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**ON THE OBSERVATIONS OF THE EARTH TIDE  
BY MEANS OF EXTENSOMETERS IN  
HORIZONTAL COMPONENTS**

**BY**

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## Abstract

The author has performed the observations of tidal strains of the earth's surface in some or several directions by means of extensometers at Osakayama observatory Kishu mine, Suhara observatory and Matsushiro observatory, and he has calculated the tide-constituents ( $M_2$ ,  $O_1$ , etc.) of the observed strains by means of harmonic analysis.

According to the results, the phase lags of  $M_2$ -constituents except one in Suhara are nearly zero, whose upper and lower limits are  $43^\circ$  and  $-29^\circ$ , respectively. That is the coefficients of  $\cos 2t$ -terms of the strains are positive value in all the azimuths, and the ones of their  $\sin 2t$ -terms are much smaller than the ones of their  $\cos 2t$ -terms, where  $t$  is an hour angle of hypothetical heavenly body at the observatory. Therefore, it is most probable that the secondary effects (due to the oceanic tide) are smaller than the primary effects (due to the tide generating force) in each observatories, and also that the value of Love's number  $h$  is about five times greater than the value of Shida's number  $l$ .

Now, the precise observations of the linear strains in the prime vertical component and in the meridional component have been performed additionally with highly sensitive extensometers of which sensitivities are  $0.37 \times 10^{-8}/\text{mm}$  and  $0.57 \times 10^{-8}/\text{mm}$ , respectively at Osakayama.

From the present observations, the value of  $h-3l$  relating to areal strain is derived as follow

$$h-3l=0.448 \pm 0.03.$$

And the amplitude ratio of  $O_1$ -constituent to  $M_2$ -constituent are obtained as follows

$$\left(\frac{O_1}{M_2}\right)_{\phi\phi}=0.85 \quad \text{in the prime vertical component,}$$

and  $\left(\frac{O_1}{M_2}\right)_{\theta\theta}=0.48$  in the meridional component.

If the secondary effects are as great as the primary effect, the amplitude ratio  $(O_1/M_2)_{\phi\phi}$  and  $(O_1/M_2)_{\theta\theta}$  will have been almost equal to each other. But on the contrary, both ratios are widely different each other. Hence we may neglect the secondary effect against the primary effect in the following process to calculate the tidal constants. The propriety of this results is shown by the comparison of the strain due to Chili tide-wave at Kishu and ones of Osakayama, and by the comparison of the strains due to the tide-wave and the semi-diurnal constituents of the tidal strain due to the earth tide at both these observatories.

And the amplitude ratio of  $M_2$ -constituent of the  $e_{\theta\theta}$ -component to one of the  $e_{\phi\phi}$ -component is obtained as follow

$$\left(\frac{e_{\theta\theta}}{e_{\phi\phi}}\right)_{M_2}=1.69,$$

where  $e_{\theta\theta}$  and  $e_{\phi\phi}$  are the strain components in the meridional direction and in the prime vertical direction, respectively.

From these results, the relations of  $h$  and  $l$  are calculated as follows

$$h=11.1l \quad \text{from} \quad \left(\frac{O_1}{M_2}\right)_{\phi\phi},$$

$$\text{and} \quad h=10.6l \quad \text{from} \quad \left(\frac{e_{\theta\theta}}{e_{\phi\phi}}\right)_{M_2}.$$

Then, from the value of  $h-3l$  and the ratio  $(O_1/M_2)_{\phi\phi}$ , the value of  $h$  and  $l$  are calculated at 0.614 and 0.055, respectively.

1. The study of earth tide is populated from the oldtimes in the both field of observation [1] and in that of theory [2], but the study in the observation of the tidal strains is not so prevalent as the ones of tilt, gravity acceleration and latitude. The observation of tidal strain of the earth was succeeded first with the Sassa type extensometer under leadings of Dr. Kenzo Sassa. Afterward, the many types of more sensitive extensometer [3], [4] have been devised and several observatories equipped with extensometers have been instituted gradually. But the distributions of these observatories are limited within a particular district.

Shida's number  $l$  of the earth tide is not able to obtained independently by the observations of only one component in ordinary latitude, and so the author has performed the observations of some or several com-

ponents of the tidal strains at Osakayama observatory, Kishu mine, Suhara observatory and Matsushiro observatory in order to derive tidal constants in earth tide and to examine the conditions of the strain in the crustal movements. According to his observations, the secondary effects (of the earth tide) due to the oceanic tide are far smaller than the primary effects of it due to the tide generating force in these observatories except Suhara observatory where is located at seashore. Hence, he has devised the new types [8] of highly sensitive extensometers and he has been performing precise observations of tidal strains in east to west and in north to south directions at Osakayama observatory. From this observations, he has calculated the value of  $h-3l$  relating to areal strain and the ratios of  $O_1$ -constituent to  $M_2$ -constituent in the prime vertical direction and in meridional direction. Then, the values of  $h$  and  $l$  are obtained by use of the value of  $h-3l$  and the ratio ( $O_1/M_2$ ).

2. The tide-generating potential  $W_2$  due to a heavenly body of which mass is  $M$  is approximated by

$$W_2 = \frac{3}{2} g \frac{M}{E} \left( \frac{a^2}{c^3} \right) r^2 \left( \cos^2 Z - \frac{1}{3} \right), \quad \dots\dots\dots(1)$$

where  $g$  is the mean gravity-acceleration of the earth at a observatory,  $E$  is the mass of the earth,  $a$  is the mean radius of the earth,  $r$  is the distance from the earth's centre to the observing point,  $c$  is the distance from the earth's centre to the centre of the heavenly body and  $Z$  is the zenith distance of the heavenly body at the observing point.

The value of  $Z$  is written as follow

$$\cos Z = \cos \theta \sin \delta + \sin \theta \cos \delta \cos(t+\phi), \quad \dots\dots\dots(2)$$

where  $\theta$  and  $\phi$  are colatitude and eastern longitude of the observing point, respectively,  $\delta$  is the declination of the heavenly body and  $t$  is the Greenwich standard time of it, and so  $t+\phi$  is the angle between the meridional plane on which heavenly body is and one on which observing point is.

From (1) and (2), it is written as

$$\begin{aligned} W_2 = \frac{3}{4} g \left( \frac{M}{E} \right) \left( \frac{a^2}{c^3} \right) r^2 & \left\{ 3 \left( \sin^2 \delta - \frac{1}{3} \right) \left( \cos^2 \theta - \frac{1}{3} \right) \right. \\ & \left. + \sin 2\delta \sin 2\theta \cos(t+\phi) + \cos^2 \delta \sin^2 \theta \cos 2(t+\phi) \right\}. \quad \dots\dots\dots(3) \end{aligned}$$

Let radial, colatitudinal and meridional components of displacement of

an any point in the earth due to the tidal force be  $u_r$ ,  $u_\theta$  and  $u_\phi$ , respectively, they are given as follows

$$\left. \begin{aligned} u_r &= \frac{H(r)}{g} W_2, \\ u_\theta &= \frac{L(r)}{g} \frac{\partial W_2}{\partial \theta}, \\ u_\phi &= \frac{L(r)}{g \sin \theta} \frac{\partial W_2}{\partial \phi}, \\ h &= H(a), \quad l = L(a), \end{aligned} \right\} \dots\dots\dots (4)$$

where  $h$  is Love's number [5],  $l$  is Shida's number [6].

The strain components respect to polar coordinate  $(r, \theta, \phi)$  are given as follows,

$$\left. \begin{aligned} e_{rr} &= \frac{\partial u_r}{\partial r}, \quad e_{\theta\theta} = \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r}, \\ e_{\phi\phi} &= \frac{1}{r \sin \theta} \frac{\partial u_\phi}{\partial \phi} + \frac{u_\theta}{r} \cot \theta + \frac{u_r}{r}, \\ e_{\theta\phi} &= \frac{1}{r} \left( \frac{\partial u_\phi}{\partial \theta} - u_\phi \cot \theta \right) + \frac{1}{r \sin \theta} \frac{\partial u_\theta}{\partial \phi}, \\ e_{\phi r} &= \frac{1}{r \sin \theta} \frac{\partial u_r}{\partial \phi} + \frac{\partial u_\phi}{\partial r} - \frac{u_\phi}{r}, \quad e_{r\theta} = \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r} + \frac{1}{r} \frac{\partial u_r}{\partial \theta} \end{aligned} \right\} \dots\dots\dots (5)$$

From (3), (4) and (5), the strain components at the earth's surface ( $r=a$ ) are obtained as follows,

| strain-component                                    | semi-diurnal component                                               | diurnal component                                                    | long period's component                                                 |
|-----------------------------------------------------|----------------------------------------------------------------------|----------------------------------------------------------------------|-------------------------------------------------------------------------|
|                                                     | $\times \frac{3}{4} \frac{M}{E} \left( \frac{a}{c} \right)^3 \times$ | $\times \frac{3}{4} \frac{M}{E} \left( \frac{a}{c} \right)^3 \times$ | $\times \frac{3}{4} \frac{M}{E} \left( \frac{a}{c} \right)^3 \times$    |
|                                                     | $\sin^2 \delta \cos 2(t+\phi)$                                       | $\sin 2\delta \cos(t+\phi)$                                          | $(3 \sin^2 \delta - 1)$                                                 |
| $e_{\theta\theta}$                                  | $h \sin^2 \theta + 2l \cos 2\theta$                                  | $(h-4l) \sin 2\theta$                                                | $h(\cos^2 \theta - 1/3) - 2l \cos 2\theta$                              |
| $e_{\phi\phi}$                                      | $h \sin^2 \theta - 2l(1 + \sin^2 \theta)$                            | $(h-2l) \sin 2\theta$                                                | $h(\cos^2 \theta - 1/3) - 2l \cos^2 \theta$                             |
| $e_{\theta\phi}$                                    | $-4l \cos \theta \tan 2(t+\phi)$                                     | $4l \sin \theta \tan(t+\phi)$                                        | 0                                                                       |
| $e_{rr}$                                            | $\left\{ a \frac{dH(a)}{dr} + 2h \right\} \sin^2 \theta$             | $\left\{ a \frac{dH(a)}{dr} + 2h \right\} \sin 2\theta$              | $\left\{ a \frac{dH(a)}{dr} + 2h \right\} \times (\cos^2 \theta - 1/3)$ |
| areal strain<br>$= e_{\theta\theta} + e_{\phi\phi}$ | $2(h-3l) \sin^2 \theta$                                              | $2(h-3l) \sin 2\theta$                                               | $2(h-3l)(\cos^2 \theta - 1/3)$                                          |
| $e_{\theta\theta} - e_{\phi\phi}$                   | $2l(1 + \cos^2 \theta)$                                              | $-2l \sin 2\theta$                                                   | $2l \sin^2 \theta$                                                      |
| $e_{r\theta}$                                       | 0                                                                    | 0                                                                    | 0                                                                       |
| $e_{r\phi}$                                         | 0                                                                    | 0                                                                    | 0                                                                       |

$$\begin{aligned} \text{cubical dilatation} &= e_{\theta\theta} + e_{\phi\phi} + e_{rr} \\ &= \left\{ a \frac{dH(a)}{dr} + 4h - 6l \right\} \times \sin^2\theta + \left\{ a \frac{dH(a)}{dr} + 4h - 6l \right\} \times \sin 2\theta + \left\{ a \frac{dH(a)}{dr} + 4h - 6l \right\} \times (\cos^2\theta - 1/3). \end{aligned} \quad (6)$$

According to formulae (6), semi-diurnal and diurnal components of  $e_{\theta\phi}$  and  $e_{\theta\theta} - e_{\phi\phi}$ , and long period's component of  $e_{\theta\theta} - e_{\phi\phi}$  are dependent on only  $l$  and are independent on  $h$ . And the long period's component of  $e_{\theta\phi}$  is equal to zero. It seems that the tidal strains in the district near to the coast are consist of the primary effect of it due directly to the tide generating force and the secondary effect of it due to the oceanic tide. The absolute quantity of the secondary effect at a place where is located from about 10 km. to score's kilometers distant from the oceanic coast has been indistinct. But as the effect of the gradient of the sea-bottom in the district where is not close to the coast may be neglected, and as the crust is not so much anisotropic in a same horizontal plane, the strain components due to the variations of oceanic tidal load are contented each other following conditions

$$e'_{\theta\theta} = -e'_{\phi\phi}, \quad \text{and} \quad e'_{rr} = 0. \quad (7)$$

Hence, it may estimate that the tide-components of horizontal areal strain, vertical strain and cubical dilatation at the earth's surface don't been influenced by oceanic tide.

3. The observations of the linear strains on the ground in some or several directions have been performed at Osakayama observatory (since 1947), Kishu mine (since 1955), Suhara observatory (since 1955) and Matsushiro observatory (since 1953), and then the tidal constituents of the observed strains at these observatories have been calculated by means of harmonic analysis. Sassa-type extensometers, H-53-type extensometer and H-59-types extensometers have been made use for these observations at Osakayama. H-53-type extensometers have been used for observation at Kishu and Suhara, and Sassa-type-extensometers have been used at Matsushiro. The some parts of the analysed values were published in some papers [4], [7] by present author. The analysed values of the horizontal tidal strains that have been observed with highly sensitive extensometers on and after 1959 at Osakayama are shown in Table 1.

The amplitudes and phase lags of  $M_2$ -constituent of the strains versus azimuth at Osakayama, Kishu and Matsushiro are shown in Fig. 1.

Table 1. Analysed values of tidal constituents of the ground strains.

Observatory: Osakayama

Location: Long. 135°51.5'E., 34°59.6'N.

Elevation: 100 m. above the sea-level

Depth: 100 m. under the ground

Instrument: H-59-type extensometer

| Azimuth of observation | Period analysed                      | Sensitivity of instrument        | M <sub>2</sub> -constituent |           | O <sub>1</sub> -constituent |           |
|------------------------|--------------------------------------|----------------------------------|-----------------------------|-----------|-----------------------------|-----------|
|                        |                                      |                                  | Amplitude                   | Phase lag | Amplitude                   | Phase lag |
| east to west           | from 1959 Jan. 27<br>to 1959 Feb. 25 | $1.81 \times 10^{-8}/\text{mm.}$ | $0.632 \times 10^{-8}$      | 4.6°      | $0.672 \times 10^{-8}$      | 22.0°     |
| east to west           | from 1959 Mar. 28<br>to 1959 Apr. 26 | 0.53 "                           | 0.758 "                     | 359.7°    | 0.735 "                     | 16.3°     |
| east to west           | from 1959 May 24<br>to 1959 Aug. 23  | 0.37 "                           | 0.880 "                     | 357.8°    | 0.760 "                     | 3.5°      |
| north to south         | from 1959 Apr. 23<br>to 1959 Jul. 22 | 0.57 "                           | 1.362 "                     | 10.9°     | 0.640 "                     | 20.8°     |

The horizontal linear strain in a given direction of which direction-cosines are  $\lambda$  and  $\mu$  against to the meridional and to the prime vertical directions, respectively is given as follow

$$e(\lambda, \mu) = e_{\theta\theta}\lambda^2 + e_{\phi\phi}\mu^2 + e_{\theta\phi}\lambda\mu. \dots\dots(8)$$

The relative values of the coefficients of cos-terms and sin-terms of tidal strain of the primary effect for some values of the parameter  $h/l$  versus the azimuths are calculated by formulae (6) and (8), and the coefficients are shown in Fig. 2 and Fig. 3. Fig. 2 shows the coefficients for the semi-diurnal component of the strain, and Fig. 3 shows the ones of diurnal component of it. The unit of the coefficient (ordinate) is  $\frac{3}{4} \frac{M}{E} \left(\frac{a}{c}\right)^3 \sin^2 \delta$  in Fig. 2 and is  $\frac{3}{4} \frac{M}{E} \left(\frac{a}{c}\right)^3 \sin 2\delta$  in

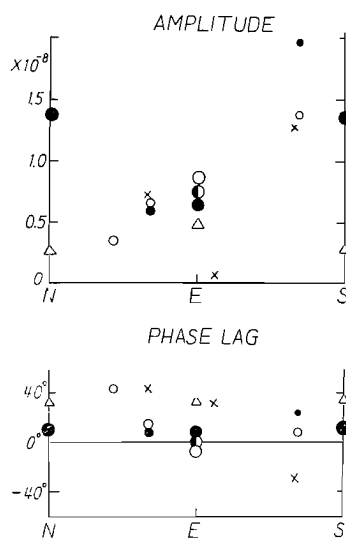


Fig. 1. Relation of amplitudes and phase lags of  $M_2$ -constituent of linear strain vs. azimuth at Osakayama, Kishu and Matsushiro.  $\circ$ ,  $\bullet$ ,  $\bullet$  show the values, obtained from the observations at Osakayama,  $\times$  shows ones at Kishu, and  $\triangle$  shows ones at Matsushiro.

Fig. 3. It is found from formulae (6) and (8), or in Fig. 2 and Fig. 3 that the ranges of the parameter  $h/l$  in which the coefficients of the cos-terms at the latitude  $34^\circ 59.6'$  should be positive quantities in all the azimuths are given as follows,

$$\left. \begin{array}{l} h/l \geq 4.98 \dots \dots \text{for semi-diurnal component,} \\ \text{and} \quad h/l \geq 4.00 \dots \dots \text{for diurnal component.} \end{array} \right\} \dots \dots \dots \text{(I)}$$

But, the range of the parameter  $h/l$  for diurnal component is a same value in all the latitude.

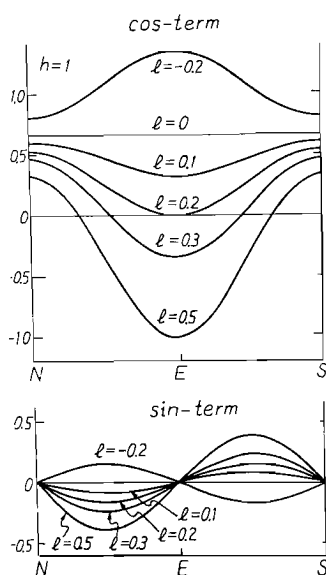


Fig. 2. The calculated coefficients of cos  $2t$ -term and sin  $2t$ -term of the tidal strain (for semi-diurnal component) vs. the azimuth.

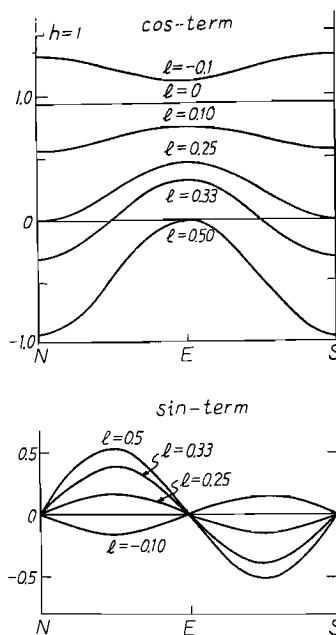


Fig. 3. The calculated coefficients of cos  $t$ -term and sin  $t$ -term of the tidal strain (for diurnal-component) vs. the azimuth.

As the cos-terms of  $e_{\phi\phi}$  and  $e_{00}$  components of the primary effect of the tidal strains are positive quantities and the sin-terms of them are nil, the cos-terms of  $e_{\phi\phi}$  and  $e_{00}$  components of the secondary effect of it should be smaller than the both  $e_{\phi\phi}$  and  $e_{00}$  components of observed tidal strains. And the sin-terms of the secondary effect are equal to the ones of the observed values. According to Fig. 1, the observed values of the cos-terms of  $M_2$ -constituents are positive quantities in all the azimuths,

and they are not particularly small in some azimuths, therefore the cos-terms of the secondary effects should be smaller than the ones of the primary effects. And also the phase lags of observed values are very small, hence the secondary effects are much smaller than the primary effect in general at these observatories. The propriety of the supposition is shown by the comparison of the strains due to Chili tide wave on May 24, 1960 at Kishu and Osakayama. This reason is that the strain amplitudes of the effect due to the tide-wave (tsunami) are much smaller than ones of the semi-diurnal component of it at these observatories in spite of that the wave length of the tsunami is as long as the radius of the main action sea-region for the secondary effect in these observatories and the tide range of the tsunami is far greater than the one of the semi-diurnal tide. And also the strain amplitude due to tsunami at Osakayama is a figure smaller than the ones at Kishu.

From the observed values on Table 1, the values of horizontal areal strains are calculated as follows

$$\left. \begin{array}{l} 2.15_1 \times 10^{-8} \cos(2t - 6.6^\circ) \text{ on } M_2\text{-constituent,} \\ 1.38_2 \times 10^{-8} \cos(t - 18.3^\circ) \text{ on } O_1\text{-constituent.} \end{array} \right\} \dots(\text{II})$$

From this obtained values and formula (6), we get as follows

$$\begin{aligned} h - 3l &= 0.45_1 \text{ on } M_2\text{-constituent,} \\ h - 3l &= 0.44_2 \text{ on } O_1\text{-constituent.} \end{aligned}$$

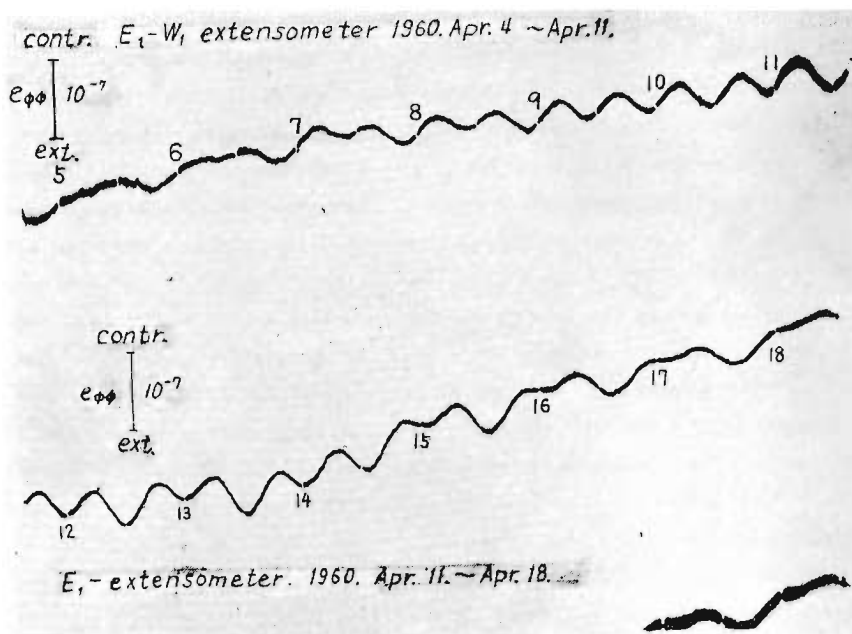
The weighted mean value of  $h - 3l$  is calculated by weighting 3 on  $M_2$ -constituent and weighting 1 on  $O_1$ -constituent as follow

$$h - 3l = 0.44_8 \pm 0.003. \quad \dots\dots\dots(\text{III})$$

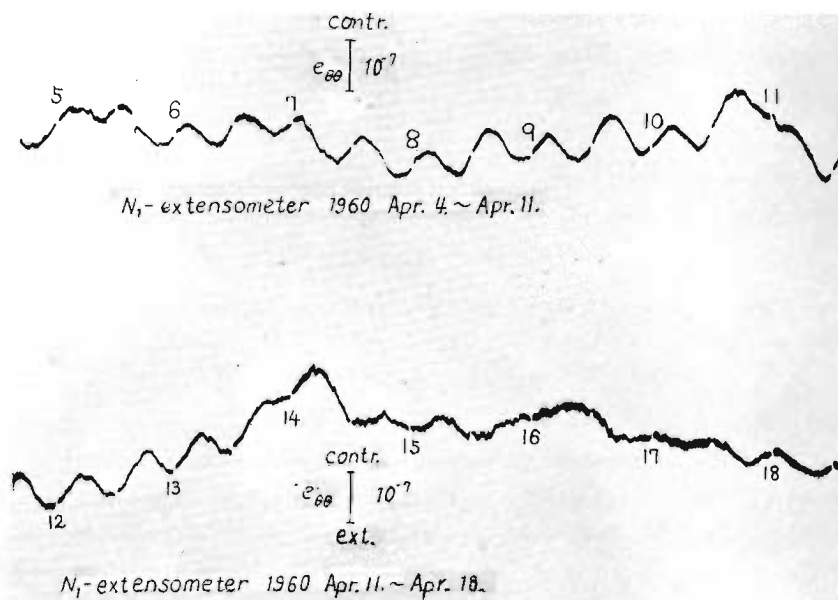
This value is nearly equal to the value that was obtained on 1954 [7] by present author as follows

$$h - 3l = 0.43_4 \pm 0.026.$$

The photographic records of  $e_{\theta\theta}$  and  $e_{\phi\phi}$  components of linear strain during twice weeks' term observed with extensometers are shown in Photo 1. The curves of reading values of the tidal strains on a crescent moon (from Jun. 14 to Jun. 15, 1959) and on a full moon (from Jun. 20 to Jun. 21, 1959) are shown in Fig. 4. According to Photo 1 and and Fig. 4, it is found that the apparent amplitude of the diurnal constituent in  $e_{\phi\phi}$ -component of the strain changes with a period of about twice weeks, and that the shape of the tidal strain curves on the full moon days and the



(a)



(b)

Photo 1 (a) and (b). The photographic tidegrams of linear strains which are recorded with the extensometers at Osakayama.  $e_{\theta\theta}$  and  $e_{\phi\phi}$  show the meridional and the prime vertical components of the ground-strains, respectively.

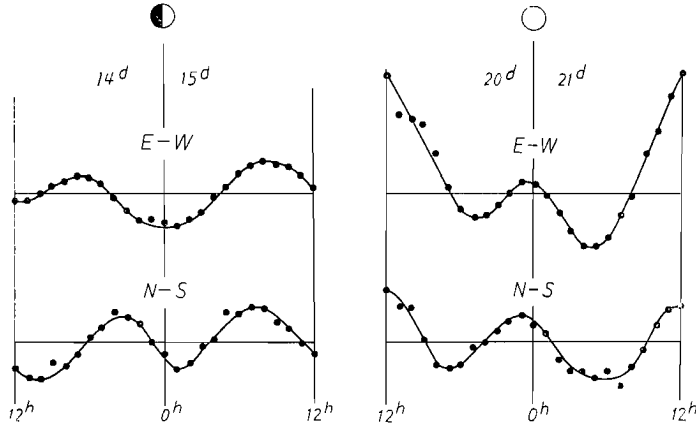


Fig. 4. Observed tide curves of the linear strains which are eliminated the secular changes, in the prime vertical component and in the meridional component at Osakayama on June 1959.

new moon are like a *W*-letter (take extensional sense to upward). But the ones in  $e_{\theta\theta}$ -component are always small.

The amplitude ratio of the analyzed values of  $O_1$ -constituent to  $M_2$ -constituent are as follows

$$\left. \begin{aligned} \left( \frac{O_1}{M_2} \right)_{\phi\phi} &= 0.85 \quad \text{in } e_{\phi\phi}\text{-component,} \\ \left( \frac{O_1}{M_2} \right)_{\theta\theta} &= 0.48 \quad \text{in } e_{\theta\theta}\text{-component.} \end{aligned} \right\} \dots\dots\dots(\text{IV})$$

On the other hand from formulae (6), the relations are obtained as follows

$$\begin{aligned} \left( \frac{O_1}{M_2} \right)_{\theta\theta} &= \frac{(h-4l)\sin 2\theta}{h \sin^2 \theta + 2l \cos 2\theta} \cdot \frac{\sin 2\delta(O_1)}{\sin^2 \delta(M_2)}, \\ \left( \frac{O_1}{M_2} \right)_{\phi\phi} &= \frac{(h-2l)\sin 2\theta}{h \sin^2 \theta - 2l(1+\sin^2 \theta)} \cdot \frac{\sin 2\delta(O_1)}{\sin^2 \delta(M_2)}, \end{aligned}$$

where  $\delta(O_1)$  and  $\delta(M_2)$  are the declinations of the hypothetical heavenly body of  $O_1$ -constituent and  $M_2$ -constituent, respectively.

The values of  $(O_1/M_2)_{\theta\theta}$  and  $(O_1/M_2)_{\phi\phi}$  are equal to each other in the case of  $l=0$  and  $h/l=3$ , but this both cases' relations don't content with the condition (I) obtained by the observations that is  $h/l \geq 4.98$ . In order that the ratio of  $(O_1/M_2)_{\theta\theta}$  and  $(O_1/M_2)_{\phi\phi}$  may be 0.56, the value of  $h/l$  should be 14.3. And the secondary effect of the strain should be far smaller than the primary effects of it, because of the values of  $(O_1/M_2)_{\theta\theta}$  and  $(O_1/M_2)_{\phi\phi}$  are very different each other. The accuracies of the numerical

values of  $(O_1/M_2)$  depend chiefly on the magnitude of the  $O_1$ -constituents in the components. And so the accuracy of  $(O_1/M_2)_{\phi\phi}$  is much higher than one of  $(O_1/M_2)_{\theta\theta}$ . In addition to this reason, irregular micro variations appear remarkably in the  $e_{\theta\theta}$ -component. Hence, using the value of  $(O_1/M_2)_{\phi\phi}$ , the ratio  $h/l$  is calculated as follow

$$\frac{h}{l} = 11.11. \quad \dots\dots\dots(V)$$

This method has a strong merit that it need not know a sensitivity of the instrument.

From (V) and (III), the values of  $h$  and  $l$  are calculated as follows,

$$\left. \begin{array}{l} h = 0.614 \\ l = 0.055. \end{array} \right\} \quad \dots\dots\dots(VI)$$

By another method, the ratios of  $e_{\theta\theta}$  to  $e_{\phi\phi}$  are obtained from the observed values of  $M_2$ -constituents and  $O_1$ -constituent in the meridional component and in the prime vertical component, respectively from formulae (6) as follows,

$$\left. \begin{array}{l} \left( \frac{e_{\theta\theta}}{e_{\phi\phi}} \right)_{M_2} = \frac{h \sin^2 \theta + 2l \cos 2\theta}{h \sin^2 \theta - 2l(1 + \sin^2 \theta)} = 1.687 \quad \text{in } M_2\text{-constituent,} \\ \left( \frac{e_{\theta\theta}}{e_{\phi\phi}} \right)_{O_1} = \frac{h - 4l}{h - 2l} = 0.867 \quad \text{in } O_1\text{-constituent.} \end{array} \right\} \quad \dots\dots\dots(VII)$$

The value of  $(e_{\theta\theta}/e_{\phi\phi})_{O_1}$  is independent on longitude, but the amplitude of  $M_2$ -constituent is much greater than that of  $O_1$ -constituent and the weight of  $M_2$ -constituent is about three times as greater as  $O_1$ -constituent. So the ratio of  $h/l$  is calculated from the value of  $M_2$ -constituent on (VII) as follow

$$\frac{h}{l} = 10.62. \quad \dots\dots\dots(VIII)$$

From (III) and (VIII), the values of  $h$  and  $l$  are obtained also as follows

$$\left. \begin{array}{l} h = 0.624 \\ l = 0.059. \end{array} \right\} \quad \dots\dots\dots(IX)$$

Now, it has the equation from the free surface condition as follow

$$a \frac{dL(a)}{dr} + h + 3l = 0. \quad \dots\dots\dots(9)$$

From (9) and (VI), the value of  $a \frac{dL(a)}{dr}$  is calculated as follow

$$a \frac{dL(a)}{dr} = -0.780. \quad \dots\dots\dots(X)$$

And the observed value of the vertical component of the tidal strain which was obtained with the extensometers in the vertical direction on 1957, it is obtained as follow

$$a \frac{dH(a)}{dr} + 2h = -0.28_{\text{s}}. \quad \dots\dots\dots(\text{XI})$$

From (VI) and (XI), it is calculated as

$$a \frac{dH(a)}{dr} = -1.50_{\text{s}}. \quad \dots\dots\dots(\text{XII})$$

According to the result of (X) and (XII) the deeper the observational point creep into the earth, the greater the values of  $L(r)$  and  $H(r)$  at neighbouring the earth's surface become. Namely, the deeper the point goes down into the earth, the greater the tidal displacement becomes at the neighbouring earth's surface.

Dr. H. Takeuchi calculated the numerical values of  $F(\xi)$ ,  $G(\xi)$ ,  $F'(\xi)$  and  $G'(\xi)$  respect to the earth tide by use of K. E. Bullen and others' models in respect to the distributions of elastic moduli and density within the earth, (where  $\xi$  is  $r/a$ ). The values of  $a \frac{dL(a)}{dr}$  and  $a \frac{dH(a)}{dr}$  are calculated by use of these values of  $F(\xi)$ ,  $G(\xi)$ ,  $F'(\xi)$  and  $G'(\xi)$  as follows

$$\left. \begin{aligned} a \frac{dL(a)}{dr} &= -0.69, \\ a \frac{dH(a)}{dr} &= -1.51. \end{aligned} \right\} \dots\dots\dots(\text{XIII})$$

and

These values agree approximately with the ones obtained by our observations of tidal strains.

These results by the observations of the tidal strains are in good agreements with theoretical values which are calculated by use of the elastic moduli which are obtained by the study of the transmissions of the seismic waves within the earth, in spite of that the our observations have been performed at Japan where is in a particular place of the eastern edge of Eurasian continent.

In order to study an anomaly of the earth tide due to the anomalous constructions of the crust, it need perform more precise observations. And also, it is one of the most important keys to resolve the problem that the secondary effects are able to be neglected against to the primary effects or not at these observatories. This assumption is proved by not only the observations of the tidal strains but also the ones due to Chili

tide wave. The author wants to study elaborately on the state of the attenuation of the strains due to tidal load versus distance from the ocean.

4. In conclusion, the author wish to express his sincere thanks to Profs. Kenzo Sassa and Eiichi Nishimura of Kyoto University for their valuable suggestions in his investigations. And also he wish to express his thanks to the members of Kishu mine who are performing the weekly observations at Kishu mine, Mr. Kenjiro Shinya who has performed the weekly observations at Suhara and the members of Matsushiro observatory. This investigation is parted by a Grant in Aid for Fundamental Scientific Research from Ministry of Education and a Grand in Aid for Science of Hattori-Hōkō-Kai. And this study has been performed chiefly at the institutions of Disaster Prevention Research Institute of Kyoto University.

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