different damper distributions and those of the equivalent shear mass systems under El Centro NS component with PGV = 0.25 [m/s] (PGA = 2.2 [m/s²]). MATLAB has been used for the numerical analyses and all the codes have been originally made. The accuracy has been checked through the comparison with the structural analysis software OpenSees especially for the building frames [46]. The details of the frame will be explained in Section 3.1. The values of $h_{add,1}$ are set to 0.1, and the values of $\bar{h}_{add,1}$ are indicated in Figure 5. In the cases of Figures 5a, b, the distributions of the maximum interstory drifts and the distributions of the maximum floor accelerations of the frame almost coincide with those of the shear mass system. This is because the uniform placement of added dampers along height reduces sufficiently the higher-mode responses of the frame and those of the shear mass system, even though the higher-modes of the frame do not coincide with those of the shear mass system. On the other hand, it can be observed from Figures 5c, d that the added dampers do not reduce the higher-mode responses of the frame and those of the shear mass system sufficiently, and this leads to the small difference between the responses of the frame and those of the shear mass system.

It is noted that Eq. (7) does not determine the distribution of added dampers in one way. In the optimization procedure, the dampers added to the specific story are preferentially given to the bays with the large value of $(\partial \bar{h}_{add,1} / \partial c_{i,j})$. It should be noted that, while the maximization of the lowest-mode damping ratio was adopted in the damper optimization of a shear mass system [47], the treatment of the lowest-mode damping ratio in the damper optimization in the horizontal direction of frame models has never been seen.



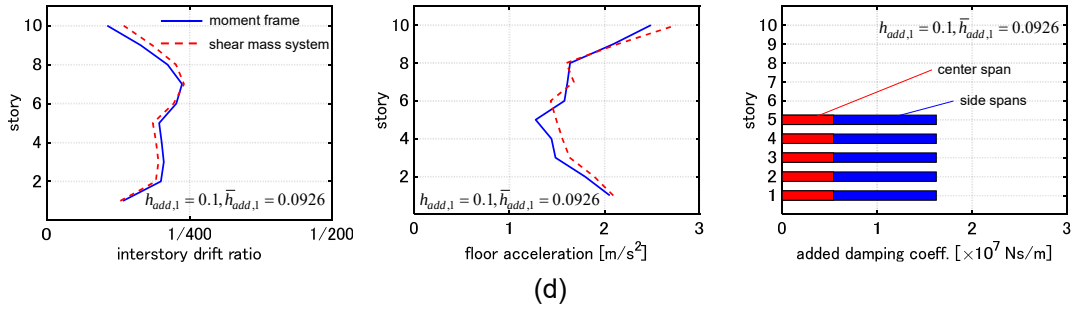


Fig. 5 Distributions of maximum interstory drifts, distributions of maximum floor accelerations, and distributions of added damping coefficients for moment frames with four different damper distributions and those for equivalent shear mass systems, (a) Uniform distribution of added dampers (at center spans), (b) Uniform distribution of added dampers (at all spans), (c) Uniform distribution of added dampers in 1-5 stories (at center spans), (d) Uniform distribution of added dampers in 1-5 stories (at all spans).

2.3 Problem of optimal damper placement

Consider the following problem of optimal damper placement for elastic-plastic frame structures.

[Problem]

$$\text{Find } \mathbf{c}_{add} = (c_{1,1}, \dots, c_{1,N_B}, \dots, c_{N_F,1}, \dots, c_{N_F,N_B})$$

$$\text{so as to minimize } d_{\max} = \max_i \{d_{\max,i}\} \quad (i = 1, \dots, N_F)$$

$$\text{subjected to } \begin{cases} \mathbf{c}_{add}^T \cdot \mathbf{1} = W_c & (\text{const.}) \\ 0 \leq c_{i,j} \leq c_{i,j}^U & (\text{for } i = 1, \dots, N_F, j = 1, \dots, N_B) \end{cases}$$

where $c_{i,j}^U, W_c, d_{\max,i}$ denote the upper bound of the damping coefficient added at the j -th bay in the i -th story, the sum of added damping coefficients, and the maximum deformation of the i -th story.

It is noted that the constraint on the sum of added damping coefficients is almost equivalent to the constraint on the total cost of added dampers since the former is almost proportional to the latter. It is also noted that the constraints on member stress are not included in this optimization problem. This is because elastic-plastic frames are treated in the problem, and the nonlinear responses of members are

311 automatically taken into account in the optimization procedure. For the same reason, the dampers
 312 added in the specific position will not be excessive even if the constraints on the upper bound of $c_{i,j}$
 313 are not included in the problem.

314

315 2.4 Solution algorithm

316 The solution algorithm may be described as follows.

317

318 [Algorithm]

319 Step 1 Transform the linear elastic bare frame into the 1st-mode equivalent shear mass system.

320 Step 2 Obtain the optimal damper placement for the elastic shear mass system using an optimization
 321 method [36, 45].

322 Step 3 Inversely transform the shear mass system with dampers into the elastic-plastic frame with
 323 added dampers so as to maximize $\bar{h}_{add,1}$. In other words, the dampers added to the i -th story
 324 are preferentially given to the bays with relatively large value of $(\partial \bar{h}_{add,1} / \partial c_{i,j})$. Put $i_{LS} \rightarrow 1$
 325 (i_{LS} : counter).

326 Step 4 Solve a linear programming problem (as stated below) and update $\mathbf{c}_{add} \rightarrow \mathbf{c}_{add} + \Delta \mathbf{c}_{add}$. If this
 327 newly obtained design totally coincides with the other obtained designs, go to Step 6.

328 Step 5 If $i_{LS} = n_{LS}$, go to Step 6 (n_{LS} : upper limit on number of repetitions of local search).

329 Otherwise, put $i_{LS} \rightarrow i_{LS} + 1$ and return to Step 4.

Step 6 Select the design which exhibits the minimum value of $d_{\max}$ from all the obtained designs,
then finalize the process.

332

333

334 **[Linear programming problem]**335 Find $\Delta \mathbf{c}_{add} = (\Delta c_{1,1}, \dots, \Delta c_{1,N_B}, \dots, \Delta c_{N_F,1}, \dots, \Delta c_{N_F,N_B})$ 336 so as to minimize $f = d_{\max,k} + \sum_{i=1}^{N_F} \sum_{j=1}^{N_B} s_{i,j} \Delta c_{i,j}$

$$\text{subjected to } \begin{cases} \Delta \mathbf{c}_{add} \cdot \mathbf{1} = 0 \\ \Delta c_{i,j}^L \leq \Delta c_{i,j} \leq \Delta c_{i,j}^U & (\text{for } i = 1, \dots, N_F, j = 1, \dots, N_B) \\ c_{i,j} + \Delta c_{i,j}^L \geq 0 & (\text{for } i = 1, \dots, N_F, j = 1, \dots, N_B) \\ c_{i,j} + \Delta c_{i,j}^U \leq c_{i,j}^U & (\text{for } i = 1, \dots, N_F, j = 1, \dots, N_B) \end{cases}$$

338 where $\Delta c_{i,j}^L, \Delta c_{i,j}^U$ denote the lower and upper bounds of $\Delta c_{i,j}$, $s_{i,j}$ denotes the finite difference339 approximation of the first-order sensitivity $(\partial d_{\max,k} / \partial c_{i,j})$, and $k = \arg \max d_{\max,i}$. The calculation340 of $s_{i,j}$ requires the time-history response analysis for the model with slightly added damping at the j -341 th bay in the i -th story. By solving the above-stated linear programming problem, update342 $\mathbf{c}_{add} \rightarrow \mathbf{c}_{add} + \Delta \mathbf{c}_{add}$ (see Appendix).

343

344 Figure 6 shows the schematic diagram of the proposed solution algorithm. In the optimization

345 procedure, a linear elastic bare frame is transformed into the 1st-mode equivalent shear mass system

346 first. Then, the optimization of the damper placement is conducted for the elastic shear mass system.

347 In this paper, the sensitivity-based algorithm [36, 45] is applied to the optimization for the shear mass

348 system. The inversely transformed damper placement obtained in Step 3 corresponds to the optimal

349 damper placement for the elastic-plastic frame under the small-amplitude input ground motions. This

350 is because the geometric nonlinearity like the P-delta effect is negligible in this case and the frame
351 responses elastically under the small-amplitude input ground motions. The procedure in Step 4
352 corresponds to the local search. The objective function in the local search is the first-order
353 approximation of $d_{\max,k}$ ($k = \arg \max d_{\max,i}$). After going to Step 5 and updating i_{LS} , k and $d_{\max,k}$
354 are also updated. Such local search works effectively because the inversely transformed damper
355 placement is employed as an initial design and the inversely transformed damper placement is close
356 to the optimal damper placement for the large-amplitude ground motions.

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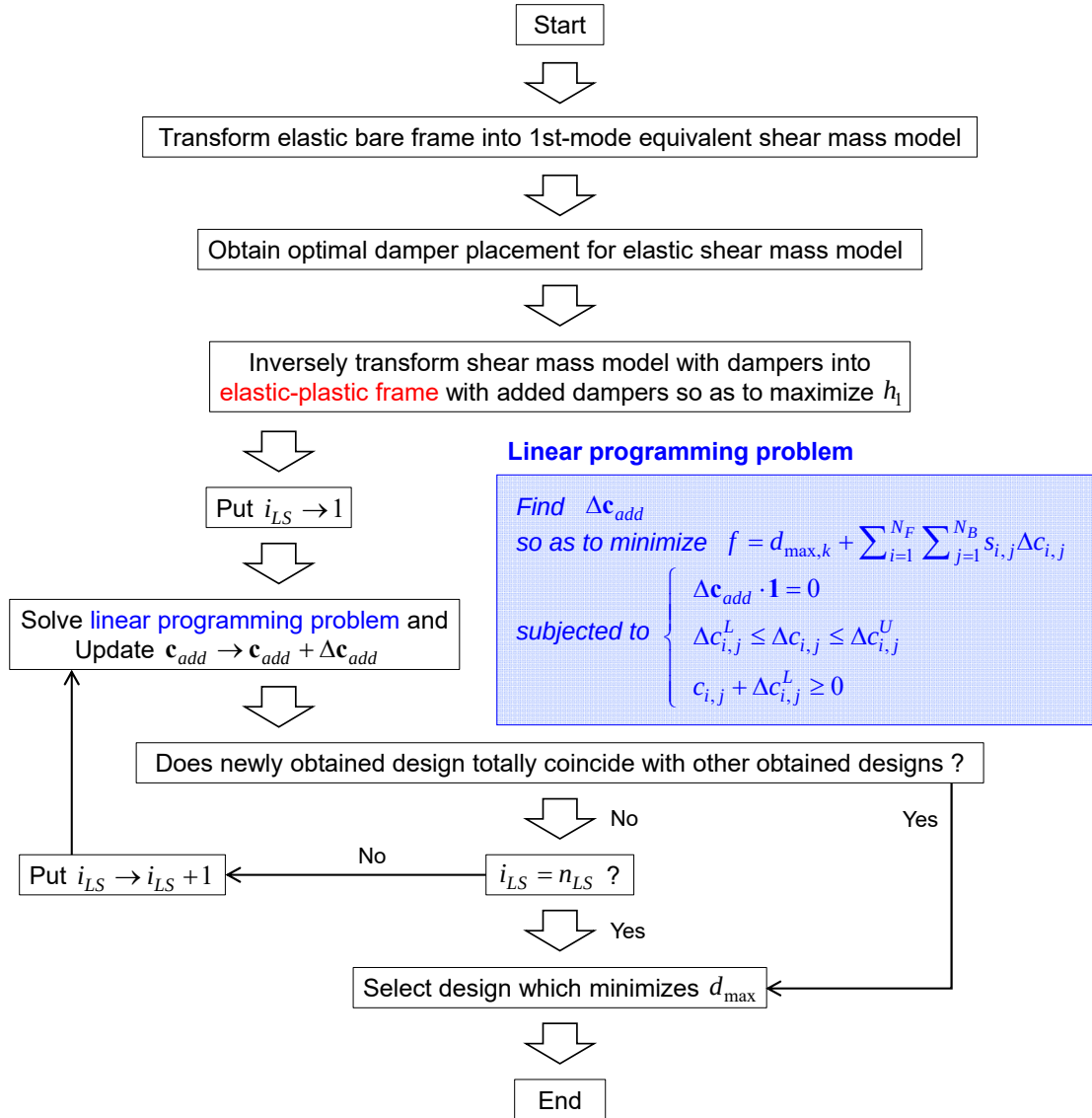


Fig. 6 Schematic diagram of proposed method

3. Numerical examples

In this section, the effectiveness of the proposed method is investigated through numerical examples. El Centro NS component during the Imperial Valley earthquake (1940) is employed. The influence of the setting of the level of the ground motion on the damper placement is investigated in Section 3.2. Incremental Dynamic Analysis (IDA) is conducted for the optimally designed models to

investigate whether the models designed under the specified-level ground motions effectively reduce the responses under the ground motions scaled to the other levels in Section 3.3.

3.1 Building frame model

Consider a 10-story 3-bay steel moment-resisting frame. The common story height is 4 [m], and the common span length is 7 [m]. All the floor masses are 100×10^3 [kg]. In relation to the vertical inertial forces, $(100 / 6) \times 10^3$ [kg] is allocated to the top nodes of the corner columns, and $(100 / 3) \times 10^3$ [kg] is allocated to the top nodes of the interior columns. W21 $\times$ 201 and W21 $\times$ 182 are employed as the sections of the columns in the 1-5th stories and those in the 6-10th stories. W33 $\times$ 130 and W30 $\times$ 99 are employed as the sections of the beams in the 1-5th stories and those in the 6-10th stories. Table 2 shows the moments of inertia I_X , the cross-sectional area A , the plastic section modulus Z_X , and the yield stress σ_y of the member sections. The columns are designed to have the larger values of the plastic moment compared with those of the beams. The column bases in the 1st story are fixed. The joints are assumed to be designed strong enough so as not to yield. Young's modulus is set to 2.05×10^5 [N/mm²]. The undamped fundamental natural period is 1.29 [s], and the structural damping ratio is 0.02 (stiffness proportional type). The linear viscous dampers are treated and the dampers can be installed at all the bays in the all stories as shown in Figure 7.

The structural analysis software OpenSees is used to conduct the time-history response analysis for the elastic-plastic frame [46]. The P-delta effect of the columns and the corotational formulation of the beams are taken into account as the geometric nonlinearity. The material Steel01 with the strain hardening ratio 0.01 is applied to all the beams and the columns. The flanges of the H-shaped cross sections are modeled by 6 $\times$ 1 fibers, and the webs are also modeled by 6 $\times$ 1 fibers.

In this paper, infills in frame structures are not considered for simplicity. However, they have a key role in the distribution of the stresses, in the fundamental period, in the interstory drifts [48].

Table 2 Cross-section of beam and column

	$I_x [\text{mm}^4]$	$A[\text{mm}^2]$	$Z_x [\text{mm}^3]$	$\sigma_y [\text{N/mm}^2]$
W21×201	2.21×10^9	3.82×10^4	8.69×10^6	320
W21×182	1.97×10^9	3.46×10^4	7.80×10^6	320
W33×130	2.79×10^9	2.47×10^4	7.65×10^6	240
W30×99	1.66×10^9	1.88×10^4	5.11×10^6	240

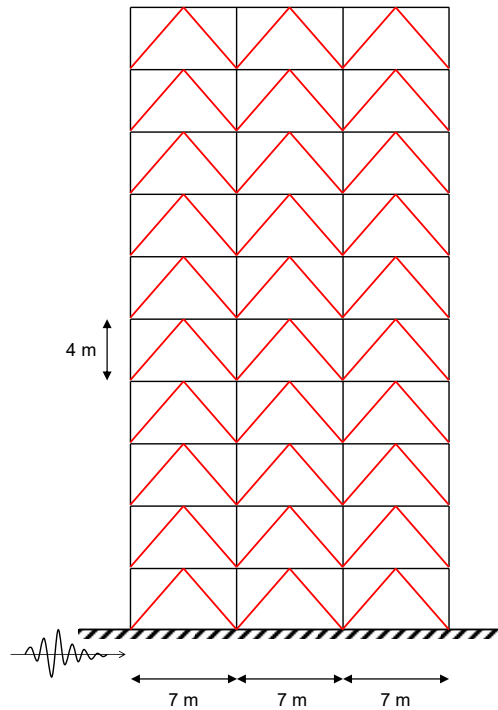


Fig. 7 Elastic-plastic moment frame

3.2 Optimization results

Figure 8a illustrates the distribution of inversely transformed added dampers for the frame structure. As stated in Section 2.4, the sensitivity-based algorithm has been applied to the optimization for the

equivalent shear mass system. In the application of the sensitivity-based algorithm, the added damping coefficient 3×10^7 [Ns/m] is given to each story at first. Then, the small damping coefficient 1×10^6 [Ns/m] is sequentially removed until the value of $h_{add,1}$ becomes 0.1. All the added dampers are allocated to the center bays through the inverse transformation because the constraints on the upper bound of $c_{i,j}$ are not taken into account. These procedures took about 300 [s] (a PC whose clock frequency is 3.60 [GHz] was used without parallel computing). Figure 8b shows the distributions of the maximum interstory drifts of the shear mass system and the moment frame with the inversely transformed added dampers. The distributions of the maximum interstory drifts of the moment frame without dampers are also plotted. It can be observed from Figure 8b that the distribution of the maximum interstory drifts of the shear mass system almost coincides with that of the moment frame. Moreover, the dampers effectively reduce the maximum interstory drifts. In other words, all the interstory drifts are almost uniform except for the stories without dampers, and those stories' drifts are smaller than the interstory drifts of the stories with dampers. It is noted that the dampers are not allocated to the 5th story since the member sections switch to the smaller ones beyond the 6th story and the stiffness of the 5th story is relatively large. It is also noted that the stiffness of the 1st story is large since the column bases in the 1st story are fixed.

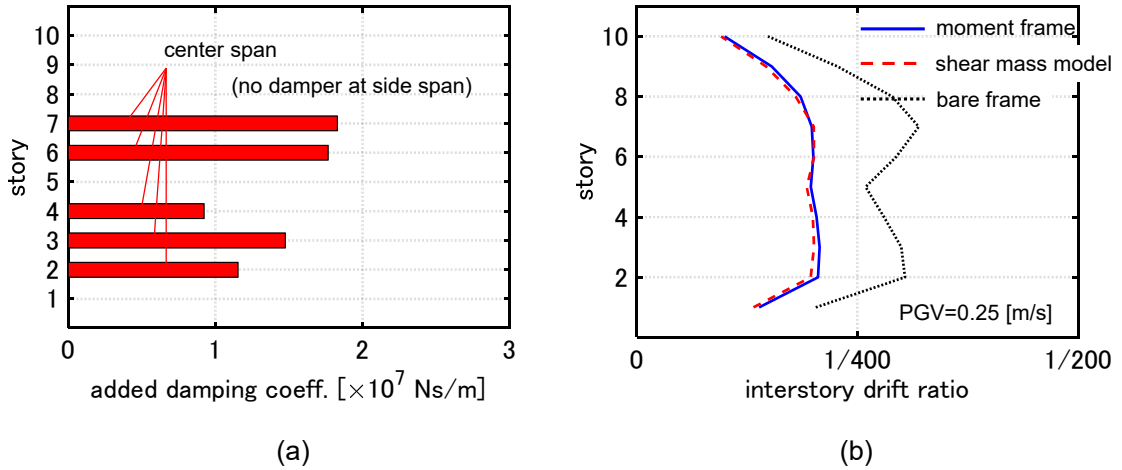


Fig. 8 Distribution of added damping coefficients and distribution of maximum interstory drift ratios of moment frame with inversely transformed added dampers, (a) Distribution of added damping coefficients, (b) Distribution of maximum interstory drift ratios.

Figure 9 shows the distribution of added damping coefficients and the distribution of the maximum interstory drifts of the moment frame optimally designed for $PGV = 1.0$ [m/s] ($PGA = 8.8$ [m/s²]), and Figure 10 shows those of the moment frame optimally designed for $PGV = 1.5$ [m/s] ($PGA = 13.3$ [m/s²]). The distributions of the interstory drifts of the bare moment frame are not plotted in Figures 9, 10 because they are extremely large. The distribution of damping coefficients shown in Figure 8 has been adopted as an initial design for the local search (the procedure of Step 4 in the proposed algorithm). $\Delta c_{i,j}^U = (1/\cos^2 \phi) \times 0.5 \times 10^6$ [Ns/m] and $\Delta c_{i,j}^L = \max\{-c_{i,j}, -\Delta c_{i,j}^U\}$ are adopted for the parameters in the linear programming problem included in the proposed method, where $\phi (= \arctan\{4/3.5\})$ is the angle of the dampers. It is noted that the dampers are allocated symmetrically, and 20-time response analyses for the elastic-plastic moment frame are required to calculate all $s_{i,j}$ introduced in Section 2.4. In the case of $PGV = 1.0$ [m/s], the design obtained after solving the linear programming problem 7 times ($i_{LS} = 7$) totally coincides with the earlier obtained design. Then, the process is finalized. In the case of $PGV = 1.5$ [m/s], the design obtained after

solving the linear programming problem 12 times ($i_{LS} = 12$) totally coincides with the earlier obtained design. Then, the process is finalized. In other words, $140 (= 7 \times 20)$ -time response analyses are required for the elastic-plastic moment frame in the case of $PGV = 1.0$ [m/s], and it took about 1600 [s]. In the case of $PGV = 1.5$ [m/s], $240 (= 12 \times 20)$ -time response analyses are required and it took about 2700 [s]. It can be observed from Figures 9, 10 that the proposed method effectively reduces the maximum interstory drifts. Namely, all the interstory drifts are almost uniform except for the stories without dampers, and those stories' drifts are smaller than the interstory drifts of the stories with dampers. It can also be observed that the increase of the design PGV gives more damping to both of the center and side bays in the lower stories. Figure 11 illustrates the location of plastic hinges. In the case of $PGV = 1.0$ [m/s], only the beams have plastic hinges and the columns do not have. In the case of $PGV = 1.5$ [m/s], only the beams have plastic hinges except for the column bases in the 1st story. In the case of $PGV = 1.0$ [m/s], the maximum rotational ductility factor among all the beams is about 2.5 and those in the 1-7th stories are larger than 2.1. In the case of $PGV = 1.5$ [m/s], the maximum value of those is about 4.3 and those in the 1-6th stories are larger than 4. In both cases, the rotational ductility factor of the beam in the side bay tends to be larger than that of the beam in the center bay on the same floor. Although the rotational ductility factors of the beams are not treated as the objective function in this paper, the proposed method can also control them by changing the sum of the added damping coefficients.

From all the above, it can be said that the proposed method efficiently provides damper designs effective for large-amplitude ground motions.

In the present numerical examples, only one recorded ground motion has been used for the simple presentation of the proposed method. It may be straightforward to use multiple recorded ground motions and employ the envelope response of the maximum interstory drifts as the target response.

Only computational load will be the principal issue to be resolved. When treating multiple recorded ground motions for the optimization, the use of the ‘active earthquake’ [29, 33], which maximizes the structural response among all the selected motions, will greatly decrease the total computational load.

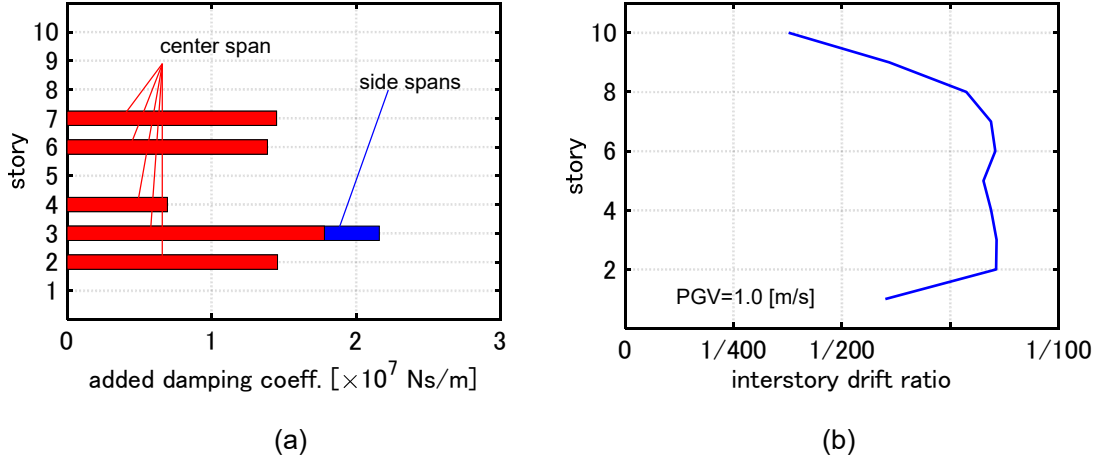


Fig. 9 Distribution of added damping coefficients and distribution of maximum interstory drift ratios (design PGV = 1.0 [m/s]), (a) Distribution of added damping coefficients, (b) Distribution of maximum interstory drift ratios.

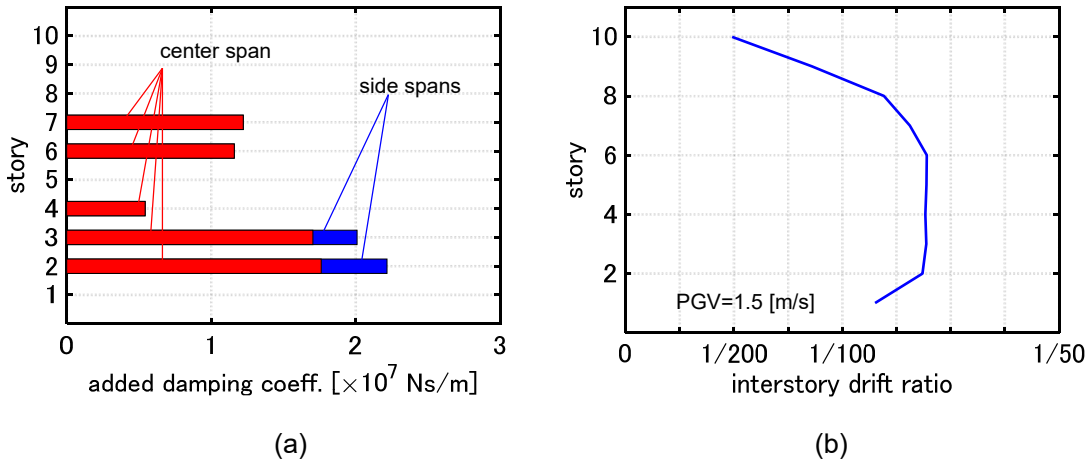


Fig. 10 Distribution of added damping coefficients and distribution of maximum interstory drift ratios (design PGV = 1.5 [m/s]), (a) Distribution of added damping coefficients, (b) Distribution of maximum interstory drift ratios.

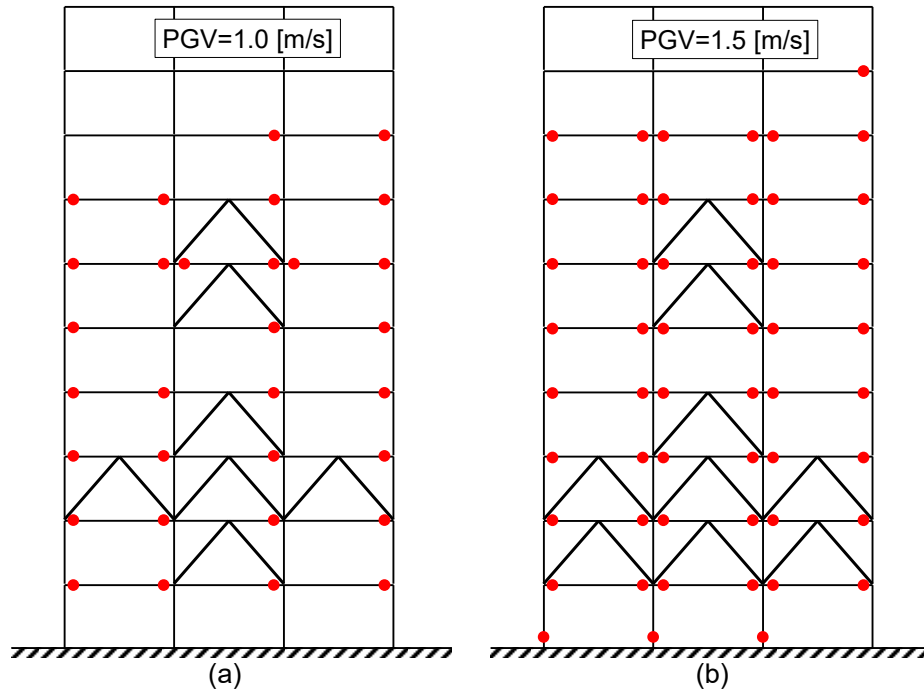


Fig. 11 Location of plastic hinges, (a) Design PGV = 1.0 [m/s], (b) Design PGV = 1.5 [m/s]

3.3 Incremental dynamic analysis and ideal drift response curve

Figure 12 represents the results of the IDA analysis for the 3 models shown in Section 3.2 (Figures 8-10). PGV in the IDA analyses is increased from 0.25 [m/s] to 1.5 [m/s] by 0.05 [m/s]. The distributions of the maximum interstory drifts are plotted for PGV = 0.25, 0.50, ..., 1.5 [m/s] in Figure 12. It can be observed that the frame with the inversely transformed dampers shows the remarkable plastic deformation concentration in the lower stories for the large-amplitude inputs. The frame optimally designed for PGV = 1.0 [m/s] also shows the remarkable plastic deformation concentration in the lower stories for the large-amplitude inputs. On the other hand, the frame optimally designed for PGV = 1.5 [m/s] effectively reduces the maximum interstory drifts for various levels of the ground motions.

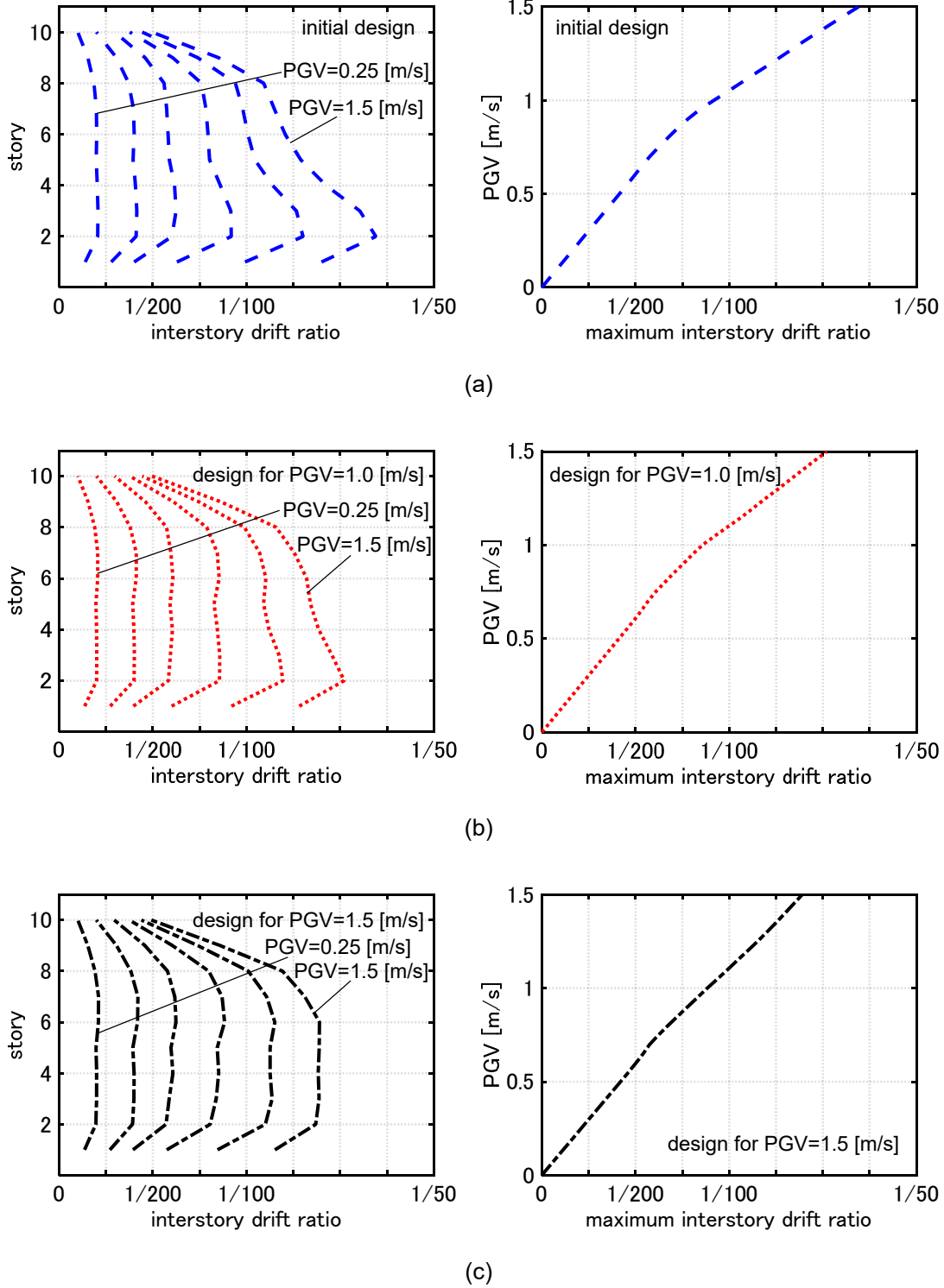


Fig. 12 IDA analysis, (a) Initial design (moment frame with inversely transformed dampers), (b) Model designed under GM with PGV = 1.0 [m/s], (c) Model designed under GM with PGV = 1.5 [m/s]

Figure 13 indicates the IDA curves and the ideal drift response curve (IDRC). The IDRC represents the ideal performance of the drift response under the constraint on the sum of added damping coefficients [45, 49]. It is noted that, since the sum of added damping coefficients is almost proportional to the total cost of added dampers, the constraint on the sum of added damping coefficients is almost equivalent to the constraint on the total cost of added dampers. In addition, the IDRC is the envelope of the IDA curves of the models optimally designed for various levels of ground motions under the same constraints on the sum of added damping coefficients. Therefore, the damper designs whose IDA curves run near the IDRC for various levels of ground motions are desirable.

The IDRC of the moment frame shown in Figure 13 has been obtained through the repetition of the small increase of PGV and the local search (Step 4 in the proposed method). The detailed procedure is explained as follows. First, it has been confirmed that the inversely transformed dampers are almost optimum for $PGV \leq 0.40$ [m/s]. Then, by setting the inversely transformed dampers as an initial design, the optimal design for $PGV = 0.45$ [m/s] has been obtained by the local search. The optimal design for $PGV = 0.50$ [m/s] has been obtained by the local search by setting the optimal design for $PGV = 0.45$ [m/s] as an initial design. The same procedure has been repeated to obtain all the optimal designs for various levels of the ground motions. Since the optimized maximum deformations for all the levels of the ground motions are simultaneously obtained, the IDRC can also be obtained by plotting all the optimized maximum deformations versus the design PGVs.

It can be observed from Figure 13 that the IDA curve for the frame with the inversely transformed dampers and that for the frame optimally designed for $PGV = 1.0$ [m/s] run away from the IDRC in the upper range of PGV. On the other hand, the IDA curve for the frame optimally designed for $PGV = 1.5$ [m/s] almost coincides with the IDRC although the IDA curve is plotted slightly away from the

IDRC in the range of $0.5 \leq \text{PGV} \leq 1.0$ [m/s]. It may be concluded that using the large-amplitude ground motions for the damper optimization is important because this leads to the design effective for various levels of the ground motions.

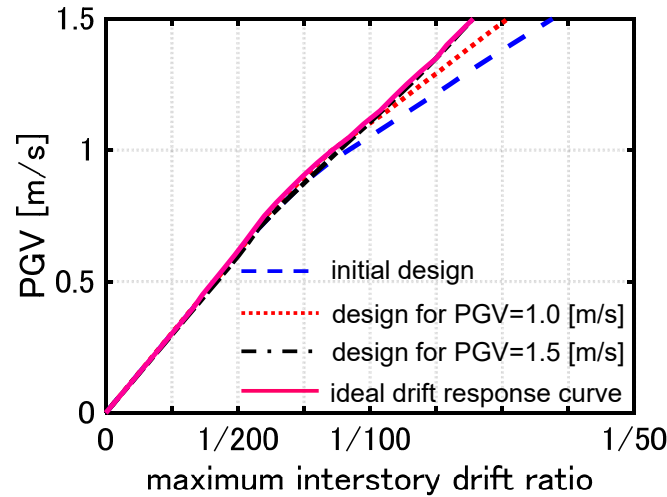


Fig. 13 IDA curves for 3 models and ideal drift response curve.

4. Conclusions

A new optimal damper design method was presented for elastic-plastic planar frame structures under large-amplitude ground motions. The main conclusions can be summarized as follows.

- (1) The proposed method effectively uses the transformation of a moment frame into the 1st-mode equivalent shear mass system to obtain the optimal damper placement for the elastic-plastic moment frame. In the optimization procedure, a linear elastic bare frame is transformed into the 1st-mode equivalent shear mass system first. Then, the optimization of damper placement **along the building height** is conducted for the elastic shear mass system. After that, the damped shear mass system is inversely transformed into the elastic-plastic frame with the added dampers. This inversely transformed damper system in the moment frame is employed as an initial design and the local search-based optimization is conducted for the efficient distribution of dampers **not only along**

the building height, but also among the different bay. It was demonstrated through numerical examples that the proposed method efficiently provides the damper designs with high accuracy.

(2) An innovative inverse transformation method of dampers was proposed. The added damping coefficient of the shear mass system is inversely transformed into the added dampers of the moment frame based on the added damping coefficient in the horizontal direction. Especially in the optimization procedure, the inverse transformation is conducted so as to maximize the approximate value $\bar{h}_{add,1}$ of the fundamental damping ratio by the dampers.

(3) The distribution of the maximum interstory drifts of the damped shear mass system almost coincides with that of the moment frame with the inversely transformed dampers. On the other hand, the distribution of the maximum floor accelerations of the shear mass model does not correspond to that of the moment frame when higher-mode responses are not negligible. This is because the higher modes of the frame do not coincide with those of the shear mass system, even though the fundamental natural mode of the frame almost coincides with that of the shear mass system.

(4) The incremental dynamic analysis (IDA) was conducted for the optimally designed models to investigate whether the models designed under the specified-level ground motions effectively reduce the responses under the ground motions scaled to the other levels. The models optimally designed under the small-amplitude ground motions tend to show the remarkable plastic deformation concentration in the specific stories for the large-amplitude ground motions. On the other hand, the models optimally designed under the large-amplitude ground motions effectively reduces the maximum interstory drifts for various levels of the ground motions. In other words, all the interstory drifts are almost uniform except for the stories without dampers and those stories'

drifts are smaller than the interstory drifts of the stories with dampers. Therefore, using the large-amplitude ground motions for the damper optimization is important.

In this paper, elastic-plastic moment-resisting frames and viscous dampers without supporting braces were treated. The applicability of the proposed design method to other type of structures and dampers should be further investigated. However, by adding some modification, the proposed design method will work well for other type of structures and dampers because the basic concept of the proposed method consists simply of the generation of the initial damper design by the optimization for elastic structures and the improvement of the design for elastic-plastic responses by local search-based optimization.

5. Acknowledgement

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Appendix

The linear programming problem included in the proposed algorithm is explained in this appendix (see Figure A1).

$M(= N_F \times N_B)$ denotes the degrees of the $\Delta \mathbf{c}_{add}$ here. Consider the end points of the feasible region. When the inner product of $-(\partial / \partial \mathbf{c}_{add}) f$ and the vector towards the end point is maximum, that point is chosen as $\Delta \mathbf{c}_{add}$ because the objective function $f = d_{\max, k} + \sum_{i=1}^{N_F} \sum_{j=1}^{N_B} s_{i,j} \Delta c_{i,j}$ is minimized.

Since the constraint $\Delta \mathbf{c}_{add}^T \cdot \mathbf{1} = 0$ is kept in the problem, the inner product of the vector towards the end point and the following vector becomes also maximum

$$-\mathbf{P} \begin{pmatrix} \frac{\partial f}{\partial c_1} & \cdots & \frac{\partial f}{\partial c_M} \end{pmatrix}^T, \quad (A1)$$

where

$$\mathbf{P} = (\mathbf{i}_1 \cdots \mathbf{i}_{M-1})(\mathbf{i}_1 \cdots \mathbf{i}_{M-1})^T = \frac{M-1}{M} \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 \end{pmatrix} - \frac{1}{M} \begin{pmatrix} 0 & 1 & \cdots & 1 \\ 1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 1 \\ 1 & \cdots & 1 & 0 \end{pmatrix}, \quad (A2)$$

In Eq.(A2), $\mathbf{i}_1, \dots, \mathbf{i}_{M-1}$ is an orthogonal set of the hyperplane $\Delta \mathbf{c}_{add}^T \cdot \mathbf{1} = 0$, and $\mathbf{P}$ denotes the projection matrix. An example of $\mathbf{i}_1, \dots, \mathbf{i}_{M-1}$ is expressed as follows.

$$\mathbf{i}_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \mathbf{i}_2 = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ 1 \\ -2 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \mathbf{i}_3 = \frac{1}{\sqrt{12}} \begin{pmatrix} 1 \\ 1 \\ 1 \\ -3 \\ 0 \\ 0 \end{pmatrix}, \dots, \mathbf{i}_{M-1} = \frac{1}{\sqrt{M(M-1)}} \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ 1 \\ -(M-1) \end{pmatrix} \quad (A3)$$

The simplex method and the interior-point method are well-known to solve linear programming problem, and they have been extended to advanced versions [50].

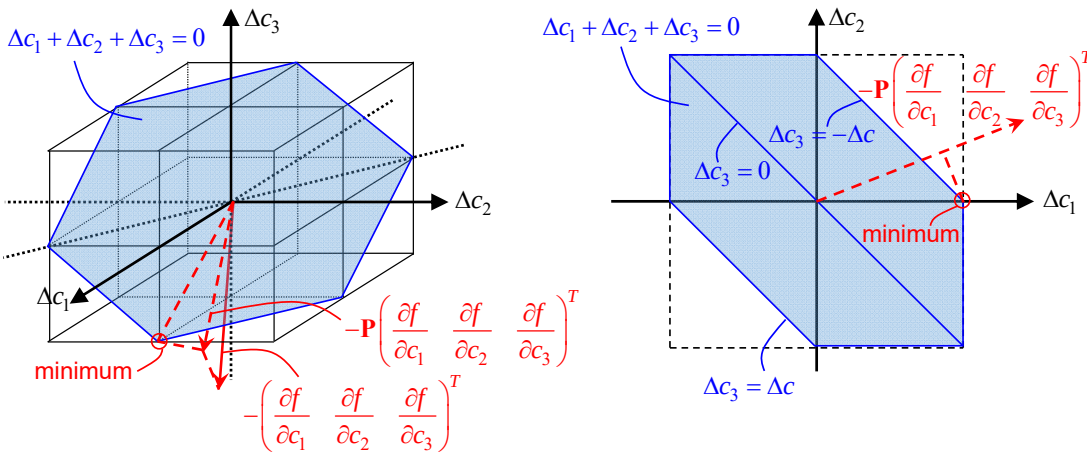


Fig. A1 Overview of linear programming problem included in proposed method ($M = 3$)