

Strongly countably complete spaces と fragment について

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Definition 1. Let X be a topological space and ρ be a metric on X . X is said to be fragmented by ρ (or ρ -fragmented) if for each $\varepsilon > 0$ and each nonempty subset A of X there is an open subset U of X such that $U \cap A \neq \emptyset$ and $\rho\text{-diam}(U \cap A) \leq \varepsilon$.

The topological space X is said to be fragmentable if there exists a metric on X which fragments X .

Definition 2. A well ordered family $\mathcal{U} = \{U_\xi \mid 0 \leq \xi < \xi_0\}$ of subsets of the topological space X is said to be a relatively open partitioning of X , if

- (1) $U_0 = \emptyset$;
- (2) U_ξ is contained in $X \setminus (\bigcup_{\eta < \xi} U_\eta)$ and is relatively open in it for every ξ , $0 < \xi < \xi_0$;

$$(3) X = \bigcup_{\xi < \xi_0} U_\xi.$$

A family $\mathcal{U}$ of subsets of X is said to be a σ -relatively open partitioning of X , if $\mathcal{U} = \bigcup_{n=1}^{\infty} \mathcal{U}^n$, where $\mathcal{U}^n, n=1,2,\dots$ are relatively open partitionings of X .

$\mathcal{U}$ is said to separate the points of X , if whenever x and y are two different elements of X there exists n such that x and y belong to different elements of the partitioning $\mathcal{U}^n$.

In this case we say that X admits a separating σ -relatively open partitioning.

In [9], N.K.Ribarska proved the following two theorems.

Theorem (N.K.Ribarska). The topological space X admits σ -relative open partitioning if and only if there exists a metric which fragments X .

Theorem A (N.K.Ribarska). Let X be a compact Hausdorff space. If X is a fragmentable then there exists a complete metric ρ on X such that X is ρ -fragmented and the topology generated by ρ is stronger than the original topology on X .

A.V.Arhangel'skii proved that a functionally complete compact Hausdorff space is an Eberlein compact space ([1]). And each Eberlein compact space is Radon-Nikodým compact space which is homeomorphic to a norm-fragmented w -compact subset of a dual Banach space ([7]). Hence we obtain the following theorem.

Theorem B (I.Namioka [7]). Let X be an Eberlein compact space. Then X is fragmented by a lower semi-continuous metric.

In this note we extend these results (Theorem A and Theorem B) to the class of the strongly countably complete spaces.

Definition 3. A topological space X is said to be strongly countably complete (s.c.c.) if there exists a strongly countably complete sequence of open coverings of X .

Theorem 1. Let X be a completely regular s.c.c. space. If X is a fragmentable then there exists a complete metric on X such that X is ρ -fragmented and the topology generated by ρ is stronger than the original topology on X .

Theorem 2. Every completely regular s.c.c. functionally complete space is fragmented by a lower semi-continuous metric.

Definition 4. Let X be a topological space. X is said to be a Namioka space if the following condition is satisfied for any compact space Y ;

(*) for any separately continuous function $f: X \times Y \rightarrow R$ there exists a dense G_δ subset A of X such that f is jointly continuous at each point of $A \times Y$ (where R is a real line).

Remark. Completely regular s.c.c. space is a Namioka space. Each closed subspace of a s.c.c. space is also s.c.c., hence Namioka space.

Theorem 3. Let X be a completely regular functionally complete space. If each closed subspace of X is a Namioka space then X is fragmented by a lower semi-continuous metric.

From Theorem 3 we get Theorem 2.

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