

多自由度における確率共鳴と
その信号処理への応用

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(3) 出版物

なし

研究成果による工業所有権の出願・取得状況

なし

研究成果

(1) 本研究の背景

Stochastic Resonance(SR) の最初の研究は R.Benzi らによってなされた。約十万年周期で氷河期と間氷期を周期的に繰り返す地球の気候の変動についての研究である。このような気候の変化の強い周期性は、地球の軌道がごく小さく周期的に変化していることだけでは説明できなかった。彼らは二つの安定な定常状態をもつ “climatic potential” を考えることによって、軌道のごく小さな周期的変動 (signal) とランダムな揺らぎ (noise) との協同現象が、観測されるような気候変動の強い周期性を説明し得るかもしれないということを示唆した。

SR は、二状態の系に対して coherent-signal(周期性を持っている) と random noise の二つの入力があるとき、系の内部の dynamics によって決まる出力にあらわれる一種の共鳴現象である。ある系を “black box” として見たとき、入力の signal と noise を変化させてやると出力はどのように変化するであろうか。線形応答理論によると、線形な dynamics

を持つような系においては、出力の signal-to-noise ratio(SN 比) は入力 SN 比に等しい。つまり入力 noise の大きさを増していけば出力 SN 比は減少していくことになる。SR においては、これとは対照的に、入力 noise の増加に伴って出力 SN 比が改善され得るという特徴が挙げられる。したがって SR は本質的に非線形現象であるといえる。SR においては、パラメタ (この場合は入力 noise の大きさ) を signal とうまく協調するような値 (二状態間の平均遷移時間が signal の周期に近くなる noise の大きさ) にとることによって SN 比を最大化できることを意味している。二状態間の遷移周期と signal の周期とが一種の共鳴を起こして、粒子の運動が signal に同期するようになって SN 比が改善されるのである。この意味で我々はこのメカニズムを Stochastic “Resonance” と呼んでいる。この現象で特筆すべきことは周期的な信号の大きさは非常に小さくとも SR は起こることである。その後、生体系、レーザー、磁性体、神経系などで SR 現象が実験的に発見され、計算機シミュレーションによっても確かめられた。理論的にも多くの研究者が取り組みその機構が明らかにされつつある。また、最近では SN 比が極大になることを利用して信号処理に対する応用の試みもなされている。

これまで、主に 1 自由度の系に置ける SR が研究されてきたが、本研究に置いては SR を起こす系を相互に結合させ、SR の性質が非結合系に対してのように改善されるかを調べる。また、入力信号として従来の周期的な信号だけではなくて非周期的な一般の信号に対して状態遷移頻度と信号の相関が適当なノイズの大きさに対して極大となり SR 現象を起こすことを示した。また、信号処理の性能を表す指標の 1 つである ROC(Receiver Operating Characteristics) を計算し、確率共鳴の信号処理への応用の可能性を示した。次に、パルスニューロンモデルを結合させた系にも SR 現象が認められることを示し、さらにヘブ則を用いてパターンを記憶させ想起を起こさせると適当なノイズに対して想起しやすくなるという SR 現象が発見された。さらにマップを用いたニューロンモデルを結合して同様に SR 現象を調べ、想起に関する性質を調べた。さらに、カオス系による出力によるノイズの生成について考察し、確率共鳴現象に関係する分子モーター系におけるノイズの影響を調べた。

(2) 本研究での研究成果

(i) 研究成果の概要

(a) 結合したダブルウエル系

確率共鳴を起こす系としてよく研究されてきたダブルウエルポテンシャルで記述される系に着目し、それらが 1 次元格子状に結合し自由度間の相互作用が線形のパネで表される系を考える。この系に入力信号を加え出力の SN 比を計算すると単独の場合と同様 SN 比が最適になるノイズの強さが存在し、それは結合の強さによって変化することがわかった。しかも、SN 比がもっとも大きくなる最適なノイズの大きさと結合の強さの組み合わせが存在することがわかった。また、その最適な SN 比は各自由度を結合しない場合よりも大きく向上した。また、SN 比が極大になるパラメーターの値を選んだ場合、入力信号と系の出力信号はよく同期していることが示された。

次に、この系の信号処理の性能を示す 1 つの指標である、ROC(Receiver Operating Char-

acteristics) を求めた。ROC とは、縦軸に Probability of detection (PD)、すなわち実際に信号が入力している時に出力を調べて信号が検出される確率。横軸に Probability of false alarm (PFA)、すなわち実際には信号が入力していないにもかかわらず、出力から信号が検出されてしまう確率をとったグラフである。もちろん、PD は 1 に近いことが望ましく、PFA は 0 に近いことが望ましい。PD を PFA の関数として書いた ROC をみてこの系の信号処理の性能を知ることができる。すなわち ROC のグラフが右上の方に寄っているほど、信号処理系としては望ましい性質を示すこととなる。結合系において、ROC を求めたところ SN 比が極大になる、結合の強さとノイズの大きさの組み合わせの時もっとも望ましいグラフとなり、パラメーターの値がそれからずれると右上の位置から ROC のグラフが離れていく。また結合していない系の SN 比が最大の場合と較べると、結合させた方が大幅に ROC のグラフが右上に近づき信号処理の能力が改善されたことを示した。

(b) 結合したパルスニューロン系の確率共鳴と想起

ニューロンの活動を表すモデルとして生理学的な実験から求められたホジキン・ハックスレイモデルを単純化したフィッツヒュー・南雲モデル (FN) を用い、それらがグローバルに結合している系、すなわちニューラルネットワークをモデル化した系を考える。すなわち、 i 番目のニューロンは 2 変数 u_i, w_i で記述され

$$\begin{aligned} \epsilon \dot{v}_i &= v_i(v_i - a)(1 - v_i) - w_i + A + I_i(t) + \eta_i(t) \\ &\quad + \frac{\gamma}{N-1} \sum_{j=1}^N W_{ij} u(t - T_j) \\ \dot{w}_i &= v_i - w_i - b \\ (i, j &= 1, \dots, N) \end{aligned} \quad (1)$$

ϵ, a, A, b は定数、 $I_i(t)$ は i 番目のニューロンへの入力、 $\eta_i(t)$ はそれへのノイズである。 N 個のニューロンは結合定数 γW_{ij} で結合しており、 j 番目のニューロンからの影響はそのニューロンが発火したとき (時刻 T_j) 以降に $u(t - T_j)$ という関数の形で受けることとした。 $u(t - T_j)$ はニューロンの発火したパルス状の信号がニューロン同士の結合を通して伝えられるとき、幅ができ、遅れも生じてパルスが伝えられる様子を表したものである。具体的には

$$\begin{cases} u(t) = 0 & (t < 0) \\ u(t) = (t/t_r) \exp(-(t - t_r)/t_r) & (0 \leq t \leq t_r) \\ u(t) = ((t + t_f - t_r)/t_f) \exp(-(t - t_r)/t_f) & (t > t_r) \end{cases} \quad (2)$$

と表せる。ここで t_r, t_f は定数であって t_f が広がったパルスの幅、 t_r が遅れを表す。最初に、結合定数 W_{ij} がすべて同一の場合を考え、計算機シミュレーションを行なって確率共鳴を調べた。特に信号が単一の周波数である場合だけでなく、一般的な場合に関しても系の出力である発火頻度と信号との相関係数および両者の相互情報量を調べ従来の SN 比で現れたと同様の確率共鳴現象 (最適なノイズの大きさに対して極大となる。) が認められた。

次に系の結合定数 W_{ij} をヘブ則によってきめることでこの系に記憶を埋め込み連想記憶の性能を調べた。すなわち入力として想起したいパターンに近い入力を与え、出力がそ

のパターンと同様の発火状態を示す確率を調べたところ、適当な大きさのノイズが加わった時、その確率がもっとも大きくなり、想起がもっとも良く起るという、確率共鳴的な性質があることがわかった。

(c) 結合したマップで表されるニューロン系の確率共鳴と想起

ニューロンのモデルとして (b) で取り上げた、微分方程式で記述されるモデルは、生理学的な知見を直接にもっとも反映したモデルであるが、多数のニューロンを結合した場合、計算量が膨大となり、取り扱いがたいへん難しくなる。そこで、モデルを単純化し、より取り扱いやすい、マップで表されるモデルを採用することとする。しかしながら、ホップフィールドモデルのように発火の有無を 2 値に置き変えて単純化されたものとは違う、内部状態を連続的に表す変数で記述されるモデルを採用する。すなわち、

$$f(x) = \begin{cases} 0 & \text{for } x < -1 \\ ax & \text{for } -1 \leq x < 0.2 \\ bx - 0.1 & \text{for } 0.2 \leq x < 0.85 \\ c(x - 1) & \text{for } 0.85 \leq x \end{cases} \quad (3)$$

(ただし、 $a = 0.5$ 、 $b = 1.5$ 、 $c = 0.04$ とする。) で定義される、関数 $f(x)$ を用い、 i 番目のニューロンの時刻 t での状態を表す変数 $x_t(i)$ が

$$x_{t+1}(i) = f \left(x_t(i) + \sum_j w_{ij} x_{t-\tau}(j) + y(i) \right), \quad (4)$$

というマップで時間発展するとした。ここで、 w_{ij} は i 番目と j 番目のニューロン同士の結合定数。 τ はニューロン同士で伝わる発火の信号の遅れを示す。

この系に対して、最初に w_{ij} がすべて同じ場合に入力信号に対する確率共鳴の性質を、シミュレーションにより詳しく調べた。また、ヘブ則によって w_{ij} を決め、パターンを記憶させて連想記憶の性能を調べたところ HN モデルと同様、ノイズの大きさを適当にとることで想起が起りやすくなることがわかった。すなわち、想起の確率が大きくなった。またこの系の記憶容量についても調べることができた。

(ii) 印刷物

Transport properties and efficiency of elastically coupled Brownian motors

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As models for biological molecular motors, Brownian motors have been studied recently by many workers, and their physical properties such as velocity, efficiency, and so on, have been investigated. They have also attracted much interest in an application to nanoscale technology. It is significant to study more complex systems, that is, coupled Brownian motors, in detail, since Brownian motors with a single particle have been mainly studied until now. In this paper, we consider Brownian motors coupled mutually with elastic springs, and investigate the dynamics of the model and the efficiency of energy conversion. In particular, we find that the center of the mass of the elastically coupled particles moves faster than the corresponding single-particle model, and also that the efficiency of the coupled-particle model is larger than that of the single-particle model.

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I. INTRODUCTION

Brownian motors have attracted much attention as models of molecular motors [1] and their physical properties such as velocity, efficiency, and so on have been studied in detail [2]. Recently, some workers have also paid attention to them in an application to nanoscale technology [3]. For example, Porto *et al.* [3] studied microscopic engines on the atomic scale that transform the fed energy into directed motion through a dynamical competition between the intrinsic length of the moving object and supporting carrier.

The molecular motors are involved in cell locomotion, some cellular transport and muscle contraction, and so forth. Many models have been proposed to comprehend theoretically the mechanism of the molecular motors. Doering *et al.* [4] investigated "single-particle" Brownian motor models, a "rocking" ratchet model where a periodic or random external force is applied to the system. A famous one among the single-particle models is a so-called "flashing" ratchet model [5]. It is shown that in the model, only thermal noise and a proper asymmetric potential are enough to produce macroscopic motion of the particle toward a particular direction that depends on the asymmetry of the potential. Next, "coupled-particle" models, where particles interact mutually, have been investigated. Csahók *et al.* [6] studied the dynamics of elastically coupled particles in a "rocking" ratchet model. On the other hand, Jülicher *et al.* [2] introduced and analyzed theoretically particles rigidly attached to a rigid backbone with equal spacing in a "flashing" ratchet. Recently, Elston and Peskin [7] investigated the characteristic of the elasticity between the motor and its cargo and showed that the elasticity allowed the motor to run faster than if they were linked rigidly. Klumpp *et al.* [8] studied the two harmonically coupled particles in the ratchet model and observed a driving mechanism different from the one in the case of a single particle, which does not need diffusion. As for the response to an external force, Reimann *et al.* [9] introduced another model of interacting Brownian particles and found some collective phenomena.

Studying the physical properties of coupled Brownian motors is interesting in itself. Moreover, it is quite significant

to investigate the coupled systems, since in an application to nanoscale technology, coupled effects should be taken into account [3]. In an application to molecular motors, it is also interesting to study coupling effects, because, for example, molecular motors in muscle have a linear structure [1] that consists of many parts.

In this paper, we consider elastically coupled particles in a flashing ratchet model, in which each particle makes transition repeatedly between two states where interactions are expressed by their respective kinds of potential, and conduct numerical simulation. Unidirectional motion of the particles is confirmed in this model and the velocity for various values of the coupling constant, the temperature, and so on, is measured. Although an isolated single particle in flashing ratchet models cannot move in nonthermal conditions, that is, without thermal noise, elastically coupled particles in the flashing ratchet model may move due to the action of the interparticle springs restored to their natural length that is incommensurable with the period of the periodic potential, even if thermal diffusion is not allowed. It is also found that the velocity of the elastically coupled model under such conditions is larger than that of the corresponding single-particle model, and that the velocity has a maximum as a function of the coupling constant of the springs. Moreover, if we restrict the region where transition may be allowed, we find that the velocity of the model is enhanced by the restriction. We also apply various loads externally to our system to investigate the efficiency of energy conversion, and find that the efficiency has a peak as a function of the load. We also find that the peak values are larger when we restrict the region of transitions and are improved by the coupling effect in comparison with the single-particle model.

II. THE MODEL

We consider elastically coupled particles (Fig. 1). In this paper, our model is described by dimensionless quantity. It is assumed that the particles are put in a heat bath represented by white noise. Particles are subjected to one of the two substrate potential stochastically. $W_j(x)$ ($j=1,2$) defines the potential in state j at point x . W_1 is a flat potential and we

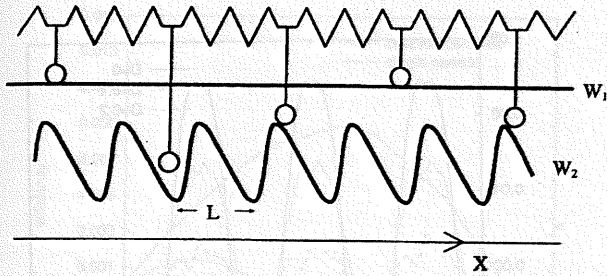


FIG. 1. In our simulation, the period of the potential is not equal to the natural length of the springs. This "incommensuration" leads to the easy movement of the particles to a particular direction (to the right direction in this figure). Some particles are subjected to W_1 and the others to W_2 .

choose the following asymmetric interaction potential W_2 :

$$W_2 = \left[\frac{1}{2} \sin\left(\frac{2\pi x}{L}\right) + \frac{1}{8} \sin\left(\frac{4\pi x}{L}\right) \right] \times U, \quad (2.1)$$

where U and L represent the depth and period of the potential, respectively. In state one, no force from the substrate is exerted on the particles because the substrate potential W_1 is flat. Therefore, state one is called the detached state. Since, in state two, the particles feel periodic substrate potential, state two is called the attached state.

The equations of motion of the particles read

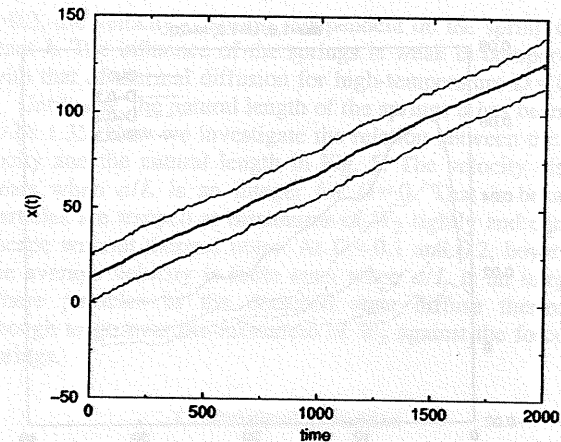


FIG. 2. x_1 , x_N , and the center of mass as a function of time. We find that finite temperature is not necessarily needed for the finite velocity of the elastically coupled particles in an asymmetric potential. This figure shows finite velocity for zero temperature ($k = 4.0, N = 20$).

$$\gamma \dot{x}_i = k(x_{i+1} - 2x_i + x_{i-1}) - h_i(t) \frac{\partial W_j(x_i)}{\partial x_i} + \sqrt{2D} \xi_i(t), \quad 2 \leq i \leq N-1, \quad (2.2)$$

where x_i denotes the position of the i th particle and we consider the overdamped case. $\xi_i(t)$ denotes white noise of zero

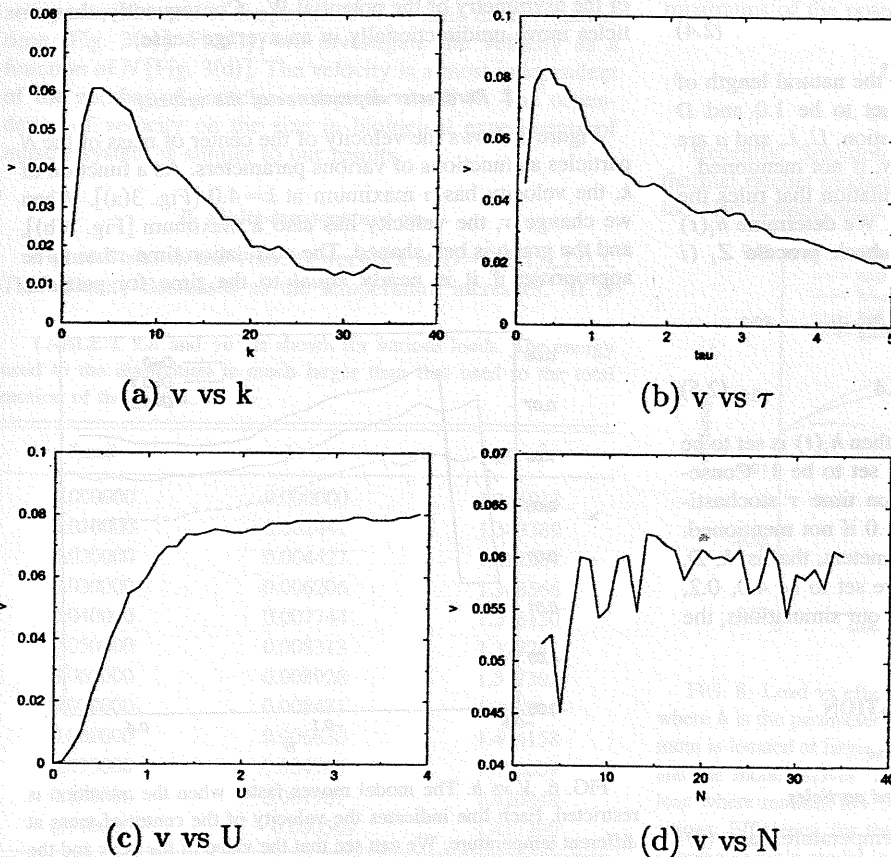


FIG. 3. Velocity as functions of various parameters. The velocity shows various dependence on the parameters. From these figures, we can understand the characteristics of the model more profoundly.

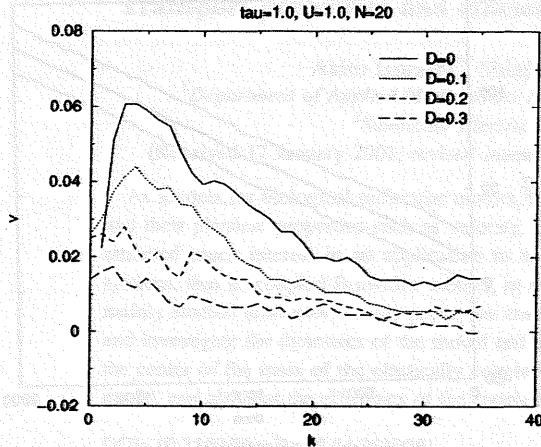


FIG. 4. V vs k . The lower the temperature is, the faster the particles move in general. For high temperature, the peak almost disappears since the particles tend to move more randomly.

mean and correlation $\langle \xi_i(t) \xi_j(s) \rangle = \delta(t-s) \delta_{ij}$. k is the spring constant and N the number of particles. Since the boundary condition of the particles is free, the forces due to springs of Eq. (2.2), the first term of the right-hand side, is

$$k(x_2 - x_1 - a) \quad (2.3)$$

for the first particle and

$$k(x_{N-1} - x_N + a) \quad (2.4)$$

for the N th particle, where a stands for the natural length of the springs. A friction constant γ is set to be 1.0 and D stands for the temperature. In our simulation, U , L , and a are set to be 1.0, 1.0, and 1.35, respectively, if not mentioned.

$h_i(t)$ is a dichotomous random modulation that rules the time-dependent change expected 0 or 1. We determine $h_i(t)$ process as follows. An Ornstein-Uhlenbeck process Z_i ($i = 1, 2, \dots, N$), where

$$\begin{aligned} \langle Z_i(t) Z_j(s) \rangle &= \delta_{ij} (D' / \tau) e^{-|t-s|/\tau}, \\ \langle Z_i(t) \rangle &= 0, \quad D' = 0.4 \end{aligned} \quad (2.5)$$

is considered and if $Z_i(t)$ is less than 0, then $h_i(t)$ is set to be 0, if $Z_i(t)$ is more than 0, then $h_i(t)$ is set to be 1. Consequently, $h_i(t)$ changes with a correlation time τ stochastically. In our simulation, τ is set to be 1.0 if not mentioned. Thereby we can change only three parameters, that is, k , D , and N . If not mentioned, k , D , and N are set to be 4.0, 0.2, and 20, respectively. At the beginning of our simulations, the i th particle is always located at $x = ia$.

III. NUMERICAL SIMULATION

A. Zero-temperature case

1. Dynamics of elastically coupled particles

First of all, we investigate a zero-temperature case ($D = 0$). Figure 2 shows the motion of two boundary (first and

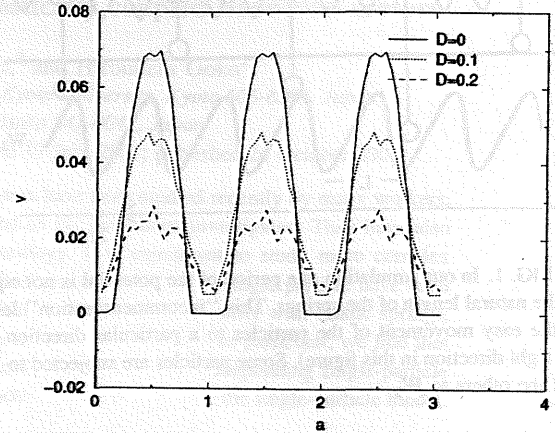


FIG. 5. V vs a . When the natural length of the spring is almost equal to an integer times the period of the potential, the model moves so slowly because each particle is caught to a minimum of the potential and cannot escape from it easily.

N th) particles (thin lines), and the center of mass of the coupled particles (thick line). A particle trapped to the potential W_2 is likely to be near a minimum of it and going to move by the elasticity when trapped to W_1 . When it is attached to the substrate again under the influence of W_2 , it goes down to a minimum of the potential W_2 to the $+x$ direction more frequently than to the $-x$ direction because of the asymmetry of the potential W_2 . Consequently, the particles move unidirectionally in an average sense.

2. Parameter dependence of the velocity

Figure 3 shows the velocity of the center of mass of the N particles as functions of various parameters. As a function of k , the velocity has a maximum at $k \approx 4.0$ [Fig. 3(a)]. When we change τ , the velocity has also a maximum [Fig. 3(b)], and the graph is bell shaped. The correlation time τ has to be appropriate if it is nearly equal to the time for particles

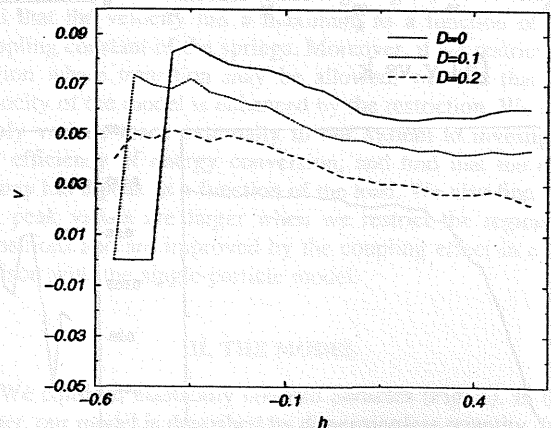


FIG. 6. V vs h . The model moves faster when the transition is restricted. Each line indicates the velocity of the center of mass at different temperature. We can see that the value of the peak and the position of the peak depend on D ($k=4.0, N=20$).

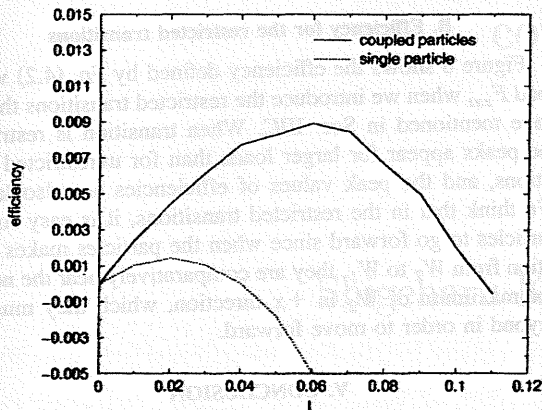


FIG. 7. Load vs efficiency at $D=0.1$ and $k=4.0$. The efficiency has a maximum at $F_{ext}=0.06$ for the coupled system, and it is less than 0 for $F_{ext} \geq 0.11$, which means that the model moves in $-x$ direction. Efficiency for a single particle is lower than for the coupled model.

enough to go beyond maximums of W_2 in the detached state and for the springs to go back to the natural length in the attached state in order to pull the particles forward before they are detached again. The larger the depth of the potential W_2 , U , becomes, the faster the particles go down toward the minimum of W_2 . Since the correlation time τ is finite, the velocity of the particles increases as the depth increases until the velocity saturates for large U such that particles probably arrive at minimums of the potential W_2 within the correlation time [Fig. 3(c)]. Finally, we investigate the velocity as a function of N [Fig. 3(d)]. The velocity is almost independent of the number of particles except for small N . The dependence of velocity on the size in biological experiments of muscle system is similar to our results.

B. Finite-temperature case

Figure 4 shows the velocity versus k at $D=0,0.1,0.2,0.3$. The velocity decreases as the temperature increases. At D

TABLE I. Lv and γv^2 is shown for various loads. The energy used to the dissipation is much larger than that used to the total motion of the model.

Load	$L \times v$	$\gamma \times v^2$
0.000000	0.000000	1.386913
0.010000	0.002441	1.383389
0.020000	0.004427	1.392557
0.030000	0.006206	1.388566
0.040000	0.007744	1.393150
0.050000	0.008313	1.397241
0.060000	0.008928	1.397302
0.070000	0.008471	1.398886
0.080000	0.006630	1.404158
0.090000	0.004963	1.412609
0.100000	0.001967	1.420848
0.110000	-0.000569	1.417309

$=0.3$, the velocity is almost independent on the spring constant k . The influence of the springs is weak in comparison with that of thermal diffusion for high-temperature $D=0.3$.

Until now, the natural length of the springs a has been set to be 1.35. Now we investigate the relation between the velocity and the natural length in Fig. 5. The velocity disappears when a/L is an integer for $D=0$. That is because particles are trapped at minimums of W_2 tightly and cannot escape without thermal noise. At $D=0.1$ and 0.2 , however, the average velocity is finite even when a/L is an integer, where particles in the detached state diffuse thermally enough to go over the maximum of W_2 against the force of springs.

C. Restricted transition

Until now, transitions between detached and attached states has been allowed to occur everywhere in the potential. Now we restrict the region where transition toward the detached state can be allowed. This restriction is defined by parameter h . That is, when a particle is located at x and if $W_2(x) < h$, transition may be allowed and if $W_2(x) \geq h$ it cannot be allowed. In our simulation, transitions from the detached state to the attached state may occur without any restriction of the positions of the particles. Figure 6 shows the velocity as a function of h . For a certain region of h , the velocity is larger than that of the unrestricted transition case, which is the right end of Fig. 6. One reason for this, we think, is that the possibility of the particles going over the maximum W_2 is larger when transition is restricted near minimums of the potential.

IV. EFFICIENCY

A. Calculation of efficiency

Recently, Sekimoto [10] has defined the efficiency for thermal ratchet models with a load F_{ext} . Derényi *et al.* [11] has also defined the efficiency in another way.

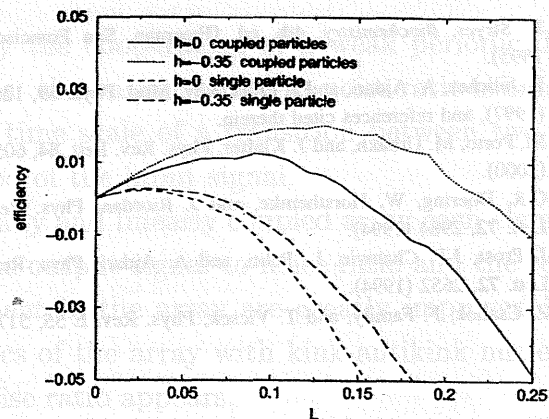


FIG. 8. Load vs efficiency at $D=0.1$, $k=4.0$, and $h=0, -0.35$ where h is the parameter to restrict the transition region. The maximum is located at larger F_{ext} than in the unrestricted case of Fig. 7, and the model moves $+x$ direction even for larger F_{ext} than the load where motions to $-x$ direction occurs in the unrestricted transitions. Efficiency for the single-particle model is lower than the coupled-particle model in this case as well as in the case of Fig. 7.

If we define v as an average velocity of the model, the conventional expression of the efficiency η is

$$\eta = \frac{F_{ext}v}{P_{in}}, \quad (4.1)$$

where P_{in} is the power input [10]. If we define P_{out} as the power output, it seems more appropriate to define it as

$$P_{out} = F_{ext}v + \gamma v^2, \quad (4.2)$$

rather than only $F_{ext}v$, the numerator of Eq. (4.2) [11]. The term γv^2 denotes, of course, the dissipation via friction. In our simulation, the load F_{ext} is dispersedly exerted to each particle, that is, $-F_{ext}/N$ is applied to each particle.

We show in Fig. 7 the efficiency defined by Eq. (4.2). It has a maximum as a function of F_{ext} and at a certain threshold it has a value less than 0, which means that the model moves in the $-x$ direction. The efficiency for a single particle is shown also in Fig. 7. The efficiency for the coupled model is higher than for the single-particle model.

In Table I, we compare $F_{ext}v$ with γv^2 . We find that γv^2 is always much larger than $F_{ext}v$ for any load, which makes sure that most of the energy of the molecular motor is used for the Brownian motion and therefore for the dissipation. This is a reason why the efficiency of the molecular motor is not so high.

Since muscle is known to have much higher efficiency than in our simulation, we do not succeed in reproducing the real situation if we apply our model to muscle contraction. This is mainly because of the simplicity of our model.

B. Efficiency for the restricted transitions

Figure 8 shows the efficiency defined by Eq. (4.2) versus load F_{ext} when we introduce the restricted transitions that we have mentioned in Sec. IIIC. When transition is restricted, the peaks appear for larger loads than for unrestricted transitions, and the peak values of efficiencies are also larger. We think that in the restricted transitions, it is easy for the particles to go forward since when the particles makes transition from W_2 to W_1 , they are comparatively near the neighbor maximum of W_2 in $+x$ direction, which they must go beyond in order to move forward.

V. CONCLUSION

We demonstrate that elastically coupled particles in a flashing ratchet model move co-operatively. An important result is the generation of directed motion in a no-thermal condition. Though an isolated particle in flashing models cannot move in a no-thermal condition, elastically coupled particles in flashing models may move by the effects of springs even if thermal diffusion is not allowed. It is also found that the velocity of the particles has a maximum at a specific coupling constant of springs. The maximum of the velocity as a function of h , the restriction parameter, appears when we restrict the region of transitions as well. Changing other various parameters, we confirm the complicated behaviors of the elastically coupled particles.

On the efficiency of energy conversion, the effects of the coupling between particles are also found to be very important. The elastically coupled particles may pull a heavier load than a single particle, and the efficiency for the coupled model is larger than the single-particle model. Most of the energy, however, is used for Brownian motion, that is, for the dissipation.

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Stochastic resonance in locally and linearly coupled systems

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Abstract

Stochastic resonance (SR) is induced by the cooperation of a weak periodic driving signal and random noise in bistable systems: the output signal-to-noise ratio is enhanced by a synchronization of the characteristic time scale of a transition between two states activated by the input noise with the period of the input signal.

We numerically demonstrate SR in a locally and linearly coupled array each element of which shows the SR phenomenon. Both the output signal-to-noise ratio and the receiver operating characteristic for an single element in the array are greatly improved by the couplings. We relate SR to global dynamics of the array with kink-antikink nucleations and show how the peak of the signal-to-noise ratio appears.

1 Introduction

Stochastic resonance (SR) is induced by a cooperation of a weak external driving signal and random noise in bistable systems : Random noise greatly enhances the periodic character of the response of the bistable systems with an input, the weak coherent signal. With the result, the output *signal-to-noise ratio* (SNR) may be maximized by treating the input noise strength as a design parameter.

SR was originally proposed to understand the model for the periodic recurrences of the earth's ice ages by Benzi *et al* [1]. They showed that the apparent periodicity of the earth's ice age, the period of which is about 100,000 years, is led from the enhancement of the weak periodic variation of the earth's orbit (input signal) by weather changes (input noise) in the bistable "climatic potential". Now SR is known to occur in a wide range of physical systems. SR was reported in a Schmitt trigger [2], a ring-laser [3], a electron paramagnetic system [4], and so on. Theoretically, Wiesenfeld and McNamara has put forward an approach based on two state master equation ([5], for a review see [6]). We discuss this theory and introduce a more realistic approach with a double-Gaussian approximation in Section 4.

More recently, it has been considered that SR plays a significant role in biological systems. The crayfish mechanoreceptor cells showed the characteristic signature of SR [7]. In this case, it is provided that a single cell is specialized to detect weak coherent water motions enhanced by additional noise. Real biological systems are, however, constructed with many neurons. From this point of view, the response of coupled nonlinear elements has been studied [8],[9].

A *locally* and *linearly* coupled chain of bistable elements is one of the simplest models of those networks, which shows a phenomenon called *array enhanced stochastic resonance* (AESR) [10]. While SR has only one design parameter : the noise strength, AESR has an additional design parameter : the coupling strength. The couplings significantly improves the maximum value of the output SNR. We numerically demonstrate AESR and compute the SNR as a function of the noise strength and the coupling strength in Section 5. Moreover, to measure the system's signal detection performance, we obtain the *receiver operating characteristic* (ROC).

As for the dynamics of the chain, F. Marchesoni *et al.* theoretically estimated some

AESR results in an overdamped ϕ^4 chain [11]. M.Löcher *et al.* first experimentally demonstrated AESR in an array of diode resonators [12] and showed the presence of kink-antikink pairs in the array. We focus on a periodic kink-antikink nucleation rate in the overdamped ϕ^4 model and apply two state theory in Section 6.

$$(1) \quad \dot{x} = -\frac{d}{dx} V(x) + A \cos(2t + \varphi) + \xi(t), \quad V(x) = \frac{x^2}{2} + \frac{x^4}{4},$$

where V is the bistable potential, $A \cos(2t + \varphi)$ is the modulation (the input signal), $\xi(t)$ is Gaussian white noise (the input noise) and x is the position of the particle (the output). We investigate a model for AESR: an array which is constructed with these bistable elements coupled to their nearest neighbors and has free boundary conditions. The model for AESR is

$$(2) \quad \dot{x}_i = -\frac{d}{dx_i} V(x_i) + A \cos(2t + \varphi) + \xi(x_{i-1} - x_i) + \xi_i(t),$$

where $\xi_i(t)$ is the Kossacki's function and denotes that noise are uncorrelated from site to site.

3.2 Two State Theory

A projecting theory for SR has been put forward by Wissensfeld and McVernan [5]. Assuming that the particle can take place in only two right and left stable states x_+ , x_- the master equations for the population in the right and left well of the bistable potential

2 Background

2.1 Underlying Model for SR and AESR

Consider a particle in a bistable system, where the potential is periodically and weakly modulated (see Fig.(1)). Here, "weak" means that the potential has still two stable states and the barrier between the states all the time in spite of the existence of the modulation. Without noise, the particle remains forever in the well where it has started. On the other hand, the presence of additional noise induces a jump process of the particle between the wells. In the absence of the periodic driving, however, the jump process is random. If the modulation synchronizes with the hopping, i.e. if the cycle of the driving synchronizes with the characteristic time scale of the jump process, the particle regularly switches between the wells.

The Underlying model for SR is given by the overdamped bistable dynamics,

$$\dot{x} = -\frac{d}{dx}V(x) + A \cos(\Omega t + \varphi) + \xi(t), \quad V(x) = -\frac{x^2}{2} + \frac{x^4}{4}, \quad (1)$$

$$\langle \xi(t)\xi(t') \rangle = 2D\delta(t - t'), \quad \langle \xi(t) \rangle = 0,$$

where V is the bistable potential, $A \cos(\Omega t + \varphi)$ is the modulation (the input signal), $\xi(t)$ is Gaussian white noise (the input noise) and x is the position of the particle (the output).

We investigate a model for AESR ; an array which is constructed with these bistable elements coupled to their nearest neighbors and has free boundary conditions. The model for AESR is

$$\dot{x}_n = -\frac{d}{dx_n}V(x_n) + A \cos(\Omega t + \varphi) + \epsilon(x_{n+1} - x_n) + \epsilon(x_{n-1} - x_n) + \xi_n(t), \quad (2)$$

$$\langle \xi_n(t)\xi_{n'}(t') \rangle = 2D\delta_{n,n'}\delta(t - t'), \quad \langle \xi_n(t) \rangle = 0, \quad (n = 1, 2, \dots, N),$$

where $\delta_{n,n'}$ is the Kronecker δ function and denotes that noises are uncorrelated from site to site.

2.2 Two State Theory

A pioneering theory for SR has been put forward by Wiesenfeld and McNamara [5].

Assuming that the particle can take place in only two right and left stable states x_+ , x_- , the master equations for the population in the right and left well of the bistable potential

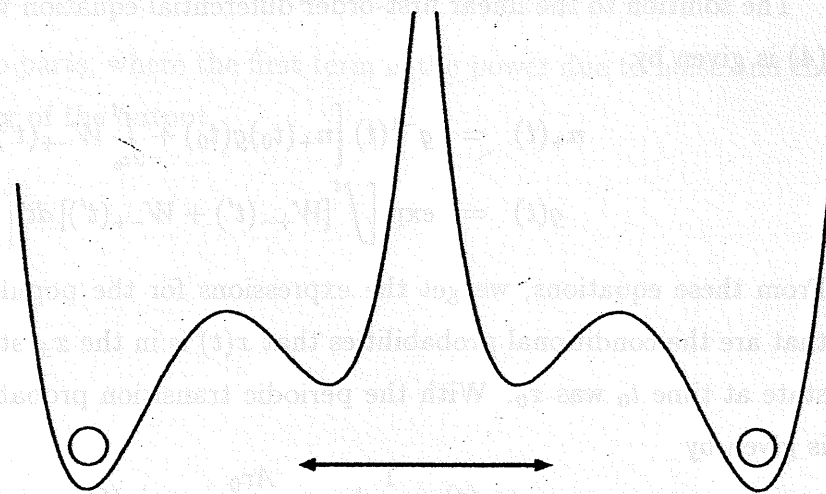


Figure 1: Sinusoidally modulated potential and behavior of a particle.

n_+, n_- , are respectively

$$\dot{n}_+(t) = W_{-+}(t)n_-(t) - W_{+-}(t)n_+(t) = -(W_{+-}(t) + W_{-+}(t))n_+(t) + W_{-+}(t), \quad (3)$$

where $W_{-+}(t)$ and $W_{+-}(t)$ are the transition probabilities from the right well to the left and from the left to the right, respectively. In Eq.(3), we use $n_+ + n_- = 1$. Applying Kramers formula, we obtain periodic transition probabilities for $A/D \ll 1$

$$W_{\pm\mp}(t) = r_0 \exp \left[\pm \frac{A}{D} \sin(\Omega t + \varphi) \right] \simeq r_0 \pm r_0 \frac{A}{D} \sin(\Omega t + \varphi), \quad (4)$$

where r_0 is the unperturbed Kramers rate

$$r_0 = \frac{1}{\sqrt{2\pi}} \exp \left[-\frac{1}{4D} \right]. \quad (5)$$

The solution to the linear first-order differential equation with periodic coefficients Eq. (4) is given by

$$\begin{aligned} n_+(t) &= g^{-1}(t) \left[n_+(t_0)g(t_0) + \int_{t_0}^t W_{-+}(t')g(t') dt' \right], \\ g(t) &= \exp \left[\int_{t_0}^t [W_{+-}(t') + W_{-+}(t')] dt' \right]. \end{aligned} \quad (6)$$

From these equations, we get the expressions for the population $n_{\pm}(t)$ and $n_{\pm}(t|x_0, t_0)$ that are the conditional probabilities that $x(t)$ is in the $x_{\pm}$ state at time t , given that the state at time t_0 was x_0 . With the periodic transition probability Eq.(4), the population is given by

$$n_{\pm}(t) = \frac{1}{2} \pm \frac{Ar_0}{(\Omega^2 + 4r_0^2)D} \sin(\Omega t + \phi + \varphi), \quad (7)$$

where $\phi = \arctan \Omega/2r_0$.

Using $n_{\pm}(t)$ and $n_{\pm}(t|x_0, t_0)$, the autocorrelation function can be computed

$$\begin{aligned} \langle x(t)x(t+\tau)|x_0, t_0; \varphi \rangle &= +n_+(t+\tau|+1, t)n_+(t|x_0, t_0) - n_+(t+\tau|-1, t)n_-(t|x_0, t_0) \\ &\quad - n_-(t+\tau|+1, t)n_+(t|x_0, t_0) + n_-(t+\tau|-1, t)n_-(t|x_0, t_0) \\ &= [2n_+(t+\tau|+1, t) - 1 + 2n_+(t+\tau|-1, t) - 1]n_+(t|x_0, t_0) \\ &\quad - [2n_+(t+\tau|-1, t) - 1]. \end{aligned} \quad (8)$$

To take into account the dependence on initial conditions, we take the limit $t_0 \rightarrow -\infty$ and average over the initial phase ; φ ,

$$\langle\langle x(t+\tau)x(t); \varphi \rangle\rangle_{\varphi} = \frac{1}{2\pi} \int_0^{2\pi} \lim_{t_0 \rightarrow -\infty} \langle x(t+\tau)x(t)|x_0, t_0; \varphi \rangle d\varphi. \quad (9)$$

According to the Wiener-Khintchine theorem, the power spectral density is given by the Fourier transform of the autocorrelation function

$$\langle S(\omega; \varphi) \rangle_{\varphi} = \frac{1}{2\pi} \int_0^{2\pi} S(\omega; \varphi) d\varphi = \int_{-\infty}^{\infty} \langle x(t+\tau)x(t); \varphi \rangle_{\varphi} e^{-i\omega\tau} d\tau. \quad (10)$$

We use the phase averaged *one-sided* power spectral density (PSD) defined for ω positive only,

$$S(\omega) = \langle S(\omega; \varphi) \rangle_{\varphi} + \langle S(-\omega; \varphi) \rangle_{\varphi}. \quad (11)$$

For the periodic transition probability Eq.(4) PSD is given by

$$\begin{aligned} S(\omega) &= \frac{4r_0}{4r_0^2 + \omega^2} \left(1 - \frac{2r_0^2 A^2}{(4r_0^2 + \Omega^2)D^2} \right) + \frac{4\pi r_0^2 A^2}{(4r_0^2 + \Omega^2)D^2} \delta(\omega - \Omega) \\ &= S_{\text{noise}}(\omega) + S_{\text{sig}}\delta(\omega - \Omega) \end{aligned} \quad (12)$$

It consists of two parts, where the first term is the power due to noise and the second due to the periodicity of the output.

3 Numerical Simulation of SR

3.1 Numerical Simulation and Results

In this section, we numerically integrated the stochastic differential equation Eq.(1) and compute a power spectral density.

A difference equation valid to order $(\Delta t)^2$ corresponding to Eq.(1) is

$$\begin{aligned} x(t + \Delta t) = & x(t) + (x(t) - x(t)^3 + A \cos \Omega t) \Delta t \\ & - \frac{1}{2} (1 - 3x(t)^2) (x(t)^3 - x(t)^3 + A \cos \Omega t) (\Delta t)^2 + A \Omega \sin \Omega t (\Delta t)^2 \\ & + \int_t^{t+\Delta t} dt' \xi(t') - (1 - 3x(t)^3) \int_t^{t+\Delta t} dt' \int_t^{t'} dt'' \xi(t''). \end{aligned} \quad (13)$$

where Δt is a time step. Here we introduce new variables $R(t)$ and $Q(t)$

$$R(t) = \int_t^{t+\Delta t} dt' \xi(t'), \quad Q(t) = \int_t^{t+\Delta t} dt' \int_t^{t'} dt'' \xi(t''). \quad (14)$$

If $R(t)$, $Q(t)$ are replaced by $R_m = \int_{m\Delta t}^{(m+1)\Delta t} dt' \xi(t')$, $Q_m = \int_{m\Delta t}^{(m+1)\Delta t} dt' \int_{m\Delta t}^{t'} dt'' \xi(t'')$, covariances $\langle R_m R_n \rangle$, $\langle R_m Q_n \rangle$ and $\langle Q_m Q_n \rangle$ can be calculated as

$$\begin{aligned} \langle R_m R_n \rangle &= 2D\Delta t \delta_{m,n}, \\ \langle R_m Q_n \rangle &= D(\Delta t)^2 \delta_{m,n}, \\ \langle Q_m Q_n \rangle &= \frac{2}{3}(\Delta t)^3 \delta_{m,n}. \end{aligned}$$

$R(t)$ and $Q(t)$ are thus given by

$$R(t) = \sqrt{\frac{3}{4}\Delta t} \theta + \sqrt{\frac{1}{4}\Delta t} \eta, \quad Q(t) = \sqrt{\frac{1}{3}(\Delta t)^2} \theta, \quad (15)$$

where θ and η are the random variables which are mutually independent and subject to Gaussian distribution with the mean 0 and the variance $\sqrt{2D}$. Using Eqs.(13),(15) with a time step $\Delta t \sim 0.012$, we obtain the time series of $x(t)$.

Figs.(2),(3) and (4) present the trajectory of the particle for $A = 0.1$, $\Omega = \pi/200$. With D increasing, the inter-well transitions (the jump process of the particle between wells which we are interested in) occurs more frequently. In Fig.(2), a particle does not always jump between the states when the potential is tilted. In Fig.(3), a particle regularly jumps between the states and an apparent periodicity can be observed. In Fig.(4), a particle frequently jumps between the states with no apparent periodicity.

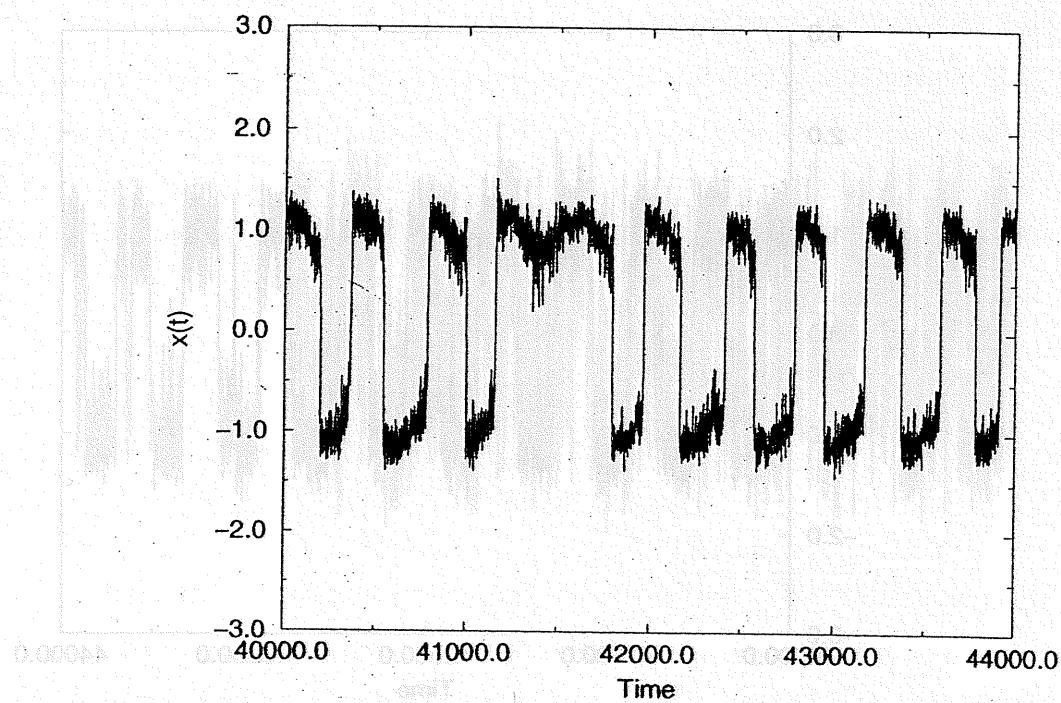


Figure 2: Trajectory of a particle in a double well potential with sinusoidal driving and additional noise. $\Omega = \pi/200$, $A = 0.1$, $D = 0.03$.

3 Numerical Simulation of SR

3.1 Numerical Simulation and Results

In this section, we numerically integrated the stochastic differential equation Eq.(1) and compute a power spectral density.

A difference equation valid to order $(\Delta t)^2$ corresponding to Eq.(1) is

$$x(t + \Delta t) = x(t) + (x(t) - x(t)^3 + A \cos(\Omega t)) \Delta t - \frac{1}{2}(1 - 3x(t)^2)(x(t)^2 - x(t)^4 + A \cos(\Omega t) \Delta t^2) + \sqrt{2D} \sin(\Omega(\Delta t))$$

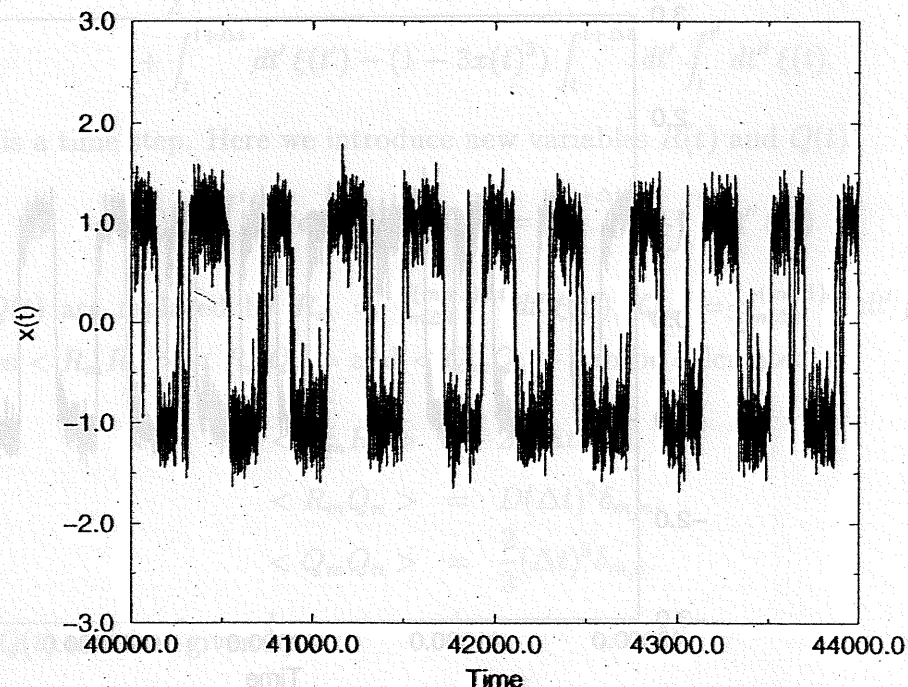


Figure 3: Trajectory of a particle in a double well potential with sinusoidal driving and additional noise. Ω and A are the same as in Fig.(2). $D = 0.12$. Note that the apparent periodicity can be observed.

As a time step $\Delta t = 0.01$, we obtain the time series of $x(t)$.

Figs(2),(3) and (4) present the trajectory of the particle for $A = 0.1, \Omega = \pi/200$. With D increasing, the inter-well transitions (the jump process of the particle between wells which we are interested in) occurs more frequently. In Fig (2), a particle does not always jump between the states when the potential is tilted. In Fig (3), a particle regularly jumps between the states and an apparent periodicity can be observed. In Fig.(4), a particle frequently jumps between the states with no apparent periodicity.

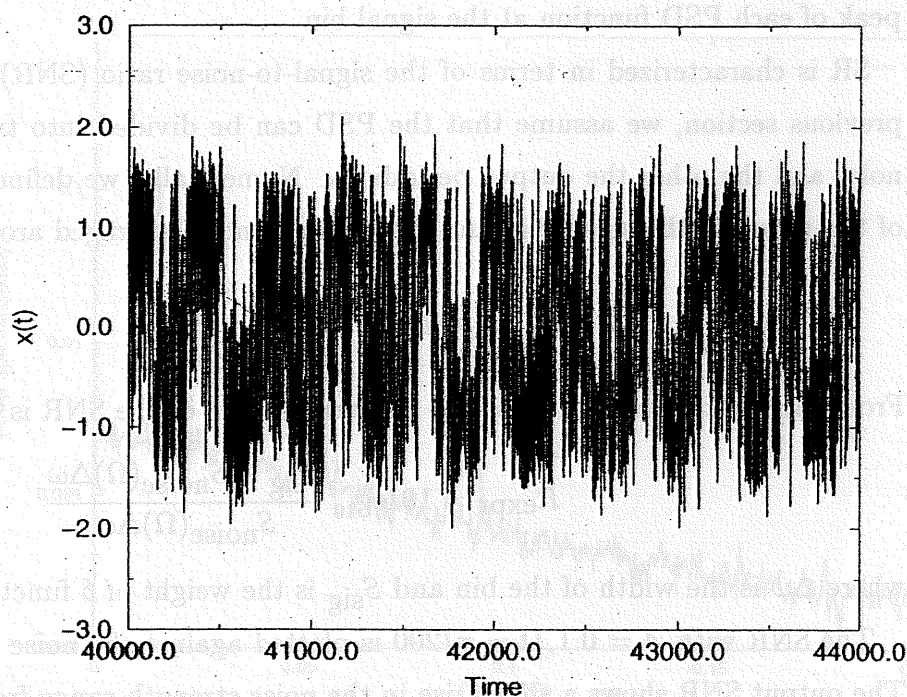


Figure 4: Trajectory of a particle in a double well potential with sinusoidal driving and additional noise. Ω and A are the same as in Fig.(2). $D = 0.4$.

With the data at every 200 steps of this time series, we compute a power spectral density (PSD); a periodgram using fast Fourier transform (FFT). To reduce bin leakage we convolve the data with a Parzen window

$$W_j = \left| \frac{j - \frac{1}{2}(N-1)}{\frac{1}{2}(N+1)} \right|, \quad (16)$$

and average PSD segments, each of 4096 samples (dataset) per segment.

Fig.(5) is a plot of the PSD as a function of the circular frequency with the same parameter. From this figure, one can observe the periodicity of the output from a sharp peak of each PSD function at the signal bin.

SR is characterized in terms of the signal-to-noise ratio (SNR). As mentioned in the previous section, we assume that the PSD can be divided into two part, one is due to noise and the other the output periodicity. Numerically, we define the SNR as the ratio of the power in the signal bin and the power of noise averaged around the signal bin.

$$R_{\text{expr}} = 10 \log_{10} \frac{S_{\text{sigbin}}}{S_{\text{noise}}}. \quad (17)$$

From theoretical point of view, the numerical value of the SNR is reconstructed with

$$R_{\text{expr}} = 10 \log_{10} \frac{S_{\text{sig}} + S_{\text{noise}}(\Omega)\Delta\omega}{S_{\text{noise}}(\Omega)\Delta\omega} \quad (18)$$

where $\Delta\omega$ is the width of the bin and S_{sig} is the weight of δ function.

The SNR with $A = 0.1, \Omega = \pi/200$ is plotted against the noise strength D in Fig.(6). The output SNR shows a sharp rise in the noise strength range from 0.03 to 0.10, above which it decreases gradually. For a system well characterized by linear-response theory the SNR of the output decreases with any increase in input noise. In constant, the significant feature of SR is an increase in the output SNR with decreased input noise.

4 Double-Gaussian Approximation

4.1 Double-Gaussian Distribution

In this section, we treat the SP phenomenon by means of Fokker-Planck equation. In the two-state theory, we assume the particle can take place only in two stable states. To take into account the intra-well behavior of the particle, we introduce a double-Gaussian distribution which models the distribution of $x(t)$ around the two stable states.

The distribution of $x(t)$ is denoted as $P(x, t)$ and Fokker-Planck equation which is equivalent to Eq. (1) is

$$\frac{\partial P(x, t)}{\partial t} = -\frac{\partial}{\partial x} [v(x) P(x, t)] + D \frac{\partial^2 P(x, t)}{\partial x^2} \quad (19)$$

$$v(x) = -\frac{\partial}{\partial x} (U(x) + A \cos(2\pi x)) + \frac{D}{2} \frac{\partial^2}{\partial x^2} \quad (20)$$

$$P(x, t) = \alpha_1(t) P_1(x, t) + \alpha_2(t) P_2(x, t) \quad (21)$$

$$P_1(x, t) = \frac{1}{\sigma_1 \sqrt{2\pi}} \exp\left(-\frac{(x - \mu_1)^2}{2\sigma_1^2}\right) \quad (22)$$

$$P_2(x, t) = \frac{1}{\sigma_2 \sqrt{2\pi}} \exp\left(-\frac{(x - \mu_2)^2}{2\sigma_2^2}\right) \quad (23)$$

$$\alpha_1(t) + \alpha_2(t) = 1 \quad (24)$$

$$\frac{d\alpha_1(t)}{dt} = -\alpha_1(t) \lambda_{12} + \alpha_2(t) \lambda_{21} \quad (25)$$

$$\frac{d\alpha_2(t)}{dt} = -\alpha_2(t) \lambda_{21} + \alpha_1(t) \lambda_{12} \quad (26)$$

If we assume $\alpha_1(t)$ and $\alpha_2(t)$ are independent with each other, we can calculate the autocorrelation function, Eq. (9)

$$\begin{aligned} \langle x(t)x(t') \rangle &= \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dx' x W(x, t | x', t') P(x', t') \\ &= \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dx' x P(x, t | x', t') P(x', t') \end{aligned} \quad (27)$$

In Eq. (27), $W(x, t | x', t')$ is the transition probability the particle which was located in the position x at time t is in x' at t' , and $W(x', t' | x, t)$ can be replaced with $P(x, t | x', t')$ which is the double-Gaussian distribution at time t' with the initial state x at time t .

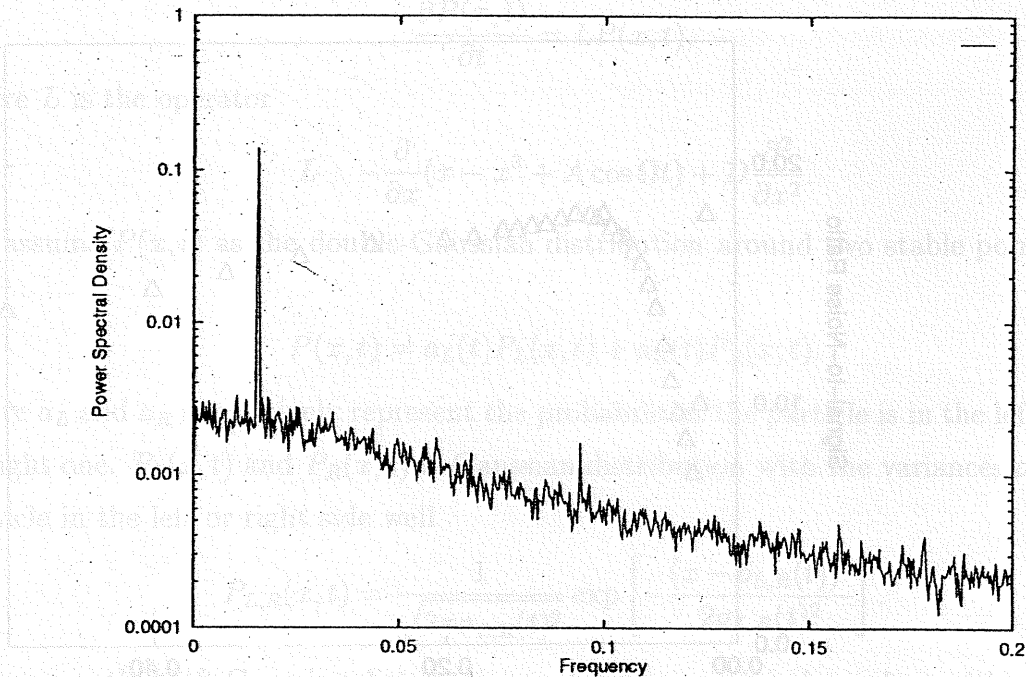


Figure 5: Typical power spectral density with $\Omega = \pi/200$, $A = 0.1$, $D = 0.12$. A sharp peak appears at signal bin the center of which is $\omega = \Omega \sim 0.0157$.

With the data at every 200 steps of the time series, we compute a power spectral density (PSD), a periodogram using fast Fourier transform (FFT). To reduce bin leakage, we convolve the data with a Parzen window

$$W_x = \left| \frac{1 - (x/N)^2}{4} \right| \quad (16)$$

and average PSD segments, each of 400 samples (divided) per segment.

Fig(5) is a plot of the PSD as a function of the excitation frequency with the same parameter. From this figure, one can observe the periodicity of the output from a sharp peak of each PSD function at the signal bin.

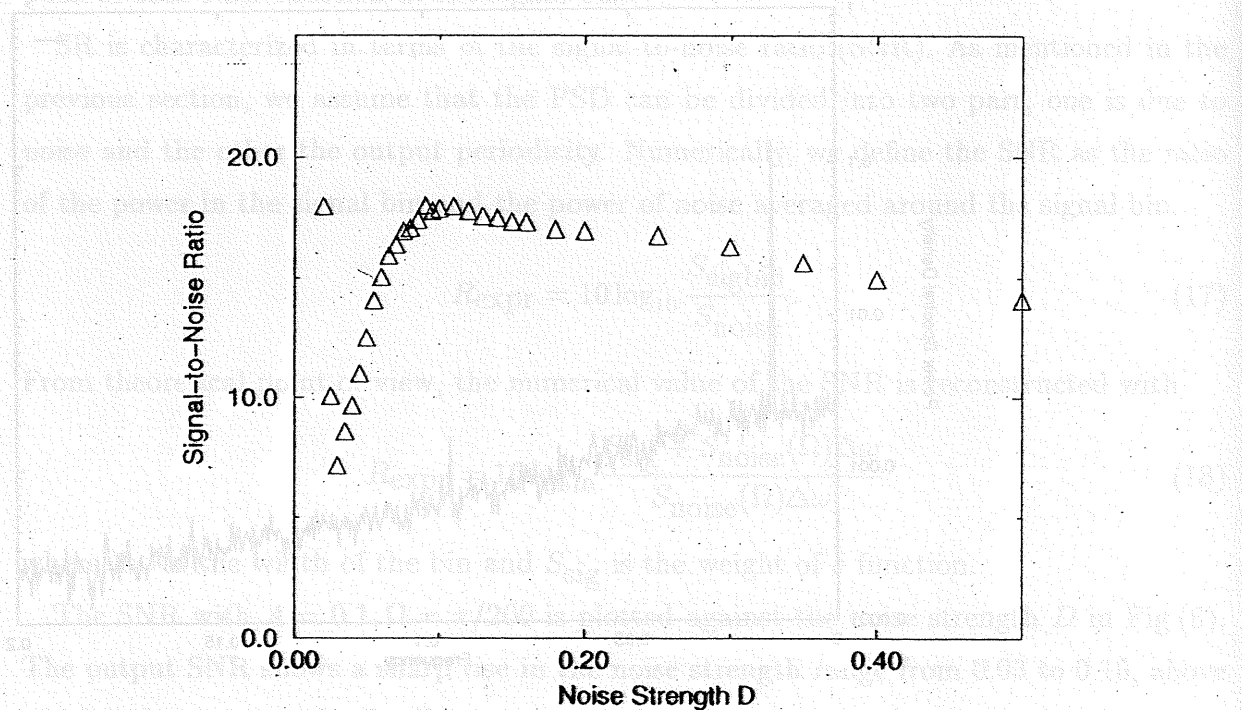


Figure 6: SNR (dB) as a function of noise strength D .

4 Double-Gaussian Approximation

4.1 Double-Gaussian Distribution

In this section, we treat the SR phenomenon by means of Fokker-Planck equation. In the two state theory, we assume the particle can take place only in the two stable states. To take into account the intra-well behavior of the particle, we introduce a double-Gaussian distribution which models the distribution of $x(t)$ around the two stable states.

The distribution of $x(t)$ is denoted as $P(x, t)$ and Fokker-Planck equation which is equivalent to Eq.(1) is

$$\frac{\partial P(x, t)}{\partial t} = LP(x, t), \quad (19)$$

where L is the operator

$$L : -\frac{\partial}{\partial x}(x - x^3 + A \cos \Omega t) + D \frac{\partial^2}{\partial x^2}. \quad (20)$$

We assume $P(x, t)$ as the double-Gaussian distribution around two stable points b_L and b_R

$$P(x, t) = a_L(t)P_L(x, t) + a_R(t)P_R(x, t). \quad (21)$$

where a_L and a_R respectively represent the probabilities the particle is in the left side well or right one. $P_L(x, t)$ and $P_R(x, t)$ is Gaussian distribution with the variances $\sigma_{L,R}$ of the particle in the left or right side well

$$P_{L,R}(x, t) = \frac{1}{\sqrt{2\pi\sigma_{L,R}(t)^2}} \exp \left[-\frac{(x - b_{L,R}(t))^2}{2\sigma_{L,R}(t)^2} \right]. \quad (22)$$

We assume that left Gaussian distribution and right one are independent with each other; *i.e.*

$$|b_L - b_R| \gg \sigma_L(t), \sigma_R(t).$$

If we know left and right Gaussian distribution ; we know $a_{L,R}$, $b_{L,R}$ and $\sigma_{L,R}$, we can calculate the autocorrelation function Eq.(8)

$$\begin{aligned} \langle x(t)x(t') \rangle &= \int_{-\infty}^{\infty} dx \int dx' x' W(x', t'|x, t) x P(x, t) \\ &= \int_{-\infty}^{\infty} dx \int dx' x' P(x', t'|x, t) x P(x, t), \end{aligned} \quad (23)$$

In Eq.(23), $W(x', t'|x, t)$ is the transition probability the particle which was located in the position x at time t is in x' at t' , and $W(x', t'|x, t)$ can be replaced with $P(x', t'|x, t)$ which is the double-Gaussian distribution at time t' with the initial state x at time t .

The way to calculate $a_{L,R}$, $b_{L,R}$ and $\sigma_{L,R}$ is as follows. First, we define a moment of order n of left (right) Gaussian distribution

$$\langle x^n \rangle_L \equiv \int_{-\infty}^0 dx x^n P(x, t) = \int_{-\infty}^0 dx x^n P_L(x, t). \quad (24)$$

Equations describing time evolution of the left (right) moments are given by

$$\frac{d}{dt} \langle x^n \rangle_L = \int_{-\infty}^0 dx \frac{d}{dt} [x^n P_L(x, t)] = \int_{-\infty}^0 dx x^n L P_L(x, t) \quad (25)$$

We apply Eq.(25) to the right and left moments for order 0,1 and 2; $\langle 1 \rangle_L$, $\langle x \rangle_{L,R}$ and $\langle x^2 \rangle_{L,R}$, and use the conservation law of probability $a_L + a_R = 1$. From these six independent equations, we can calculate $a_{L,R}$, $b_{L,R}$ and $\sigma_{L,R}$.

Calculation of the autocorrelation function Eq.(23) can be put forward

$$\begin{aligned} \langle x(t)x(t') \rangle &= \int dx \int dx' P(x', t' | x_0, t) x' \int dx_0 P(x_0, t) \delta(x - x_0) x_0 \\ &= \sum_{x_0} P(x_0, t) x_0 \Delta x_0 \int dx' P(x', t' | x_0, t) x' \\ &= \sum_{x_0} P(x_0, t) x_0 \Delta x_0 (a_L b_L + a_R b_R). \end{aligned} \quad (26)$$

Note that $a_L b_L + a_R b_R$ depend on the initial state x_0 and the initial phase of the driving φ .

From Eqs.(9),(10),(11) and (26), the power spectral density is given by

$$S(\omega) = \frac{1}{\pi} \sum_{x_0} P(x_0, t_0) x_0 \int_0^\infty dt \cos \omega t \int_0^{2\pi} d\varphi (a_L b_L + a_R b_R). \quad (27)$$

We numerically compute $a_L b_L + a_R b_R$ for 10 initial phases each of which contains 60 initial states, and obtain the power spectral density from Eq.(27).

4.2 Comparison with Numerical Results

Numerical solutions to the moment equations Eq.(25) with $A = 0.1$, $\Omega = \pi/200$, $D = 0.15$ are shown in Fig.(7) in a shape of the double-Gaussian distribution Eq.(21). Left and right Gaussian has a sharp peak around the left and right stable states respectively and the height of the peak oscillate periodically as a function of time. We can easily reduce that the particle's jump processes shows the apparent periodicity.

Fig.(8) is a plot of the SNR with $A = 0.1$, $\Omega = \pi/200$. A solid line with diamonds represents the SNR obtained from the double-Gaussian approximation Eqs.(27),(17) and

triangles the SNR from the numerical simulation. Though the difficulty of numerical Fourier transform of nearly complete periodic function of time, the theoretical values are in quantitative agreement with the simulated ones.

We consider that the double-Gaussian approximation has some advantages over the two-state theory. First, this approximation can be applied to the case to which the two-state theory is not applicable, such as the case in which noise adds to the driving frequency Ω . This approximation is applicable even to a case with a large driving amplitude A . In this range of the amplitude, the periodic transition probability of the particle in the two-state theory cannot be expanded into Taylor series like Eq. (4). Another advantage is that the double-Gaussian approximation can predict an SNR decrease in the low noise strength range. The two-state theory, because of the neglect of ultra-low motion of the particle, cannot predict this feature.

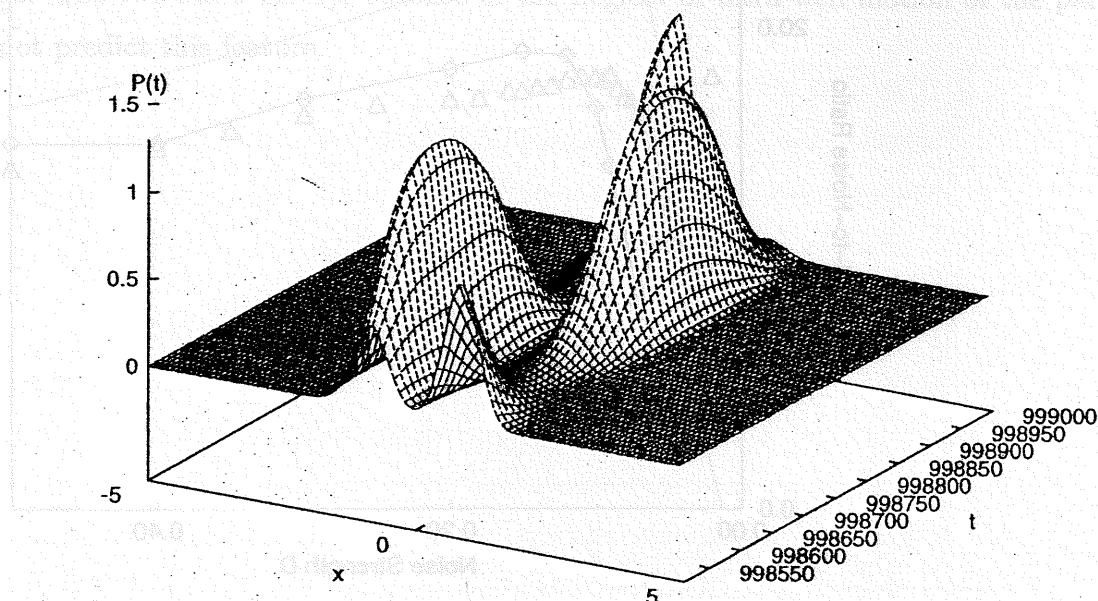


Figure 7: Double-Gaussian distribution. $\Omega = \pi/200$, $a = 0.1$, $D = 0.15$, $x_0 = -1.0$.

The way to calculate $\langle x^n \rangle_t$ and $\langle x^n \rangle_{ss}$ is as follows. First, we define a moment of order n of left (right) Gaussian distribution

$$\langle x^n \rangle_L = \int_{-\infty}^0 dx x^n P_L(x, t) = \int_{-\infty}^0 dx x^n P_L(x, t) \quad (24)$$

Equations describing time evolution of the left (right) moments are given by

$$\frac{d}{dt} \langle x^n \rangle_L = \int_{-\infty}^0 dx \frac{d}{dt} x^n P_L(x, t) = \int_{-\infty}^0 dx x^n \frac{d}{dt} P_L(x, t) \quad (25)$$

We apply Eq.(25) to the right and left moments for order 0,1 and 2, $\langle x \rangle_L$, $\langle x \rangle_R$ and $\langle x^2 \rangle_L$, $\langle x^2 \rangle_R$, and use the conservation law of probability $P_L + P_R = 1$. From these independent equations, we can calculate $\langle x \rangle_t$, $\langle x^2 \rangle_t$ and P_L .

Calculations of the autocorrelation function, Eq.(23) can be put forward

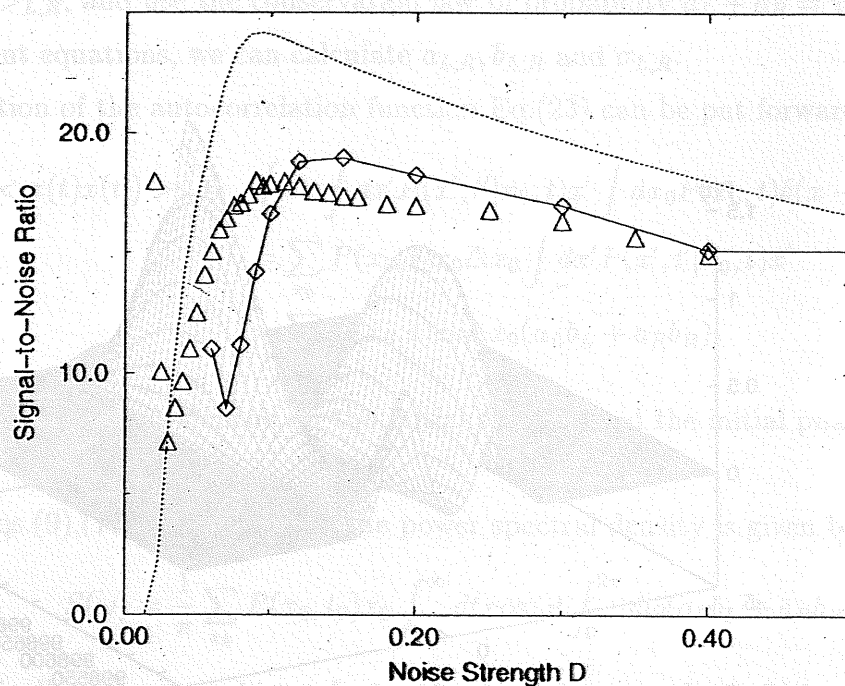


Figure 8: SNR as a function of D . Solid line with diamonds represents the SNR obtained from double-Gaussian approximation. Triangles represents the SNR from numerical simulation, dotted line represents the prediction of two-state theory.

Numerical solutions to the moment equations Eq.(25) with $A = 0.1$, $\Omega = \pi/200$, $D = 0.15$ are shown in Fig.(7) in a shape of the double-Gaussian distribution Eq.(21). Left and right Gaussian has a sharp peak around the left and right stable states respectively and the height of the peak oscillate periodically as a function of time. We can easily reduce that the particle's jump processes shows the apparent periodicity.

Fig.(8) is a plot of the SNR, with $A = 0.1$, $\Omega = \pi/200$. A solid line with diamonds represents the SNR obtained from the double-Gaussian approximation, Eqs.(27),(17) and

triangles the SNR from the numerical simulation. Though the difficulty of numerical Fourier transform of nearly complete periodic function of time, the theoretical values are in quantitative agreement with the simulated ones.

We consider that the double-Gaussian approximation has some advantages over the two-state theory. First, this approximation can be applied to the case to which the two-state theory is not applicable, such as the case in which noise adds to the driving frequency Ω . This approximation is applicable even to a case with a large driving amplitude A . In this range of the amplitude, the periodic transition probability of the particle in the two-state theory cannot expand into Taylor series like Eq.(4). Another advantage is that the double-Gaussian approximation can predict an SNR decrease in the low noise strength range. The two-state theory, because of the neglect of intra-well motion of the particle, cannot predict this feature.

5 Numerical Simulation of AESR

5.1 Signal-to-Noise Ratio

In this section, we numerically demonstrate the AESR phenomenon with $N = 9$ and $N = 31$ and compute the output SNR and the receiver operating characteristic (ROC) of a single elements in the array $n = 5$ and $n = 16$ respectively. The method of computation is given in Section.3.

Figs.(9),(10) are plots of the output SNR as a function of the coupling strength ϵ and the noise strength D . These plots are characterized by the following features.

1. The SNR of a given elements in the array shows a unique maximum on the plane (D, ϵ) . This unique maximal value of the SNR increases with N increase.
2. The SNR plotted against D with given ϵ shows typical SR features. The maximum value of the SNR as a function of D for any $\epsilon > 0$ is always enhanced than that for the uncoupled element ($\epsilon = 0$). The value of D which maximize the SNR for given ϵ is a monotonous increasing function of ϵ .

These features of AESR is considered to be due to the couplings between the elements.

In a range of low coupling strength, each particle in the element can behave fairly independently in the same side well and easily switches between the wells. When some particles in the array jump between the states, others obey these behavior because of its couplings. With N increase, the number of the particle which first switches between the wells increases. With the result, the maximum SNR value increases with both ϵ and N increase.

In a range of large coupling strength, the strong couplings prevent each particle to jump between the wells. Furthermore in a large coupling strength limit, the array behave as if it were a single particle mass of which is N multiplied by mass of the single particle in the element. The value of D which maximize the SNR for given ϵ increases with ϵ increase. The maximum SNR value decrease with ϵ increase, which approach one for the uncoupled element in the large coupling strength limit.

4.3 Receiver Operating Characteristics

In order to determine whether the SNR effect has signal processing applications, we cannot rely on the SNR alone. For signal detection, we must consider detection statistics: probability of detection and probability of false alarm. Probability of detection (PD) is the probability that the system will report that a signal is detected when in fact a signal is present in the input. The receiver operates by comparing the output of the system to a threshold. If the output is greater than the threshold, the system reports that a signal is present.

To compute the ROC curves, we need to know the distribution of the test statistic. For a given SNR, the test statistic is a function of the coupling strength ϵ and the noise strength D . The ROC curve is a plot of PD versus PFA. The ROC curve is a function of the coupling strength ϵ and the noise strength D . The ROC curve is a function of the coupling strength ϵ and the noise strength D .

Each of Fig. (11) presents the ROC curves with $N = 9$ for various coupling strengths and a given noise strength. Each inset shows the SNR versus D for the given ϵ with a vertical line which presents the value of D used in the ROC curve. The ROC curve is a function of the coupling strength ϵ and the noise strength D . The ROC curve is a function of the coupling strength ϵ and the noise strength D .

The ROC curve is a function of the coupling strength ϵ and the noise strength D . The ROC curve is a function of the coupling strength ϵ and the noise strength D . The ROC curve is a function of the coupling strength ϵ and the noise strength D .

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The ROC curve is a function of the coupling strength ϵ and the noise strength D . The ROC curve is a function of the coupling strength ϵ and the noise strength D . The ROC curve is a function of the coupling strength ϵ and the noise strength D .

Figure 9: SNR as a function of coupling strength ϵ and noise strength D for a single element $n = 5$ in the array with $N=9$. $\Omega = \pi/200$, $A = 0.1$. Above one is a 3D plot and the below is its contour plot.

5 Numerical Simulation of ACSR

5.1 Signal-to-Noise Ratio

In this section, we numerically demonstrate the ACSR phenomenon with $N = 9$ and $N = 31$ and compute the output SNR and the receiver operating characteristic (ROC) of

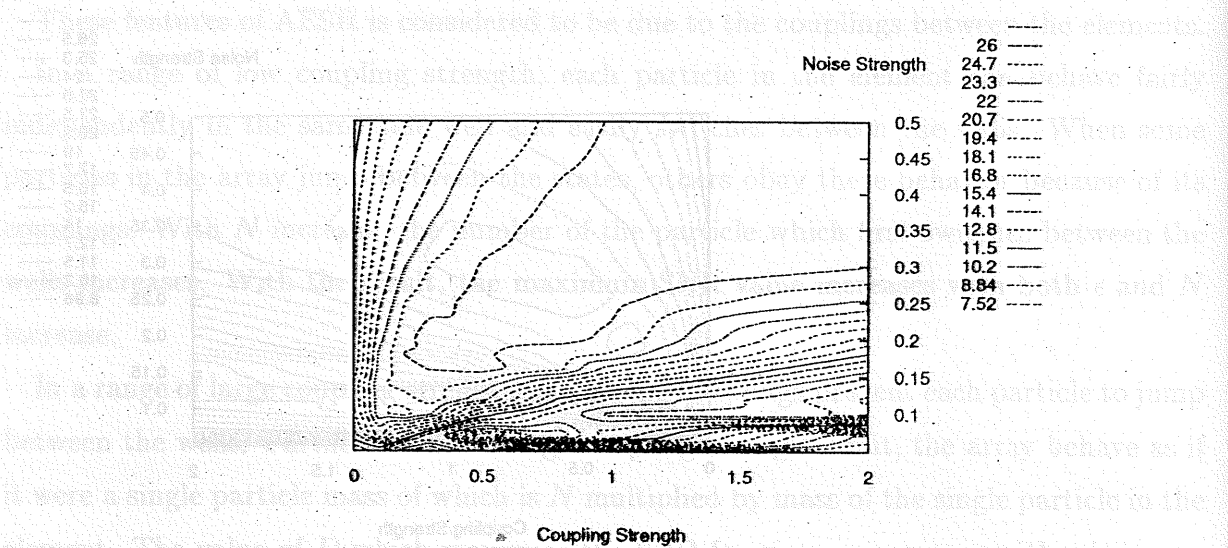
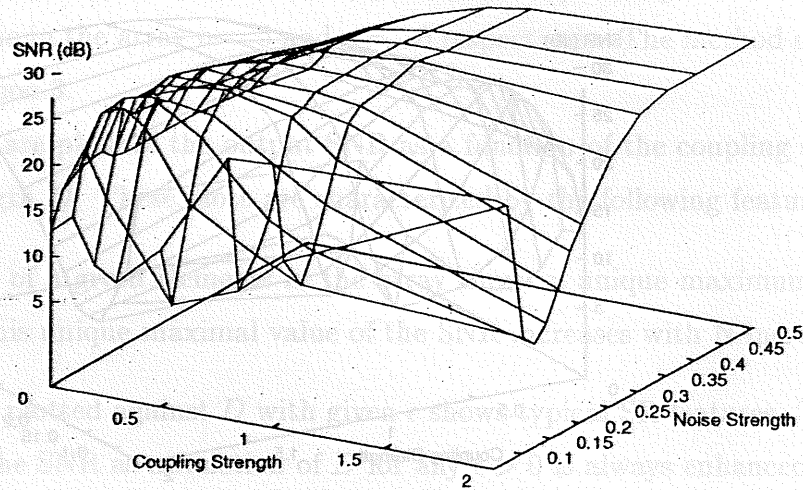


Figure 10: SNR as a function of coupling strength ϵ and noise strength D for a single element $n = 16$ in the array with $N=31$. $\Omega = \pi/200$, $A = 0.1$. Above one is a 3D plot and the below is its contour plot.

5.2 Receiver Operating Characteristic

In order to determine whether the SNR effect has signal processing applications, we cannot rely on the SNR alone. For signal detection, we must consider detection statistics : probability of detection and probability of false alarm. Probability of detection (PD) is the probability that the system will report that a signal is detected when in fact a signal is present in the input. Probability of false alarm (PFA) is the probability that the system will report that a signal is detected when in fact a signal is *not* present in the input. The receiver operating characteristic is well known as a plot of probability of detection versus probability of false alarm.

To compute the ROC, we repeatedly record the power in the signal bin of the PSD for both with and without the external driving. For a given threshold, if the system's output is exceeds it, that means that the system reports that a signal is present. A low threshold leads to high PD and high PFA. A high threshold leads to low PD and PFA. A better system will gives higher PD and lower PFA.

Each of Fig.(11) presents the ROC curves with $N = 9$ for different coupling strengths and a given noise strength. Each inset shows the SNR against D for the given ϵ with a vertical line which presents the value of D used in the computation of the ROC. The ROC and the SNR for $\epsilon = 0$ (uncoupled), $\epsilon = 0.3$, and $\epsilon = 1.0$ are represented with dotted lines, solid lines and dashed lines in each graph and inset, respectively. In each figures, the ROC with $\epsilon = 0.3$ is much better than that for uncoupled element. With $\epsilon = 1$, when D leads to the sufficiently high SNR value, the ROC is much better than that for uncoupled element (see Figs.(11.c),(11.d)). In Fig.(11.c), the systems with $\epsilon = 0.3$ and 1 give low PFA and high PD.

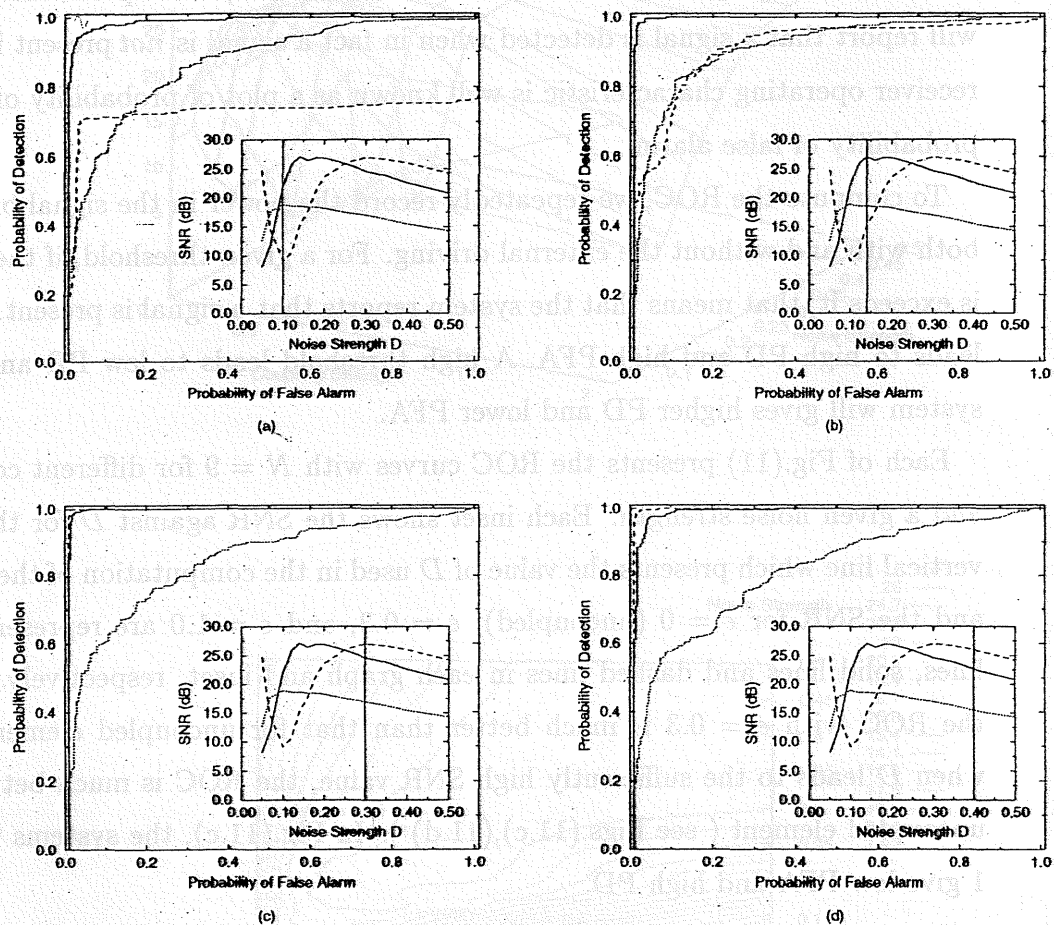


Figure 11: ROC (probability of detection *vs* probability of false alarm) curves for an single element $n = 5$ in the array with $N = 9, \Omega = \pi/200, A = 0.1$. Each inset represent the SNR for a given ϵ with a vertical line which show the noise strength D used in a computation of the ROC ((a) $D = 0.1$, (b) $D = 0.14$, (c) $D = 0.3$, (d) $D = 0.4$). Dotted line represents the ROC with $\epsilon = 0.0$ (uncoupled), Solid line $\epsilon = 0.3$, Dashed line $\epsilon = 1.0$.

6 ϕ^4 Model and Two State Theory

6.1 Behavior of Array

In this section, we study about the dynamics of the array theoretically.

Figs.(12),(13) and (14) show behavior of the array. When a particle is in the left-side well of the potential, a triangle is plotted. In a range of low noise strength (Fig.(12)), the array can hardly switch from one stable state to the other. In constant, in a range of large noise strength (Fig.(14)), each particle behaves as if it were uncoupled. In the middle range (Fig.(13)), the apparent periodicity can be observed.

We can consider that the behavior of the particle in the element which we focus on consists of two parts ; under the control of the global dynamics of the array and the local excitation of the particle itself.

6.2 Low Noise Strength Range

In the low noise strength and the large coupling strength range, because of these couplings, the behavior of each particle is under the control of the global dynamics of the array. The mechanism of the array's switch process relate to the nucleation of kink-antikink pairs [11]. Kinks and antikinks are well known as localized (soliton) solutions to ϕ^4 model. Thermal fluctuations are expected to activate a critical nucleus of kink-antikink pairs and trigger the switch process of the array. The presence of kink-antikink pairs in the array was experimentally reported in a recent paper [12].

In the $N \rightarrow \infty$ limit the array can be considered as an overdamped ϕ^4 chain of length 1. With the transformation $x_n(t) \rightarrow \phi(n/N, t)$, Eq.(2) takes the form

$$\phi_t = \phi - \phi^3 + \epsilon_N \phi_{xx} + A \cos \Omega t + \zeta(x, t), \quad (28)$$

$$\langle \zeta(x, t) \zeta(x', t') \rangle = 2D_N \delta(x, x') \delta(t, t'), \quad \langle \zeta(x, t) \rangle = 0,$$

with homogeneous von Neumann boundary conditions, $\phi_x(0, t) = \phi_x(1, t) = 0$, where ϵ_N and D_N are respectively related to ϵ and D by $\epsilon = \epsilon_N N^2$ and $D = D_N N$. Localized solutions to the ϕ^4 theory can be well approximated to an appropriate linear superposition of moving kinks ϕ_+ and antikinks ϕ_-

$$\phi_{\pm} = \tanh \left[\pm \frac{1}{2d} \frac{x - X_{\pm}(t)}{\sqrt{1 - \dot{X}_{\pm}^2(t)/\epsilon_N}} \right]. \quad (29)$$

In this section, we study about the dynamics of the array theoretically. Figs (12), (13) and (14) show behavior of the array. When a particle is in the left-side well of the potential, a triangle is plotted. In a range of low noise strength (Fig (12)), the array can easily switch from one stable state to the other. In constant, in a range of large noise strength (Fig (14)), each particle behaves as if it were uncoupled. In the middle range (Fig (13)), the apparent periodicity can be observed.

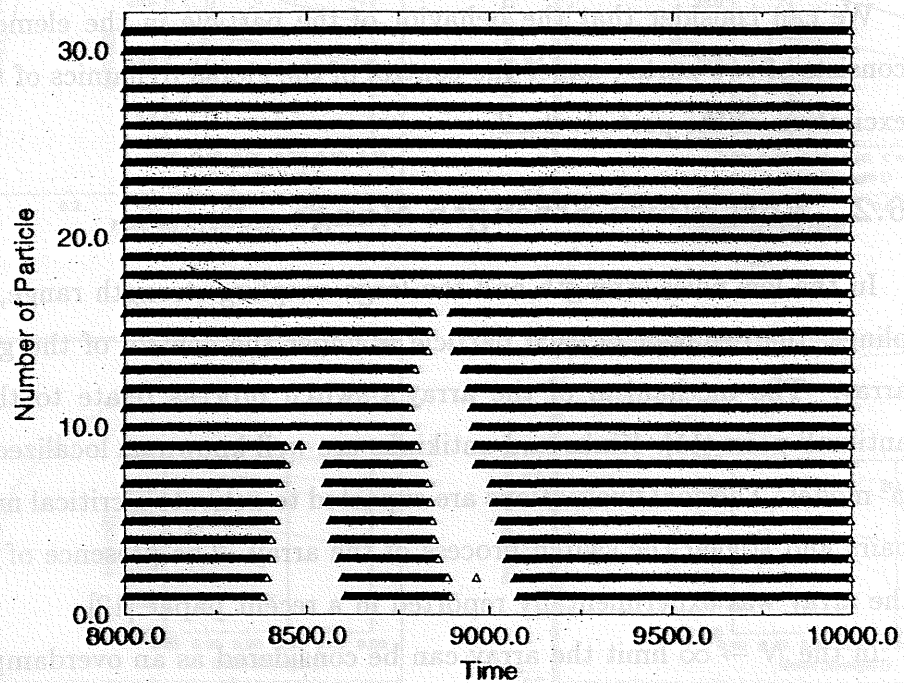


Figure 12: Behavior of the array with $N = 31$. Triangle are plotted if a particle of number n exists in the left-side well. $\Omega = \pi/200, A = 0.1, \epsilon = 0.3, D = 0.5$.

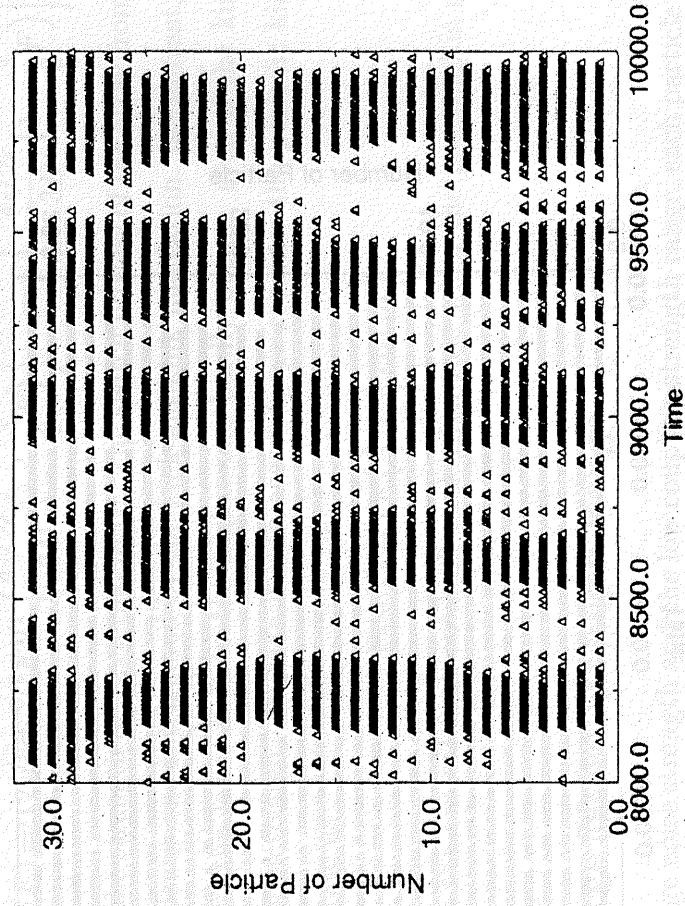


Figure 13: Behavior of the array with $N = 31$. Triangle are plotted if a particle of number n exists in the left-side well. Parameters Ω, A and ϵ is the same as in Fig.(12). $D = 0.18$.

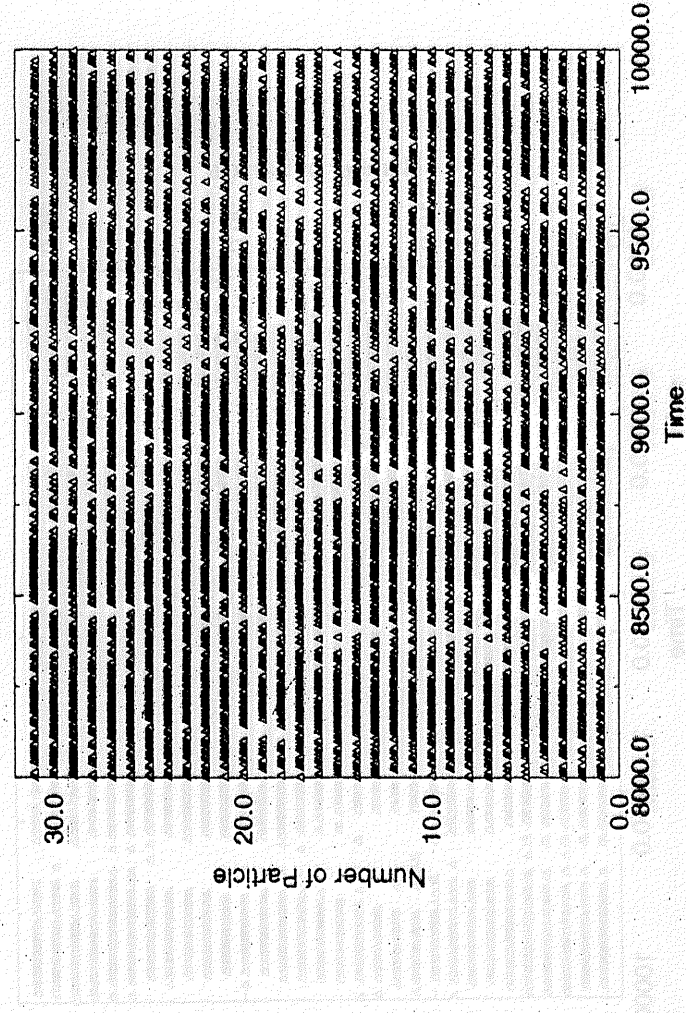


Figure 14: Behavior of the array with $N = 31$. Triangle are plotted if a particle of number n exists in the left-side well. Parameters Ω, A and ϵ is the same as in Figs.(12). $D = 0.4$.

where $d = \sqrt{\epsilon_N/2}$ is the width of a kink and $X_{\pm}(t)$ the location of a kink.

As mentioned in section 4, in the SR mechanism, the relation between the characteristic time scale of the jump process and the cycle of the external driving is very important. When the kink-antikink nucleation process in the array is compared with the jump process in the uncoupled single element, it is clear that a critical kink-antikink nucleation rate strongly relates to the AESR mechanism.

In the low noise strength and the large coupling strength range, the kink-antikink nucleation rate is well approximated with

$$\Gamma(F) = 12\sqrt{6}\pi^{-3/2}F^{1/2} \left(\frac{4\sqrt{2}\epsilon}{3D} \left(1 - \frac{3}{8}F \right) \right)^{1/2} \exp \left[-\frac{4\sqrt{2}\epsilon}{3D} \left(1 - \frac{3}{8}F \right) \right], \quad (30)$$

where F is the strength of a external force. The switch process of the array from the higher well to the lower one is triggered by the kink-antikink nucleation. In contrast, the switch process from the lower well to the higher one hardly occurs, even if the kink-antikink nucleation is activated. To take it into account, we consider the periodic kink-antikink nucleation rate as

$$\Gamma_{\pm\mp}(t) = \Gamma_c \left(\frac{1}{2} \pm \frac{1}{2} \cos \Omega t \right), \quad (31)$$

where Γ_c is the critical kink-antikink nucleation rate when the potential is most tilted, $\Gamma_c = \Gamma(A)$.

6.3 Large Noise Strength Range

In the large noise strength and the low coupling strength range, each particle can behave almost independently. According to Kramers formula, the transition rate between two states of the particle depends on the activation energy, which is constructed with the height of the potential barrier and an interaction potential energy due to the couplings.

Configurations of nearest neighbor particles of the one which we focus on are presented in Fig.(15). It is considered that the configuration Fig.(15.a) appears most frequently in the middle range of the noise strength, however, in the large noise strength limit, each of four configurations can be realized equally.

Thus the periodic transition probability is approximated with

$$W_{\pm\mp}(t) = r_0 \exp \left[\pm \frac{A}{D} \cos \Omega t - \frac{4\epsilon n_{\pm}(t)^2}{D} \right], \quad (32)$$

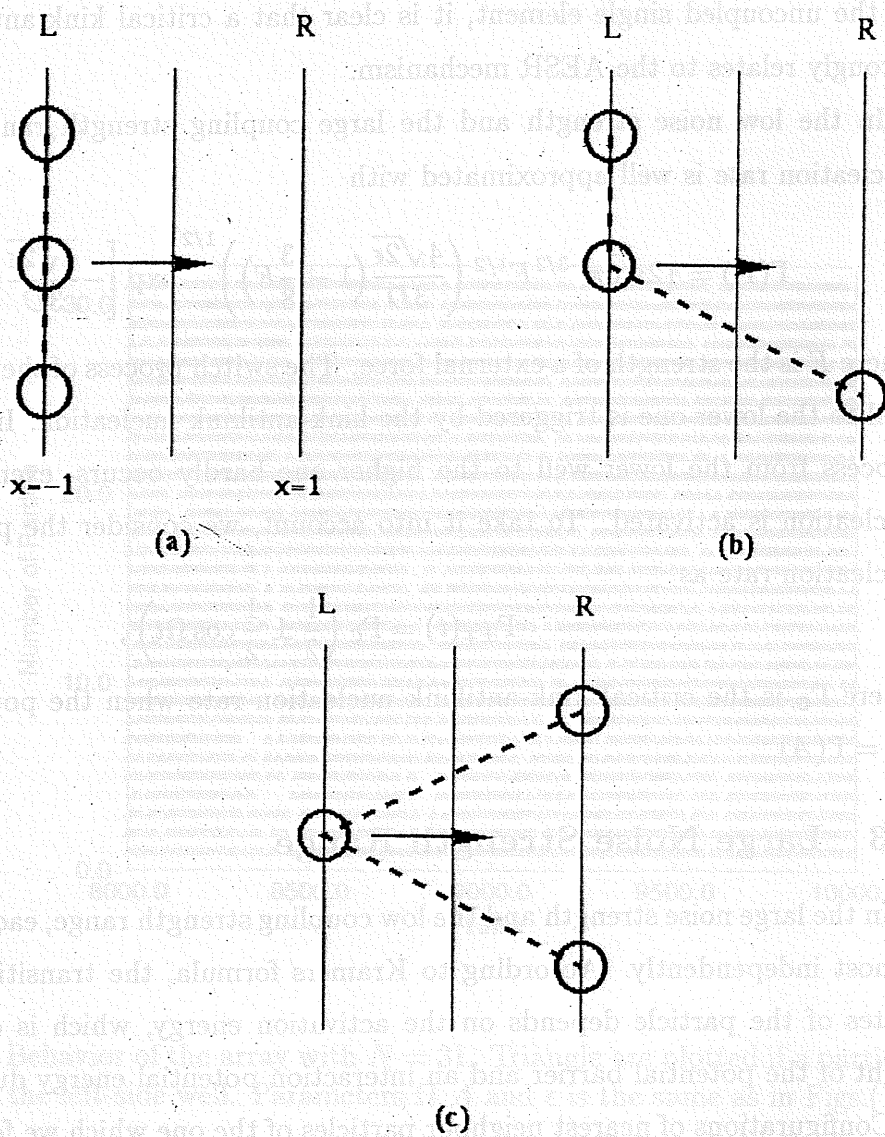


Figure 15: Configurations of nearest neighbor particles of the one we focus on. An activation energy due to the couplings is (a) 4ϵ , (b) 3ϵ , (c) -4ϵ , respectively.

where $n_{\pm}$ is given by Eq.(7). In Eq.(32), 4ϵ is the activation energy due to the couplings ; $\epsilon x^2/2$ (see Fig.(15)).

6.4 Results

We assume that the particle which we focus on can take place in only the two stable states and applying two-state theory. In the low noise strength range, the periodic transition probability is replaced by the periodic kink-antikink nucleation rate

$$W_{\pm\mp}^{\text{low}}(t) = \frac{1}{2}\Gamma_c \pm \frac{1}{2}\Gamma_c \cos \Omega t. \quad (33)$$

In the large noise strength range, the periodic transition probability is given by Eq.(32)

$$W_{\pm\mp}^{\text{large}}(t) = r_0 e^{-\frac{\epsilon}{D}} \pm r_0 e^{-\frac{\epsilon}{D}} \left[\left(\frac{(\Omega^2 + 4r_0^2)AD - 16r_0^2 A}{(\Omega^2 + 4r_0^2)D^2} \right)^2 + \left(\frac{8r_0 A \Omega}{(\Omega^2 + 4r_0^2)D^2} \right)^2 \right] \cos(\Omega t + \psi), \quad (34)$$

where $\psi = ((\Omega^2 + 4r_0^2)D - 16r_0^2)/8r_0\Omega$.

For a given point on (ϵ, D) plane, we compute the SNR from Eq.(18) with each case and choose the lower value as the SNR at the point. The SNR is showed as a function of the coupling strength and the noise strength in Fig.(16). In the low coupling and the large noise strength range, our approximation is not applicable. Though the height and the location of the peak are different from the ones numerically simulated, the qualitative features mentioned in Section.5 are observed.

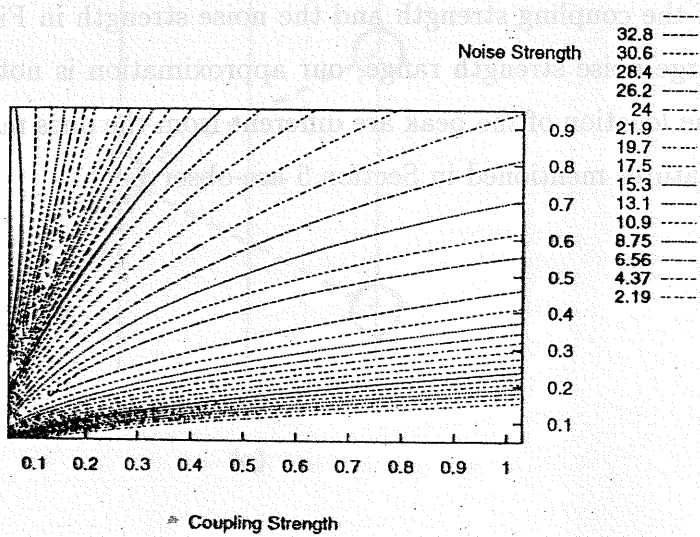
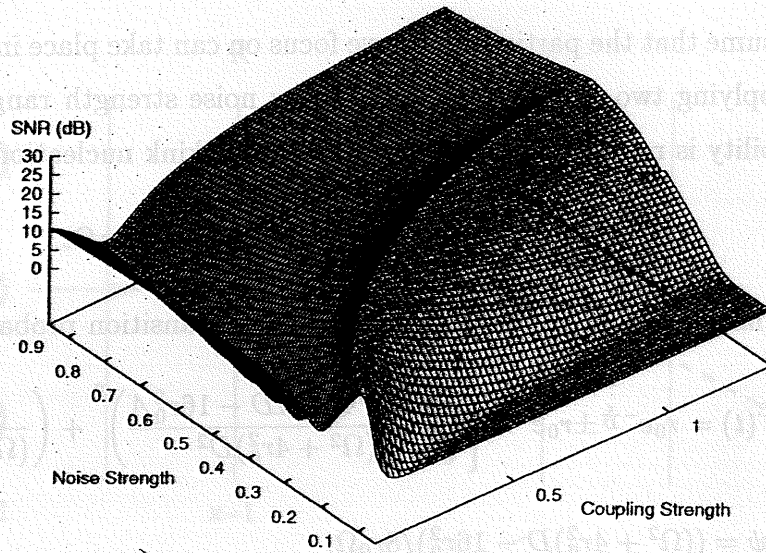


Figure 16: SNR as a function of coupling strength ϵ and D obtained from two-state theory applied to the global dynamics of the array and the excitation of the particle itself. $\Omega = \pi/200$, $A = 0.1$, $N \rightarrow \infty$. Above one is a 3D plot and the below is its contour plot.

7 Conclusion

In this paper, we numerically simulate SR and AESR and apply an expansion of two-state theory.

As for SR, double-Gaussian approximation is useful in the range of the noise strength and the amplitude of the external driving even in which two-state theory is not applicable. The SNR plot obtained from double-Gaussian approximation is in quantitatively good agreement with numerical one. The difficulty on numerically treating δ function in Eq.(26) prevents us to compute the SNR which is in both quantitative and qualitative agreement with simulation result. We believe that double-Gaussian approximation is applicable to wide range of models for SR : for example, the model in which noise add to the driving frequency not only the driving amplitude.

The SNR and the ROC for a locally and linearly coupled array each element of which shows the SR phenomenon are greatly improved comparing to those for the uncoupled element. The couplings play an important role in the mechanism of AESR. The behavior of a particle in the single element depends on the global dynamics of the array with kink-antikink nucleations in the low noise strength range, and the excitation of the particle itself in the large noise strength range. We approximate the periodic transition probability of the particle with the periodic kink-antikink nucleation rate and the one with the potential energy due to the couplings, respectively, and apply two-state theory. Under these assumption, the SNR plot as a function of the coupling strength and the noise strength is in qualitative agreement with numerical one. We consider that the SNR peak locate in the range of the coupling and noise strength where the picture that the global dynamics of the array with kink-antikink nucleations control the behavior of the single particle begin to break down.

In SR, the synchronization of the characteristic time scale of the jump process with the cycle of the external driving force enhances the SNR of the output. In AESR, the SNR is a monotonously increasing function of the noise strength even above the value of that where the synchronization is considered to be observed, because we assume that the switch process triggered by the kink-antikink nucleation against the external force hardly occurs in Eq.(31). The relation between the AESR mechanism and the synchronization of the characteristic time scale of the switch process with the cycle of the driving is not

clear in our picture. It remains to the future works.

We emphasize that one can improve the signal detection performance of systems which shows the SR phenomenon by coupling them into an array and that AESR may be applicable to design and operation of sensory systems.

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Stochastic resonance in coupled neuron models and its application to an associative memory

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Abstract

Stochastic resonance is a phenomenon that the response of a nonlinear system to a weak periodic input signal is optimized by the presence of noise with a particular level. In such noisy and nonlinear threshold systems as sensory neurons, a phenomenon of stochastic resonance is also shown by the optimum enhanced coherence of a firing rate with a input signal for a particular level of noise. In this paper, we investigate stochastic resonance by numerical simulations using FitzHugh-Nagumo neuron models with a noisy input. As an application of the coupled FitzHugh-Nagumo neuron models, we introduce a retrieving model of the Hopfield neural network, which has been studied as brain models. A performance as an associative memory is also examined with the model from numerical simulations.

1 Introduction

Stochastic resonance is a phenomenon that the coherence of a nonlinear system to a weak periodic signal can be enhanced by the presence of noise with a particular level. It was originally proposed as a possible explanation for the observed switching of the Earth's climate between ice ages and periods of relative warmth with a period of about

100,000 years [1]. In order to explain the climate dynamics, they introduced a double-well model [2], where a periodic input signal is weak one which can not by itself cause transitions between wells. It characterizes the effect of stochastic resonance that noise with particular magnitude produces the maximum coincidence of the noise-induced transition rate between wells with the periodic signal.

Stochastic resonance was also examined experimentally in sensory neurons [3], a bidirectional ring laser [4], a magnetic system [5], and so on. In particular, the characteristics of stochastic resonance were observed in sensory neurons, which are noisy and nonlinear threshold systems. In such systems, where a input signal is set to be too weak to cause firings by itself, threshold crossings which mean the firings of the neurons are caused by noise. Stochastic resonance is characterized by the optimum enhanced coherence of the response of the system (firing rate) with the input signal for a particular level of noise.

Recently, Collins *et al.* [6] have examined the characteristic of stochastic resonance in FitzHugh-Nagumo neuron model with a slowly varying aperiodic signal. They proposed "power norm" C_0 and "normalized power norm" C_1 as stochastic resonance measures. These quantities measure the correlation between the input signal and the firing rate. It shows stochastic resonance-like behavior that, with increasing noise, the correlations C_0 and C_1 rise sharply to a peak and drop thereafter for the FitzHugh-Nagumo neuron model. In another paper [7], a model with multiple FitzHugh-Nagumo neurons with the same aperiodic input signal were introduced, and the aforementioned C_1 computed with the input signal and ensemble averaged firing rate of these neurons was enhanced by increasing the number of the neurons.

In this paper, we examine the effect of stochastic resonance in the FitzHugh-Nagumo neuron model with aperiodic signals and high frequency periodic ones, because the previous studies of stochastic resonance were well performed for this model with low frequency periodic signals. We treat models of a single neuron and coupled neurons, and compute normalized power norm C_1 and mutual information [8, 9] as stochastic resonance measures. Using the mutual information, we also examine the effect of stochastic resonance from the point of view of information channel. Furthermore, as an application of stochastic resonance to coupled neuron models, we introduce a Hopfield-type network as a model for an associative memory [10]. The Hopfield-type neural networks have been studied as brain models which can perform recognition and retrieving. Seung *et al.* [11] proposed a retrieving model that bistable state systems were coupled in a Hopfield-type network. We examine the more realistic biological retrieving model using the FitzHugh-Nagumo neurons as nodes of the Hopfield-type network.

In Sec. 2, the FitzHugh-Nagumo neuron model and stochastic resonance measures are introduced. The results of numerical simulations with the two aforementioned signals are shown in Sec. 3, where a model for coupled neurons with high frequency periodic signal is also explained and the results of the numerical simulations are shown. In Sec. 4, the retrieving model is introduced, and the performance of an associative memory is examined. The paper concludes in Sec. 5.

2 Neuron model

2.1 the FitzHugh-Nagumo neuron model

We consider the FitzHugh-Nagumo neuron model [12] driven by an input signal and noise on condition that the input signal is a subthreshold signal. Subthreshold means that the amplitude of the input signal alone is not sufficient to cause the neuron to fire. The dynamical behavior is governed by the following differential equations

$$\begin{aligned}\epsilon \dot{v} &= v(v - a)(1 - v) - w + A + S(t) + \eta(t) \\ \dot{w} &= v - w - b\end{aligned}\quad (1)$$

where ϵ fixes the time scale of v which is the fast (voltage) variable, w is the slow (recovery) variable, and A denotes a subthreshold constant of activation set to be 0.03. The parameters are chosen as in a previous study [6], $\epsilon = 0.005$, $a = 0.5$, and $b = 0.15$. $S(t)$ stands for the input signal, and taken to have zero mean. The concrete form of $S(t)$ is explained in Sec. 3. $\eta(t)$ is the noise given by an Ornstein-Uhlenbeck stochastic process of the form

$$\dot{\eta}(t) = -\lambda_n \eta(t) + \lambda_n \xi(t) \quad (2)$$

where $\xi(t)$ is Gaussian white noise with zero mean and the autocorrelation, $\langle \xi(t)\xi(s) \rangle = 2D\delta(t - s)$. The autocorrelation of the Ornstein-Uhlenbeck process is $\langle \eta(t)\eta(s) \rangle = (D/\tau_n) \exp(-|t - s|/\tau_n)$, and its variance is D/τ_n . We choose correlation time $\tau_n (= \lambda_n^{-1})$ to be 0.001. The noise can be approximated to Gaussian white noise by choosing τ_n which is sufficiently small size.

We integrate Eq. 1 combined with Ornstein-Uhlenbeck process, Eq. 2 with a time step $\Delta t = 0.001$. Fig. 1 shows simulation results for the membrane potential $v(t)$ and a firing rate $R(t)$, which is defined later, for time 60, where the noise intensity is $D = 3.0 \times 10^{-6}$, and the signal is the sinusoidal function whose period is 20 and amplitude is 0.015. From this figure, it is observed that the membrane potential fluctuates around stable state normally, and, occasionally, the membrane potential crosses over the firing threshold (set to be 0.5) by the effect of the noise, that is, the neuron has a firing spike. After the firing threshold has been crossed, the membrane potential returns to the stable state deterministically within some refractory period (≈ 0.4). The firing spike train is convolved with a symmetric Hanning window (window width = 20) in order to obtain a firing rate. Hanning window function is defined by $\frac{1}{2}(1 - \cos 2\pi x)$ for the range of $(0 \leq x \leq 1)$. The obtained firing rate is the output $R(t)$.

2.2 stochastic resonance measures

In order to measure the coherence between the signal $S(t)$ and the output $R(t)$, we first introduce the “power norm” C_0 and the “normalized power norm” C_1 which are proposed in [6]. These quantities are defined as

$$C_0 = \frac{S(t)[R(t) - \overline{R(t)}]}{C_0} \quad (3)$$

$$C_1 = \frac{C_0}{[S^2(t)]^{\frac{1}{2}} \{ [R(t) - \overline{R(t)}]^2 \}^{\frac{1}{2}}} \quad (4)$$

where the over line means time average. C_1 is normalized to be divided by the standard deviations of the signal and of the firing rate. Since $S(t)$ has zero mean, C_0 is the covariance and C_1 is correlation coefficient. C_0 depends on the noise intensity because of the dependence of $R(t)$ on the noise intensity. As a measure of the coherence between $S(t)$ and $R(t)$, the correlation coefficient C_1 is more suitable than the covariance C_0 because C_1 is independent of the noise intensity.

Secondly, we introduce mutual information as another stochastic resonance measure. The mutual information $I(x, y)$ between the input bit series x and the output bit series y is defined as [8]

$$I(x, y) = H(y) - H(y|x) \quad (5)$$

$$H(y) = -P_y \log_2 P_y - P_{\bar{y}} \log_2 P_{\bar{y}} \quad (6)$$

$$H(y|x) = P_x(-P_{yx} \log_2 P_{yx} - P_{\bar{y}x} \log_2 P_{\bar{y}x}) + P_{\bar{x}}(-P_{y\bar{x}} \log_2 P_{y\bar{x}} - P_{\bar{y}\bar{x}} \log_2 P_{\bar{y}\bar{x}}) \quad (7)$$

$H(y)$ is the entropy of y and $H(y|x)$ is the conditional entropy of y for given x . Here a bit value in the input bit series x is +1 with probability P_x and is -1 with probability $P_{\bar{x}} = 1 - P_x$. Similarly a bit value in the output bit series y is +1 with probability P_y and is -1 with probability $P_{\bar{y}} = 1 - P_y$. The conditional probability P_{yx} is that y is +1 under the condition that x is +1 and the other conditional probabilities are defined in a similar way. When $P_x = P_{\bar{x}} = P_y = P_{\bar{y}} = 1/2$, $I(x, y)$ takes a value in the range of $[0, 1]$ and, if the output y has high correlation with the input x , $I(x, y)$ takes a high value. In this paper, we transform $S(t)$ to an input bit series x , that is, x is set to be +1 or -1 if the average of $S(t)$ during each period τ is positive or negative. Similarly, $R(t) - \overline{R(t)}$ is transformed to the output bit series y . Using the obtained bit series, we calculate $I(x, y)$.

3 Numerical simulation

3.1 an aperiodic signal

The aperiodic input signal $S(t)$ is now constructed according to an Ornstein-Uhlenbeck stochastic process Eq. 2 by

$$\dot{S}(t) = -\lambda_s S(t) + \lambda_s \xi(t) \quad (8)$$

where $\xi(t)$ is Gaussian white noise and the correlation time $\tau_s (= \lambda_s^{-1})$ is set to be 20. The signal has zero mean and a variance 1.5×10^{-5} . Hanning window width for $R(t)$ is set to be 20. Fig 2 shows C_1 at different noise intensities D computed by the numerical simulation for time 3000×20 , where the result shows the characteristic of stochastic resonance-like behavior that as a function of noise intensity D rapidly rises to a peak and then slowly drops for higher values of noise intensity. For $D = 3.0 \times 10^{-6}$, the output $R(t)$ has maximum correlation with the input $S(t)$, and C_1 obtains the peak. Fig. 3 shows mutual information $I(x, y)$ at different noise intensities D . The input bit series x and the output y are calculated from the obtained $S(t)$ and $R(t)$ for every period, $\tau = 1$. Similarly, a peak is shown for $D = 3.0 \times 10^{-6}$, and optimum information channel is obtained.

3.2 a high frequency signal

First, we analyze the properties of the FitzHugh-Nagumo neuron model on the effect of a noise alone [14]. The sign of firings is found from the firing spikes on the neuron model induced by the noise. Fig. 4 shows the interspike interval histograms for different values of noise intensities. For small noise intensity, the interspike interval distribution is very broad, while for large noise intensity the peak of interspike interval histogram is more pronounced. With increase of the noise intensities, while the mean value of the distribution is variable and changes smaller, the most probable value is almost constant. This result denotes that the neuron has the internal clock of the system with the period ≈ 1.2 .

From this result, we examine the FitzHugh-Nagumo neuron model including a forcing periodic input signal of period $T = 1.2$. The amplitude of the signal $S(t)$ is set to be 0.015. The Hanning window width for $R(t)$ is set equal to the period of $S(t)$. Fig. 5 shows C_1 and Fig. 6 shows mutual information $I(x, y)$ at different noise intensities D computed by the numerical simulation. The input bit series x and the output y are calculated from $S(t)$ and $R(t)$ be every period $\tau = 0.2$. Both results show the characteristic of stochastic resonance-like behavior. Fig. 7 shows interspike interval histograms for different values of noise intensities. For a noise intensity $D = 3.0 \times 10^{-6}$ where a maximum value of C_1 is obtained, the first left peak of the histogram is highest. Fig. 8, for an example, shows $v(t)$ and $S(t)$ for time 20. For $D = 3.0 \times 10^{-6}$, where the neuron does not occasionally fire, it fires near the top of the sinusoidal wave and the firing spikes are synchronized to the input signal. For $D = 10.0 \times 10^{-6}$, however, the neuron frequently fires off the top of the input and the firing spikes are not exactly synchronized to the input signal. Fig. 9 shows histograms of the input phase where the neuron fires. Although the phase of the top of the input sinusoidal wave is 0.25, for both histograms in this figure, most probable values are smaller than 0.25 and the distribution for $D = 10.0 \times 10^{-6}$ is broader than that for $D = 3.0 \times 10^{-6}$. We find that the neuron rather frequently fires near the bottom of the input sinusoidal wave for $D = 10.0 \times 10^{-6}$.

Secondly, we examine the effect of stochastic resonance varying the frequency of the periodic input signal. Fig. 10 show C_1 computed by the numerical simulation in the cases of $T = 1.2, 1.8, 2.4$ and $T = 5, 10, 20$. We find the characteristic of stochastic resonance-like behavior in the all cases. The noise intensity D which produce the maximum value of C_1 is variable as a function of T . For high frequencies, the effect of stochastic resonance is enhanced in the case of $T = 1.2$ near to the internal clock of the neuron. For low frequencies, the effect of stochastic resonance is quite enhanced in the case of $T = 20$ which is the most adiabatic condition.

3.3 coupled neurons

Here, we consider coupled FitzHugh-Nagumo neuron models. In neuron i , the dynamical behavior is governed by the following differential equations

$$\begin{aligned} \epsilon \dot{v}_i = & v_i(v_i - a)(1 - v_i) - w_i + A + S(t) + \eta_i(t) \\ & + \frac{\gamma}{N-1} \sum_{j=1}^N u(t - T_j) \end{aligned} \quad (9)$$

$$\begin{aligned}\dot{w}_i &= v_i - w_i - b \\ (i, j &= 1, \dots, N)\end{aligned}$$

where v_i is the voltage variable and w_i is the recovery variable in neuron i , and the parameters are equal to those of Eq. 1. N denotes the number of the neurons, γ the coupling constant set to be 0.02 and T_j the last firing time of neuron j ($j \neq i$). When neuron j fires at T_j , neuron i receives an excitatory input $u(t - T_j)$, where $u(t)$ is given by [13]

$$\begin{cases} u(t) = 0 & (t < 0) \\ u(t) = (t/t_r) \exp(-(t - t_r)/t_r) & (0 \leq t \leq t_r) \\ u(t) = ((t + t_f - t_r)/t_f) \exp(-(t - t_r)/t_f) & (t > t_r) \end{cases} \quad (10)$$

where $t_r = 0.06$ and $t_f = 0.05$. The shape of $u(t)$ is presented in Fig. 11. It is resemble to a realistic spiking of a neuron whose characteristic is rapidly rising and then slowly dropping as the time goes by. The input signal $S(t)$ is chosen to be the such periodic input signal as that in Sec. 3.2. Fig. 12 shows C_1 computed by the numerical simulation in the cases of $N = 1, 2, 5, 10$. $R(t)$ is calculated taking notice of one neuron. We find that the effect of stochastic resonance is enhanced increasing numbers of N . Fig. 13 shows interspike interval histograms for different values of noise intensities for coupled 5 neurons. For a noise intensity $D = 2.0 \times 10^{-6}$ which give a maximum value of C_1 , the first peak is highest, and higher and sharper than that for the single neuron model in Fig. 7. These results show the enhanced effect of stochastic resonance in coupled FitzHugh-Nagumo neuron models due to coupling.

4 Associative memory

4.1 retrieving model

In Sec. 3, we examined the effects of stochastic resonance in single and coupled neuron models. In this section, we construct a Hopfield-type neural network [10] whose nodes are the FitzHugh-Nagumo neuron models, and apply it to retrieving,

$$\begin{aligned}\epsilon \dot{v}_i &= v_i(v_i - a)(1 - v_i) - w_i + A + I_i(t) + \eta_i(t) \\ &\quad + \frac{\gamma}{N-1} \sum_{j=1}^N W_{ij} u(t - T_j) \\ \dot{w}_i &= v_i - w_i - b \\ (i, j &= 1, \dots, N)\end{aligned} \quad (11)$$

where N is the number of the coupled neurons, and the parameters are $A = 0$ and coupling constant γ takes several values in numerical simulation. The values of the other parameters are equal to those of Eq. 9. $I_i(t)$ is the input signal including a input pattern and is given as the following function,

$$I_i(t) = I_0 \sin \omega_0 t + \alpha p(i) \quad (12)$$

where $p(i)$ is the value of the node i in a input pattern. The parameters are chosen to be $I_0 = 0.015$ and $\omega_0 = 2\pi/1.2$ equal to those in Sec. 3.2. The input constant α takes several values in numerical simulation. W_{ij} is the weight of coupling between neurons i and j , and is given by Hebb's rule [15] for $i \neq j$,

$$W_{ij} = \sum_{p=1}^Q (\zeta_i^p - a^p)(\zeta_j^p - a^p) \quad (13)$$

where Q stands for the number of memory patterns, ζ_i^p the value of node i in the p -th memory pattern ζ^p , and $a^p = \langle \zeta_i^p \rangle$, averaged over all nodes. Here W_{ii} is set to be 0.

In this paper, we consider a simple network example consisting of 4×4 nodes. Two patterns ζ^1 and ζ^2 shown in Fig. 14 are stored in the memory. The nodes are labeled by numbers 1, 2, ..., 16. Patterns ζ^p are represented by 4×4 binary data. The black node denotes a firing state ($\zeta_i^p = 1$), and the white node denotes a resting state ($\zeta_i^p = 0$). Therefore, under the Hebb's rule given by Eq. 13, the way of the coupling between neurons representing a same state is excitatory and the way of coupling between neurons representing a different state is inhibitory. As the way to examine the retrieving model, we use two input patterns A_1 and A_2 shown in Fig. 15, which coincide with the memory pattern ζ^1 except for one node. The black node denotes the presence of a input ($p(i) = 1$), and the white node denotes no input ($p(i) = 0$). We also consider a network consisting of 8×8 nodes. Fig. 16 shows memory patterns ζ^p and Fig. 17 shows two input patterns B_1 and B_2 in this case.

In order to measure the performance as an associative memory, first, we define the association ratio R_a as the averaged ratio of the number of the successful memory retrieval to the number of the input peaks. The successful memory retrieval of the network is defined that the neurons belonging to the black node of the memory pattern ζ^1 fires and the others does not fire within the every half period where the input sinusoidal signal is positive. Secondly, we define that R_f is the averaged rate of firing within the every half period where the input sinusoidal signal is positive in the firing neurons $\{1, 2, 3, 4\}$, R_r in the resting neurons $\{5 - 15\}$ and R_e in the excessive neuron $\{16\}$ for the pattern A_1 . As other measures of the performance as an associative memory, we compute the difference of the averaged rate of firing between firing neurons and resting neurons, $R_1 = R_f - R_r$, and between firing neurons and excessive neurons, $R_2 = R_f - R_e$. For the pattern A_2 , similarly, we define that R_f is the averaged rate of firing in the firing neurons $\{1, 2, 3\}$, R_r in the resting neurons $\{5 - 16\}$ and R_m in the missing neuron $\{4\}$, and compute the difference of the averaged rate of firing between firing neurons and resting neurons, $R_1 = R_f - R_r$, and between firing neurons and missing neurons, $R_3 = R_f - R_m$.

4.2 numerical simulation

First, We carry out the numerical simulation fixing the parameters $\alpha = 0.03$ and $\gamma = 0.6$. Fig. 18 shows a time series of $v_i(t)$ ($i = 1, 2, \dots, 16$) and $S(t)$ in the cases of two input patterns A_1 and A_2 . For the pattern A_1 , the firings of the excessive neuron $\{16\}$ is suppressed by the inhibitory coupling with neurons $\{1, 2, 3, 4\}$. For the pattern A_2 , the firings of the missing neuron $\{4\}$ is encouraged by the excitatory coupling with neurons $\{1, 2, 3\}$. Fig. 19 shows R_a computed by the numerical simulation in the cases

of two input patterns A_1 and A_2 . These results show the characteristic of stochastic resonance-like behavior that rapidly rises to a peak and then slowly drops for higher values of noise intensity. Fig. 20 shows R_1 and R_2 for the pattern A_1 . Both results show the characteristic of stochastic resonance-like behavior. Fig. 21 shows R_1 and R_3 for the pattern A_2 . R_1 shows the characteristic of stochastic resonance-like behavior. R_3 is monotonously increasing at small values, which is expected in this model. Secondly, we carry out the numerical simulations for several values of α and γ in the case of the input pattern A_1 . Fig. 22 shows R_a for several values of $\alpha = 0.02, 0.03, 0.04, 0.05$. For $\alpha = 0.02$ and 0.03 , the input signal $I_i(t)$ are subthreshold modulations, so stochastic resonance-like behavior is found. While, for $\alpha = 0.04$ and 0.05 , neurons $\{1, 2, 3, 4, 16\}$ have deterministic firings, so stochastic resonance-like behavior is not found. From this result, $\alpha = 0.03$ is more suitable. Fig. 23 shows R_a for several values of $\gamma = 0.4, 0.6, 0.8$. For $\gamma = 0.4$ and 0.6 , the values of R_a becomes large increasing the values of γ . For $\gamma = 0.8$, the value of γ is too large, so the membrane potential of neuron $\{16\}$ which has been stimulated by inhibitory coupling with neurons $\{1, 2, 3, 4\}$ is widely off the stable state, which causes the neuron $\{16\}$ to easily fire reversely. So the values of R_a is a little smaller than that for $\gamma = 0.6$.

Fig. 24 shows R_a computed by the numerical simulation in the cases of two input patterns B_1 and B_2 for the network of 8×8 nodes. These results show the characteristic of stochastic resonance-like behavior but R_a for the pattern B_1 is smaller than that for the pattern A_1 in the case of 4×4 nodes. Fig. 25 shows R_1 and R_2 for the pattern B_1 . Both results in this case are larger than those for the pattern A_1 in the case of 4×4 nodes. From those results, we find that the performance as an associative memory is enhanced for large networks if we use the averaged rate of firing. Fig. 26 shows R_1 and R_2 for the pattern B_2 . Both results in this case are larger than those for the pattern B_1 , and the performance as an associative memory is enhanced.

5 Conclusion

We demonstrated the characteristic of stochastic resonance, that the response of the system is enhanced for a particular level of noise, in the FitzHugh-Nagumo neuron model with aperiodic signals and high frequency periodic ones. In the coupled neuron models, the effect of stochastic resonance was enhanced due to the coupling. Then, as the application of a coupled neuron models, we investigated the retrieving model of the Hopfield-type network, which performed as an associative memory with the use of the effect of stochastic resonance.

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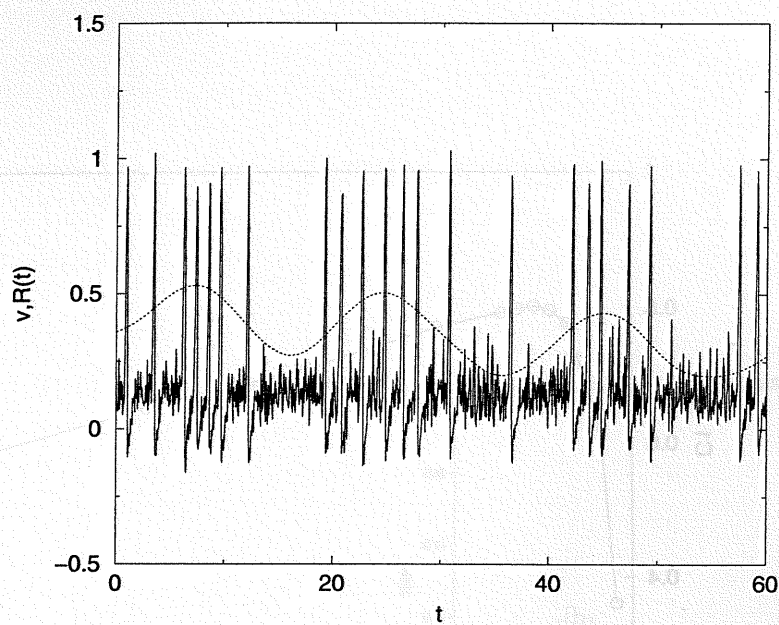


Figure 1: Membrane potential $v(t)$ (solid line) and a firing rate $R(t)$ (dotted line)

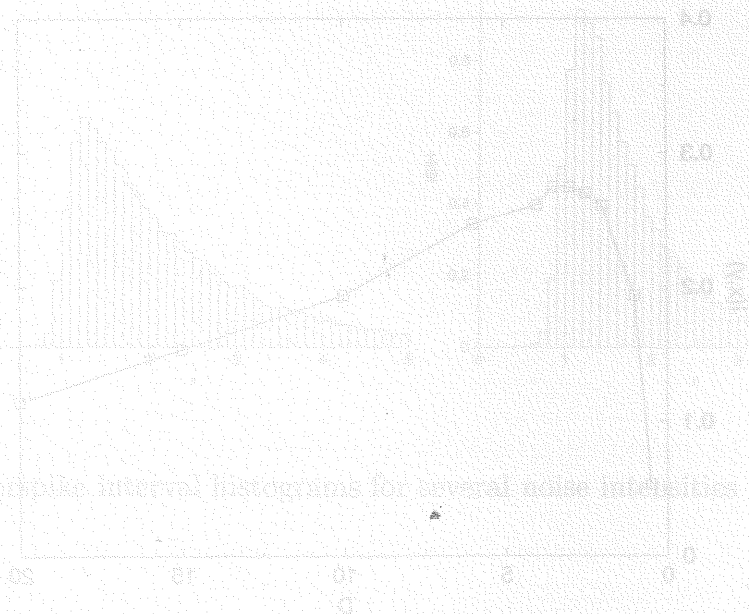


Figure 3: (a) $v(t)$ vs. t for single neuron with the aperiodic input

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Figure 2: C_1 vs. $D \times 10^6$ for single neuron with the aperiodic input

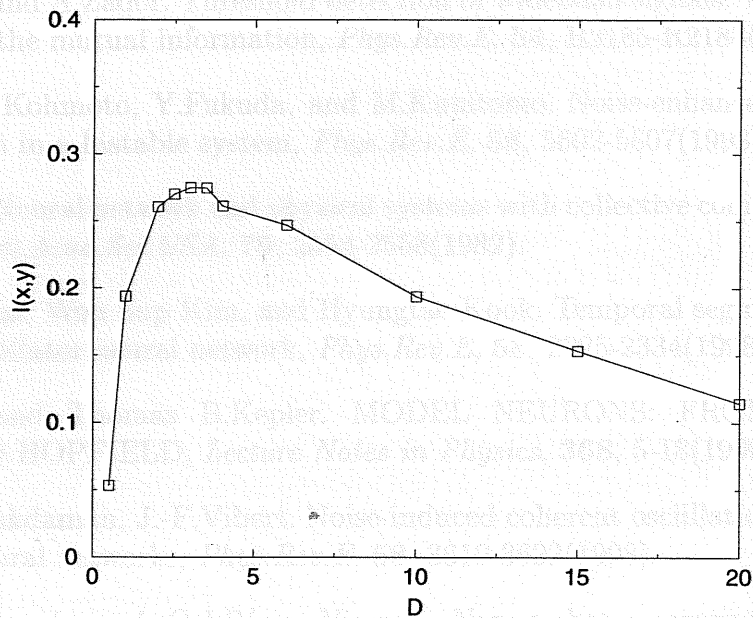


Figure 3: $I(x,y)$ vs. $D \times 10^6$ for single neuron with the aperiodic input

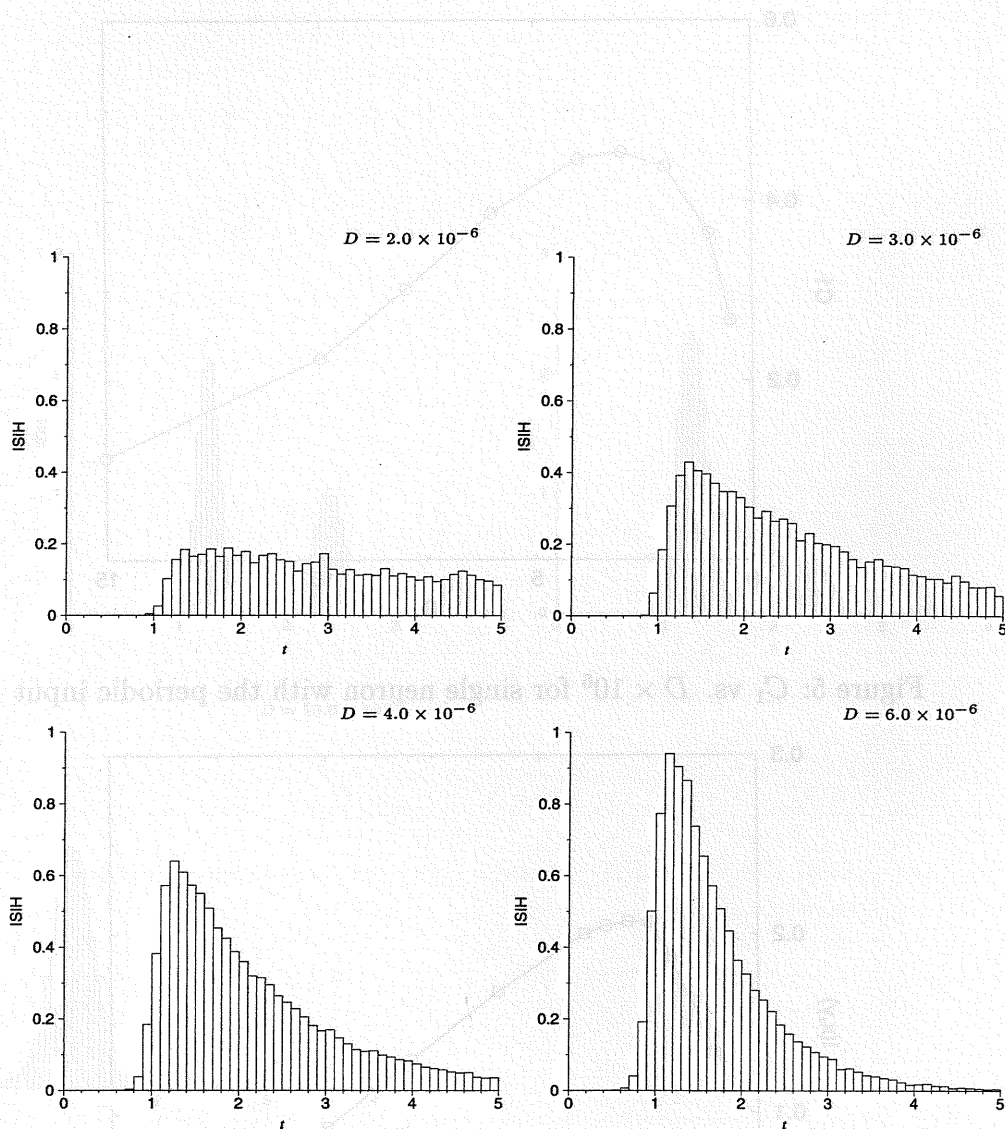


Figure 4: Interspike interval histograms for several noise intensities D for single neuron

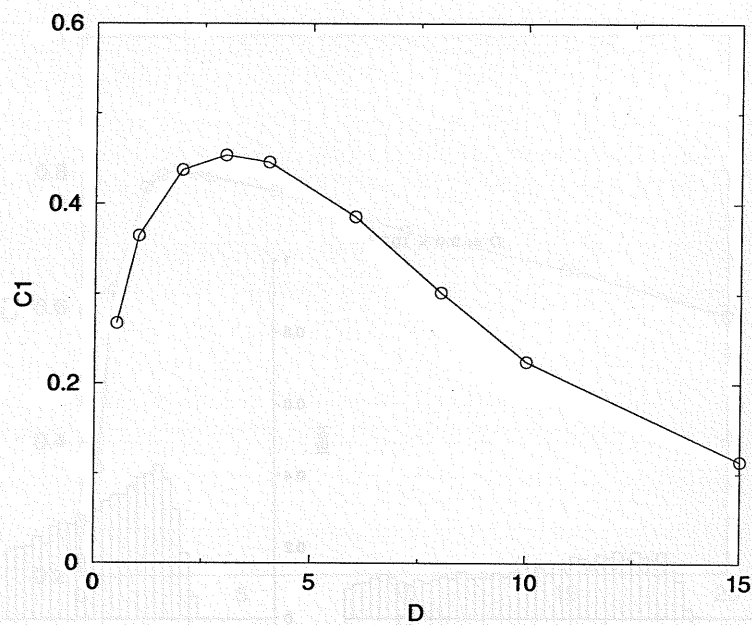


Figure 5: C_1 vs. $D \times 10^6$ for single neuron with the periodic input

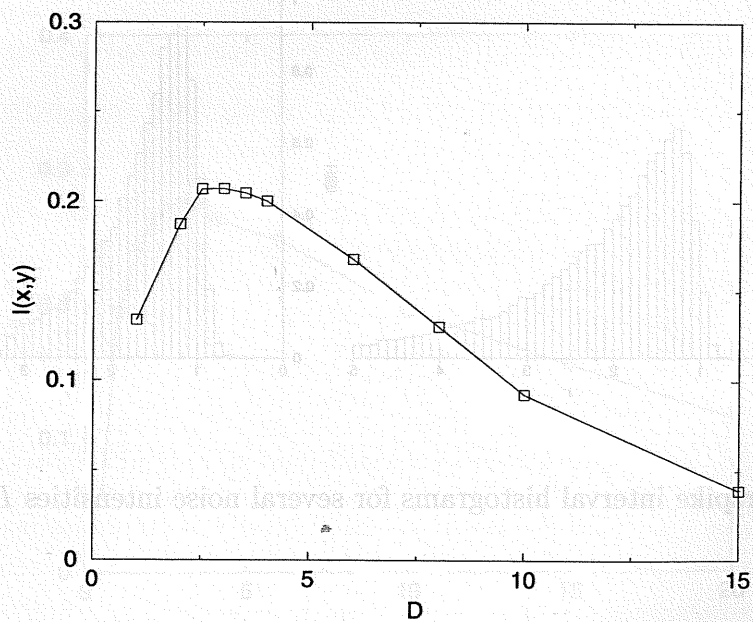


Figure 6: $I(x,y)$ vs. $D \times 10^6$ for single neuron with the periodic input

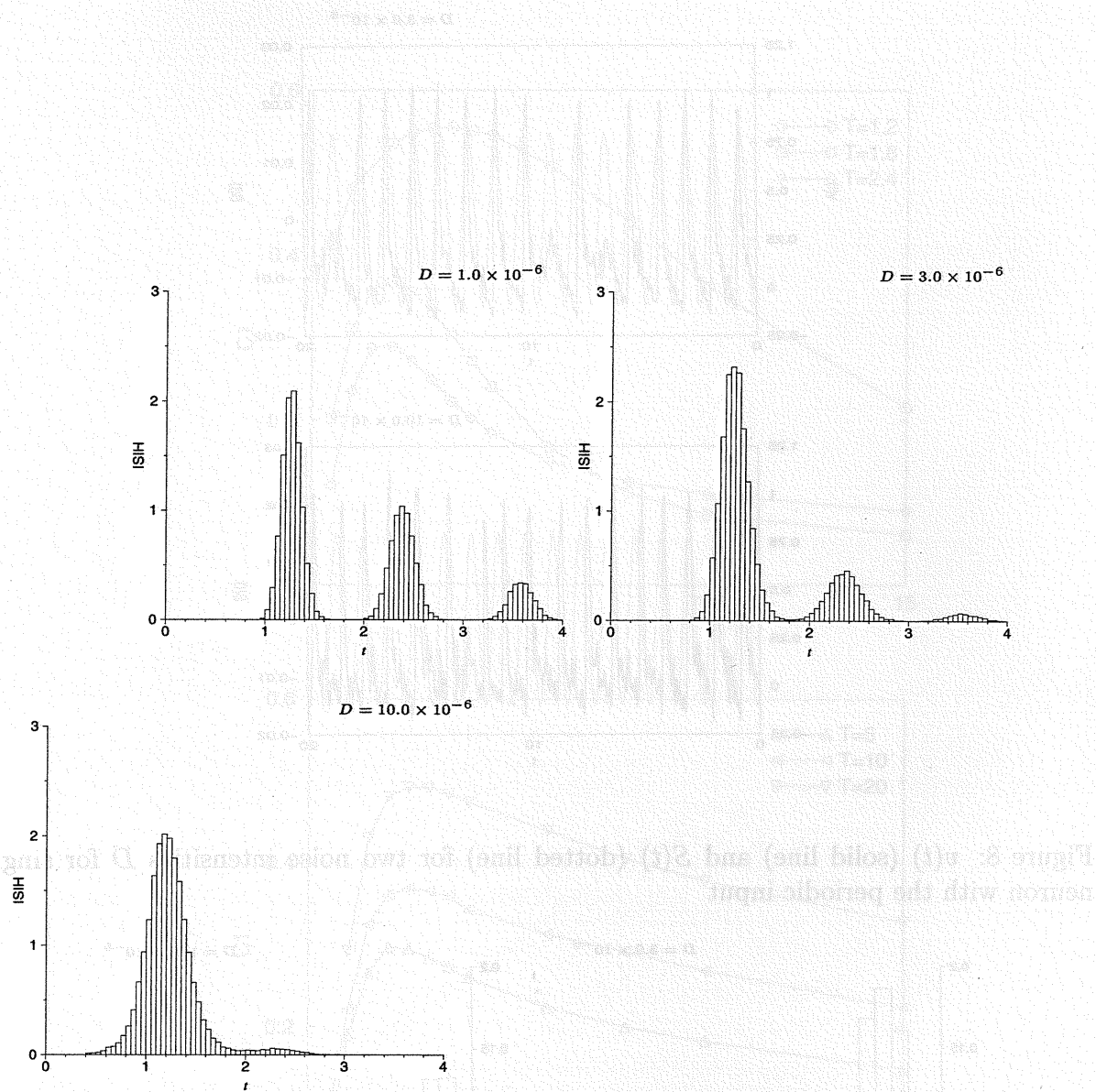


Figure 7: Interspike interval histograms for several noise intensities D for single neuron with the periodic input

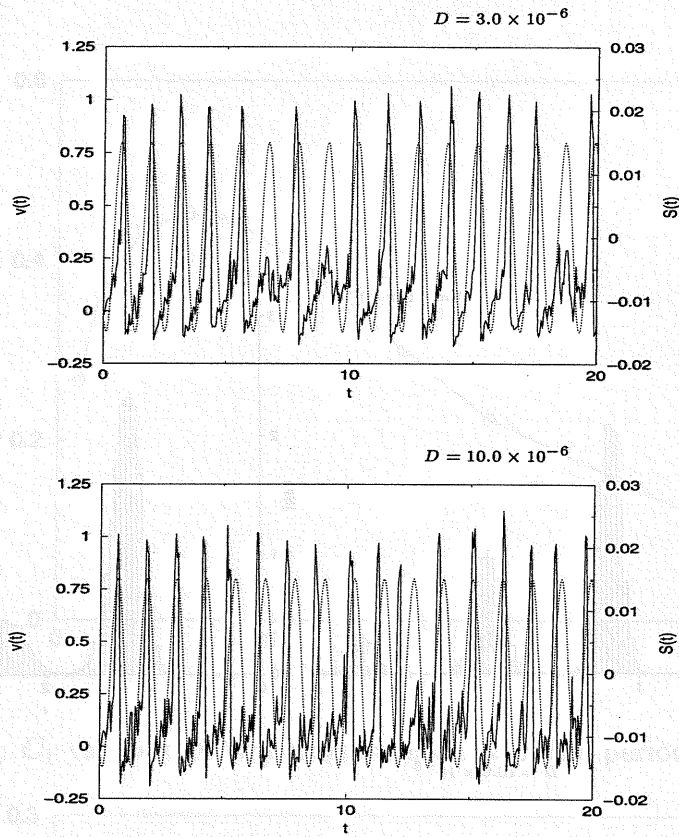


Figure 8: $v(t)$ (solid line) and $S(t)$ (dotted line) for two noise intensities D for single neuron with the periodic input

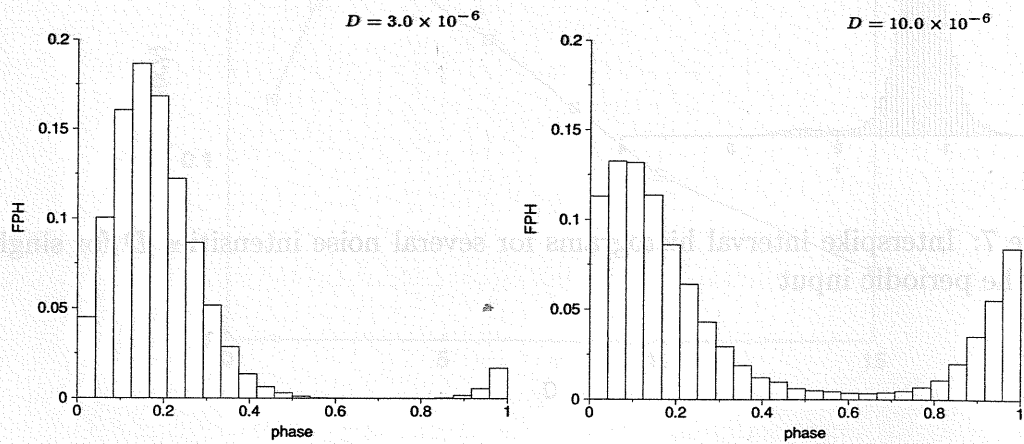


Figure 9: Firing phase histograms for two noise intensities D for single neuron with the periodic input

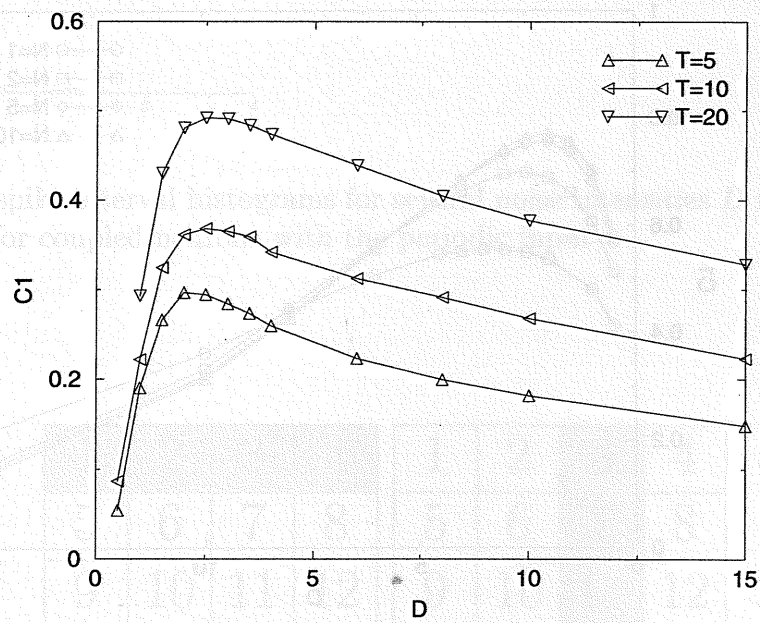
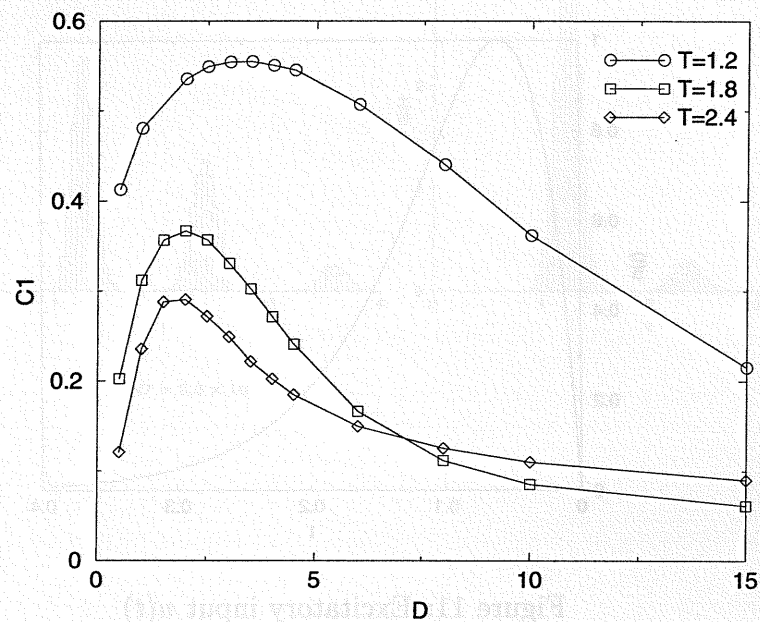


Figure 10: C_1 vs. $D \times 10^6$ for several periods T of input signal for single neuron with the periodic input

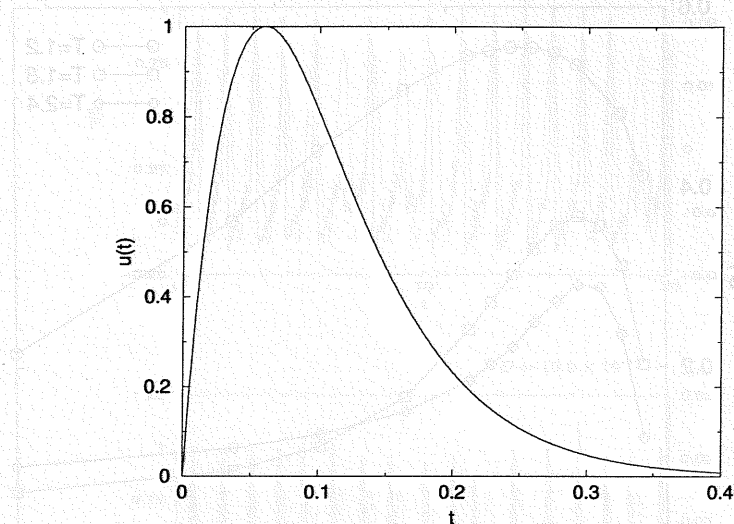


Figure 11: Excitatory input $u(t)$

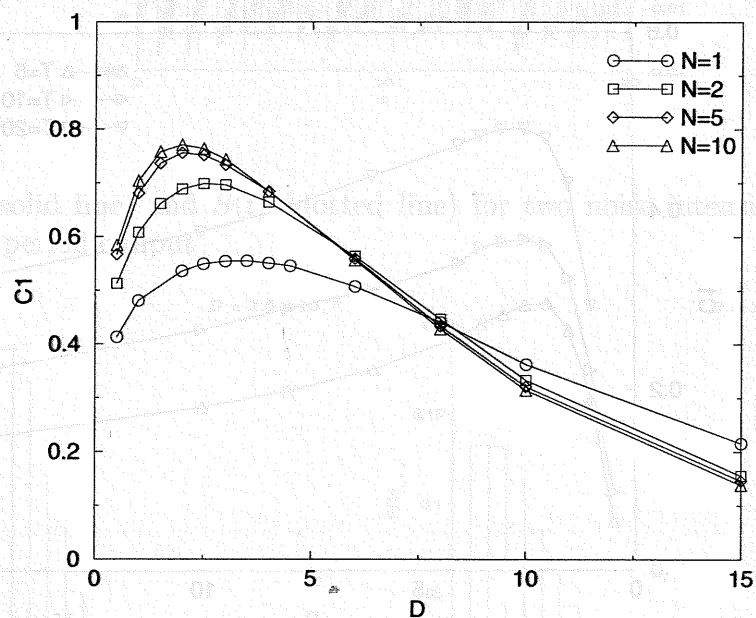


Figure 12: C_1 vs. $D \times 10^6$ for the number of neurons $N = 1, 2, 5, 10$ for coupled neurons with the periodic input

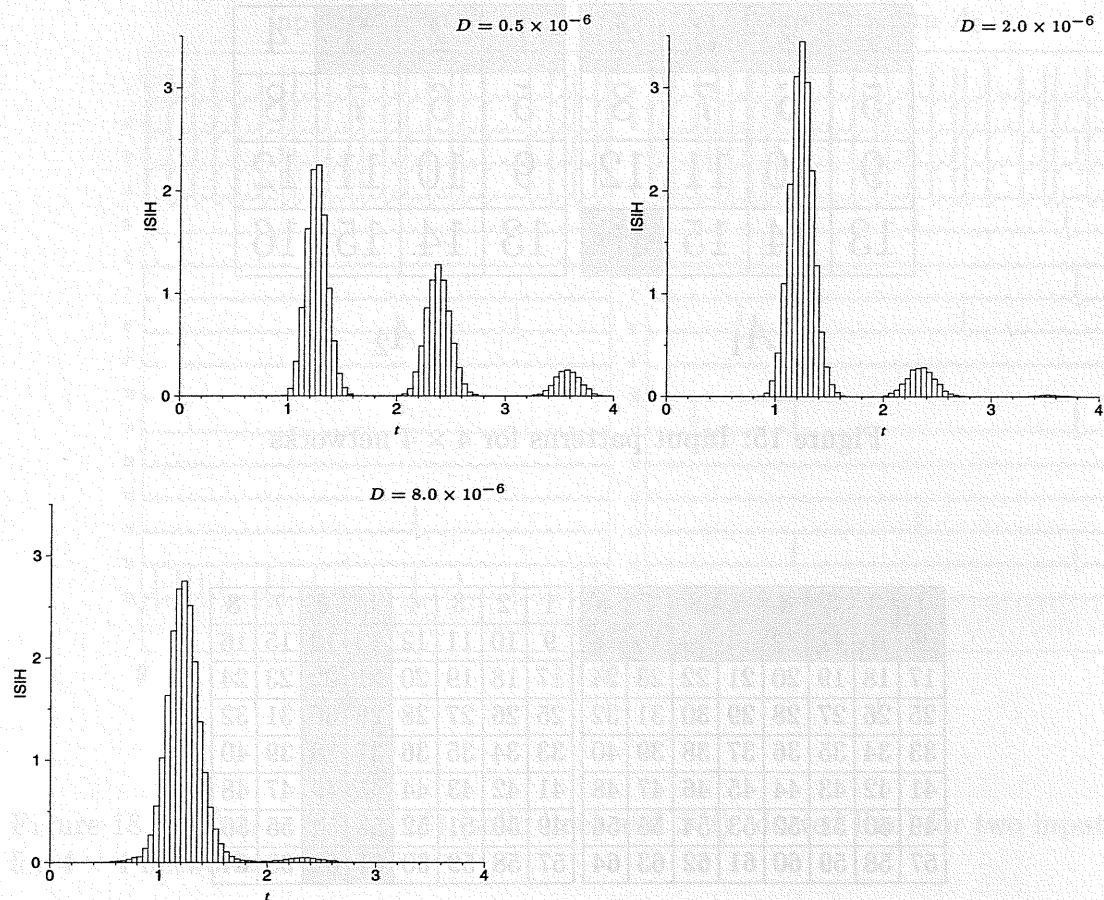


Figure 13: Interspike interval histograms for several noise intensities D for the number of neurons $N = 5$ for coupled neurons with the periodic input

1	2	3	4	1	2	3	4
5	6	7	8	5	6	7	8
9	10	11	12	9	10	11	12
13	14	15	16	13	14	15	16

ζ^1
 ζ^2

Figure 14: Memory patterns for 4×4 networks

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

A_1

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

A_2

Figure 15: Input patterns for 4×4 networks

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56
57	58	59	60	61	62	63	64

ζ^1

ζ^2

Figure 16: Memory patterns for 8×8 networks

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56
57	58	59	60	61	62	63	64

B_1

B_2

Figure 17: Input patterns for 8×8 networks

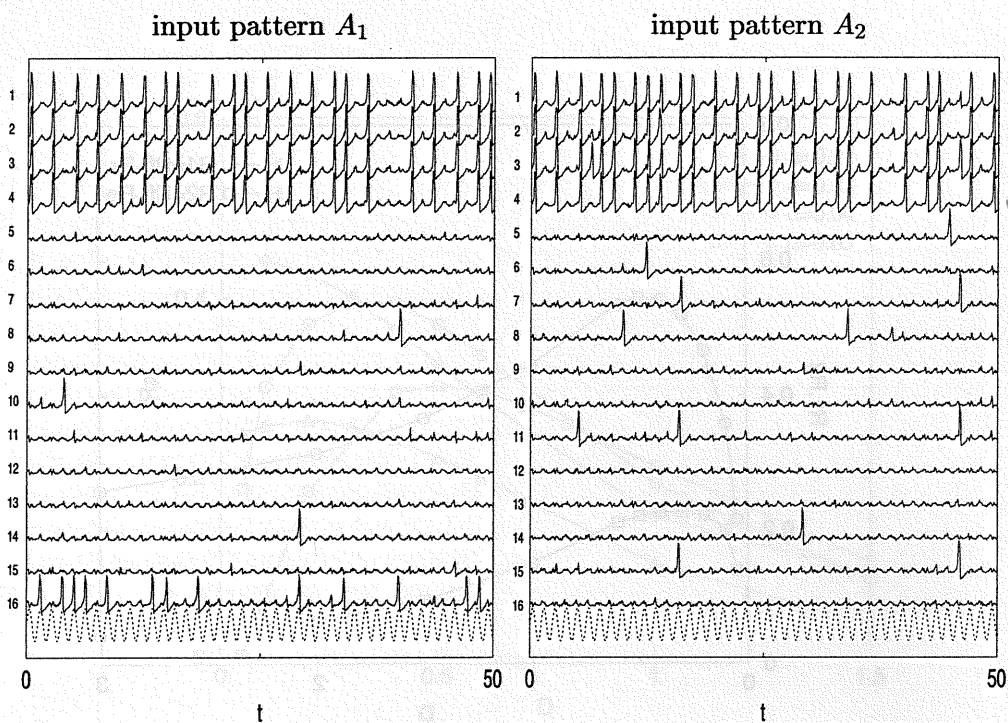


Figure 18: $v_i(t)$ ($i = 1, 2, \dots, 16$) (solid line) and $S(t)$ (dotted line) for two input patterns for 4×4 networks

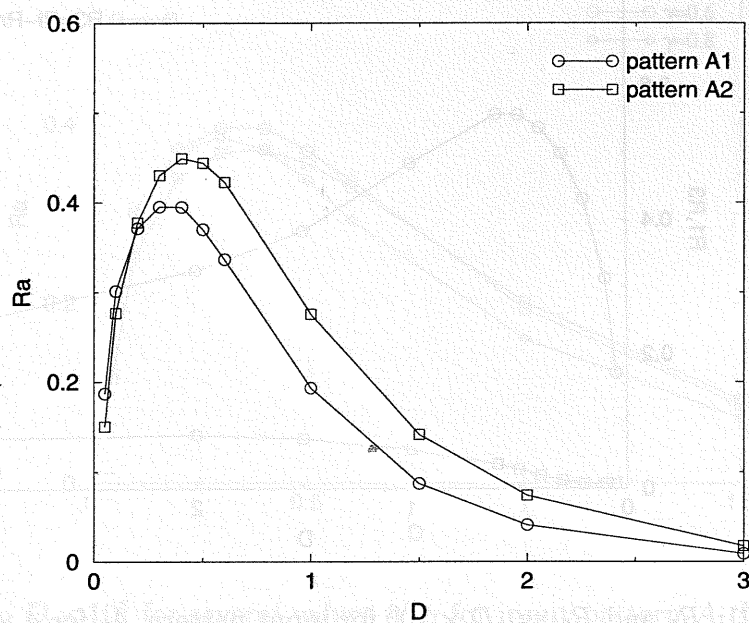


Figure 19: R_a vs. $D \times 10^6$ for input patterns A_1 and A_2 for 4×4 networks

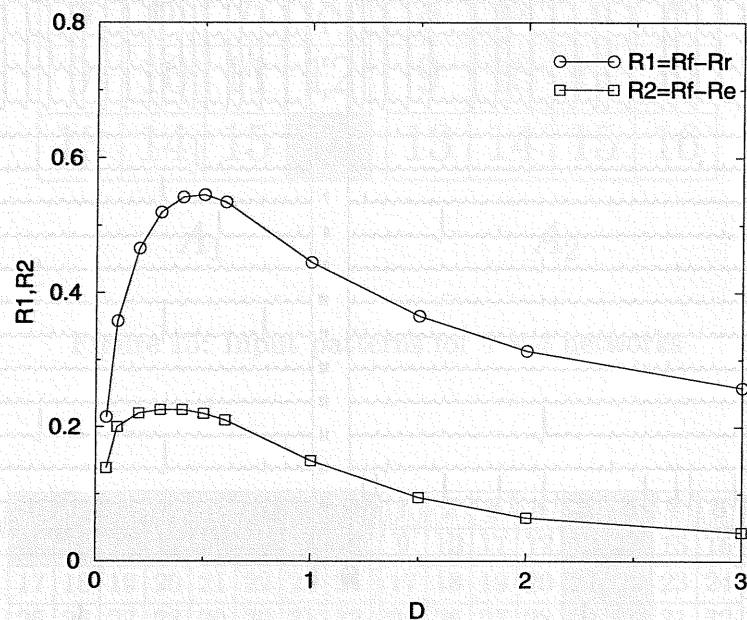


Figure 20: R_1 and R_2 vs. $D \times 10^6$ for input pattern A_1 for 4×4 networks

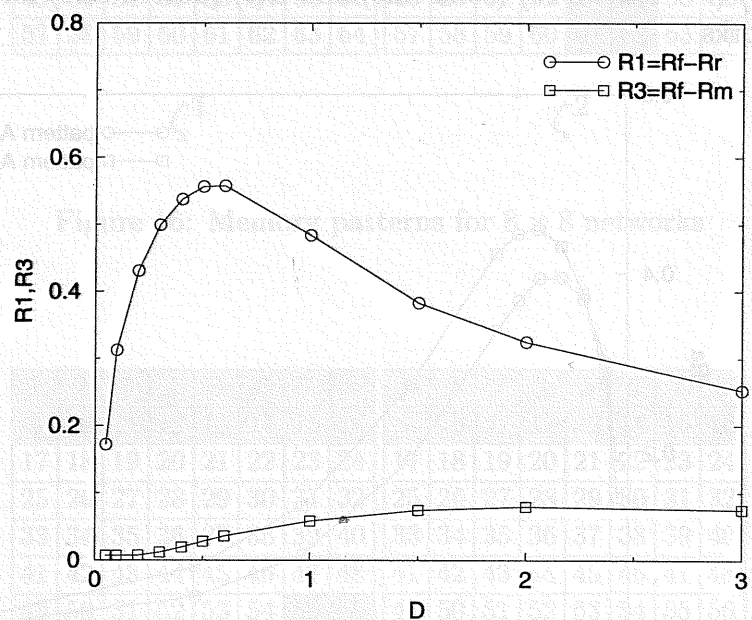


Figure 21: R_1 and R_3 vs. $D \times 10^6$ for input pattern A_2 for 4×4 networks

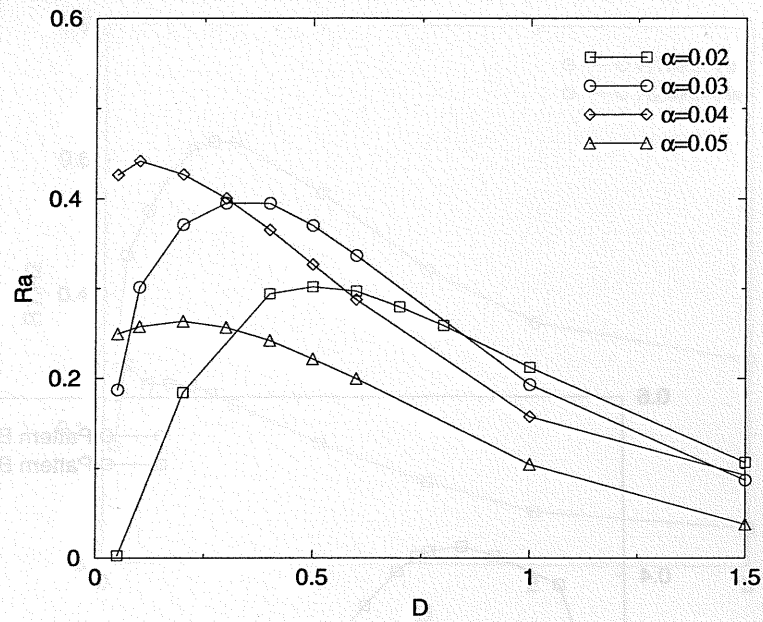


Figure 22: R_a vs. $D \times 10^6$ for several values of α for input pattern A_1 for 4×4 networks

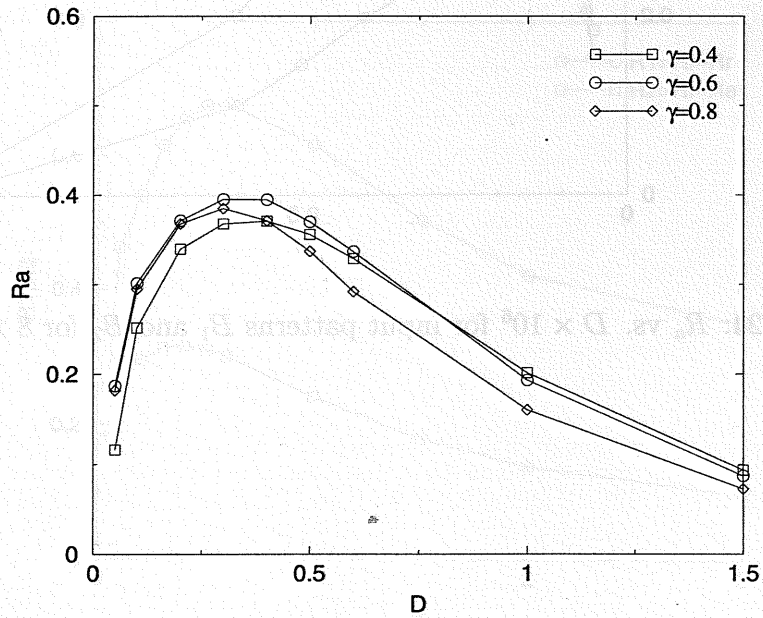


Figure 23: R_a vs. $D \times 10^6$ for several values of γ for input pattern A_1 for 4×4 networks

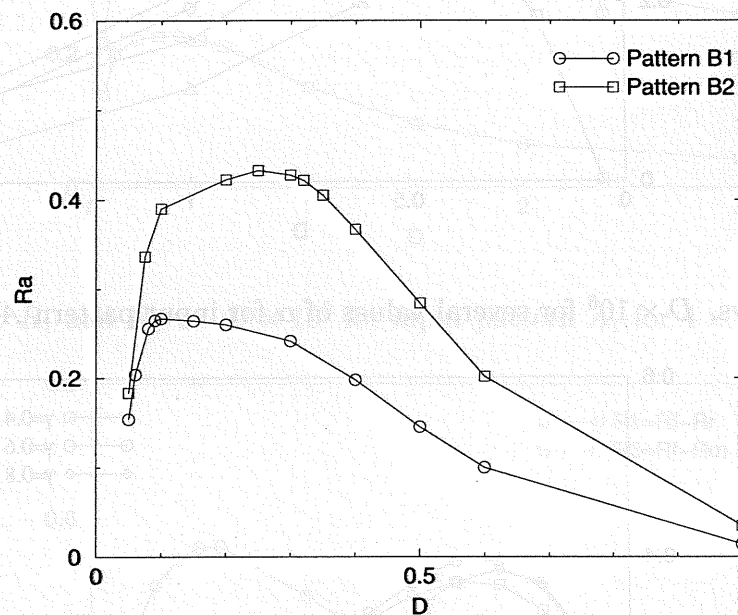


Figure 24: R_a vs. $D \times 10^6$ for input patterns B_1 and B_2 for 8×8 networks

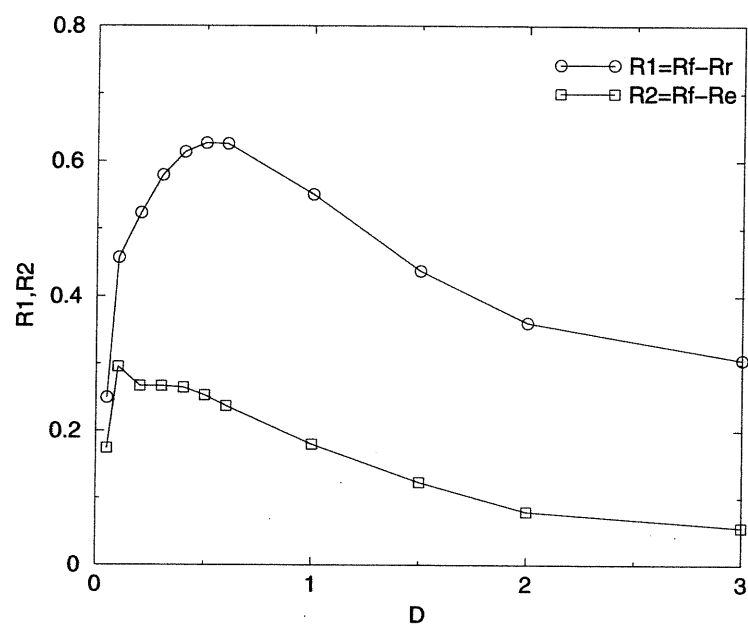


Figure 25: R_1 and R_2 vs. $D \times 10^6$ for input pattern B_1 for 8×8 networks

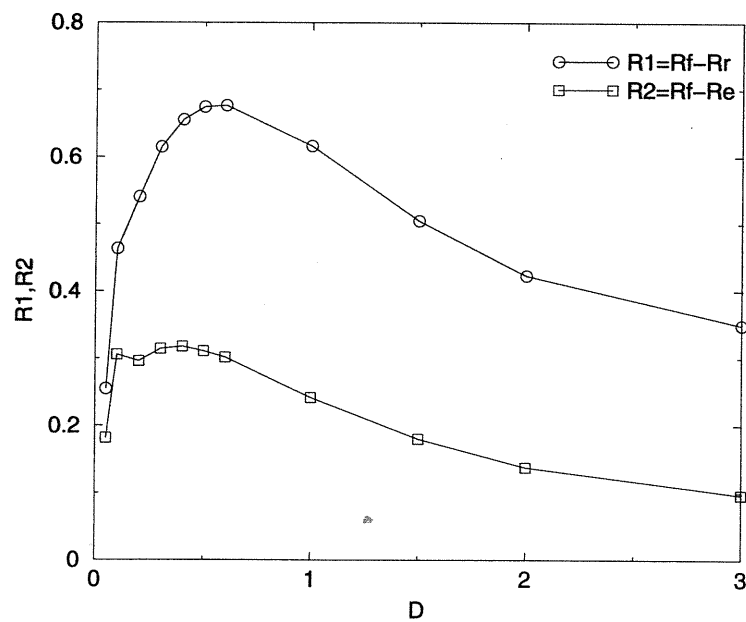


Figure 26: R_1 and R_2 vs. $D \times 10^6$ for input pattern B_2 for 8×8 networks