ON THE TIME-DEPENDENT DEFORMATION OF A DROPLET UNDER SHEAR FLOW

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A perturbation approach is presented to derive a kinetic equation for a droplet under shear flow. A time-dependent deformation of the droplet is calculated to show an overshooting behavior in approach to a steady state. An application of our approach is also discussed.

We start with the interface equation of motion[1] for a droplet under shear flow $\mathbf{u(r)} = Sx\mathbf{e}_z$,

$$v(a,t) = Sx(a)\mathbf{n}(a)\cdot \mathbf{e}_z + \int da'\ \mathbf{n}(a)\cdot \mathrm{T}(\mathbf{r}(a)-\mathbf{r}(a'))\cdot \mathbf{n}(a')\sigma K(a') \quad , \tag{1}$$

where $\mathrm{T}(\mathbf{r}) = \frac{1}{8\pi\eta}\left(\frac{1}{r} + \frac{\mathbf{rr}}{r^3}\right)$. Here S is the shear rate, $\mathbf{e}_z$ is the unit vector along the z-axis, a stands for the position on the interface of the droplet, $v(a)$ is the normal component of the interface velocity, $\mathbf{n}(a)$ is the normal unit vector at the interface. $K = -\nabla\cdot\mathbf{n}$ is the mean curvature, and σ is the surface tension. We have neglected inertia effects[1] and assumed the common shear viscosity η in the two fluids.

When a dimensionless parameter $\varepsilon = \eta SR/\sigma$ is sufficiently small, the interfacial profile $r(\Omega,t) = R\{1+f(\Omega,t)\}$ would be described by a superposition of the spherical harmonics $Y_l^m(\Omega)$.

At the first order in ε, (1) reduces to the well-known Taylor's solution[2], which constitutes the basis of our theory. The $O(\varepsilon^2)$ theory was considered in a different point of view from ours.[3,4]. Here we develop a third order theory in a systematic way.

First, we multiply (1) by $Y_l^m(\Omega)^*$ and integrate over the interfacial area. Then we expand the resultant equation in terms of $f(\Omega,t)$ and evaluate the equation up to $O(\varepsilon^3)$ by assuming the following form for the profile :

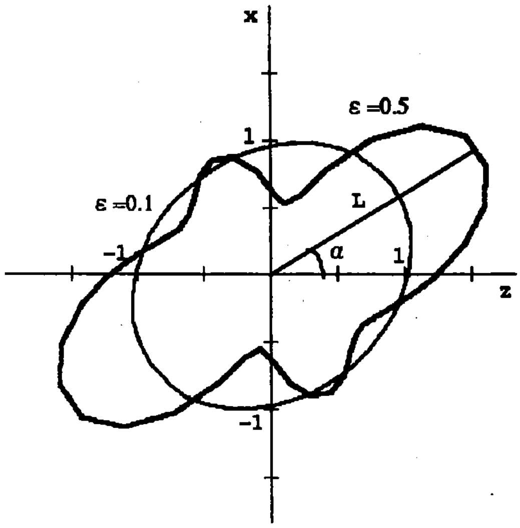
$$f(\Omega,t) = \varepsilon x_{21}(t)f_{21}(\Omega) + \varepsilon^2\left\{\sum_{nm}{}^{(2)}x_{nm}(t)f_{nm}(\Omega) - r_2(t)\right\} + \varepsilon^3\sum_{nm}{}^{(3)}x_{nm}(t)f_{nm}(\Omega)$$

, where $\sum_{nm}^{(2)} = \sum_{n=2,4}\sum_{m=0,2}$ and $\sum_{nm}^{(3)} = \sum_{n=4,6}\sum_{m=1,3}$, being $f_{nm} = 2\mathrm{Re}Y_n^m$ for $m\neq 0$ and $f_{n0} = \mathrm{Re}Y_n^0$. Here the term $r_2(t)$ has been added to conserve the volume of the droplet at this order.

Thus we have a closed set of equations for x_{nm} with(n,m)=(2,1),(2,0),(2,2),(4,0),(4,1),(4,2),(4,3),(6,1),(6,3). The steady state solution of the equations can readily obtained (Fig.1). There three sets of the solution are obtained for all ε. The linear stability analysis shows that one solution is always stable and two are always unstable. Thus our successive approximation does not reproduce the breakup[1], even though the profile for ε = 0.5 is reminiscent of that at the incipient breakup of type B2[1,5].

In Fig.2, we have shown a time change of the orientation angle α and L = r($\theta=\alpha,\phi=0$). At t = 0, a spherical droplet starts to deform under the shear ε = 0.1. After a steady state is attained, the shear is jumped to ε = 0.5. A marked result is an overshooting of the profile in approach to a new steady state.

Finally,as an application of our approach we have derived expressions of the lateral and the slip migration velocity of a droplet in close vicinity of a wall. Our results are in good agreement with a recent computer simulation[6].



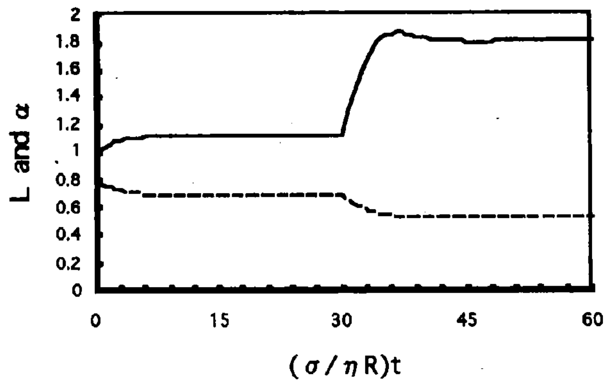


Fig.1 : The steady state profile at x-z plane for ε= 0.1 and 0.5.
Fig.2 : Time evolution of the orientation angle α and L.

References

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