

Enriques surfaces covered by Jacobian Kummer surfaces

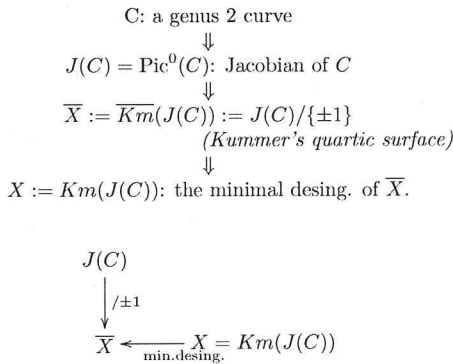
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1 Introduction

Jacobian Kummer surface X



Definition

X is Picard-general if $\rho(X) = 17$, which we assume in what follows.

$\text{Aut}(X)$ has been studied by many authors.

One definitive result is the following

Theorem (S. Kondo, 1998)

$\text{Aut}(X)$ is generated by

- $\begin{cases} 16 \times 4 & \text{Klein's involutions } (t_\alpha, \sigma_\beta, p_\alpha, p_\beta), \\ 60 & \text{Hutchinson's involutions } (\sigma_G), \\ 192 & \text{Keum's automorphisms } (\phi_{W, W'}). \end{cases}$

Where

$$\begin{aligned}
 \alpha &\in \{2\text{-torsion pts of } J(C)\}, \\
 \beta &\in \{\text{theta characteristics of } C\}.
 \end{aligned}$$

Corollary of the Main Theorem

$\text{Aut}(X)$ is generated by

- $\begin{cases} 16 \times 4 & \text{Klein's involutions } (t_\alpha, \sigma_\beta, p_\alpha, p_\beta), \\ 60 & \text{Hutchinson's (HG) involutions } (\sigma_G), \\ 192 & \text{Hutchinson-Weber (HW) involutions } (\sigma_W). \end{cases}$

Where did σ_W come from ?

2 Main Result

Main Theorem There are $31 = 6 + 10 + 15$ fixed-point-free involutions on X , up to the isomorphism of the quotient Enriques surfaces.

They are exactly as follows.

3 free involutions on X

Switches

$$\Theta_\beta = \{p - \beta | p \in C\}.$$

$$\text{For } p \in J(C), (\Theta_\beta + p) \cap (\Theta_\beta - p) = \{q, -q\}.$$

$$\sigma_\beta: \pm p \mapsto \pm q.$$

$$\sigma_\beta \in \text{Bir}(\overline{X}) = \text{Aut}(X).$$

β runs over even theta characteristics of C ; we obtain 10 free switches.

HG involutions

Restriction of the Cremona involution to $\overline{X}$:

$$\sigma_G: (x, y, z, t) \mapsto \left(\frac{1}{x}, \frac{1}{y}, \frac{1}{z}, \frac{1}{t}\right).$$

G : four points of $\overline{X}$, called Göpel subgroup.

σ_G is well-defined, because

Theorem[Hutchinson] If we choose the four points of G as the reference points of $\mathbb{P}^3$, the equation of $\overline{X}$ becomes

$$\begin{aligned}
 &A(x^2t^2 + y^2z^2) + B(y^2t^2 + z^2x^2) + C(z^2t^2 + x^2y^2) + Dxyz \\
 &E(yt + zx)(zt + xy) + G(zt + xy)(xt + yz) + H(xt + yz)(yt + zx) \\
 &= 0.
 \end{aligned}$$

There are 15 Göpel subgroups.

HW involutions

Restriction of the Cremona involution $\sigma_W: (s_i) \mapsto (s_i^{-1})$ of $\mathbb{P}^4$ to X_W , where W : a Weber hexad (definition omitted), X_W : another quartic model of X .

$$\overline{X} \xrightarrow{|\mathcal{O}_{\mathbb{P}^3}(2) - W|} X_W \subset \mathbb{P}^4.$$

Theorem[Hutchinson] The equation of X_W is

$$\sum_{i=1}^5 s_i = \sum_{i=1}^5 \lambda_i / s_i = 0, \quad \lambda_i \in \mathbb{C}^*.$$

We obtain 6 HW involutions.

4 Sketch of the Proof

We compute certain invariant, the patching subgroups of free involutions. For our X , it exactly classifies the isom. classes of quotient Enriques surfaces. The definition of it uses Nikulin's lattice theory.