

Dynamic Modeling of Magnetic Core Materials for Sub-MHz Range using Cauer Circuit

Ichiro Okuda¹, *Member, IEEE*, and Tetsuji Matsuo¹, *Member, IEEE*

¹ Graduate School of Engineering, Kyoto University, Kyoto 615-8510, Japan

In this study, the frequency response of nanocrystalline soft magnetic materials for sinusoidal and pulse inputs in the sub-MHz range was simulated using a combination of a play model and the Cauer circuit. The resistance and static hysteretic properties of the inductors in the circuit were determined from B-H loop measurements at two frequencies. For the convenience of parameter identification, several analytical solutions of the eddy current field were introduced and compared. The proposed model could accurately represent the loss for the sub-MHz range.

Index Terms— Cauer circuit, dynamic modeling, hysteresis, play model.

I. INTRODUCTION

THE demand for magnetic material modeling in the high-frequency range is growing with the widespread use of power electronic devices with the feature of high-frequency switching. High-precision design for power electronic devices requires the modeling of iron losses [1] such as the eddy current and hysteresis losses of the magnetic core materials. Hysteresis losses are modeled using play models [2]-[4]. The play model requires less memory space and can be used to perform the finite element method due to its low computational cost. The play model is a static hysteresis model that does not describe the dynamic characteristics under AC excitation. To represent the dynamic hysteretic properties of a silicon steel sheet, the play model can be combined with the Cauer circuit. The Cauer circuit is a ladder circuit that sequentially connects inductors in parallel and resistors in series to represent an eddy-current field. The combination of the play model and Cauer circuit successfully represents dynamic hysteretic properties including kHz-order harmonics. The Cauer circuit facilitates nonlinear time-dependent analysis and can handle any input waveform such as PWM input. The finite element analysis of magnetic field can be incorporated using the Cauer circuit and the play model. In this study, we investigated a combination of a four-stage Cauer circuit and a play model, and calculated the iron loss of the core material in the frequency range of 10 to 1000 kHz. Several simple formulae are known for Cauer circuit elements for simple shaped materials such as plate, sphere and cylinder. Those formulae result from the frequency characteristics of eddy-current field, which means that the elements of ladder circuit are expected to obey a rule to reconstruct the behavior of eddy-current field. From this point of view, this paper applies or modifies those simple formulae for an efficient determination of circuit elements. We obtained equation (11) to determine the inductor and resistor constants of the Cauer circuit for modeling magnetic materials used at sub-MHz.

II. MODELING OF LOSSES IN MAGNETIC MATERIALS

A. Play Model

The hysteretic characteristics of the magnetic field intensity

H with the magnetic flux density B as the input are expressed by (1).

$$H(B) = \sum_{n=1}^{n_p} f_n(\zeta_n, p_{\zeta n}(B)) \quad (1)$$

where n_p is the number of play hysterons and f_n is the shape function. $p_{\zeta n}$ is the value of the n -th play hysteron, and ζ_n is the width of the n -th play hysteron. The play hysterons expressed by (2) are used to describe the hysteretic characteristics of the play model.

$$p_n(B) = \max(\min(p_n^0, B + \zeta_n), B - \zeta_n) \quad (2)$$

where p^0 denotes the previous value of the play hysteron.

B. Cauer Circuit

As shown in Fig. 1, the AC characteristics of the magnetic material are represented with a Cauer circuit by connecting an inductor in parallel and a resistor in series to a circuit with an input voltage of dB/dt and an input current of H . Although the accuracy of the modeling decreases as the frequency approaches MHz, adding L and R in the latter stages improves the accuracy. In this study, the number of stages four was chosen to be large enough so that the number of stages did not affect the accuracy. The circuit is terminated with an inductor as shown in Fig. 1.

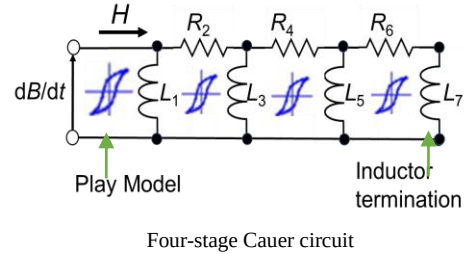


Fig. 1. Cauer circuit with inductor termination.

The magnetic properties of a plate such as an electromagnetic steel plate can be represented by a Cauer circuit. The elements can be expressed as follows:

$$R_{2n} = (4n-1) R, \quad L_{2n+1} = L_1 / (4n+1) \quad (n = 1, 2, \dots) \quad (3)$$

$$R = 4 / (\sigma d^2) \quad (4)$$

The unit of resistances is Ω/m and the unit of inductors is H/m , where σ is the electrical conductivity and d is the thickness of the steel sheet. In the case of linear magnetic properties, $L_1 = \mu$ (μ : magnetic permeability) [6][7]. For a spherical magnetic material, R_{2n} and L_{2n+1} are given by (5).

$$R_{2n} = (4n+1) R_2 / 5, L_{2n+1} = 2 L_1 / (4n+3) \quad (n = 1, 2, \dots) \quad (5)$$

The circuit element of a cylindrical magnetic material [6][8][9] in uniform axial magnetic field is expressed as follows:

$$R_{2n} = nR_2, L_{2n+1} = L_1 / (2n+1) \quad (n = 1, 2, \dots) \quad (6)$$

$$R_2 = 4 / (\sigma a^2) \quad (7)$$

where a denotes the radius of the cylinder.

Fig. 2. summarizes the above, showing the coefficients of the four-stage Cauer circuit where the material shapes are plate, sphere, and cylinder.

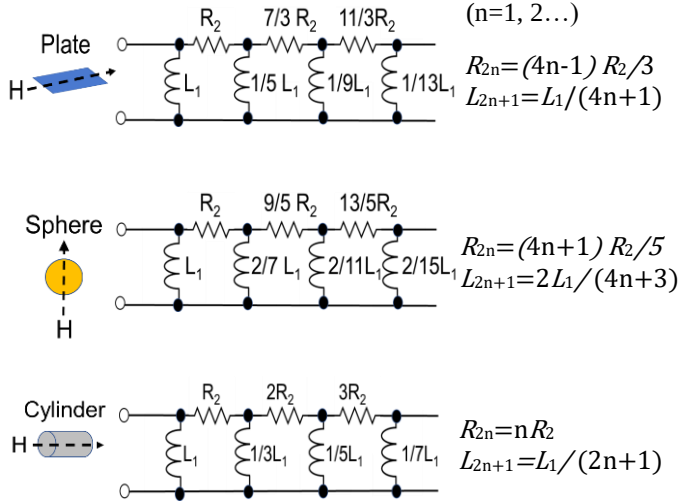


Fig. 2. Circuit elements of the plate [6], spherical, and cylindrical [6] magnetic conductors and the analytical solutions.

Core materials used for high-frequency applications often consist of small insulated particles/cells. If the magnetic field in the particles/cells is approximated by one of the above-mentioned analytical solutions [10]-[12], the Cauer circuit elements can be easily identified, where only the first inductor L_1 and resistor R_2 should be determined based on the measurement.

C. Identification of Circuit Elements

The alternating magnetic field H_{ac} in magnetic materials is assumed to be the sum of the static magnetic field H_{dc} and the AC magnetic field owing to the eddy current.

$$H_{ac} = H_{dc} + \frac{1}{R_2} \frac{dB}{dt} \quad (8)$$

R_2 represents classical eddy current losses. $1/R_2$ can be called the eddy current loss coefficient. If sinusoidal inputs $B = B_m$

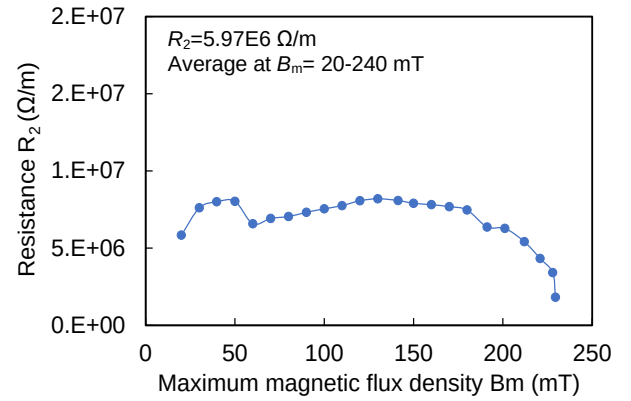
$\sin(2\pi ft)$ are used at two frequencies f_1 and f_2 ($f_2 > f_1$), the resistance R_2 is obtained as

$$\frac{1}{R_2} = \frac{\text{Loss}(f_2) - \text{Loss}(f_1)}{2\pi^2 B_m^2 (f_2 - f_1)} \quad (9)$$

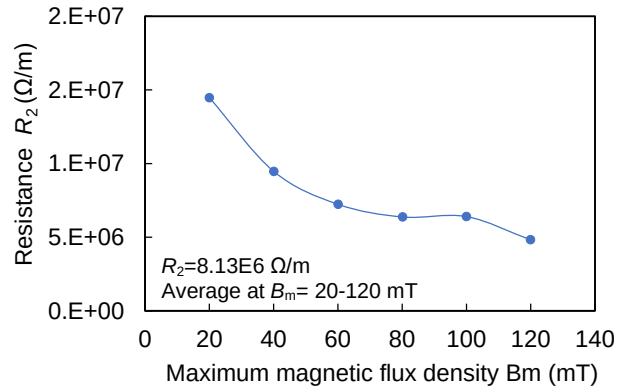
DC hysteresis loops are obtained from the measured $H_{ac}(B)$ as follows:

$$H_{dc}(B) = H_{ac}(B) - \frac{1}{R_2} \frac{dB}{dt} \quad (10)$$

R_2 is the resistance per unit length and is the same as that of the Cauer circuit described in the section II B. The iron losses at frequencies f_1 (10 kHz) and f_2 (20 kHz) are obtained by calculating the area of the measured B-H loop. For a nanocrystalline soft magnetic material, Finemet®, and a dust core ring material, Sendust, the resistance R_2 is calculated as described above. Fig. 3(a) shows the calculation results of R_2 for Finemet® (5.97E6 Ω/m), which is the average in the range of $B_m = 20$ -240 mT. Fig. 3(b) shows these for Sendust (3.0E6 Ω/m), which is the average in the range of $B_m = 20$ -120 mT.



(a) Finemet® ($f_1 = 10\text{ kHz}$, $f_2 = 20\text{ kHz}$)



(b) Sendust ($f_1 = 10\text{ kHz}$, $f_2 = 100\text{ kHz}$)

Fig. 3. Eddy current magnetic field R_2 at two frequencies f_1 and f_2 .

A flowchart for the iron loss calculation is shown in Fig. 4. R_2 is calculated by (9) using the iron loss obtained measured by the B-H analyzer. The property of L_1 is represented by the play model, which is identified from H_{dc} given by (10).

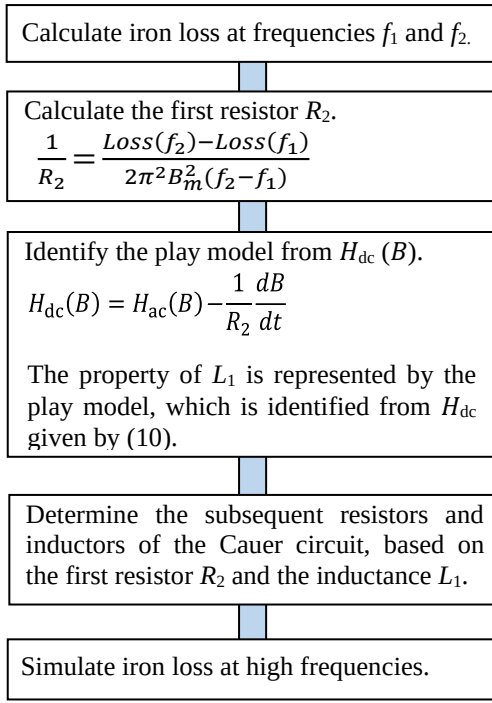


Fig. 4. Flowchart of iron loss calculation of magnetic material using Cauer circuit.

III. COMPUTATIONAL RESULTS

The iron loss of ring-shaped Finemet® (effective cross-sectional area: 46 mm², effective magnetic path length: 32.2 mm, effective volume: 1461 mm³, outer diameter: 16 mm, inner diameter: 5 mm, thickness: 8 mm) is measured under the condition of sinusoidal excitation at 10–300 kHz. Figs. 5 and 6 show the iron losses per cycle, which were simulated using the circuit elements shown in Fig. 2.

Several versions of the hysteretic inductors L_3 , L_5 , etc., exist. The simple and reversible approximations proposed in [7] are illustrated in Figs. 5 and 6, respectively. In Fig. 5, the calculated losses obtained using a simple approximation are in agreement with the measured losses for an amplitude $B_m \leq 180$ mT for the case in which the circuit elements are spherical. This is probably because the crystalline grains were approximately spherical. We also calculated losses using reversible approximation to verify the agreement between calculated and measured losses and found that the modeling accuracy was not improved. The reversible approximation method is a powerful method for investigating nonlinearity. The broken red lines labeled “proposed” in Fig. 6 show the loss when the inductances and resistances are given as

$$L_{2n+1} = L_1 / (4n-3), R_{2n} = nR_2 \quad (n = 1, 2, \dots), \quad (11)$$

which improves the accuracy of the loss simulation. However, the physical background for obtaining (11) is not yet known. Thus, the loss characteristics under sub-MHz fields are mathematically modeled using the four-stage Cauer circuit.

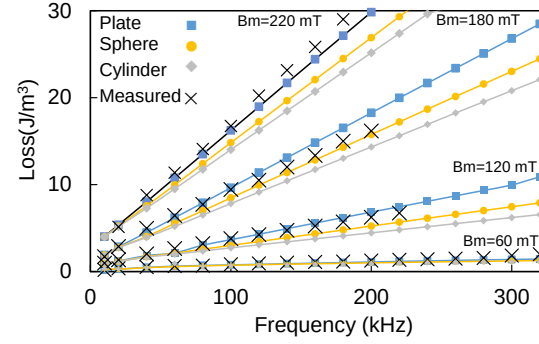


Fig. 5. Loss per cycle simulated using simple approximation method.

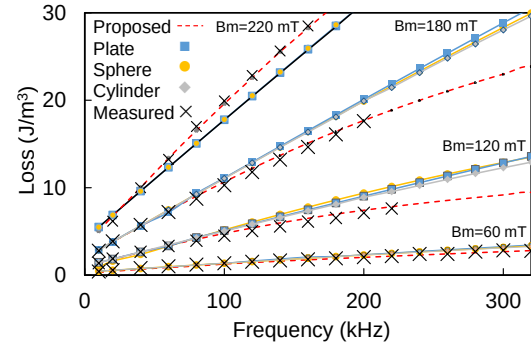
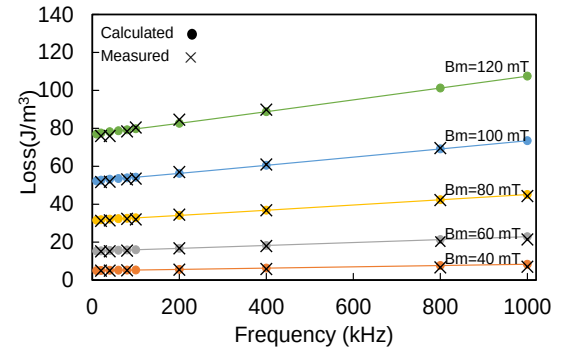
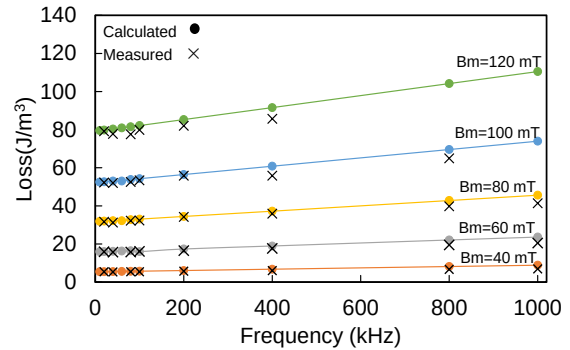


Fig. 6. Loss per cycle simulated using reversible approximation method.



(a) Sinusoidal



(b) Pulse

Fig. 7. Loss per cycle of Sendust powder core simulated using simple approximation method.

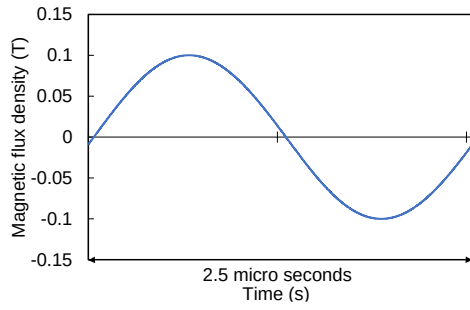
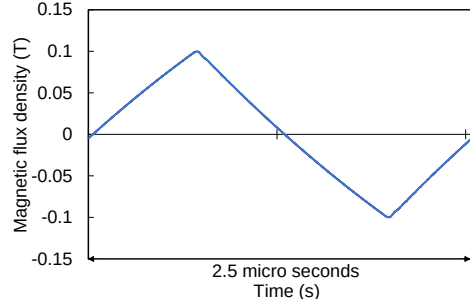
(a) Waveform due to sinusoidal voltage ($f=400$ kHz, $B_m=100$ mT)(b) Waveform due to pulse voltage ($f=400$ kHz, $B_m=100$ mT)

Fig. 8. Sinusoidal and pulse input waveform to Sendust powder core.

Next, sinusoidal and pulse voltages to confirm the possibility of PWM input of 10-1000 kHz were applied to a dust core material of Sendust (effective cross-sectional area: 12.26 mm², effective magnetic path length: 31.73 mm, effective volume: 388.9 mm³). We determined the inductor and resistor values other than L_1 and R_2 for the four-stage Cauer circuit with the inductor constant in (11).

Fig. 7. shows the loss per cycle of the Sendust dust core simulated using a simple approximation method. Fig. 8 shows sinusoidal and pulse input waveforms fed to the Sendust dust core. Calculated and measured values were almost the same. Although the circuit elements are determined under sinusoidal excitation conditions, the loss under the pulse voltage agrees with the measured value.

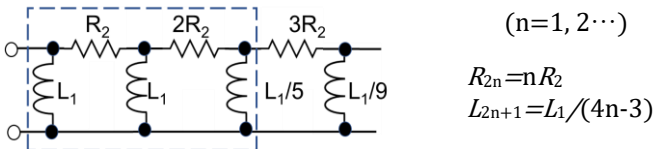


Fig. 9. Loss per cycle simulated using reversible approximation method.

We examined the one- to four-stage Cauer circuits for modeling of magnetic core materials. We confirmed that the calculated iron loss using the three-stage circuit is almost equal to that using the four-stage circuit. The constants of the Cauer circuit of Finemet® and Sendust are shown in Fig. 9. The dashed lines represent the three-stage Cauer circuit.

IV. CONCLUSION

The iron losses of a nanocrystalline material and dust core were simulated in the high-frequency range of 10-1000 kHz, and the results obtained were in agreement with the measured values.

Cauer circuits facilitate nonlinear time-dependent analysis and can handle arbitrary input waveforms such as PWM inputs. Using the Cauer circuit as a bulk material to express the macroscopic magnetic properties of the iron core material, the material properties can be incorporated into a finite element magnetic field analysis. Therefore, in the future, we will perform simulations using finite element method analysis.

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