

Complex Dynamics of Pendulums for Energy Harvesting

M. Wiercigroch, B. Horton, J. Sieber, J.M.T. Thompson and E. Pavlovskaya

Centre for Applied Dynamics Research, School of Engineering, University of Aberdeen, Aberdeen, ABB24 3UE, Scotland, UK, m.wiercigroch@abdn.ac.uk

Complex dynamics of pendulums is being studied with a view of vibrational energy harvesting and vibration isolation. The main focus of the paper is a new concept of energy conversion [1], where a predominantly vertical oscillatory motion is being converted into rotations by means of a pendulum or pendulums system. Numerical, analytical and experimental studies undertaken on a parametric pendulum [2-4] and a pendulum excited by planar motion [5] suggest significant potentials for energy harvesting.

Pendulum Driven by Planar Excitation

A new concept of energy extraction, where a predominantly vertical oscillatory motion is being converted into rotations by means of a pendulum or pendulums system has been proposed by Wiercigroch [1]. In this paper the nonlinear dynamics of pendulums exhibiting complex responses is investigated with a view to its application to energy harvesting and vibration isolation.

The dynamics of pendulums focusing on this application has been extensively studied by the Centre for Applied Dynamics Research (CADR) and its collaborators [2-5]. Through these studies comprising numerical, analytical and experimental investigations, it has been understood that the rotary motion offers major gains in terms of the efficiency of energy transferred and motion stability when compared to its oscillatory counterpart.

In the first instance pendulums driven by a planar excitation as shown in Figure 1 is considered. The dimensionless equation of motion for all three cases can be presented as

$$\ddot{\theta} + \gamma \dot{\theta} + (1 + p \cos \omega \tau) \sin \theta + ep \sin(\omega \tau + \kappa) \cos \theta = 0, \quad (1)$$

where θ is angular displacement of the pendulum, dot denotes differentiation with respect to dimensionless time $\tau = \omega_n t$, $\gamma = c_0 / \sqrt{m / \omega_n}$ is dimensionless damping coefficient, $\omega_n = \sqrt{g / l}$ is the natural frequency, $p = Y \Omega^2 / g$ and $\omega = \Omega / \omega_n$ are the dimensionless amplitude and frequency of the external excitation, $e = X / Y$ is a parameter controlling the ratio between the magnitudes of horizontal and vertical components of excitation and parameter κ is responsible for the shape of the forcing. The angular displacement of the pendulum is positive when moving in anti-clockwise direction as shown in Figure 1.

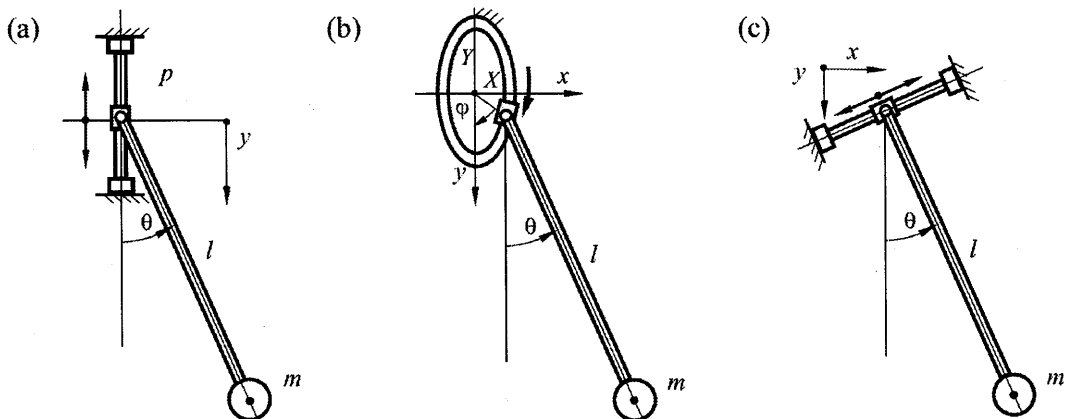


Figure 1: Physical model of the pendulum excited at the pivot point (a) vertically; (b) elliptically ($\kappa = 0$) and (c) along a tilted axis ($\kappa = \pi/2$). Here m is the mass of the pendulum bob, l is the length of the pendulum arm, Y and X are vertical and horizontal amplitudes of forcing.

Equation 1 has been extensively studied analytically and numerically by the CADR and here we present the results of bifurcation analysis undertaken for the rotary motion. Figures 2(a) and 2(b) show the bifurcations of period-one rotations for $e = 0.1$ and $e = 0.5$. The bifurcations of rotations in the parametric case $e = 0$ are included (J and G, in black and grey) in both panels to show the effect of the nonzero ellipticity e . In all

cases the stable period-one rotations are born, for increasing forcing p , in a fold bifurcation (curves J , J^p and J^n) and lose their stability in a period doubling (curves G , G^p and G^n) as p increases further. For even higher forcing the period-one rotation regains its stability (in the period doublings E , E^p and E^n). For $e > 0$ the curve J^n is shifted downwards from J by $2\gamma\omega e/(1+e)$ and the curve J^p is shifted upwards from J by $2\gamma\omega e/(1-e)$.

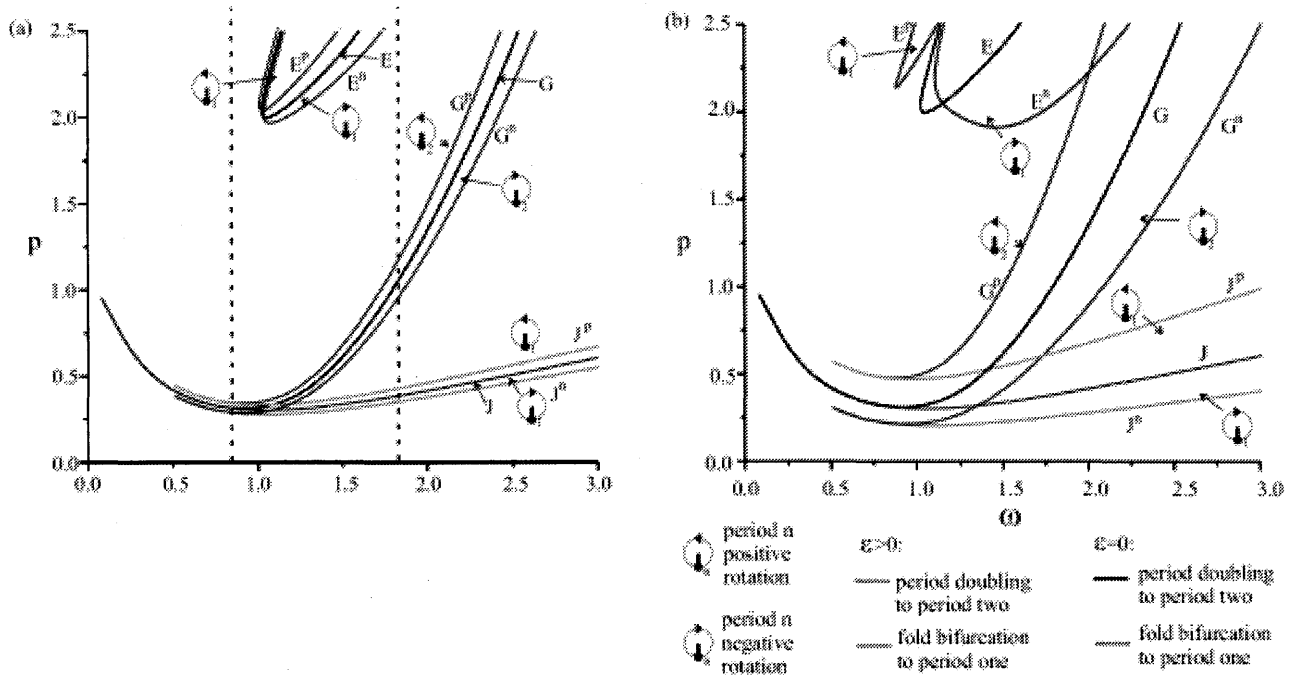


Figure 2: Bifurcation diagrams in the (ω, p) plane comparing the bifurcations of the period-one rotations for $e = 0.1$ (panel (a)) and $e = 0.5$ (panel (b)) with the classical case $e = 0$. The damping ratio γ is 0.1 .

Closing Remarks

In this paper complex dynamics of pendulums excited by a planar forcing was studied with a view of energy extraction and vibration isolation. The most notable effect of the planar forcing is the fact that for a nonzero X component of the external excitation all bifurcations are shifted toward lower forcing for clockwise rotations. The bifurcations of anti-clockwise are shifted upward.

The presented analysis has been applied to a system of identical pendulums supported on elastic foundations. In the lecture, I will show the most recent work on the pendulums systems, which is focus on synchronization and control. Specifically, we have applied various control algorithms to synchronize, initiate and maintain of stable rotary motion of the two identical counter rotating pendulums. Details of the numerical analysis and experimental studies will be presented.

References

- [1] M. Wiercigroch, Energy conversion and extraction for oscillatory motion using pendula systems operating in rotational motion, *Patent pending*, 2009.
- [2] X. Xu, and M. Wiercigroch, Approximate analytical solutions for oscillatory and rotational motion of a parametric pendulum, *Non. Dyn.* vol. 47, pp. 311-320, 2007.
- [3] X. Xu, E.E. Pavlovskaja, M. Wiercigroch, F. Romeo, and S. Lenci, Dynamic interactions between parametric pendulum and electrodynamical shaker: Effect of non-ideal excitation source, *Z. Angew. Math. Mech.* vol. 87, pp. 172-187, 2007
- [4] B. Horton, B., M. Wiercigroch, M., and X. Xu, Transient tumbling chaos and damping identification for parametric pendulum, *Phil. Trans. Roy. Soc. A* vol. 366: 767-784, 2008.
- [5] B. Horton, J. Sieber, J.M.T. Thompson, and M. Wiercigroch, Dynamics of the nearly parametric pendulum, *Int. J. Non-Linear Mech.*, doi:10.1016/j.ijnonlinmec.2010.11.003.