

Multi-port Model Order Reduction with Multiple Expansion Points

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Multiple expansion points are introduced to the multi-port Cauer ladder network method to realize an efficient model order reduction of an eddy-current field. The proposed method can set multiple target frequencies to cover a wide frequency range with a small number of multi-port electric network elements. A time-dependent response to pulse-width-modulation (PWM) excitation is accurately reconstructed by setting the target frequencies at both fundamental and carrier frequencies.

Index Terms—Cauer ladder network, expansion point, model order reduction, multiple ports

I. INTRODUCTION

THE DESIGN of electric machinery sometimes requires a large number of electromagnetic field computations to be made repeatedly. As an example, the PWM excitation with a high carrier frequency requires time-dependent analysis with a large number of time steps. The motor design for electric vehicles requires evaluation of the motor performance under various operating conditions. Even though the finite element method (FEM) allows an accurate electromagnetic field analysis, its computational cost is not low enough for a large number of repeated executions. To avoid a heavy computational burden, model order reduction (MOR) methods [1]–[4] have been intensively studied.

The Cauer ladder network (CLN) method [5] is an efficient and convenient MOR method for an eddy-current (EC) field. While most MOR methods are mathematically oriented, the CLN method provides a clear physical meaning owing to the electric network representation. The CLN method has been extended rapidly on the basis of the network representation. As an example, a multi-port version [6] was successfully applied to an MOR of an induction motor [7]. Multiple expansion points [8] were introduced to the single-port CLN method to cover a wide frequency range with a small number of network elements.

This paper develops a multiport CLN method with multiple expansion points to realize a more efficient multi-port reduced representation of the EC field. In spite of the apparent similarity between the single-port and multi-port CLNs, the extension to the multi-port CLN is not straightforward because the product of matrices, such as network element matrices and basis vector matrices, is noncommutative. After formulating

the multiport CLN with expansion points, the paper discusses the mathematical basis including the derivation of network equations, which were not sufficiently presented in [8]. The orthogonality among basis vectors generated with different expansion points is shown to reconstruct the electric and magnetic fields based on the orthogonal expansion.

II. MULTIPORT CLN WITH EXPANSION POINTS

A. Multiport CLN with a single expansion point

Vector potential $\mathbf{A}$ and electric field $\mathbf{E}$ are expressed in finite element (FE) space as

$$\mathbf{A} = \sum_i a_i \mathbf{w}_i^1, \quad \mathbf{E} = \sum_i e_i \mathbf{w}_i^1, \quad (1)$$

where a_i and e_i are line integrals of $\mathbf{A}$ and $\mathbf{E}$ on edge i and $\mathbf{w}_i^1$ is the edge element. The EC field is described in FE space as

$$\mathbf{C}^T \mathbf{v} \mathbf{C} \mathbf{a} = \boldsymbol{\sigma} \mathbf{e}, \quad \mathbf{C} \mathbf{e} = -s \mathbf{C} \mathbf{a}, \quad (2)$$

where $\mathbf{a} = [a_1, a_2, \dots]^T$, $\mathbf{e} = [e_1, e_2, \dots]^T$, $\mathbf{C}$ is the edge-face incidence matrix, $\mathbf{C}^T \mathbf{v} \mathbf{C}$ is the stiffness matrix for FE EC analysis, and s means d/dt . The conductivity matrix $\boldsymbol{\sigma}$ and reluctivity matrix $\mathbf{v}$ are given as

$$\mathbf{v} = \{v_{kl}\}, \quad v_{kl} = \int_{\Omega} \frac{1}{\mu} \mathbf{w}_k^2 \cdot \mathbf{w}_l^2 d\Omega, \quad (3)$$

$$\boldsymbol{\sigma} = \{\sigma_{ij}\}, \quad \sigma_{ij} = \int_{\Omega} \sigma \mathbf{w}_i^1 \cdot \mathbf{w}_j^1 d\Omega, \quad (4)$$

where μ is the permeability and σ is the conductivity while $\mathbf{w}_k^2$ is the facial element of face k . The magnetic and electric properties are assumed to be linear in this paper.

Let a target frequency be f_i . An expansion point $s_i = 2\pi f_i$ is introduced to the CLN method by defining

$$\mathbf{K}_i = \mathbf{C}^T \mathbf{v} \mathbf{C} + s_i \boldsymbol{\sigma}. \quad (5)$$

Let the number of ports be M . In a similar manner to the single-port version [8], a multiport CLN is constructed by recurrence relations as

$$\mathbf{K}_i (\mathbf{a}_{2n+1} - \mathbf{a}_{2n-1}) = \boldsymbol{\sigma} \mathbf{e}_{2n} \mathbf{R}_{2n}, \quad (6)$$

$$\mathbf{e}_{2n+2} - \mathbf{e}_{2n} = -\mathbf{a}_{2n+1} \mathbf{L}_{2n+1}^{-1}, \quad (7)$$

$$\mathbf{R}_{2n}^{-1} = \mathbf{e}_{2n}^T \boldsymbol{\sigma} \mathbf{e}_{2n}, \quad \mathbf{L}_{2n+1} = \mathbf{a}_{2n+1}^T \mathbf{K}_i \mathbf{a}_{2n+1}, \quad (8)$$

where $\mathbf{R}_{2n}$ and $\mathbf{L}_{2n+1}$ ($n = 0, 1, \dots$) are $M \times M$ resistance and inductance matrices while $\mathbf{e}_{2n} = (\mathbf{e}_{1,2n}, \dots, \mathbf{e}_{M,2n})$ and $\mathbf{a}_{2n+1} = (\mathbf{a}_{1,2n+1}, \dots, \mathbf{a}_{M,2n+1})$ are the n -th basis vector matrices. The m -th columns $\mathbf{e}_{m,2n}$ and $\mathbf{a}_{m,2n+1}$ are field basis vectors when source m

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has a unit voltage/current source and the other sources are zero.

Suppose that a unit voltage is given to port m ($m = 1, \dots, M$) as the boundary condition and the resultant scalar potential distribution is represented as ϕ_m . The initial fields are then given by $\mathbf{a}_{-1} = 0$ and by

$$\mathbf{e}_0 = -\mathbf{G}\phi - s_0\mathbf{a}_0, \quad (9)$$

where $\phi = [\phi_1, \dots, \phi_M]$, $\mathbf{G}$ is the node-edge incidence matrix that corresponds to the grad operator satisfying $\mathbf{C}\mathbf{G} = 0$, and $\mathbf{a}_0$ is obtained by solving

$$\mathbf{K}_0\mathbf{a}_0 = -\sigma\mathbf{G}\phi. \quad (10)$$

When the first expansion point s_0 is nonzero, after generating $\mathbf{e}_0$, additional network elements are obtained as

$$\mathbf{L}_{-1} = (\mathbf{e}_0^T \mathbf{K}_0 \mathbf{e}_0)^{-1}, \quad \mathbf{a}_{-1}^* = -\mathbf{L}_{-1}\mathbf{e}_0. \quad (11)$$

After power sources are given in the first step, equation (6) is solved for $\mathbf{a}_{2n+1}$ with the Dirichlet or Neumann boundary condition without power inputs in the following steps.

Similar to the case for the single-port version, it can be proven that procedure (6)–(8) generates basis vectors that have blockwise orthogonality:

$$\mathbf{e}_{2i}^T \sigma \mathbf{e}_{2j} = \delta_{ij} \mathbf{R}_{2i}^{-1}, \quad \mathbf{a}_{2i+1}^T \mathbf{K}_i \mathbf{a}_{2j+1} = \delta_{ij} \mathbf{L}_{2i+1}, \quad (12)$$

where δ_{ij} is the Kronecker delta.

B. Change in the expansion point

Similar to other MOR methods, the CLN method requires a high-order expansion to cover a wide frequency range if a single expansion point is used. To realize a more efficient multi-port MOR, this study introduces multiple expansion points to the multi-port CLN.

The target frequency f_i can be changed after several generations of ladder elements. As an example, when the target frequency changes from f_0 to f_1 in the P -th stage, the network connection between the two expansion points is achieved as follows. Generate $\mathbf{a}_{2P-1}$, $\mathbf{a}'_{2P-1}$ and $\mathbf{e}_{2P}$ as

$$\mathbf{K}_1 \mathbf{a}_{2P-1} - \mathbf{K}_0 \mathbf{a}_{2P-3} = \sigma \mathbf{e}_{2P-2} \mathbf{R}_{2P-2}, \quad (13)$$

$$\mathbf{L}_{2P-1} = \mathbf{a}_{2P-1}^T \mathbf{K}_1 \mathbf{a}_{2P-1}, \quad (14)$$

$$\mathbf{a}'_{2P-1} = \mathbf{a}_{2P-1} + \sum_{i=1}^{P-1} \mathbf{a}_{2i-1} \mathbf{p}_{2i-1}, \quad (15)$$

$$\mathbf{e}_{2P} - \mathbf{e}_{2P-2} = -\mathbf{a}'_{2P-1} \mathbf{L}_{2P-1}^{-1}, \quad (16)$$

where $\mathbf{p}_{2i-1}$ is determined to maintain the orthogonality among basis vectors $\mathbf{e}_{2n}$. To ensure $\mathbf{e}_{2P}$ is orthogonal to $\mathbf{e}_{2i}$ ($i < P$), $\mathbf{p}_{2i-1}$ is given as

$$\mathbf{p}_{2i-1} = -\mathbf{L}_{2i-1}^{-1} \mathbf{a}_{2i-1}^T \mathbf{K}_0 \mathbf{a}_{2P-1}. \quad (17)$$

The relations

$$\mathbf{a}'_{2P-1}^T \mathbf{K}_0 \mathbf{a}_{2i-1} = 0 \quad (i < P), \quad (18)$$

$$\mathbf{a}_{2P-1}^T \mathbf{K}_1 \mathbf{a}_{2i-1} = 0 \quad (i < P), \quad (19)$$

$$\begin{aligned} \mathbf{a}_{2P-1}^T \mathbf{K}_1 \mathbf{a}_{2i-1} &= \mathbf{a}_{2P-1}^T \mathbf{K}_0 \mathbf{a}_{2i-1} + (s_1 - s_0) \mathbf{a}_{2P-1}^T \sigma_0 \mathbf{a}_{2i-1} \\ &= \begin{cases} 0 & (i < P-1) \\ -(s_1 - s_0) \mathbf{L}_{2P-1} \mathbf{R}_{2P-2}^{-1} \mathbf{L}_{2P-3} & (i = P-1) \end{cases}, \end{aligned} \quad (20)$$

$$\mathbf{a}_{2P-1}^T \mathbf{K}_1 \mathbf{a}'_{2P-1} = \mathbf{L}_{2P-1} - (s_1 - s_0) \mathbf{L}_{2P-1} \mathbf{R}_{2P-2}^{-1} \mathbf{L}_{2P-3} \mathbf{p}_{2P-3} \quad (21)$$

are used to derive network equations. For the convenience of notation, $\mathbf{a}'_{2n+1}$ other than that for $n = P-1$ is defined as

$$\mathbf{a}'_{2n+1} = \mathbf{a}_{2n+1} \quad (n \neq P-1), \quad (22)$$

whereas $\mathbf{a}'_{2P-1}$ is defined by (15) and (17). The blockwise orthogonality between $\mathbf{a}_{2n+1}$ and $\mathbf{a}'_{2n+1}$ with respect to $\mathbf{K}_0$ and $\mathbf{K}_1$ is presented in Table I. Using the orthogonality, a set of $\mathbf{a}'_{2n+1}$ is used to derive the network equations and to expand the magnetic field.

C. Derivation of the network equation

When $s_0 \neq 0$, an excitation inductance $\mathbf{L}_{-1}$ is additionally required to start the CLN construction as (11). It is a reasonable choice to choose $s_0 = 0$ because this value improves the accuracy at low frequency and gives a simple start from the magnetostatic field without computing $\mathbf{L}_{-1}$. In this case, the electromagnetic fields are expanded as

$$\mathbf{e} = \sum_{n=0} \mathbf{e}_{2n} \mathbf{V}_{2n}, \quad \mathbf{a} = \sum_{n=0} \mathbf{a}'_{2n+1} \mathbf{I}_{2n+1}, \quad (23)$$

where $\mathbf{I}_{2n+1} = (I_{1,2n+1}, \dots, I_{M,2n+1})^T$ and $\mathbf{V}_{2n} = (V_{1,2n}, \dots, V_{M,2n})^T$ are coefficient vectors.

Table I. Orthogonality between $\mathbf{a}_{2n+1}$ and $\mathbf{a}'_{2n+1}$ with respect to $\mathbf{K}_0$ and $\mathbf{K}_1$, where “*” means a nonzero matrix.

$\mathbf{K}_0$	...	$\mathbf{a}_{2P-5}$	$\mathbf{a}_{2P-3}$	$\mathbf{a}_{2P-1}$	$\mathbf{a}_{2P+1}$	$\mathbf{a}_{2P+3}$	...
$\vdots$	*	0	0	*	0	0	0
$\mathbf{a}'_{2P-5}$	0	$\mathbf{L}_{2P-5}$	0	*	0	0	0
$\mathbf{a}'_{2P-3}$	0	0	$\mathbf{L}_{2P-3}$	*	0	0	0
$\mathbf{a}'_{2P-1}$	0	0	0	*	*	0	0
$\mathbf{a}'_{2P+1}$	0	0	0	*	*	*	0
$\mathbf{a}'_{2P+3}$	0	0	0	0	*	*	*
$\vdots$	0	0	0	0	0	*	*

$\mathbf{K}_1$	...	$\mathbf{a}_{2P-5}$	$\mathbf{a}_{2P-3}$	$\mathbf{a}_{2P-1}$	$\mathbf{a}_{2P+1}$	$\mathbf{a}_{2P+3}$	...
$\vdots$	*	*	0	0	0	0	0
$\mathbf{a}'_{2P-5}$	*	*	*	0	0	0	0
$\mathbf{a}'_{2P-3}$	0	*	*	0	0	0	0
$\mathbf{a}'_{2P-1}$	0	0	*	$\mathbf{L}_{2P-1}$	0	0	0
$\mathbf{a}'_{2P+1}$	0	0	0	0	$\mathbf{L}_{2P+1}$	0	0
$\mathbf{a}'_{2P+3}$	0	0	0	0	0	$\mathbf{L}_{2P+3}$	0
$\vdots$	0	0	0	0	0	0	*

The substitution of (23) into (2) yields

$$\mathbf{C} \sum_{n=0} \mathbf{e}_{2n} \mathbf{V}_{2n} = -s \sum_{n=0} \mathbf{C} \mathbf{a}'_{2n+1} \mathbf{I}_{2n+1}, \quad (24)$$

$$\mathbf{C}^T \mathbf{v} \mathbf{C} \sum_{n=0} \mathbf{a}'_{2n+1} \mathbf{I}_{2n+1} = \sigma \sum_{n=0} \mathbf{e}_{2n} \mathbf{V}_{2n}. \quad (25)$$

On the basis of (24), the electric field is expressed as

$$\sum_{n=0} \mathbf{e}_{2n} \mathbf{V}_{2n} = -s \sum_{n=0} \mathbf{a}'_{2n+1} \mathbf{I}_{2n+1} + \mathbf{e}_0 \mathbf{V}_S, \quad (26)$$

where $\mathbf{V}_S = (V_{S1}, \dots, V_{SM})$ denotes the voltages of power sources while $\mathbf{e}_0$ is the electrostatic field given by (9) for which $s_0 = 0$ and $\mathbf{C}\mathbf{e}_0 = 0$. It follows from (25) and (26) that

$$\mathbf{K}_i \sum_{n=0} \mathbf{a}'_{2n+1} \mathbf{I}_{2n+1} = (1 - \frac{s_i}{s}) \sigma \sum_{n=0} \mathbf{e}_{2n} \mathbf{V}_{2n} + \frac{s_i}{s} \mathbf{e}_0 \mathbf{V}_S. \quad (27)$$

Multiplying (26) by $\sigma \mathbf{e}_{2k} \mathbf{R}_{2k}$ and using (6) and (13) yields

$$\mathbf{R}_{2k} \mathbf{e}_{2k}^T \sigma \left(\sum_{n=0} \mathbf{e}_{2n} \mathbf{V}_{2n} - \mathbf{e}_0 \mathbf{V}_S \right) \quad (28)$$

$$= -s (\mathbf{a}_{2k+1}^T \mathbf{K}_i - \mathbf{a}_{2k-1}^T \mathbf{K}_j) \sum_{n=0} \mathbf{a}'_{2n+1} \mathbf{I}_{2n+1}$$

where $(i, j) = (0, 0), (1, 0)$ and $(1, 1)$ when $k < P-1$, $k = P-1$,

and $k \geq P$, respectively. It follows from (28) and orthogonality that

$$\begin{aligned} V_0 &= V_S - sL_1 I_1 \\ V_{2k} &= s(L_{2k-1} I_{2k-1} - L_{2k+1} I_{2k+1}) \quad (k=1, \dots) \end{aligned} \quad (29)$$

Multiplying (27) by $a'_{2k+1} L_{2k+1}^{-1}$ and using (7) and (16) yields

$$\begin{aligned} L_{2k+1}^{-1} a'_{2k+1} K_i \sum_{n=0} a'_{2n+1} I_{2n+1} \\ = -(1 - \frac{s_i}{s})(e_{2k+2} - e_{2k})^T \sigma \sum_{n=0} e_{2n} V_{2n} \end{aligned} \quad (30)$$

When $k < P-1$, $i=0$, and $s_i = s_0 = 0$, it follows from (30) and orthogonality that

$$I_{2k+1} = R_{2k}^{-1} V_{2k} - R_{2k+2}^{-1} V_{2k+2} \quad (k < P-1). \quad (31)$$

When $k \geq P$ and $i=1$, it follows from (30) and orthogonality that

$$I_{2k+1} = (1 - \frac{s_1}{s})(R_{2k}^{-1} V_{2k} - R_{2k+2}^{-1} V_{2k+2}) \quad (k \geq P). \quad (32)$$

When $k = P-1$ and $i=1$, it follows from (20), (21), (30) and orthogonality that

$$\begin{aligned} [1 - (s_1 - s_0)R_{2P-2}^{-1}(L_{2P-1} + L_{2P-3} p_{2P-3})] I_{2P-1} \\ = (1 - \frac{s_0}{s})R_{2P-2}^{-1} V_{2P-2} - (1 - \frac{s_1}{s})R_{2P}^{-1} V_{2P} \end{aligned} \quad (33)$$

where $\mathbf{1}$ is the unit matrix. Equation (33) includes the case where $s_0 \neq 0$. As a result, the EC field is represented by the CLN shown in Fig. 1(a), where L'_{2P-1} is given by

$$L'_{2P-1} = -L_{2P-1}(L_{2P-1} + L_{2P-3} p_{2P-3})^{-1} R_{2P-2} / (s_1 - s_0). \quad (34)$$

When $P=1$, (34) is reduced to $L'_{2P-1} = -R_0/(s_1 - s_0)$ and $a'_1 = a_1$.

D. Further change in the expansion point

The expansion point can be changed from f_1 to f_2 in the Q -th stage again. The CLN procedure to connect the two expansion points is similarly obtained by replacing (15) and (17) respectively with

$$a'_{2Q-1} = a_{2Q-1} + \sum_{i=1}^{Q-1} a'_{2i-1} q_{2i-1}, \quad (35)$$

$$q_{2i-1} = \begin{cases} -L_{2i-1}^{-1} a_{2i-1}^T K_0 a_{2Q-1} & (i < P) \\ -L_{2i-1}^{-1} a_{2i-1}^T K_1 a_{2Q-1} & (P \leq i < Q) \end{cases} \quad (36)$$

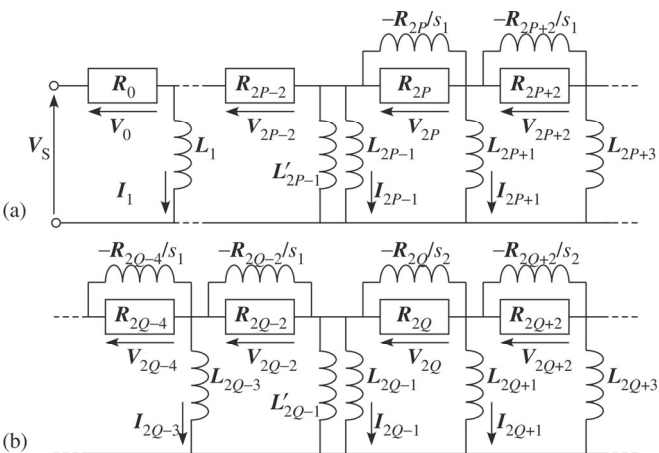


Fig. 1. Connection of a multi-port CLN having multiple expansion points: (a) changing $s_0 = 0$ to $s_1 = 2\pi f_1$ in the P -th stage and (b) changing s_1 to s_2 in the Q -th stage.

The connection part of the CLN is shown in Fig. 1(b).

III. NUMERICAL RESULTS

A transformer with a bulk core [Fig. 2] is analyzed, where the permeability and conductivity of the core are respectively $4\pi \times 10^{-4}$ H/m and 10^6 S/m while the conductivity of the coil is 4×10^7 S/m.

First, the frequency dependence of the impedance matrix is reconstructed as in Figs. 3 and 4 using the two-port CLN and the two-port elements terminated by R_8 with (a) a single expansion point $f_0 = 0, 10^2$, or 10^4 Hz and (b) multiple expansion points $(f_0, f_1) = (10, 10^3)$ or $(10^2, 10^4)$ Hz with $P=2$, and $(f_0, f_1, f_2) = (0, 10^2, 10^4)$ Hz with $(P, Q) = (1, 3)$. The impedance is accurately represented around the expansion frequency. The multiple expansion points achieve accurate impedance in a wide frequency range, which is more efficient than increasing the number of stages.

When $s_0 \neq 0$, the vector potential can be expanded as

$$a = \sum_{n=-1} a'_{2n+1} I_{2n+1}, \quad (37)$$

where a'_{-1} is obtained by removing the electrostatic field $-\mathbf{G}\phi$ from a_{-1} as $a'_{-1} = -L_{-1}(e_0 + \mathbf{G}\phi) = s_0 L_{-1} a_0$. Figure 5 depicts magnetic flux lines near the core region under the sinusoidal excitation of V_{S1} at 1 kHz, which are obtained adopting the FEM [Fig. 5(a)] and the two-port CLN terminated by R_8 with expansion points $f_0 = 0$ [Fig. 5(b)], $f_0 = 10^3$ [Fig. 5(c)], and $(f_0, f_1) = (10, 10^3)$ Hz with $P=2$ [Fig. 5(d)]. The expansion point effectively improves the accuracy of flux representation.

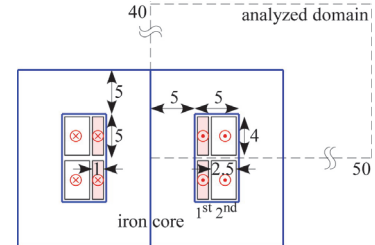


Fig. 2. Transformer with a bulk core (unit: mm).

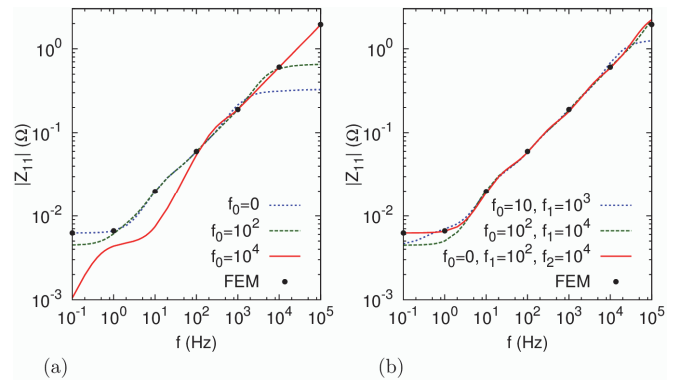


Fig. 3. Frequency dependence of impedance $|Z_{11}|$: (a) $f_0 = 0, 10^2, 10^4$ Hz and (b) $(f_0, f_1) = (10, 10^3), (10^2, 10^4)$ and $(f_0, f_1, f_2) = (0, 10^2, 10^4)$ Hz.

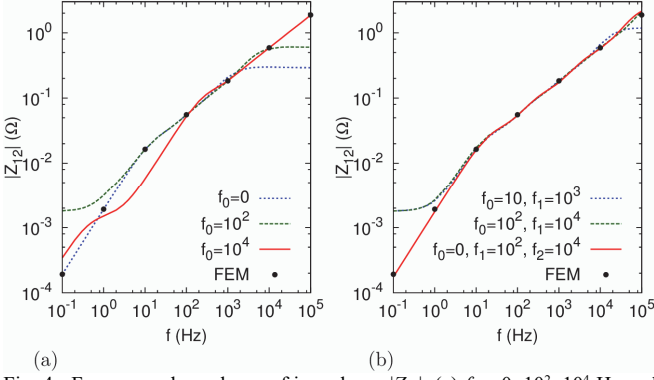


Fig. 4. Frequency dependence of impedance $|Z_{12}|$: (a) $f_0 = 0, 10^2, 10^4$ Hz and (b) $(f_0, f_1) = (10, 10^3), (10^2, 10^4)$ and $(f_0, f_1, f_2) = (0, 10^2, 10^4)$ Hz.

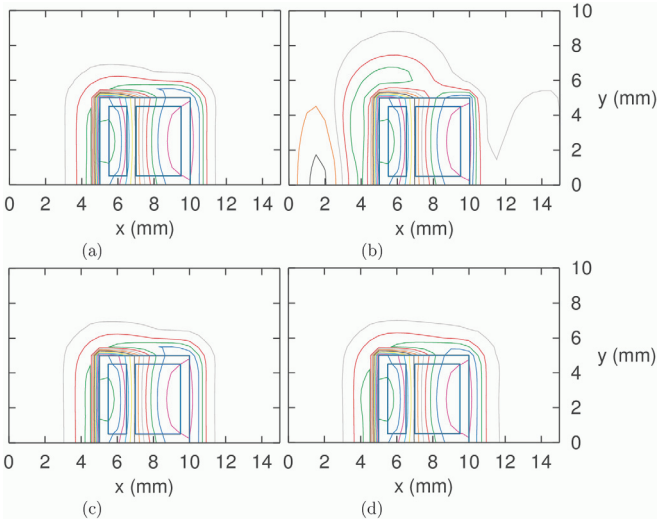


Fig. 5. Magnetic flux lines reconstructed using a 4 stage CLN at a frequency of 10^3 Hz: (a) FEM, (b) $f_0 = 0$, (c) $f_0 = 10^3$, and (d) $(f_0, f_1) = (10, 10^3)$ Hz.

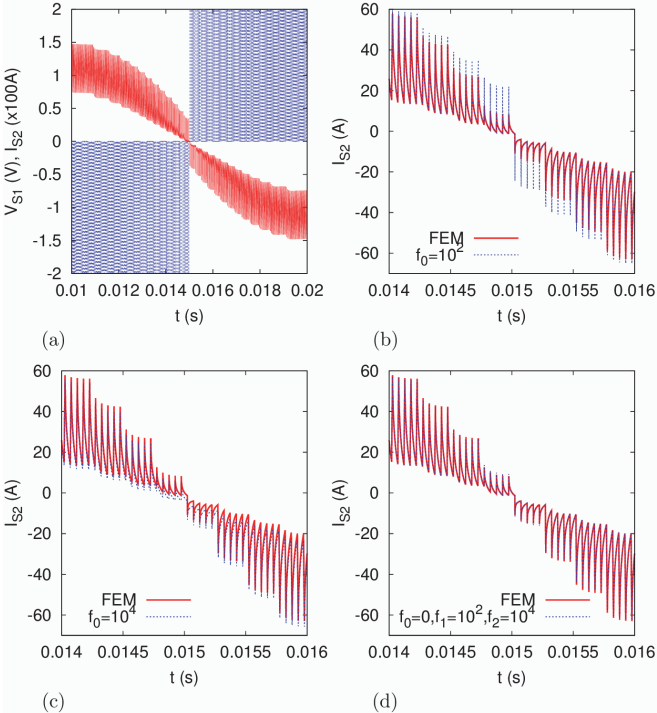


Fig. 6. Current I_{S2} under PWM excitations of V_{S1} : (a) voltage and current waveforms, (b) $f_0 = 10^2$, (c) $f_0 = 10^4$, and (d) $(f_0, f_1, f_2) = (0, 10^2, 10^4)$ Hz.

A PWM excitation with a fundamental frequency of 50 Hz and a carrier frequency of 10 kHz [Fig. 6(a)] is fed to port 1. The current of the short-circuited port 2 is simulated adopting the two-port CLN terminated by R_8 with expansion points $f_0 = 10^2$ [Fig. 6(b)], $f_0 = 10^4$ [Fig. 6(c)], and $(f_0, f_1, f_2) = (0, 10^2, 10^4)$ Hz with $(P, Q) = (1, 3)$ [Fig. 6(d)]. The current waveform is not accurate when the single expansion point is set around the fundamental or carrier frequency only. The current waveform is accurately reconstructed by setting two expansion points around the two frequencies. The expansion points are real numbers, and the solution to (6) thus costs roughly M times as much as one time step of the FE EC computation. In the above case with $M = 2$, the construction cost of the CLN is roughly equal to only eight time steps of the FE EC computation to generate L_1 to L_7 when $f_0 = 0$ and 10 steps when $f_0 \neq 0$ using L_{-1} to L_7 . The cost to solve the network equation is negligible.

IV. CONCLUSION

This paper introduced multiple expansion points to the multi-port CLN method. By setting multiple target frequencies, the reduced model covers a wide frequency range with only a small number of electric network elements. Not only the frequency dependence of the impedance matrix but also the field distribution and time-dependent response of multiport device are reconstructed accurately and efficiently.

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REFERENCES

- [1] P. Feldmann and R.W. Freund, "Efficient linear circuit analysis by Padé approximation via the Lanczos process," *IEEE Trans. Comput.-Aided Design Integr. Circuits Syst.*, vol. 14, pp. 639–649, May 1995.
- [2] Y. Sato and H. Igarashi, "Generation of equivalent circuit from finite-element model using model order reduction," *IEEE Trans. Magn.*, vol. 52, 1100304, Mar. 2016.
- [3] A. Pierquin, T. Henneron, S. Clénet, and S. Brisset, "Model-order reduction of magnetoquasi-static problems based on POD and Arnoldi-based Krylov methods," *IEEE Trans. Magn.*, vol. 51, 7206204, Mar. 2015.
- [4] T. Henneron and S. Clénet, "Model-order reduction of multiple-input non-linear systems based on POD and DEI methods," *IEEE Trans. Magn.*, vol. 51, 7207104, Mar. 2015.
- [5] T. Matsuo, A. Kameari, K. Sugahara, and Y. Shindo, "Matrix formulation of the Cauer ladder network method for efficient eddy-current analysis," *IEEE Trans. Magn.*, vol. 54, 7205805, Nov. 2018.
- [6] T. Matsuo, K. Sugahara, A. Kameari, and Y. Shindo, "Model order reduction of an induction motor using a Cauer ladder network," *IEEE Trans. Magn.*, vol. 56, 7514704, Jun. 2020.
- [7] K. Kuriyama, A. Kameari, H. Ebrahimi, T. Fujiwara, K. Sugahara, Y. Shindo, and T. Matsuo, "Cauer ladder network with multiple expansion points for efficient model order reduction of eddy-current field," *IEEE Trans. Magn.*, vol. 55, 7203404, Jun. 2019.
- [8] T. Matsuo, T. Fujiwara, K. Kuriyama, K. Sugahara, A. Kameari, T. Tokumasu, Y. Shindo, "Multi-port model order reduction using a matrix Cauer ladder network," *IEEE Trans. Magn.*, vol. 56, 7506905, Feb. 2020.