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Functions Measuring the Centrality (or Mediality) of a point in a Network

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Abstract:

A new theory of functions measuring the centrality (or mediality) of a point in a network is developed.

Summary:

It is one of the fundamental problems in network theory with applications to study the centrality (or mediality) of a point in a network and properties measuring the centrality (or mediality). In this paper, a new theory of functions mesuring the centrality (or mediality) of a point in a network is developed. Kajitani and Maruyama's theory and our old theory are a special case of the theory developed herein.

Consider a connected network N with vertex set V(N) and edge set E(N). The network may be either directed or undirected. With each edge e of N, two kinds of non-negative real numbers $l(e)$ and $c(e)$, called the edge-length and the edge-capacity of e, respectively, are associated, and with each vertex v of N, a non-negative real number $\sigma(v)$, called the vertex-weight of v, is associated. Let S be a set of points of N where a point of N can be either a vertex or an interior point of an edge. For any two points s_i and s_j in S, we define two kinds of non-negative real numbers $\rho(s_i,s_j)$ and $\gamma(s_i,s_j)$, called the di-distance from s_i to s_j and the di-capacity from s_i to s_j, respectively, by using the underlying graph, edge-lengths and edge-capacities of N. A typical example of a di-distance from a point s_i to a point s_j in N is the length of a shortest path from s_i to s_j in N and a typical example of a di-capacity from a point s_i to a point s_j in N is the value of the maximum flow from s_i to s_j in N. The concepts of di-distance and di-capacity are different but are fundamentally important in evaluating the degree of closeness of one point to another. Next, we introduce the concept of monotone modification as a natural unification of network modifications such as adding new edges, coalescencing some verties, shortening edge-lengths and increasing edge-capacities. The monotone modification consists of two kinds of network modifications, called a monotone contraction and a monotone expansion, where the monotone contraction is a network modification with respect to di-distance and the monotone expansion is a network modification with respect to di-capacity. Next, in order to measure the centrality (or mediality) of a point r in N, we consider a real-valued function $f(r,\rho,\gamma,\sigma)$ defined on N where $\rho| S\times S \to \bar{R}_+$, $\gamma| S\times S \to \bar{R}_+$ and $\sigma| V(N) \to \bar{R}_+$ and characterize the centrality of a point r in N by using the tendency of the change of the functional value of $f(r,\rho,\gamma,\sigma)$ under a monotone modification of N. Next, in the case where we restrict the form of f to

$$f(r,\rho,\gamma,\sigma) = \sum_{s_i \varepsilon V(N) \subseteq S} \psi(\rho(r,s_i),\gamma(r,s_i),\sigma(s_i)),$$

we give a necessary and sufficient condition for f to be a function measuring the centrality (or mediality) of a point in N in our sense, and study some relationships between the monotone modification of N and the convexity or concavety of ψ.

A precise description of this paper will be included in a full paper of the coming Trans. of IECE of Japan.

<u>References:</u>

[1] Kajitani, Y. and Maruyama, T.: Functional expression of the centrality of a vertex in a graph, Trans.IECE of Japan, Vol.59A, 531-538, 1976.

[2] Shinoda, S. and Sengoku, M.: Axiomatic foundations of the theories of functions expressing the mediality of a point in a metric space, ibid., Vol.J66-A, 352-359, 1983.