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Multiple-Conclusion System as Communication Calculus*

- Informal meaning of proofs as communication terms -

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Abstract

Following the so-called inversion principle of Gentzen-Prawitz, we demonstrate that multiple-conclusion system of classical logic can be naturally regarded as a communication calculus. For the motivation, we first introduce a hierarchical structure for representing an abstract structure of classical proofs. The hierarchical structure describes that classical proofs consist of intuitionistic proofs as the components and the structural connections between those elements. We next provide proof term assignment rules based on the the hierarchical structure. The resulting term calculus is a natural extension of λ-calculus and can be considered as a communication calculus. It is found that the hierarchical structure can be regarded as a network structure for the communication, based on which a term can be passed on to distributed terms. We also show the fundamental properties of the communication terms.

1 Introduction

On the basis of the Curry-Howard-de Bruijn isomorphism [14], proof reductions can be regarded as computational rules. The computational meaning of proofs has been investigated by numerous researchers, not only in intuitionistic logic but also in classical logic and modal logic [15]. In the area of classical logic, there have been a number of noteworthy investigations including those of Griffin [10], Murthy [18], Parigot [22], Barbanera&Berardi [1], Rehof&Sørensen [25], de Groote [11], Ong [21], and Hofmann&Streicher [13]. A classical logic can be obtained from intuitionistic logic by adding an intrinsically classical theorem [12], such as the excluded middle or the double negation elimination rule. Griffin [10] has discovered that the double negation elimination can be interpretated computationally as a control operator.

This paper demonstrates a communication-like computational property of proofs of classical logic. We introduce a classical system not by adding a classical theorem to intuitionistic logic but by allowing multiple-conclusion; "classicality" depends on multiple-conclusion. Although there already exist multiple-conclusion systems such as [5], [6], [26],

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[22], and [8], our approach is essentially different from the existing ones in the following sense:

1. From the viewpoint of the inversion principle, a communication-like interpretation is given to multiple-conclusion system.

2. The hierarchical structure is introduced to represent abstract structures of proofs in classical logic.

3. Communication-terms as a natural extension of λ-terms are directly assigned to classical proofs based on the hierarchical structure.

In multiple-conclusion system, we will investigate the traditional idea of Gentzen-Prawitz [9, 23], the so-called inversion principle. Following the principle, it will be found that a classical proof of $A \to B$ in multiple-conclusion system can be interpreted as a sending (multicast) operator among multiple-conclusion. This result naturally extends the functional interpretation of intuitionistic proofs via λ-terms to a communication-like interpretation of classical proofs.

For this motivation, we first introduce a new structure for classical proofs, called a hierarchical structure for classical proofs. The concept of the hierarchical structure comes from the observation that classical proofs can be obtained from a certain combination of intuitionistic proofs by means of the right structural rules. In other words, classical proofs can be separated into intuitionistic components and the structural connections in the sense of the right structural rules, between those elements. The hierarchical structure for classical proofs are defined as ordered trees labeled with sets of intuitionistically valid formulae. This set of intuitionistic provable formulae is treated as a world, and the binary relations between worlds constitute an abstract form of the right structural rules. We will prove that there exists a classical proof if and only if there exists a finite hierarchical structure. The hierarchical structure can be regarded as a certain model of classical proofs in terms of intuitionistic provability and the right structural rules.

On the basis of this hierarchical structure, we next introduce a proof term assignment to establish "multiple-conclusion system as communication calculus". We discuss the dynamic aspect of the hierarchical structure from the viewpoint of a communication-like interpretation among worlds. Here, the hierarchical structure is regarded as the network structure within which the communication takes place. On the basis of this structure, roughly speaking, λ-terms are passed from one world to other worlds.

2 Outline of the Idea

In this section, we observe the distinction of "meaning" of the implications ($\to$) between intuitionistic system and classical system. Following Genzten [9] and Prawitz [23, 24], the introduction rules represent the definition of the logical constants concerned, and the elimination rules are justified by the meaning of the logical constants given by the introduction rules. This reinterpretation gives the constructive meaning of the logical constants. In the case of intuitionistic logic, the introduction rule for $\to$ is given by

$$\Gamma, x{:}A \vdash_I M : B \;\Rightarrow\; \Gamma \vdash_I \lambda x.M : A \to B$$

Then the elimination rule is justified by the meaning of $A \to B$, that is, a function $\lambda x.M$ which to each proof of A gives a proof of B. On the other hand, in classical system with

multiple-conclusion, assume that we have proof terms assigned to each conclusion. Then the introduction rule for classical $\rightarrow$ can be defined as follows:

$$\Gamma, x{:}A \vdash M_1 : B_1, M_2 : B_2, \cdots, M_n : B_n \\ \Rightarrow\ \Gamma \vdash \lambda^* x.M_1 : A \rightarrow B_1, M_2 : B_2, \cdots, M_n : B_n$$

Here, not only with B_1 but also with B_i $(2 \leq i \leq n)$, one can derive $\lambda^* x.M_i : A \rightarrow B_i$, even if the assumption $x{:}A$ is not actually used for deducing $M_i{:}B_i$, that is, $x \in FV(M_j)$ for some $j \neq i$. (Although Nakano [19, 20] proposed an intuitionistic system with multiple-conclusion, the intuitionistic system has a side condition for $\rightarrow$-introduction such as $x \notin FV(M_j)$ for all $j \neq i$.) Now the elimination rule can be justified by considering $\lambda^* x.M_i$ as a certain function that communicates each proof of A to proofs of every B_i in $\{B_1, \cdots, B_n\}$ whose element actually depends on the proof of A. This meaning is a natural extension on the case of a single conclusion. Following the natural observation, a classical proof of $A \rightarrow B$ can be reinterpreted as a multicast (broadcast) operator among multiple-conclusion, and proof terms assigned to conclusions serve as a communication entity, where the whole deduction tree is regarded as a network structure for the communication.

In order to demonstrate this observation, we first illustrate the idea of a hierarchical structure for classical proofs by using an example. On the basis of this hierarchical structure, proof term assignment rules will be defined to realize "multiple-conclusion system as communication calculus". In a natural deduction system with multiple conclusions, one can prove Peirce's law as follows:

$$\dfrac{\dfrac{[(A \rightarrow B) \rightarrow A]^2 \quad \dfrac{\dfrac{[A]^1}{B, A}\,(w)}{A \rightarrow B, A}\,1}{\dfrac{A, A}{A}\,(c)}\,2}{((A \rightarrow B) \rightarrow A) \rightarrow A}$$

Here, we consider that the above proof consists of the following three intuitionistic proofs, and that these intuitionistic proofs are combined by means of the right contraction or the right weakening rules:

$$[\alpha]\ \boxed{A^1}$$

$$\downarrow R(\,)$$

$$[\beta]\ \boxed{\dfrac{(A \rightarrow B) \rightarrow A^2 \quad \dfrac{B^*}{A \rightarrow B}\,1}{A}}$$

$$\downarrow R(\alpha, \beta)$$

$$[\gamma]\ \boxed{\dfrac{A^*}{((A \rightarrow B) \rightarrow A) \rightarrow A}\,2}$$

Here, the names $[\alpha]$, $[\beta]$, and $[\gamma]$ are given to each intuitionistic proofs, and the formulae A^* and B^* are introduced by the right contraction and the right weakening, respectively. We can now regard the names as names of worlds wherein intuitionistic proofs are carried out, and the formulae A^* and B^* are induced from the name-indexed relations between worlds (see Fig. 1).

In the minimal world $[\alpha]$, we execute intuitionistic proofs under the assumption A. There is a relation $R(\ )$ without names between the minimal $[\alpha]$ and the second minimal $[\beta]$. The relation means that we use an arbitrary formula as an assumption in the world $[\beta]$, e.g., B^*. Then in the second minimal world $[\beta]$, we carry out intuitionistic proofs under the assumptions $(A \to B) \to A$ and B^*. We have the relation $R(\alpha, \beta)$ between the second minimal world $[\beta]$ and the greatest world $[\gamma]$. This relation means that we use a formula X as an assumption in the top world $[\gamma]$, such that the formula X is the common conclusion in both $[\alpha]$ and $[\beta]$, e.g., A^*.

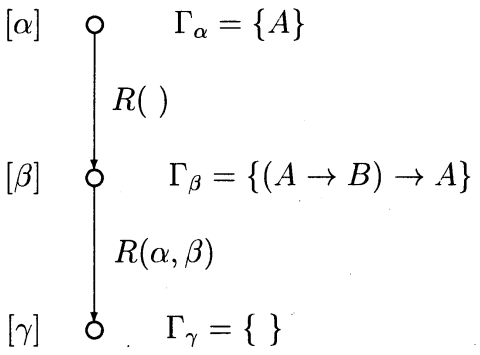


Fig. 1

The above hierarchical structure for proofs represents an abstract structure for classical proofs, and this structure contains more information than that of the proof term for Peirce's formula, since the existence of intuitionistic proofs is herein abstracted as a world. Here, we regard the above Kripke-like structure as follows: The minimal world $[\alpha]$ denotes a set of formulae X such that X is provable from Γ_α in intuitionistic logic; strictly speaking, in minimal logic. The second minimal world $[\beta]$ represents a set of formulae X such that $\Gamma_\beta \vdash_I X$ is intuitionistically admissible from $\Gamma_\alpha \vdash_I Y$ for an arbitrary Y. The greatest world $[\gamma]$ denotes a set of formulae X such that $\Gamma_\gamma \vdash_I X$ is intuitionistically admissible from $\Gamma_\beta \vdash_I Y$ for some $Y \in \inf\{[\alpha], [\beta]\}$, where $\inf\{[\alpha], [\beta]\}$ denotes the greatest lower bound of the two denotations $[\alpha]$ and $[\beta]$, i.e., $[\alpha] \cap [\beta]$.

3 Informal Meaning of Proof Terms as Communication Terms

In this section, we give some examples and informal meaning of proof terms as communication terms.

The proof term of the form $\Box(\alpha_1, \ldots, \alpha_n)$ is a distributing operator in the sense that the term distributes its argument to a term in every world $[\alpha_i]$ $(1 \leq i \leq n)$. Here, the communication channels are a set of pairs of names $\{\langle \alpha, \alpha_1 \rangle, \ldots, \langle \alpha, \alpha_n \rangle\}$. In terms of the right structural rules of sequent calculi, $\Box(\alpha_1, \ldots, \alpha_n)$ codes the application of the right weakening rule if $n = 0$, the right exchange if $n = 1$, and the right contraction if $n \geq 2$. $\Box(\ )$ is simply denoted by $\Box$.

The term of the form $\lambda^*(x^{\alpha_1(m_1)}, \ldots, x^{\alpha_n(m_n)}).M$ is a multicast (broadcast) operator which sends its argument to m_i-occurrences x in each world $[\alpha_i]$ where $1 \leq i \leq n$. Here, the communication channels consist of $\{\langle \alpha, \alpha_1 \rangle, \ldots, \langle \alpha, \alpha_n \rangle\}$ where α denotes the name of the world of the multicast operator. In the case of a single channel, a term $(\lambda^*(x^{\alpha(m)}).M)N$ in the world $[\beta]$, denoted by $(\lambda^*(x^{\alpha(m)}).M)N \in [\beta]$ where $\alpha \not\equiv \beta$, maby be roughly represented as $\overline{\langle \beta, \alpha \rangle} N.M$ in terms of higher-order π-calculus. Then some term $M_1 \in [\alpha]$ would be read as $\langle \beta, \alpha \rangle(x).M_1$. The operator λ^* plays both roles of a classical

and an intuitionistic codes for $\rightarrow$, i.e., $(\lambda^*(x^{\alpha(m)}).M)N \in [\beta]$ where $\alpha \not\equiv \beta$ is a proof code for classical $\rightarrow$, and $(\lambda^*(x^{\alpha(m)}).M)N \in [\alpha]$, simply written by $\lambda x.M$, is a proof code for intuitionistic $\rightarrow$. Hence, this computation is a natural extension of the β-reduction; the induced substitution is applied not only to M but also to distributed terms in other worlds.

The first example is a proof Peirce's law. From the following structure together with proof terms (Fig. 2):

$$\boxed{x^{\alpha(1)} : A \in [\alpha]}$$

$$\downarrow R(\,)$$

$$\boxed{\dfrac{y^{\beta(1)} : (A \rightarrow B) \rightarrow A \in [\beta] \qquad \dfrac{\Box : B \in [\beta] \mid x : A \in [\alpha]}{\lambda^*(x^{\alpha(1)}).\Box : A \rightarrow B \in [\beta] \mid x : A \in [\alpha]}}{y(\lambda^*(x^{\alpha(1)}).\Box) : A \in [\beta] \mid x : A \in [\alpha]}}$$

$$\downarrow R(\alpha, \beta)$$

$$\boxed{\dfrac{\Box(\alpha,\beta) : A \in [\gamma] \mid y(\lambda^*(x^{\alpha(1)}).\Box) : A \in [\beta] \mid x : A \in [\alpha]}{\lambda^*(y^{\beta(1)}).\Box(\alpha,\beta) : ((A \rightarrow B) \rightarrow A) \rightarrow A \in [\gamma] \mid y(\lambda^*(x^{\alpha(1)}).\Box) : A \in [\beta] \mid x : A \in [\alpha]}}$$

the term assignment can be obtained such that

$$x \in [\alpha] \mid y(\lambda^*(x^{\alpha(1)}).\Box) \in [\beta] \mid \lambda^*(y^{\beta(1)}).\Box(\alpha,\beta) \in [\gamma]$$

The subterm $\lambda^*(x^{\alpha(1)}).\Box$ is a communication operator along channel $\langle \beta, \alpha \rangle$. In terms of π-calculus, $(\lambda^*(x^{\alpha(1)}).\Box)N$ can be read as $\overline{\langle \beta, \alpha \rangle} N.\Box$, and $x \in [\alpha]$ as $\langle \beta, \alpha \rangle(x).x$. The term $\lambda^*(y^{\beta(1)}).\Box(\alpha, \beta) \in [\gamma]$ is a communication operator along channel $\langle \gamma, \beta \rangle$, and the subterm $\Box(\alpha, \beta) \in [\gamma]$ is a distributing operator along two channels $\{\langle \gamma, \alpha \rangle, \langle \gamma, \beta \rangle\}$.

To give a simple and small example, assume that the following recursive program G computes the product of all integers in an integer list l:

$$\begin{aligned}&\texttt{fun } G\ l = \lambda k.\texttt{let fun } f\ \texttt{nil} = 1\\ &\qquad\qquad\qquad\qquad \mid f\ (hd :: tl) = \texttt{if } hd{=}0 \texttt{ then } (k\ 0) \texttt{ else mult } hd\ (f\ tl)\\ &\qquad\qquad \texttt{in } f\ l \texttt{ end}\end{aligned}$$

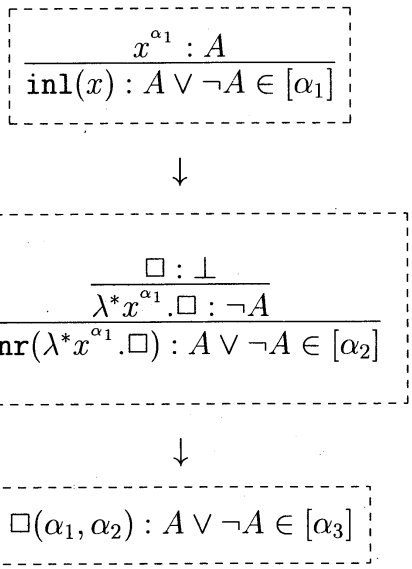
Then, applying the proof of Peirce's law (see Fig. 2), we obtain the following computation:

$$\begin{aligned}&x \in [\alpha] \mid y(\lambda^*(x^{\alpha(1)}).\Box) \in [\beta] \mid (\lambda^*(y^{\beta(1)}).\Box(\alpha,\beta))(G\ [1,2,0,4]) \in [\gamma]\\ \triangleright\ &x \in [\alpha] \mid (\lambda k.\texttt{mult } 1\ (\texttt{mult } 2\ (k\ 0)))(\lambda^*(x^{\alpha(1)}).\Box) \in [\beta] \mid \Box(\alpha,\beta) \in [\gamma]\\ \triangleright\ &x \in [\alpha] \mid \texttt{mult } 1\ (\texttt{mult } 2\ ((\lambda^*(x^{\alpha(1)}).\Box)\ 0)) \in [\beta] \mid \Box(\alpha,\beta) \in [\gamma]\\ \triangleright\ &0 \in [\alpha] \mid \texttt{mult } 1\ (\texttt{mult } 2\ \Box) \in [\beta] \mid \Box(\alpha,\beta) \in [\gamma]\end{aligned}$$

where the continuation with $\Box$ is preserved in the world $[\beta]$, and this continuation is obtained at the time when the answer 0 is communicated to x in $[\alpha]$ without executing the multiplications any more.

The second example is the law of excluded middle; $EM : A \vee \neg A$. We define $A \vee B =$

$\neg A \to \neg\neg B$, and $\mathtt{inl}(M) = \lambda f.\lambda v.fM$ and $\mathtt{inr}(M) = \lambda v.\lambda f.fM$. Then from the following structure:



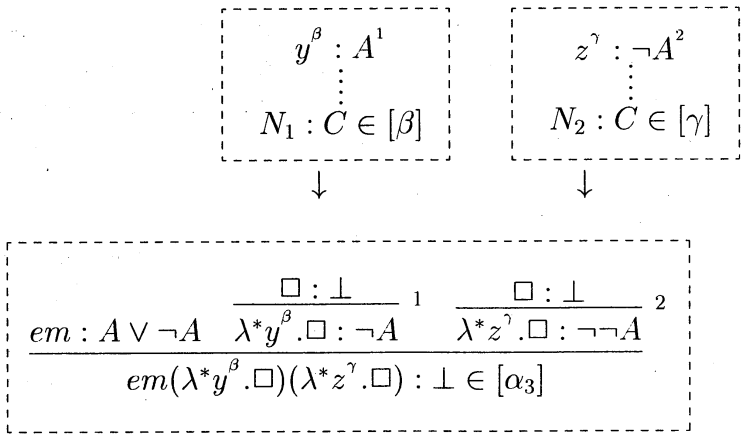
we obtain the proof term EM of $A \vee \neg A$ as follows:

$$EM = \lambda fv.fx \in [\alpha_1] \mid \lambda vf.f(\lambda^* x^{\alpha_1}.\Box) \in [\alpha_2] \mid \Box(\alpha_1, \alpha_2) \in [\alpha_3]$$

The elimination rule:

$$\frac{EM : A \vee \neg A \qquad \begin{matrix} y : A \\ \vdots \\ N_1 : C \end{matrix} \qquad \begin{matrix} z : \neg A \\ \vdots \\ N_2 : C \end{matrix}}{C}$$

can be here interpreted in the following:



where $em = \Box(\alpha_1, \alpha_2)$.

Let $!_n M \in [\alpha]$ denote n-occurrences of $M \in [\alpha]$ where $!_0 M \in [\alpha] \equiv \Box \in [\alpha]$. Now the composition of the proof terms gives the following computation:

$$\begin{aligned} &\lambda fv.fx \in [\alpha_1] \mid \lambda vf.f(\lambda^* x^{\alpha_1}.\Box) \in [\alpha_2] \mid (\Box(\alpha_1, \alpha_2))(\lambda^* y^{\alpha}.\Box)(\lambda^* z^{\beta}.\Box) \in [\alpha_3] \\ &\qquad\qquad \mid N_1 \in [\beta] \mid N_2 \in [\gamma] \\ \triangleright\ &(\lambda fv.fx)(\lambda^* y^{\beta}.\Box)(\lambda^* z^{\gamma}.\Box) \in [\alpha_1] \mid (\lambda vf.f(\lambda^* x^{\alpha_1}.\Box))(\lambda^* y^{\beta}.\Box)(\lambda^* z^{\gamma}.\Box) \in [\alpha_2] \\ &\qquad\qquad\qquad\qquad\qquad\qquad \mid \Box(\alpha_1, \alpha_2) \in [\alpha_3] \mid !_2 N_1 \in [\beta] \mid !_2 N_2 \in [\gamma] \end{aligned}$$

where N_1 and N_2 are copied because of copies of $(\lambda^* y^{\beta}.\Box)(\lambda^* z^{\gamma}.\Box)$ which is derived from $N_1 \in [\beta]$ and $N_2 \in [\gamma]$.

$\triangleright$ $(\lambda^* y^{\beta}.\Box)x \in [\alpha_1] \mid (\lambda^* z^{\gamma}.\Box)(\lambda^* x^{\alpha_1}.\Box) \in [\alpha_2]$
$\mid \Box(\alpha_1, \alpha_2) \in [\alpha_3] \mid N_1 \in [\beta] \mid \Box \in [\beta] \mid N_2 \in [\gamma] \mid \Box \in [\gamma]$

where one occurrence of N_1 becomes $\Box$ as well as N_2 becomes, because of the use of β-reductions by vacuous discharge.

Let N_2 contain $\#z$-occurrences of the variable z. Then one comes to

$!_{\#z} N_1[y := x] \in [\beta] \mid N_2[z := \lambda^* x^{\beta}.\Box] \in [\gamma]$

via communications, under a structural congruence relation such that $\Box \in [\alpha_1] \mid \Box(\alpha_1, \alpha_2) \in [\alpha_3] \equiv \Box(\alpha_2) \in [\alpha_3]$ and that $\Box \in [\alpha] \mid M \in [\beta] \equiv M \in [\beta]$ where M has no occurrences of $\Box(\alpha)$. Roughly speaking, $\Box \in [\alpha]$ is an identity among multiple-conclusions.

On the basis of this result, a proof of $\neg A$ is waiting for a proof of A. If one has a proof of A, then it is passed on to x in $[\beta]$ from N_2 in $[\gamma]$, and moreover N_2 involving $\Box$ (i.e., the remainder of the computation, called continuation) is still preserved in the world $[\gamma]$.

4 Concluding Remarks

In this article, we described only outline of the idea and informal meaning of proof terms as communication terms by using simple examples. For the following subjects:

- Soundness and completeness of hierarchical structures
- Deterministic and non-deterministic communications
- Strong normalization property
- Church-Rosser property for the deterministic version

forthcoming papers are in preparation, and see also *Electronic Notes in Theoretical Computer Science* Vol. 31, No. 1, pp. 167–182 (2000), Computing: The Australasian Theory Symposium (CATS 2000), `http://www.elsevier.nl/locate/entcs/volume31.html`.

Following Gentzen-Parwitz [9, 23], we have demonstrated that a proof code for classical implications can be naturally reinterpreted as a sending communication operator among multiple-conclusion. The sending operator is obtained as a natural extension of λ-terms, and this higher-order communication can be regarded as a computational content of classical proofs.

λ-terms can be interpreted as processes, as in Milner [17], and hence processes can be indirectly assigned to classical proofs as CPS-translated codes. On one hand, the computational behaviour of classical proofs has been interpreted as a process based on an analysis of the non-confluence of classical proofs in [3]. With respect to a hierarchical structure, on the other hand, intuitionistic proofs are still interpreted as λ-terms, and a communication-like interpretation is given only to classical part of proofs. From the viewpoint of classical terms-as-processes, the finite tree of the hierarchical structure can be regarded as the network structure of the communication, in which a term is passed on to terms distributed in other worlds. The theorem of soundness and completeness of hierarchical structures gives a basis to a translation from $\lambda\mu$-terms to the communication-terms of a hierarchical structure, in other words, a program translation from a *sequential* style to a *distributed* style representation. Although we observed the connection between $\lambda\mu$-calculus and the hierarchical structure, the distinction between a sequential style and a distributed style is important in the sense that the first example in Section 3 results in

`mult` 1 (`mult` 2 ($\mu\beta.[\alpha]0$)) by the use of call-by-name $\lambda\mu$-calculus [22]. The distributed style makes it possible to separate the answer from the result in the sequential style.

Since classical logic has both the weakening and the contraction rules, the non-linear property makes the computation (communication) rules complex and interesting, involving a copy operator $!_n$. The hierarchical structure can also be defined for proofs of classical substructural logics [7]. For instance, in the case of BCI-logic plus the double negation elimination, the proof terms are in the form of $\lambda^*(x^{\alpha(1)}).M$ and $\Box(\beta)$, and we need no $!_n$ because of $n = 1$.

It is worth pursuing the following problems as further directions: In this paper, we defined computation rules for well-typed communication-terms based on the hierarchical structure. It is also possible to define a type-free version of the communication-terms. A finite tree was used to define the heirarchic structure. The hierarchical structure could be defined with an infinite tree, and then the validity relation is given by the use of bisimulation relation [16].

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References

[1] Barbanera, F. and S. Berardi: *Extracting Constructive Context from Classical Logic via Control-like Reductions*, Lecture Notes in Computer Science **664** (1993), 45–59

[2] Barbanera, F. and S. Berardi: *A Strong Normalization Result for Classical Logic*, Annals of Pure and Applied Logic **76** (1995), 99-116

[3] Barbanera F., Berardi S., and M. Schivalocchi: *"Classical" Programming-with-Proofs in* λ_{PA}^{Sym}: *An Analysis of Non-confluence*, Lecture Notes in Computer Science **1281** (1997), 365–390

[4] Baba K., Hirokawa S., and K. Fujita: *Parallel Reduction in Type-Free* $\lambda\mu$*-Calculus*, The 7th Asian Logic Conference, 1999.

[5] Boričić, B. R.: *On Sequence-Conclusion Natural Deduction Systems*, Journal of Philosophical Logic **14** (1985), 359–377

[6] Cellucci, C.: *Existential instantiation and normalization in sequent natural deduction*, Annals of Pure and Applied Logic **58** (1992), 111–148

[7] Fujita, K.: *On Proof Terms and Embeddings of Classical Substructural Logics*, Studia Logica **61** (1998), 199–221

[8] Fujita, K.: *A Binary-Conclusion Natural Deduction System*, Logic Journal of the Interest Group in Pure and Applied Logics, Vol. 7, No. 4 (1999), 517–545

[9] Gentzen, G.: *Untersuchungen über das logische Schliessen*, Mathematishe Zeitschrift **39** (1935), 176–210, 405–431 ("The Collected Papers of Gerhard Gentzen", edited by M.E. Szabo, North-Holland, 1969)

[10] Griffin, T. G.: *A Formulae-as-Types Notion of Control*, Proc. 17th Annual ACM Symposium on Principles of Programming Languages (1990), 47–58

[11] de Groote, Ph.: *A Simple Calculus of Exception Handling*, Lecture Notes in Computer Science **902** (1995), 201-215

[12] Hanazawa, M.: *A Characterization of Axiom Schema Playing the Rôle of Tertium non Datur in Intuitionistic Logic*, Proc. Jap. Acad., Vol. 42, No. 9 (1966), 1007–1010

[13] Hofmann, M. and T. Streicher: *Continuation models are universal for $\lambda\mu$-calculus*, Proc. 12th Annual IEEE Symposium on Logic in Computer Science (1997), 387–395

[14] Howard, W.: *The Formulae-as-Types Notion of Constructions*, 479–490, "To H.B. Curry: Essays on combinatory logic, lambda-calculus, and formalism", edited by J.P. Seldin and J.R. Hindley, Academic Press, 1980

[15] Kobayashi, S.: *Monads as modality*, Theor. Comput. Sci. **175** (1997), 29–74

[16] Milner, R.: "Communication and Concurrency", Prentics Hall, 1989

[17] Milner, R.: *Functions as processes*, Math. Struct. in Comp. Science **2** (1992), 119–141

[18] Murthy, C. R.: *An Evaluation Semantics for Classical Proofs*, Proc. 6th Annual IEEE Symposium on Logic in Computer Science (1991), 96–107

[19] Nakano, H.: *The Non-deterministic Catch and Throw Mechanism and Its Subject Reduction Property*, Lecture Notes in Computer Science **792** (1994), 61–72

[20] Nakano, H.: "Logical Structures of the Catch and Throw Mechanism", PhD thesis, University of Tokyo, 1995

[21] Ong, C. -H. L.: *A Semantic View of Classical Proofs: Type-Theoretic, Categorical, and Denotational Characterizations*, Linear Logic '96 Tokyo Meeting, 1996

[22] Parigot, M.: *$\lambda\mu$-Calculus: An Algorithmic Interpretation of Classical Natural Deduction*, Lecture Notes in Computer Science **624** (1992), 190–201

[23] Prawitz, D.: "Natural Deduction. A Proof-Theoretical Study", Almqvist&Wiksell, 1965

[24] Prawitz, D.: *Ideas and Results in Proof Theory*, 235–307, "Proceedings of the Second Scandinavian Logic Symposium", edited by J.E. Fenstad, North-Holland, 1971

[25] Rehof, N. H. and M. H. Sørensen: *The λ_Δ-Calculus*, Lecture Notes in Computer Science **789** (1994), 516–542

[26] Ungar, A. M.: "Normalization, Cut-Elimination and the Theory of Proofs", Center for the Study of Language and Information, 1992