

Classification of non-symplectic automorphisms of order 3 on $K3$ surfaces

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In this poster, we study non-symplectic automorphisms of order 3 on algebraic $K3$ surface X over $\mathbb{C}$ which act **trivially** on the Néron-Severi lattice S_X . In particular we shall characterize their fixed locus in terms of the invariants of 3-elementary lattices.

This poster is devoted to study of **non-symplectic** automorphism φ on X . i.e. $\varphi^*\omega_X = (1 + \sqrt{-3}/2)\omega_X$. (ω_X : nowhere vanishing holomorphic 2-form)

Main Theorem

Let ρ be the Picard number of X and let s be the minimal number of generators of S_X^*/S_X .

$$(1) \quad 22 - \rho - 2s < 0 \Rightarrow \nexists \varphi.$$

$$(2) \quad 22 - \rho - 2s \geq 0 \Rightarrow \exists \varphi.$$

Moreover $X^\varphi := \{x \in X | \varphi(x) = x\}$ has the form

$$\begin{cases} \coprod_i^3 \{P_i\} \coprod C^{(1)} & \text{if } S_X = U(3) \oplus E_6^*(3) \\ \coprod_i^M \{P_i\} \coprod C^{(g)} \coprod_j^{N-1} E_j & \end{cases}$$

where $M = \rho/2 - 1$, $g = (22 - \rho - 2s)/4$, $N = (6 + \rho - 2s)/4$.

Lattices

$U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$: hyperbolic lattice.

A_2 , E_6 or E_8 an even negative definite lattice associated with the Dynkin diagram of type A_2 , E_6 or E_8 .

L : lattice. $L^* = \text{Hom}(L, \mathbb{Z})$.

L : **3-elementary lattice** $\Leftrightarrow L^*/L \simeq (\mathbb{Z}/3\mathbb{Z})^s$.

An even indefinite 3-elementary lattices were **classified** by A.N. Rudakov and I.R. Shafarevich.

An even indefinite 3-elementary lattice of rank $n(\geq 2)$ is uniquely determined by the number s .

$\Rightarrow$ We can classify even indefinite 3-elementary lattices admitting primitive imbedding in $U^3 \oplus E_8^2$. $\downarrow$

Let L be an even indefinite 3-elementary lattice admitting primitive imbedding in $U^3 \oplus E_8^2$.
 $22 - \text{rank } L - 2s < 0 \Rightarrow L$ has no non-trivial isometry of order 3 which acts trivially on L^*/L .

Fixed locus

♠ Let P be an isolated fixed point of φ on X . Then φ^* can be written as $\begin{pmatrix} \zeta^2 & 0 \\ 0 & \zeta^2 \end{pmatrix}$ under some appropriate local coordinates around P .

♠ Let C be a fixed irreducible curve and Q a point on C . Then φ^* can be written as $\begin{pmatrix} 1 & 0 \\ 0 & \zeta \end{pmatrix}$ under some appropriate local coordinates around Q . In particular, fixed curves are smooth.

Hence $X^\varphi = \{P_1\} \amalg \cdots \amalg \{P_M\} \amalg C_1 \amalg \cdots \amalg C_N$, where P_j is an **isolated point** and C_k is a **smooth curve**.

♡ Toward a proof of Main Theorem ♡

Lefschetz formula + Hurwitz formula + classification of singular fibers of elliptic pencils by Kodaira + classification tables of 3-elementary lattices.

$\downarrow \quad \downarrow \quad \downarrow$

We can calculate M , N and the genus of C_k .

Examples

We give affine equations of elliptic $K3$ surfaces $X : z^3 = y \left(y^2 \prod_{i=1}^{12} (u - a_i) - x^2 \right)$ where a_i ($i = 1, 2, \dots, 12$) are distinct complex numbers.

$\varphi(x, y, z, u) := (x, y, \zeta z, u)$ (ζ : primitive 3-th root of 1).

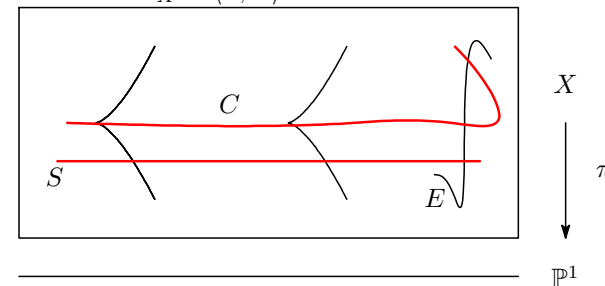
$\pi : X \xrightarrow{u} \mathbb{P}^1$: projection.

$\pi^{-1}(a_i)$: singular fiber of type II.

$X^\varphi = \{y = 0\} \amalg \{y^2 \prod_{i=1}^{12} (u - a_i) - x^2 = 0\}$.

$C := y^2 \prod_{i=1}^{12} (u - a_i) - x^2$: smooth curve.

Now we remark that X has a section. Let S be the section defined by $y = 0$ and E an class of general elliptic curves. Then $S_X = \langle S, E \rangle \simeq U$.



The automorphism φ induces non trivial automorphism of order 3 on E and $\pi^{-1}(a_i)$. Thus we calculate the genus of C by the Hurwitz formula

$$2g(C) - 2 = 2(2g(\mathbb{P}^1) - 2) + 12(2 - 1).$$

Therefore we can express the fixed locus X^φ as $C^{(5)} \amalg \mathbb{P}^1$.

Remark

$22 - \rho - 2s \geq 0 \Rightarrow$ There are some examples of $K3$ surface X with S_X s.t. $\text{rank } S_X = \rho$, $S_X^*/S_X \simeq (\mathbb{Z}/3\mathbb{Z})^s$.