

Investigation of Geomorphological Properties Using Voronoi Discretization

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Synopsis

Geomorphological properties derived from watersheds play a major role in many hydrological analysis and modeling. Area-distance function is an example of geomorphological property. The present work compared time-area functions derived from area-distance function applying a single velocity value and applying different velocity values. This method discretizes the watershed using Voronoi cells, constructed from triangulated irregular networks (TIN's). Through the comparison between the two velocity criteria was possible to state that the use of spatially distributed velocities seems to be a more coherent approach to derive time-area functions. Therefore, multi-velocities approach seems reasonable for hydrological modeling.

Keywords: Area-distance function, time-area function, Voronoi, hydrological modeling

1. Introduction

Geomorphological properties derived from watersheds play a major role in many hydrological analysis and modeling. Basin area, drainage density, river length and slope and width function are some of the information that can be derived from this data. This information is applied to the simplest hydrological models, such as the Rational Method, and to the most complex physically-based distributed models.

The number of links, or points, in a watershed are related to their respective distances to the outlet by the so called width function (Rodriguez-Iturbe, 1997). The area-distance function is a particular case of the width function since it uses total area instead of the number of links. These functions are of great importance when investigating a watershed hydrological behavior.

The Geomorphological Unit Hydrograph (Rinaldo and Rodriguez-Iturbe, 1996) can be

derived from a width function. The hydrological model TOPMODEL (Beven and Kirkby, 1979) uses the area-distance function for flow routing. Area-distance function is usually used for distributing the hydrograph in time. This task is done through the transformation of the area-distance function in a time-area function applying a velocity parameter.

Early field studies showed that processes taking place, respectively, in the hillslopes and in the channel network are characterized by distinct time scales of transport (Emmet, 1978), suggesting that the velocities of the overland and subsurface flow and of the channel streamflow can differ by order of magnitude. This has long been recognized as a primary source of the overall variance of the hydrograph (Lazzaro, 2008).

Robinson et al. (1995), cited by Lazzaro (2008), investigated the issue of the relative contribution of hillslope processes and network geomorphology to the hydrologic response of natural catchments over

a range of catchment sizes, using geomorphology based models of runoff routing. In their work hillslope and network processes are assumed entirely independent. They concluded that the dispersion originated by network response becomes largely predominant as a threshold of about 10 km² is exceeded.

Rinaldo et al. (1995) observed that the typical long-tail observed in natural hydrographs is a dynamic effect resulting from the delay introduced by hillslope transport processes.

Saco and Kumar (2004), cited by Lazzaro (2008), confirmed that as hillslope velocity becomes smaller enough if compared with channel velocity, the variance, duration and peak discharge of the generated hydrograph are strongly affected by the distribution of hillslope lengths.

In this sense, the present work compared time-area functions derived from area-distance functions applying a single velocity value and applying different velocity values. The method discretizes the watershed in Voronoi cells. Voronoi cells are constructed from triangulated irregular networks (TIN's) and have the advantage of providing a natural framework for finite-difference modeling and a better representation of topographic surface through the TIN structure (Tucker et al., 2001).

A graphical framework was implemented in Matlab in order to import watershed coordinate points from raster files, create TIN and Voronoi networks, solve for pits and flat areas, define drainage network, classify rivers according to the Horton-Strahler method and extract area distance function relating it to river order.

This method was tested in three watersheds located in the south part of Brazil: (1) Pequeno River watershed with an area of 104 km², (2) Cubatão River watershed with 394 km² and (3) Pinus I watershed with 0.16 km².

2. Materials and Methods

Study area

The Cubatão River watershed is inserted into the basin of the river Cubatão North (BHRC)(Fig. 1). The BHRC comprises the municipalities Garuva and Joinville, where 80% of the basin is in the city

of Joinville. The region is located in the northeastern state of Santa Catarina, a distance of 180 km from Florianópolis, capital city. The Cubatão River watershed outlet is defined by a dam, located near the federal highway BR-101. This watershed has an area of 394.23 km² (80.13% of the total area of BHRC) and the main channel extension is about 61.22 kilometers (62.56% of the total length of the River Cubatão North).

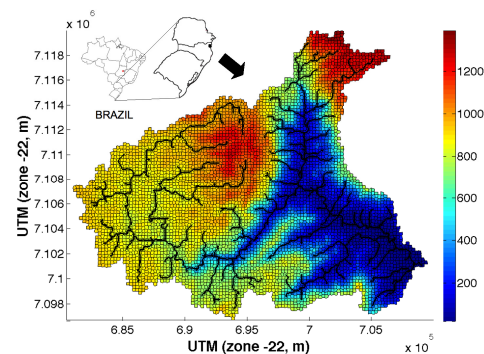


Fig. 1 Cubatão River watershed location, elevation meters

The Pequeno River catchment (104 km²) is located in São José dos Pinhais city, Curitiba metropolitan region, Paraná State, Brazil (Fig. 2). The topography is characterized by moderate slopes and its elevation varies from 895 m to 1270 m. The land use of this catchment comprises urban area (4%), agriculture and exposed areas (3%), forest (54%), grassland (35%), wetland (3%) and others (1%). At least 15% of the catchment is permanently saturated (Santos and Kobiyama, 2008).

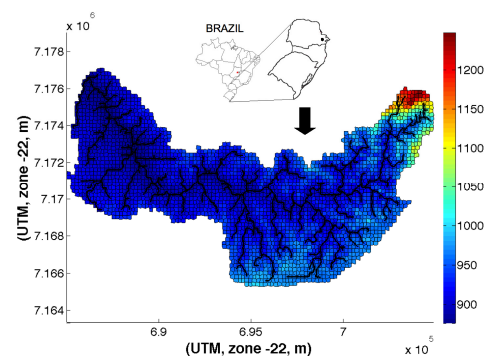


Fig. 2 Pequeno River watershed location. Elevations in meters

In the Rio Negrinho city, Santa Catarina state, is located an experimental reforestation of *Pinus sp.*

watershed, Pinus I, with 0.16 km² (Fig. 3).

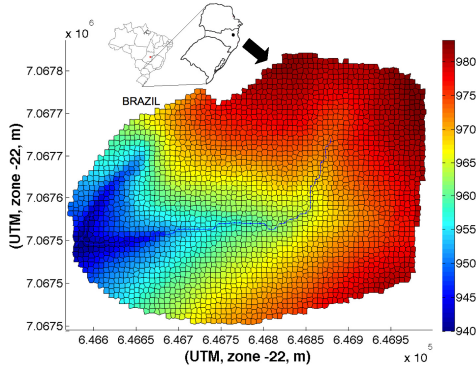


Fig. 3 Pinus I watershed. Elevations in meters

Model description and application

A graphical framework was implemented in Matlab in order to: (1) import watershed coordinate points from raster files, (2) create TIN and Voronoi networks, (3) solve for pits and flat areas, (3) define drainage network, (4) classify rivers according to the Horton-Strahler method, (5) extract area distance function, (6) distribute spatially the area distance function and (7) derive a time area function from the distance area function applying velocity parameters.

According to the raster map resolutions, different Voronoi mesh resolutions were chosen. The values were 200 m, 50 m and 6 m for Cubatão River, Pequeno River and Pinus I watersheds, respectively.

As a means to extract the river network for each watershed, the river initiation threshold (Montgomery and Dietrich, 1988, 1992, 1993) was set to 1.2×10^4 km², 6×10^5 km² and 6×10^5 km² for the Pinus I, Pequeno River and Cubatão River watersheds, respectively. These values were calibrated by means of comparison with the topographic map river network.

Strahler river classification was carried out using the river network automatically extracted from the DTMs. Pequeno River and Cubatão River watersheds were classified as 4th order, and Pinus I watershed as 2nd order watershed.

The limit of 50 classes to derive the area-time function was chosen. The time-area function for each watershed was derived through the application of two velocity criteria. The first one, hereafter, mean-velocity, multiplies the area-distance function

by an average velocity, calculated from all velocity values for each river order and hillslope. The second one, multi-velocities, applies different velocities values according to the hillslope and river order. Higher order rivers have higher velocities values taking into account a power law relationship (Rodriguez-Iturbe & Rinaldo, 1997).

3. Results

The Figs. 4, 5 and 6 show the area-distance function for each watershed.

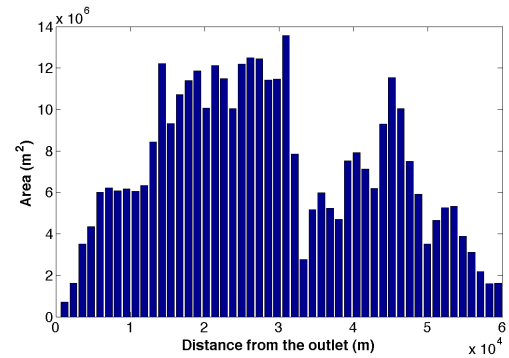


Fig. 4 Area-distance function for Cubatão River watershed

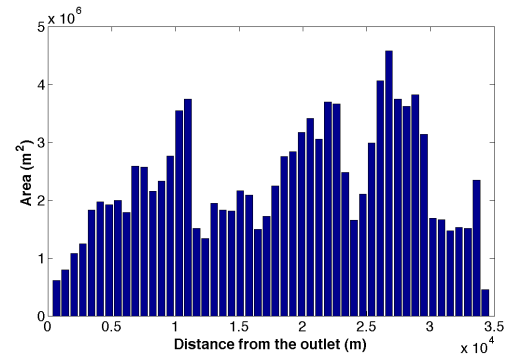


Fig. 5 Area-distance function for Pequeno River watershed

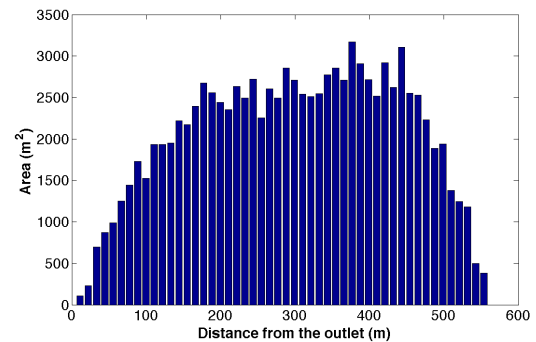


Fig. 6 Area-distance function for Pinus I watershed.

In those figures, it is observed that the area-distance function reflects the watershed shape. The Pequeno River watershed has a narrow region located in the middle of the total distance from the outlet. For each side of this narrowing, there are two distinct regions. In the Cubatão River watershed is observed two regions. The first one, the largest one, nearest the outlet corresponds to the 4th and 3rd river orders. The Pinus I watershed has a roundish shape. The same characteristics can be visualized observing the area-distance functions.

The Figs. 7, 8 and 9 show the comparison between time-area function derived from mean velocity and time-area function derived from multi-velocity for each watershed.

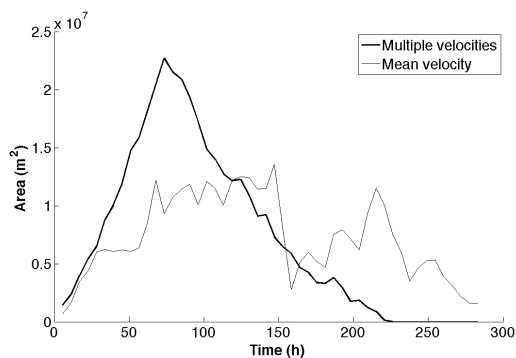


Fig. 7 Comparison between time-area functions, Cubatão River watershed

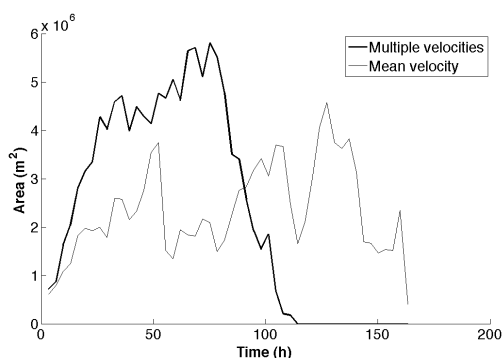


Fig. 8 Comparison between time-area functions, Pequeno River watershed

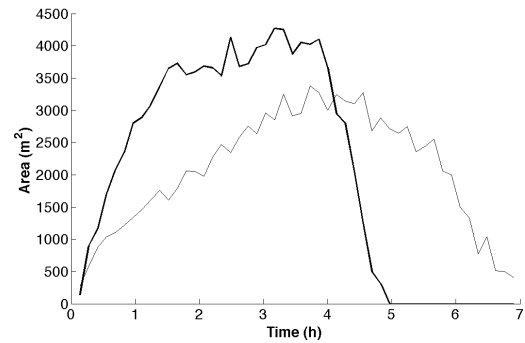


Fig. 9 Comparison between time-area functions, Pinus I watershed

It is observed that the time-area functions are quite different for all watersheds. Even so, a Kolmogorov-Smirnov (Massey, 1951) statistical test was carried out to analyze the two time-area distributions. The H_0 hypothesis, the distributions are equal, is rejected at the 0.05 significant level.

The total time for all watersheds, considering multi-velocities, is lower than the total time considering just one velocity value.

The shape of multi-velocity time-areas seems to agree on the shape of natural hydrograph.

The Figs. 10 to 15 show the spatially distributed time classes for mean and multi-velocities.

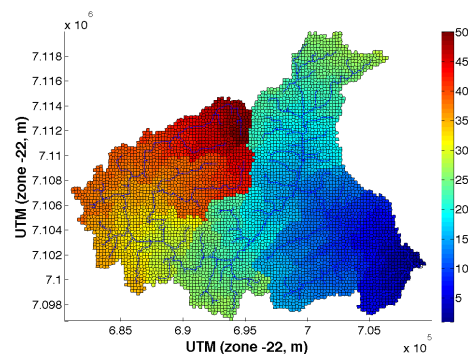


Fig. 10 Time classes spatially distributed using mean velocity, Cubatão River watershed

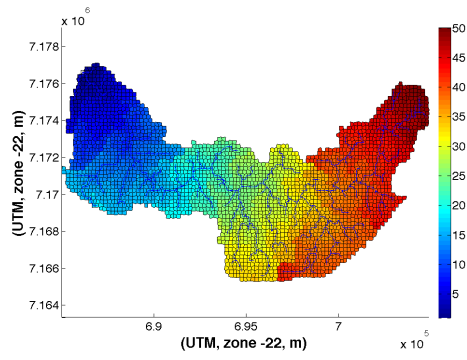


Fig. 11 Time classes spatially distributed using mean velocity, Pequeno River watershed

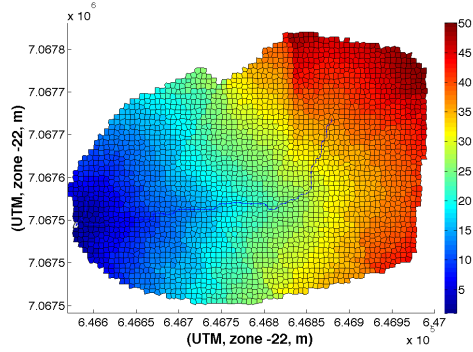


Fig. 12 Time classes spatially distributed using mean velocity, Pinus I watershed

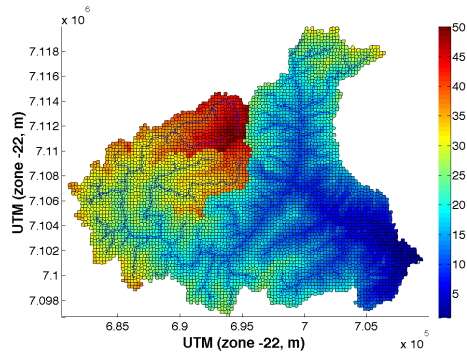


Fig. 13 Time classes spatially distributed using multi-velocities, Cubatão River watershed

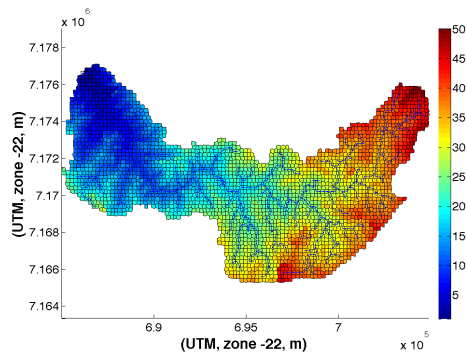


Fig. 14 Time classes spatially distributed using multi-velocities, Pequeno River watershed

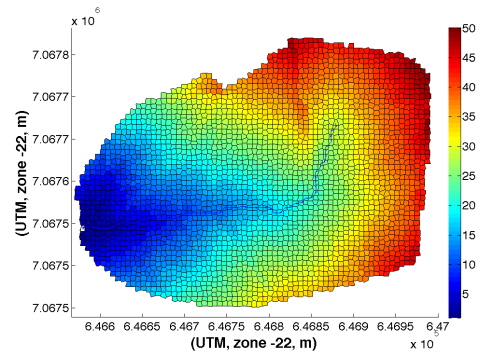


Fig. 15 Time classes spatially distributed using multi-velocities, Pinus I watershed

According to the Figs. 10 to 15, it is possible to realize that the spatially distribution of time classes using multi-velocities takes into account not only the distances from the outlet but the velocities in each river order. As river velocities are function of cumulative area, the use of spatially distributed velocities seems to be a more coherent approach to derive time-area functions.

4. Conclusions

In this work was presented a method to derive a time-distance function from an area-distance function using Voronoi cell discretization.

Time-area function for each watershed was derived through the application of two velocity criteria.

Through the comparison between the two criteria was possible to state that the use of spatially distributed velocities seems to be a more coherent approach to derive time-area functions. Therefore, multi-velocities approach seems reasonable for hydrological modeling.

This work gives support for implementation of a new TOPMODEL approach. This new version will be applied in global modeling. Recently, variable velocity has received special attention for global modeling, citing as example the work of Ngo-Duc et al. (2007).

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Voronoi離散化法を用いた地形情報処理

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要 旨

流域水文解析および水文モデル構築において、流域から抽出された地形学的特性は非常に重要な働きを示す。面積距離関数は中でも重要な地形学的特性である。本研究においては流域水文特性における面積距離関数より導かれた時間面積関数を、流域全体において平均化された単一流速を適用した場合と地点毎流速を適用した場合について比較を行なった。本手法においては、不規則三角形網を用いて作成されたボロノイセルを用いて流域を区分している。双方の流速の比較より、地点毎流速を用いた場合により時間面積関数に整合性が見られ、水文モデルにおける地点毎流速の適用が単一流速より適切であると考えられる。

キーワード： 面積距離関数，時間面積関数，ボロノイ，水文モデル