

Investigation of aerodynamic effects of rotation on 6-DOF motion of flat square plate based on free flight experiments

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ABSTRACT:

Plate-like debris is common windborne debris in windstorms. Although the 3-degree-of-freedom (3-DOF) motion (two translational DOFs and one rotational DOF) of flat plates has been widely investigated in previous studies, the 6-DOF motion is more likely to occur in real storms. We investigate the 6-DOF motion by performing free flight experiments of a square plate with different initial orientations under uniform flow in a wind tunnel. Two cameras are used to record the flight motion of the plate. Then the recordings are processed in an image analysis software to estimate the kinematics of the plate. We first investigate the characteristics of the rotational motion. According to the rotational characteristics, the plate trajectories are categorized into four flight patterns: flutter, tumbling, transition, and quasi-translation. Then we explore the rotational effects on the aerodynamics. We propose a hypothesis that the magnitude and direction of the rotational forces depend on the angular velocity and the relative wind velocity. We also propose a hypothesis that the rotational moment has negligible influences on the direction of the instantaneous moment. These hypotheses are supported by the experimental results. These verified hypotheses may be helpful for modelling the aerodynamics of the 6-DOF motion of flat plates.

Keywords: aerodynamics, square plates, 6-degree-of-freedom, three-dimensional motion, windborne debris

1. Introduction

Impact by windborne debris can cause considerable damage to building surfaces. Plate-like objects such as roof tiles and shingles are very common in strong wind events.

Researchers have widely investigated the aerodynamics and flight characteristics of the 3-degree-of-freedom (3-DOF) motion of flat plates in horizontal winds. Here 3-DOF refers to motion where the mass center of the plate moves in the plane that contains the wind direction and the gravity direction, and the angular velocity is perpendicular to the plane. The 3-DOF motion is also called two-dimensional motion. Tachikawa (1983) first conducted systematic research on the 3-DOF motion of flat plates. He presented the non-dimensional equations of motion. He proposed an aerodynamic model for numerical calculation using experimentally obtained lift and drag coefficients. He conducted free flight experiments of flat plates in a wind tunnel and then compared the experimental results and numerical results. Tachikawa's experimental and simulation methods were also used by many other researchers including Holmes et al. (2006), Lin et al. (2006), Baker (2007), and Kordi and Kopp (2009).

The studies of the 6-DOF motion (also called the three-dimensional motion) are fewer. Tachikawa (1988) carried out free flight experiments of the 6-DOF motion of flat plates focusing on the distribution range of the windborne debris trajectories. All those early experiments were conducted in uniform winds. To simulate more realistic flight conditions, Visscher and Kopp (2007), and Kordi and Kopp (2011) made the plates mounted on the roof of a house model and launched in a turbulent flow. Their studies show that the 6-DOF motion is more common than

the 3-DOF motion. Richards et al. (2008) proposed an aerodynamic model for the 6-DOF motion. However, the rotational effect was not fully considered. Yet Kakimpa et al. (2012) used a coupled CFD–RBD (Computational Fluid Dynamics–Rigid Body Dynamics) model to reveal more details of the aerodynamics of plates. Although it has been known that the rotational motion has a great influence on the aerodynamics of flat plates from the studies of the 3-DOF motion, little is known about the rotational effects of the 6-DOF motion. Even the characteristics of rotational motion of plates has not been studied in detail.

To estimate the damage caused by windborne debris, it is important to know how fast and how long debris items are likely to travel. Experimental methods and numerical methods can be used to address this problem. Simulating actual flow conditions and the flight motion of windborne debris in reduced-scale experiments is difficult. On the other hand, Numerical simulations using an aerodynamic model offer practical approaches to predict the trajectories of windborne debris. To establish aerodynamic models of plate-type debris, the rotational effects on aerodynamics should first be fully investigated.

The rotational motion influences the aerodynamic forces acting on a plate, and then the aerodynamic forces influence the flight characteristics (flight velocity and flight distance, etc.) of the plate. The main objective of this study is to examine how the rotational motion influences the aerodynamic forces and moments. However, it is difficult to directly measure these forces and moments during flight. Therefore, as an alternative way, we investigate the effects of rotation on the flight characteristics and use these observations to infer the rotational effects on the aerodynamic forces and moments.

We perform free flight experiments of a square plate because this shape is typical. In the free flight experiments of most previous studies, the angular velocity was not measured. Therefore, the rotational effects could not be studied in those experiments. By applying the motion analysis techniques, we can measure not only the velocity but also the angular velocity of the plate. Then we can investigate the aerodynamic effects of rotation of the 6-DOF motion.

2. Free-flight experiments of 6-DOF motion

The experiments were performed under a uniform wind flow in a wind tunnel. Figure 1(a) shows the experiment setup. First, the flight trajectories were recorded using two digital cameras (CASIO EX-100PRO) from two different directions. Then the recordings were input into a motion analysis software (DIPP-MotionV/3D) to estimate the time series of the position, orientation, velocity, and angular velocity of the plates in flight.

Techniques of photogrammetric 3D tracking can be briefly explained as follows: 2D coordinates of a marker can be determined by one camera; if this marker is recorded by two or more cameras, then its 3D spatial coordinates can be determined. In this study, we followed the instructions of the software DIPP-MotionV/3D to track coordinates of markers. As a pre-process, a calibration was conducted to construct the mapping between the image coordinates and the spatial coordinates. In the calibration process, we first selected eight points in the space which formed a cubic frame that covered the flying range of the square plate. We then measured the spatial coordinates of these eight points and specified their locations (pixels) on the images. Once the calibration was conducted, i.e., the mapping between image coordinates and spatial coordinates was established, the spatial coordinates of a marker at each time can be estimated by specifying its locations on the images of the two cameras.

The initial orientation of the square plate was varied in the experiments. Figure 1(b) shows the

control system for the initial orientation of the plate. The suction cup at the tip of the support rod was connected to a vacuum pump by a pipe. The way of setting up the initial orientation is as follows: First, the flat plate was stuck on the suction cup in such a way that the directions of the global reference frame $X'Y'Z'$ fixed in space coincided with the directions of the body reference frame xyz fixed on the plate. Next, Rod 1 was rotated by an angle θ_1 in the horizontal plane, and then Rod 2 was rotated by an angle θ_2 in the vertical plane, and finally the plate was rotated by an angle θ_3 around the x -axis. The angles θ_1 , θ_2 , and θ_3 are called initial orientation angles in this paper. We determined the parameter settings mainly by the experiences and knowledge from previous studies. In the 3-DOF motion, the initial orientation is characterized by only one parameter, θ_2 . From the previous study on the 3-DOF motion by Tachikawa (1983), it has been known that when θ_2 takes 0° , 30° , 60° and 75° , the plate will show different flight patterns. On the other hand, both θ_1 and θ_2 are angles around an axis in the plate plane. Therefore, we set θ_1 and θ_2 at these four angles in the experiments. For θ_3 , because a square plate has a high degree of symmetry around the x axis that is normal to the plate plane, we inferred that θ_3 around the x axis does not have a great influence on the flight trajectories. Therefore, we only set θ_3 at two angles, 0° or 45° . This inference is supported by the experiment results shown in Section 4.1. Table 1 shows the experiment cases and their case labels; that is, 32 ($= 4 \times 4 \times 2$) cases with different initial orientations were considered in the experiments. The flat plate was released once the air valve of the vacuum pump was opened. The height of Rod 2 was adjusted so that the plate was released from a height of 0.55 m in all cases.

The four corners of the plate were painted white and were used as markers for 3D tracking. To facilitate the recognition of the markers, the other part of the plate except the markers was painted black. Each camera took 240 pictures per second. The spatial coordinates of the four markers in every picture were extracted using the motion analysis software, and the time series of the position and orientation of the plate were estimated by the coordinates of the markers. Then the time series of the translational velocity and angular velocity were estimated using the difference method. To reduce the random error, a moving average of five time points was performed for the estimated time series of the velocity and angular velocity.

An important non-dimensional parameter related to the flight motion of a flat plate is Tachikawa number, which was introduced by Tachikawa (1983). Tachikawa number (Ta) represents the ratio of aerodynamic force to gravity and is calculated by the following Equation:

$$Ta = \rho U_w^2 A / 2mg \quad (1)$$

where ρ is the fluid density, U_w is the wind velocity, A is the area of the plate, m is the plate mass, and g is gravitational acceleration.

The scale model of debris used in these experiments was based on a prototype plate with dimensions $300 \times 300 \times 15$ mm and mass 4 kg. If the wind velocity U_w is 40 m/s, and the air density ρ is 1.225 kg/m^3 , then Ta is about 2.25 under these assumptions. The parameters of the free flight experiments were adjusted so that Tachikawa number in the wind tunnel experiment close to that of the assumed actual flight motion of the windborne debris. Therefore, we used a square wooden plate with a side length of 60 mm, a thickness of 3 mm, and a mass of 6.6 g. The uniform wind velocity in the wind tunnel experiment was 8 m/s. Ta for the experiment is 2.18.

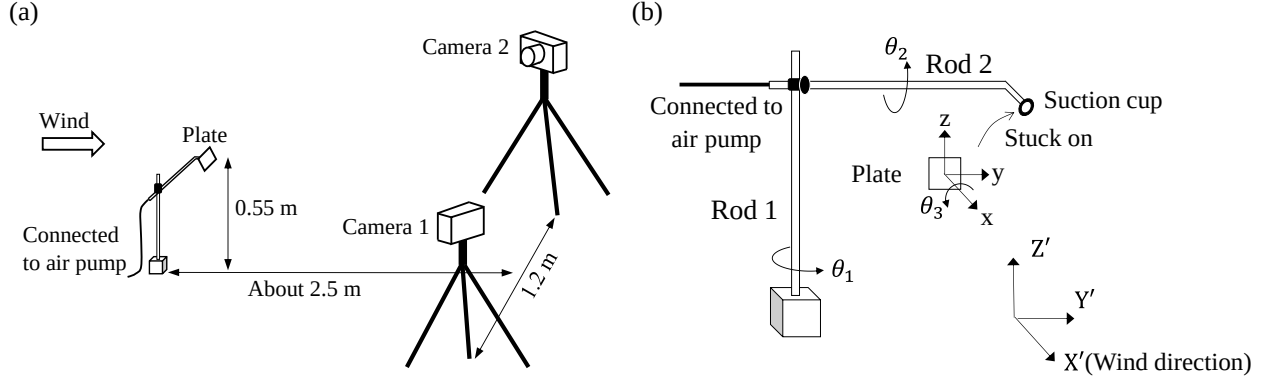


Figure 1. (a) Experiment setup; (b) Control system of the initial orientation. The global reference frame $X'Y'Z'$ is fixed in space, and the origin is at a point on the ground. The body reference frame xyz is fixed on the plate.

Table 1. Experiment cases and case labels.

(a) $\theta_3=0^\circ$					(b) $\theta_3=45^\circ$				
$\theta_1 \backslash \theta_2$	0°	30°	60°	75°	$\theta_1 \backslash \theta_2$	0°	30°	60°	75°
0°	A1	A2	A3	A4	0°	B1	B2	B3	B4
30°	A5	A6	A7	A8	30°	B5	B6	B7	B8
60°	A9	A10	A11	A12	60°	B9	B10	B11	B12
75°	A13	A14	A15	A16	75°	B13	B14	B15	B16

3. Review of rotational effects of 3-DOF motion

The aerodynamics of the 3-DOF motion (two-dimensional motion) of flat plates demonstrates basic properties that can help the understanding of the flight characteristics of the 6-DOF motion.

The aerodynamic force can be divided into drag and lift. The drag is the force along the direction of the relative wind velocity (the wind velocity minus the plate velocity), and the lift is the force perpendicular to the direction of the relative wind velocity. Figure 2 shows the positive directions of the drag, lift, as well as moment. The aerodynamic force and moment acting on a plate with no rotational motion are called translational force and translational moment, respectively. The translational drag $\mathbf{D}_t$, lift $\mathbf{L}_t$, and moment $\mathbf{M}_t$ are calculated by the following equations:

$$\mathbf{D}_t = (1/2)\rho|\mathbf{U}|^2 AC_{D_t} \mathbf{e}_D \quad (2)$$

$$\mathbf{L}_t = (1/2)\rho|\mathbf{U}|^2 AC_{L_t} \mathbf{e}_L \quad (3)$$

$$\mathbf{M}_t = (1/2)\rho|\mathbf{U}|^2 AlC_{M_t} \mathbf{e}_M \quad (4)$$

where ρ is the fluid density; $\mathbf{U}$ is the relative wind velocity; A is the area of the plate; l is the length of the plate. C_{D_t} , C_{L_t} and C_{M_t} are the translational drag, lift and moment coefficients, respectively, and dependent on the angle of attack (see Figure 2); $\mathbf{e}_D$, $\mathbf{e}_L$ and $\mathbf{e}_M$ denote unit vectors in the positive direction of the drag, lift and moment shown in Figure 2, respectively.

The translational force and moment coefficients of a square plate have been measured by fixing the angle of attack in wind tunnels in several previous studies. Holmes et al. (2006) presented a comparison of these results. Figure 3 shows the values measured by Tachikawa and

Fukuyama (1981). The values of C_{Lt} and C_{Mt} are antisymmetric about the angle of attack of 90° , while C_{Dt} is symmetric.

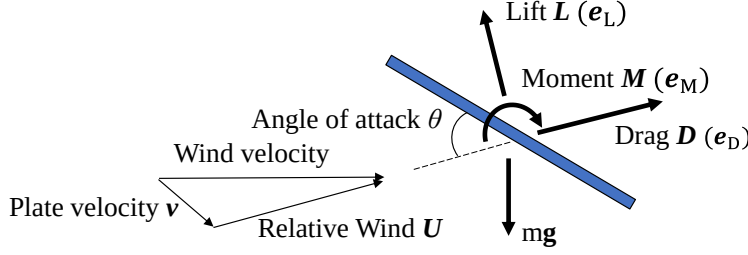


Figure 2. Schematic diagram of the 3-DOF motion

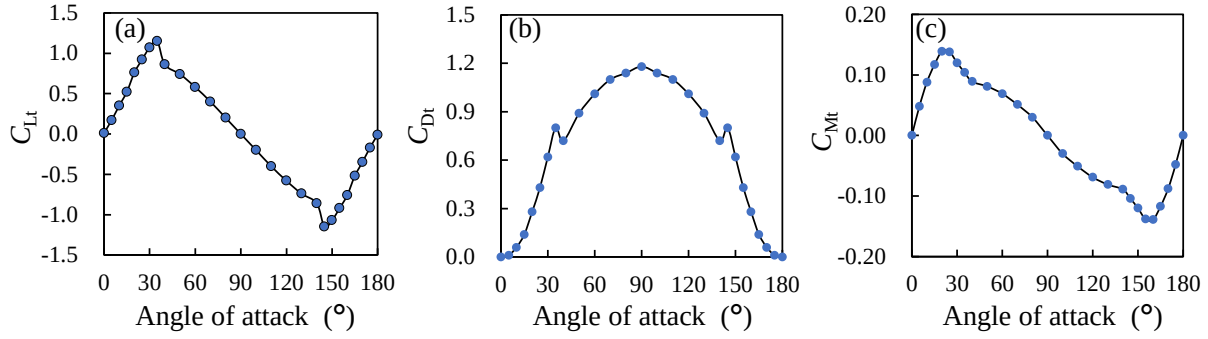


Figure 3. Translational aerodynamic coefficients. (a) Lift coefficient; (b) Drag coefficient; (c) Moment coefficient. (Reproduced from Tachikawa and Fukuyama (1981))

If a plate rotates in wind, the aerodynamic force and moment are different from the translational situation. The aerodynamic force and moment on a rotating plate can be divided into two parts: one is induced from the translational motion, and the other is induced from the rotational motion:

$$\mathbf{F}_{inst} = \mathbf{F}_t + \mathbf{F}_r \quad (5)$$

$$\mathbf{M}_{inst} = \mathbf{M}_t + \mathbf{M}_r \quad (6)$$

where $\mathbf{F}_{inst}$ and $\mathbf{M}_{inst}$ denote the instantaneous forces ($\mathbf{D}$ and $\mathbf{L}$) and moment, respectively; the subscript t denotes the translational part; the subscript r denotes the rotational part.

Tachikawa (1983) measured the instantaneous force on a rotating square plate through an autorotating test. In this test, the wind drives the plate to rotate about a fixed axis. The average angular velocity over one rotation cycle increases to reach a peak value and maintains a steady state. The instantaneous force during the test oscillates periodically corresponding to the periodical change of the angle of attack. By taking the average of equations (5) and (6) over one cycle, the equations become:

$$\overline{\mathbf{F}_{inst}} = \overline{\mathbf{F}_t} + \overline{\mathbf{F}_r} \quad (7)$$

$$\overline{\mathbf{M}_{inst}} = \overline{\mathbf{M}_t} + \overline{\mathbf{M}_r} \quad (8)$$

The average rotational force and moment coefficients are obtained via dividing the average rotational force $\overline{\mathbf{F}}_r$ by $1/2\rho|\mathbf{U}|^2A$ and $\overline{\mathbf{M}}_r$ by $1/2\rho|\mathbf{U}|^2Al$, respectively. Figure 4 shows the results of the nondimensional average rotational force coefficients obtained by Tachikawa (1983). In the figure, $\overline{\omega}_0$, $\overline{C}_{Dr0}$ and $\overline{C}_{Lr0}$ are the average angular velocity, rotational drag coefficient and rotational lift coefficient in the steady state, respectively. The rotational lift and drag coefficients increase with the increase of $\overline{\omega}/\overline{\omega}_0$. The rotational moment coefficient decreases to 0 as $\overline{\omega}/\overline{\omega}_0$ becomes 1.

Besides the magnitude, the direction of the rotational lift also depends on the angular velocity. When wind blows from left to right, an upward average rotational lift is generated on a plate rotating clockwise, while a downward average rotational lift is generated on a plate rotating counterclockwise. This relation is represented by the following equation:

$$\mathbf{e}_{Lr} = \mathbf{e}_U \times \mathbf{e}_\omega \quad (9)$$

where $\mathbf{e}_{Lr}$ denotes a unit vector in the direction of the rotational lift, $\mathbf{e}_U$ denotes a unit vector in the direction of the relative wind velocity, and $\mathbf{e}_\omega$ denotes a unit vector in the direction of the angular velocity.

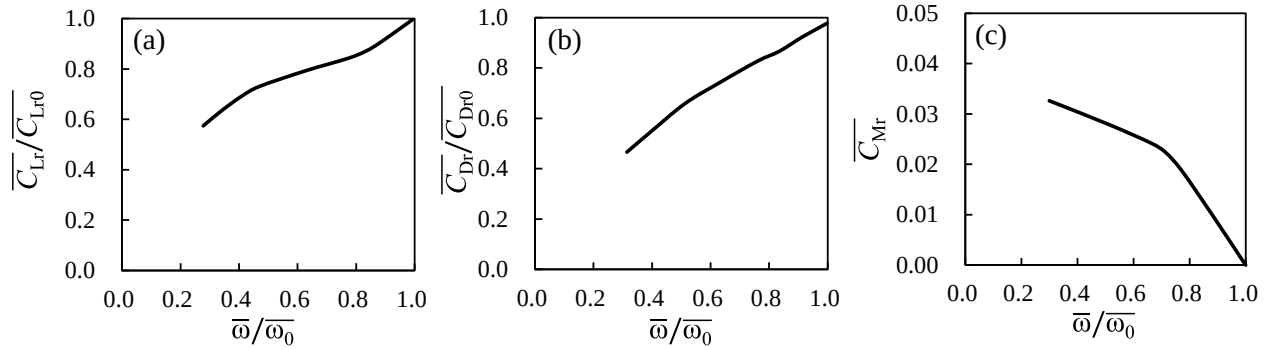


Figure 4. Variation of average rotational lift, drag and moment coefficients over one cycle. (Reproduced from Tachikawa (1983))

The flight characteristics of the 3-DOF motion of square plates are investigated by Tachikawa (1983), and Kakimpa et al. (2012). Both of them observed that as the initial angle of attack changes, the plate demonstrates different flight patterns. Kakimpa et al. (2012) used CFD-RBD method to simulate the flight motion and categorized the plate trajectories into three patterns: flutter, autorotation, and transition. The flutter pattern is characterized by that the angular velocity of the plate oscillates between positive and negative values. The plates of the autorotation pattern rotate in only one direction. The plates of transition pattern flutter at the beginning and then tumble in one direction eventually. Tachikawa (1983) categorized the plate trajectories into three patterns. However, he called them translational, autorotational and intermediate patterns. The latter two correspond to the autorotation and flutter pattern, respectively. The translational pattern named by Tachikawa (1983) and the transition pattern by Kakimpa et al. (2012) are different. The release position of plates in Tachikawa's experiments is not high enough, so he did not observe the transition pattern. On the other hand, some plates in his experiments rotated too slowly that they neither flutter nor autorotate before reaching the ground. Tachikawa (1983) called the trajectories of these plates translational pattern. Yet Kakimpa et al. (2012) could make the release position of the plates as high as possible in their numerical simulation. Kakimpa et al. (2012) observed that the amplitudes of the angular velocity of the flutter pattern plates increase from one cycle to the next. Thus, they postulated that given a

long enough flight time the flutter pattern would eventually transit into the autorotation. Therefore, the flight time (or the release height) is crucial for the classification of the flight pattern. This may be the reason why Tachikawa (1983) and Kakimpa et al. (2012) made different categories of the flight patterns.

At last, it should be mentioned that according to Iversen (1979) the aerodynamic forces of a square plate are also dependent on the thickness ratio and Reynolds number. However, the effects of those parameters are not considered in the present study, because the present study aims to investigate the general characteristics of rotational effects of the 6-DOF motion.

4. Rotational characteristics of 6-DOF motion

4.1 Flight patterns

In the 3-DOF motion, a plate begins to rotate from rest driven by the aerodynamic moment. At the beginning, the angular velocity becomes greater because the aerodynamic moment and the angular velocity have the same direction. This is called supporting period by Lugt (1980). As the angle of attack changes with the rotation, the aerodynamic moment turns to resist the rotation, and the angular velocity becomes smaller. This is called retarding period by Lugt (1980). If the angular momentum of the plate has not become great enough during the supporting period, the angular velocity would become zero, and then the plate would begin to rotate reversely. This is called flutter motion. If the angular momentum of the plate has become great enough during the supporting period, its angular velocity would not become zero and then enter the next supporting period. This is called tumbling motion.

The concepts of flutter and tumbling motion in the 3-DOF motion can be extended to the 6-DOF motion. Note that Tachikawa (1983) and Kakimpa et al. (2012) referred to tumbling as autorotation. It should be noted that the terms of flight patterns (for example, the flutter pattern) are used to describe the whole plate trajectory from release to landing, whereas the terms flutter motion and tumbling motion are used to describe parts of the motion of the plate trajectory.

In the 3-DOF motion, because the axis of the angular velocity is fixed, the direction of angular velocity only has positive and negative directions. Therefore, the flutter motion can be easily defined as a motion in which the direction of angular velocity changes. However, in the 6-DOF motion, because the axis of the angular velocity is not fixed, its direction can be always in change. Therefore, we propose a more general way of defining the flutter motion, which can be used for both the 3-DOF and 6-DOF motion. First, we introduce the angle between the relative wind velocity and the normal direction of the plate plane represented by the x axis of the body reference frame. This angle is denoted by β_w , as shown in Figure 5, and its range is $[0^\circ, 180^\circ]$. Figure 5 shows how angle β_w changes with time in two examples of the flutter motion. In Figure 5(a), the initial value of angle β_w is under 90° . First, angle β_w decreases to 0° because the aerodynamic moment tends to make the plate normal to the relative wind velocity $\mathbf{U}$. Then angle β_w begins to increase with the plate continuing to rotate clockwise. As the angular velocity reverses its direction, angle β_w reaches a local maximum value. In Figure 5(b), when the initial value of angle β_w is above 90° , the reversing of angular velocity corresponds to a local minimum value of angle β_w . Therefore, the flutter motion can also be defined as a motion when angle β_w reaches a local maximum value under 90° or a local minimum value above 90° . It should be noted that β_w may never pass through 0° or 180° in the 6-DOF motion, although

β_w passes through these two angles in Figure 5. Similarly, the tumbling motion can be defined as a motion over a period in which angle β_w crosses the line of 90° twice and finally returns to the original value without flutter motion occurring. Figure 6 shows an example of the tumbling motion.

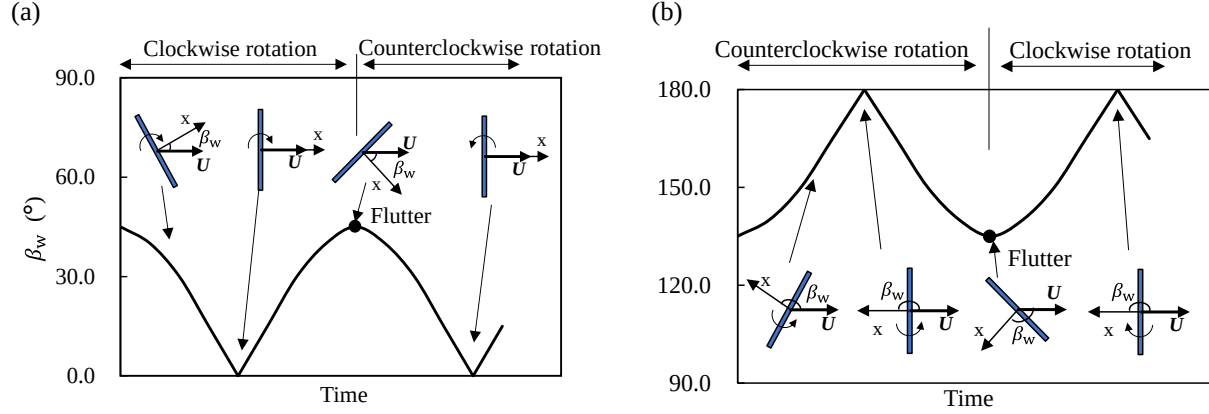


Figure 5. The change of angle β_w of flutter examples. (a) an example in which the initial value of angle β_w is under 90° . (b) an example in which the initial value of angle β_w is above 90°

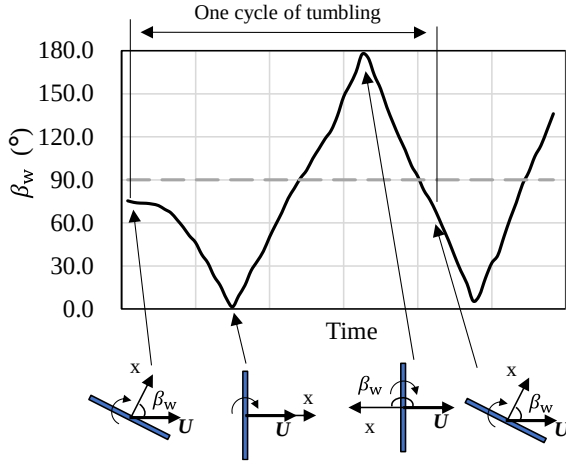


Figure 6. The change of angle β_w of a tumbling example

Using the above definitions of the flutter and tumbling motion, any plate trajectory is classified as one of the following four patterns: (1) a plate trajectory in which only the flutter motion occurs, which we call flutter pattern; (2) a plate trajectory in which only the tumbling motion occurs, which we call tumbling pattern; (3) a plate trajectory in which both the flutter motion and tumbling motion occur, which we call transition pattern (most likely, the transition is from flutter to tumbling); (4) the last is the trajectory in which the angular velocity is too small that neither flutter nor tumbling occurs, which we call quasi-translation pattern.

The plate trajectories of our free flight experiments were classified using the above definition, and Table 2 shows the results of the flight patterns. The flight patterns do not change whether the initial orientation angle θ_3 is 0° or 45° . The transition pattern and the quasi-translation pattern were not observed in the present experiments. As shown in Table 2, as the angles θ_1 and θ_2 increase, the flight pattern transforms from the flutter pattern to the tumbling pattern.

Table 2. Flight patterns. F, the flutter pattern; T, the tumbling pattern.

θ_1	θ_2			
	0°	30°	60°	75°
0°	F	F	T	T
30°	F	F	T	T
60°	F	T	T	T
75°	T	T	T	T

4.2 Magnitudes and directions of angular velocities

In the 6-DOF motion, the angular velocity ω is a vector, which has a magnitude and a direction. Figure 7 shows the magnitudes of the angular velocities $|\omega|$ of all the flutter cases and all the tumbling cases in the experiments, respectively. Figure 7 shows that in all the cases of the flutter pattern and tumbling pattern, the angular velocities tend to increase at the beginning. Then the angular velocities decrease since the aerodynamic moment begins to resist the rotation. Then, the flutter pattern and tumbling pattern show different tendencies. In the flutter pattern cases, the angular velocities tend to decrease to small values with the occurring of flutter motion. In the tumbling pattern cases, there are two types. The angular velocities of Type 1, as shown in Figure 7(b1), first decrease to small values and then tend to increase quickly. The angular velocities of Type 2, as shown in Figure 7(b2), first decrease slightly and then tend to increase to reach the steady state.

We introduce the translating reference frame XYZ that translates with the plate, however, does not rotate. The origin of the translating reference frame XYZ is placed at the mass center of the plate, and the directions of the axes coincide with those of global reference frame X'Y'Z'. Under this setting, the components of angular velocity ω of the plate have the same values in both the reference frame X'Y'Z' and XYZ. Figure 8 shows the components of ω of selected cases in both the body reference frame xyz and the translating reference frame XYZ. Here we use the initial orientation angles (θ_1 , θ_2 , θ_3) to represent each case. There are two common features in all the cases including the cases not shown here. First, in the body reference frame, ω_x is not significant compared to the other components. This means the angular velocity almost lies in the plate plane. Second, in the translating reference frame, ω_x is not significant compared to the other components. This means the angular velocity almost lies in the plane perpendicular to the wind direction.

If ω_x is considered as 0, i.e., negligibly small, then the direction of the angular velocity in the translating reference frame can be represented by a single parameter. This parameter is denoted by angle α_A , defined as in Figure 9. Here, we call α_A rotation direction angle. Table 3 shows the time average values of α_A of all the cases. For the flutter pattern cases, the average values of α_A before and after the flutter motion are presented separately. In Section 5, we will show that the rotation direction angle α_A has a significant impact on the flight velocity and flight time of a plate.

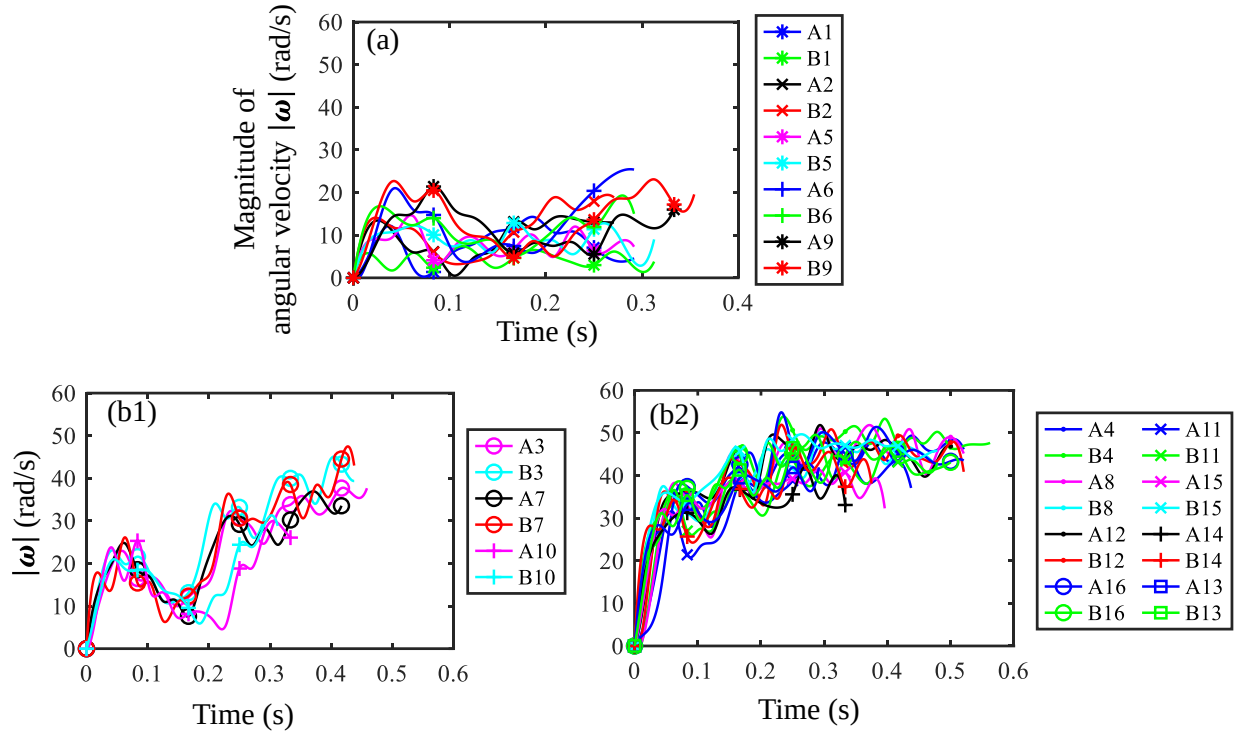


Figure 7. Magnitudes of the total angular velocities of (a) the flutter pattern cases, (b1) the tumbling pattern cases of Type 1, and (b2) the tumbling pattern cases of Type 2. (The line colours in this figure have no correlation with those in other figures in this paper)

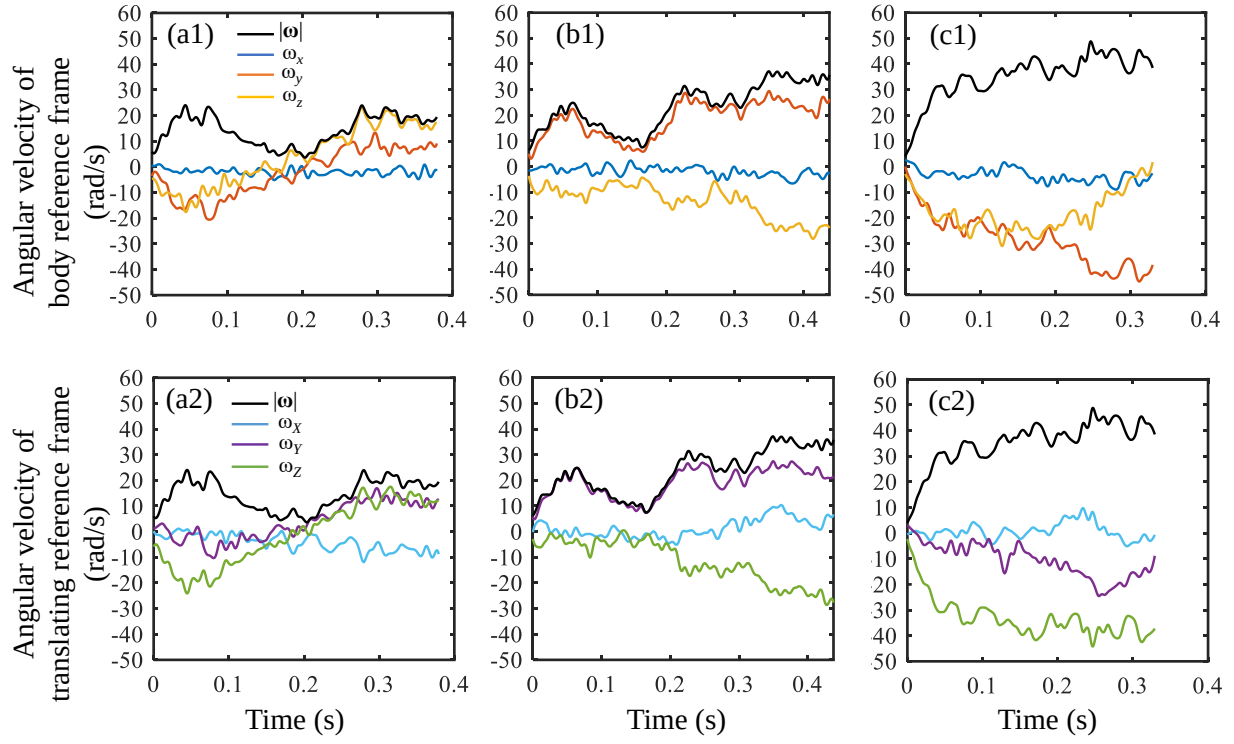
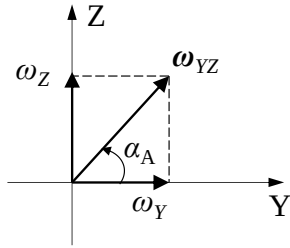


Figure 8. Angular velocities of typical cases in the body reference frame and translating reference frame. (a) a flutter pattern case ($60^\circ, 0^\circ, 45^\circ$); (b) a tumbling pattern case ($30^\circ, 60^\circ, 0^\circ$); (c) a tumbling pattern case ($70^\circ, 0^\circ, 45^\circ$).



If $\omega_Z > 0$, $\alpha_A > 0$; If $\omega_Z < 0$, $\alpha_A < 0$

Figure 9. Rotation direction angle α_A

Table 3. Time average values of rotation direction angle $\overline{\alpha}_A$ of the all the cases. For the flutter pattern, two values are shown. The upper values are $\overline{\alpha}_A$ before the flutter motion, and the lower values are $\overline{\alpha}_A$ after the flutter motion.

(a) $\theta_3=0^\circ$					(b) $\theta_3=45^\circ$				
$\theta_1 \backslash \theta_2$	0°	30°	60°	75°	$\theta_1 \backslash \theta_2$	0°	30°	60°	75°
0°	0° 180°	0° 180°	5°	-4°	0°	180° 0°	0° 180°	14°	9°
30°	-104° 83°	-50° 113°	-28°	-12°	30°	-80° 80°	-34° 127°	-19°	-29°
60°	-97° 45°	-74°	-44°	-28°	60°	-101° 43°	-85°	-35°	6°
75°	-108°	-88°	-62°	-28°	75°	-105°	-71°	-34°	-19°

5. Rotational effects on aerodynamic forces

5.1 Proposed hypotheses about rotational effects on aerodynamic forces

We have observed in Section 4.2 that the angular velocity almost lies in the plate plane in the 6-DOF motion. On the other hand, it is known that in the 3-DOF motion, the angular velocity always lies in the plate plane. This similarity implies that the rotational effects on aerodynamics observed in the 3-DOF motion may still exist in the 6-DOF motion. Although we cannot directly measure the rotational force and moment, we can make hypotheses on the rotational force and moment at first and then use the plate trajectories to verify these hypotheses. In this section, we first decompose the rotational force into two components, and then propose hypotheses about the rotational effects on the components.

The ratio of thickness to side length of a square plate is called thickness ratio. Richards et al. (2008) measured the translational forces acting on flat plates with small thickness ratios and found that the only significant force is the force normal to the plate plane. For a plate with a small thickness ratio (usually less than or equal to 0.05), the aerodynamic force parallel to the plate plane is small compared to the force normal to the plate plane, because the area of the lateral surface is small compared to the area of the plate plane. For the same reason, the rotational force $\mathbf{F}_r$ is also considered to be normal to the plate plane. Besides, it has been shown in Section 4.2 that the angular velocity almost lies in the plate plane. Therefore, it is reasonable to assume that the rotational force $\mathbf{F}_r$ is normal to the angular velocity, as shown in Figure 10. A decomposition method is to divide the aerodynamic force into a drag and a lift that are forces along the direction of the relative wind velocity $\mathbf{U}$ and the direction normal to $\mathbf{U}$, respectively. According to the definition, lift lies in the plane normal to $\mathbf{U}$ in the 6-DOF motion. The direction of the translational lift, however, depends on the shape and orientation of the flying object. The problem is that we do not know the actual direction of the rotational lift a-priori. Therefore, decomposing the rotational force $\mathbf{F}_r$ in the 6-DOF motion into a drag and lift is not straightforward. Instead, the rotational force $\mathbf{F}_r$ can be decomposed into the following two forces: the one force, $\mathbf{F}_p$, is parallel to the plane P that contains the relative wind velocity $\mathbf{U}$ and the angular velocity $\boldsymbol{\omega}$; the other force, $\mathbf{F}_n$, is normal to the plane P, as shown in Figure 10. In this decomposition, $\mathbf{F}_n$ could take one of the two possible directions, $\mathbf{e}_U \times \mathbf{e}_\omega$ or

1 $-\mathbf{e}_U \times \mathbf{e}_\omega$. In the 3-DOF motion, because $\mathbf{U}$ is always perpendicular to $\boldsymbol{\omega}$ the forces $\mathbf{F}_n$ and
 2 $\mathbf{F}_p$ are the lift and drag, respectively. However, in the 6-DOF motion, the forces $\mathbf{F}_n$ and $\mathbf{F}_p$
 3 may be different from the lift and drag, because $\mathbf{U}$ is not necessarily perpendicular to $\boldsymbol{\omega}$, as
 4 shown in Figure 10.

5 Based on the knowledge of aerodynamics of the 3-DOF motion, we propose two hypotheses
 6 on the force $\mathbf{F}_n$. First, we assume that the magnitude of the force $\mathbf{F}_n$ increases as the angular
 7 velocity increases, and the other parameters do not have significant influences on the magnitude
 8 of the force $\mathbf{F}_n$. Second, like the rotational lift in the 3-DOF motion, we assume that the
 9 direction of the force $\mathbf{F}_n$, denoted by $\mathbf{e}_n$, takes the direction calculated by the following
 10 equation (note that $\mathbf{F}_n$ could take one of the two possible directions, $\mathbf{e}_U \times \mathbf{e}_\omega$ or $-\mathbf{e}_U \times \mathbf{e}_\omega$),
 11

$$\mathbf{e}_n = \mathbf{e}_U \times \mathbf{e}_\omega \quad (10)$$

12
 13 Based on the above hypotheses, we consider two plate conditions, as shown in Figure 11. In
 14 this figure, the X direction of the translating reference frame XYZ is the wind direction, which
 15 points out of the paper. Because the plate velocity is small compared to the wind velocity in the
 16 experiments, the relative wind velocity deviates little from the wind velocity. Therefore, as a
 17 simplified situation we may approximate that the direction of relative wind velocity coincides
 18 with the wind direction, i.e., the X direction in Figure 11. The angular velocities $\boldsymbol{\omega}$ of the two
 19 plates in Figure 11 are assumed to have the same magnitudes yet different directions. We
 20 consider the flight velocity and the flight time of the two plates. According to the first hypothesis
 21 on the force $\mathbf{F}_n$, the magnitudes of the force $\mathbf{F}_n$ of the two plates are the same because of the
 22 same magnitudes of angular velocities. According to Equation (10), when the rotation direction
 23 angle $\alpha_A < 0^\circ$, as α_A increases to be close to 0° , the Z component of the force $\mathbf{F}_n$ increases;
 24 since the positive Z component contributes to resisting gravity, so the plate falls more slowly,
 25 and the flight time becomes longer.

26 Besides the effect of rotational force, the plate velocity and flight time are also affected by the
 27 translational force. However, according to the results of Richards et al. (2008), the translational
 28 force along the direction of $\mathbf{F}_n$ is almost antisymmetric about angle β_w of 90° (see the
 29 definition in Section 4.1), so the effect of translational force almost cancels out in one cycle of
 30 tumbling.

31 The above inference is verified in the next section using the results of the free flight
 32 experiments.
 33
 34

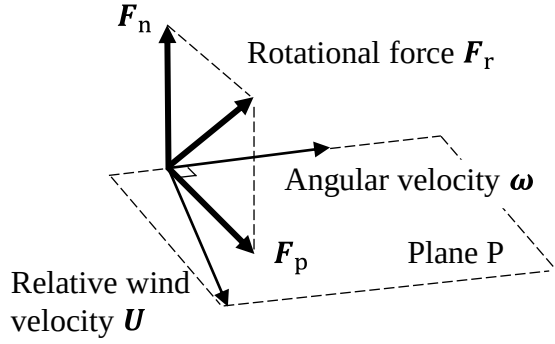


Figure 10. Decomposition of the rotational force

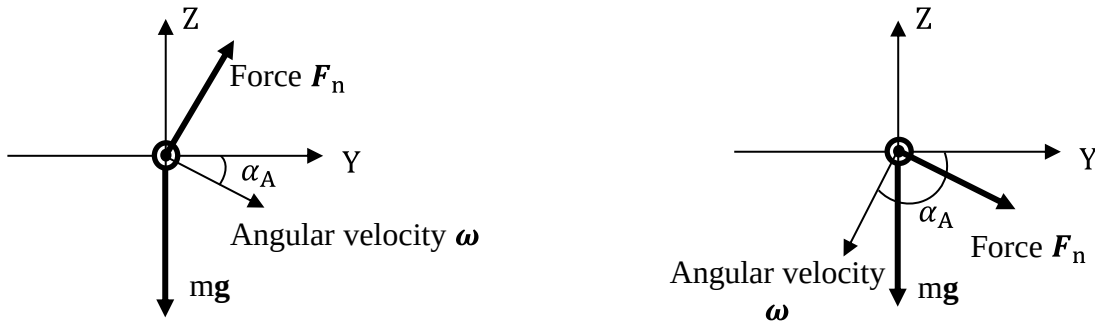


Figure 11. Relation between angle α_A and rotational force F_n based on the hypotheses in Section 5.1.

5.2 Verification of the hypotheses

To verify the inference stated in the previous Section 5.1, we need to find the cases with similar magnitudes of angular velocities $|\omega|$ and different values of α_A in the free flight experiments. Figure 12 shows the flight time of all the 32 cases versus the time average angular velocity. The cases are divided into 5 groups, Group A, B, C, D and E. In general, the plate with a greater average angular velocity has a longer flight time except the plates in Group D. Although the plates in Group D have similar average angular velocities, their flight times show great variations. The symbols for the tumbling pattern in Figure 12 are coloured according to the time average values of the rotation direction angle $\bar{\alpha}_A$. It is clear that the values of $\bar{\alpha}_A$ of Group D show greater variations than Group B, C and E.

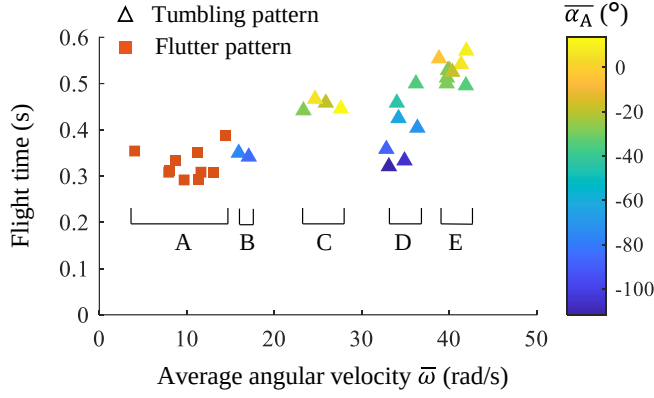


Figure 12. Flight time versus average angular velocity

We further consider the plates in Group D. It should be mentioned that although α_A of the plates in Group D changes with time, the change is not great. Therefore, if the inference in Section 5.1 holds, the flight time is longer when the time average value of the rotation direction angle is closer to 0° . Figure 13 plots the flight times of the plates in Group D versus the time average rotation direction angle $\bar{\alpha}_A$. It was shown in Figure 13 that the plate with a greater $\bar{\alpha}_A$ has a longer flight time, which supports the inference. i.e., the plate with α_A closer to 0° falls more slowly. In addition, the dotted line in Figure 13 denotes the flight time when assuming the aerodynamic force in Z direction is 0, and only gravity acts on the plate. It was shown that the flight times of the cases with $\bar{\alpha}_A$ close to -90° (-88° and -105°) are close to the flight time under the situation with only gravity. This is because when α_A is -90° , there is no aerodynamic force in Z direction according to the inference in section 5.1.

The flight time only reflects the aerodynamic force in Z direction. We need to check the plate velocity both in Y and Z directions to further verify the inference in section 5.1. Figure 14 shows the plate velocities v_Y and v_Z of the plates in Group D. As shown in Figure 14(a), v_Z of the plate with a greater $\bar{\alpha}_A$ decreases more slowly, because the Z component of the force $\mathbf{F}_n$ that resists gravity is greater, while as shown in Figure 14(b), v_Y of the plate with a greater $\bar{\alpha}_A$ increases more slowly, because the Y component of the force $\mathbf{F}_n$ is smaller. By the way, we also check the plate velocity in X direction and show the result in Figure 15, because the plate velocity along the wind velocity is important when considering the damage that it may cause. Figure 15 shows that $\bar{\alpha}_A$ does not have a significant influence on v_X . This is because the angular velocities of these plates in Figure 15 are similar.

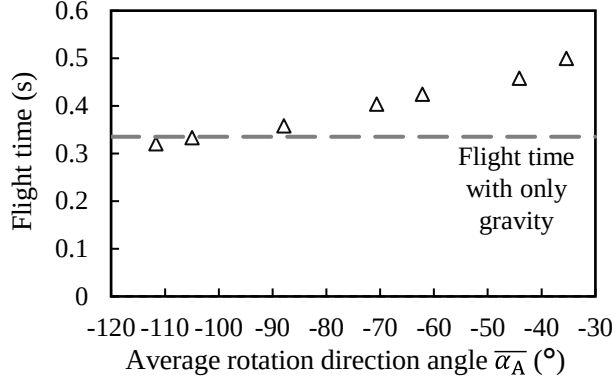


Figure 13. Flight time versus average rotation direction angle of the cases in Group D in Figure 12.

1

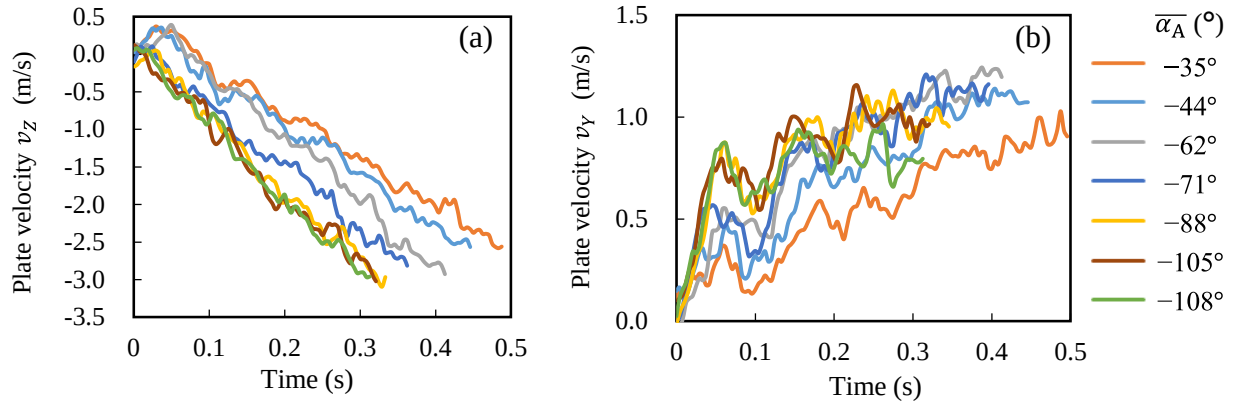


Figure 14. Plate velocities in Z direction and Y direction of the translating reference frame of the cases in Group D in Figure 12.

2

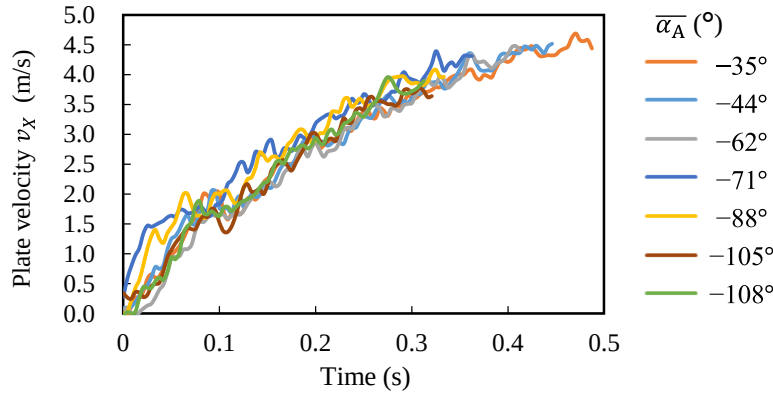


Figure 15. Plate velocities in X direction of the translating reference frame of the cases in Group D in Figure 12.

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The hypotheses about the rotational force component $\mathbf{F}_n$ in Section 5.1 can also explain the flight time of the other groups in Figure 12. Figure 12 shows that for the groups B, C and E, the plates of the same group have similar magnitudes of angular velocities, so the magnitudes of the force $\mathbf{F}_n$ are similar. Table 4 shows the time average values of $\bar{\alpha}_A$ of each group in Figure 12. It is shown that the ranges of $\bar{\alpha}_A$ of the three groups are relatively narrow, so the directions of

the force $\mathbf{F}_n$ are also similar. Therefore, the flight times in the same group have small variations. The cases in Group A are all the flutter pattern cases. Although Group A has a wide range of $\overline{\alpha}_A$, the average angular velocities are small, so the magnitudes of the force $\mathbf{F}_n$ are small. This is why all the plates in Group A have short flight times.

Table 4. Average rotation direction angle $\overline{\alpha}_A$ of the groups in Figure 12. For Group A, the values in the brackets are $\overline{\alpha}_A$ after the flutter motion.

Group	A	B	C	D	E
$\overline{\alpha}_A$	-104°~180° (43°~180°)	-85°~-76°	-28°~14°	-112°~-35°	-34°~9°

6. Rotational effects on aerodynamic moment

6.1 Change of rotation direction angle α_A

The change of the direction of the angular velocity of a plate depends on the aerodynamic moment acting on the plate. The time series of rotation direction angle α_A represents the change of the direction of the angular velocity. Figure 16 show two types of time series of α_A of tumbling pattern cases. In Figure 16(a), the time series of α_A of three cases whose α_A do not change significantly are shown, whereas in Figure 16(b), the time series of α_A of three cases whose α_A tend to decrease after being released are shown. Next, we find the mechanism behind the change of α_A .

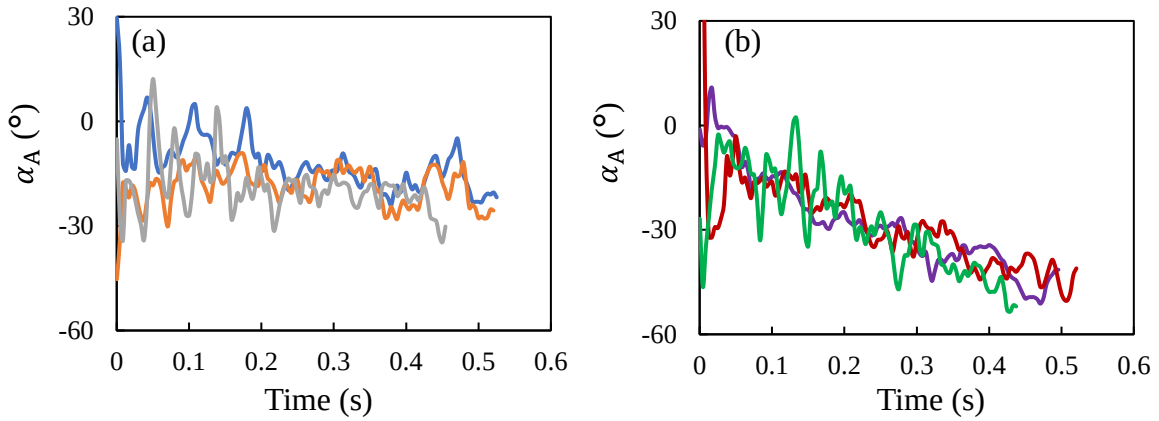


Figure 16. Time series of α_A of selected cases. (The line colours in this figure have no correlation with those in other figures)

6.2 Proposed hypothesis about rotational effects on aerodynamic moments

The aerodynamic moment $\mathbf{M}_{\text{inst}}$ on a rotating plate can be divided into the translational moment $\mathbf{M}_t$ and the rotational moment $\mathbf{M}_r$. The experimental data of the 3-DOF motion shows that the maximum value of the average rotational coefficient $\overline{C_{Mr}}$ is about 0.03, while the translational coefficient C_{Mt} is greater than 0.03 for most angles of attack (see Figure 3(c) and Figure 4(c)). Therefore, in most time, the direction of $\mathbf{M}_{\text{inst}}$ is the same as the direction of $\mathbf{M}_t$.

in the 3-DOF motion. We may propose a hypothesis that the rotational moment $\mathbf{M}_r$ has a negligible influence on the direction of $\mathbf{M}_{\text{inst}}$ in the 6-DOF motion as well. Next, we will explain the two types of time series of α_A shown in Section 6.1 based on this hypothesis about $\mathbf{M}_r$.

6.3 Verification of the hypothesis

In classical mechanics, Euler's rotation equations provide the governing equations of describing the rotation motion of a rigid body

$$\mathbf{I} \frac{d\boldsymbol{\omega}}{dt} + \boldsymbol{\omega} \times \mathbf{I} \boldsymbol{\omega} = \mathbf{M}_{\text{inst}} \quad (11)$$

where $\mathbf{I}$ is the moment inertia tensor.

Equation (11) is often expressed in the body's principal reference frame, which is exactly the body reference frame in this study. In the body's principal reference frame, the components of $\mathbf{I}$ do not change with time and are written as

$$\mathbf{I} = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix} \quad (12)$$

The x, y, z components of Equation (11) are then

$$\begin{aligned} I_x \frac{d\omega_x}{dt} + (\omega_y I_z \omega_z - \omega_z I_y \omega_y) &= M_x \\ I_y \frac{d\omega_y}{dt} + (\omega_z I_x \omega_x - \omega_x I_z \omega_z) &= M_y \\ I_z \frac{d\omega_z}{dt} + (\omega_x I_y \omega_y - \omega_y I_x \omega_x) &= M_z \end{aligned} \quad (13)$$

For a square plate, since $I_x \neq I_y = I_z$, Equation (13) becomes

$$\begin{aligned} \frac{d\omega_x}{dt} &= \frac{M_x}{I_x} \\ \frac{d\omega_y}{dt} &= \frac{(I_z - I_x)\omega_z\omega_x + M_y}{I_y} \\ \frac{d\omega_z}{dt} &= \frac{(I_x - I_y)\omega_x\omega_y + M_z}{I_z} \end{aligned} \quad (14)$$

We use $\mathbf{w}$ to denote the time derivative of $\boldsymbol{\omega}$, whose components in the body reference frame are $(d\omega_x/dt, d\omega_y/dt, d\omega_z/dt)$. The right hand of Equation (14) can be divided into two parts, a kinematic term $\mathbf{w}_k$ and a moment term $\mathbf{w}_M$ whose components in the body reference frame are $(0, (I_z - I_x)\omega_z\omega_x/I_y, (I_x - I_y)\omega_x\omega_y/I_z)$ and $(M_x/I_x, M_y/I_y, M_z/I_z)$, respectively. We

will separately discuss the effects of the kinematic term $\mathbf{w}_k$ and moment term $\mathbf{w}_M$ on the direction of $\boldsymbol{\omega}$.

The kinematic term $\mathbf{w}_k$ has the effect of making angular velocity $\boldsymbol{\omega}$ tend to rotate about the angular momentum $\mathbf{L}$ ($=\mathbf{I}\boldsymbol{\omega}$) (see Appendix A). This effect is difficult to explain the tendency of decrease of α_A shown in Figure 16(b): If the moment term $\mathbf{w}_M$ is zero, the kinematic term $\mathbf{w}_k$ would make the angular velocity $\boldsymbol{\omega}$ rotate periodically, and then α_A representing the direction of $\boldsymbol{\omega}$ would also change periodically. Therefore, the tendency of decrease of α_A does not result from the kinematic term $\mathbf{w}_k$.

Richards et al. (2008) measured the translational moment $\mathbf{M}_t$ acting on a square flat plate for the 6-DOF motion, and their results indicate that only the components of $\mathbf{M}_t$ in the yz plane (plate plane), $M_{y,t}$ and $M_{z,t}$, are significant. We also have shown in Section 4.2 that ω_x is not significant compared to the other two components. Therefore, we only discuss the aerodynamic moment components in the yz plane in this paper. $\mathbf{M}_{yz,t}$ and $\mathbf{U}_{yz}$ are the projected vectors of the translational moment $\mathbf{M}_t$ and the relative wind velocity $\mathbf{U}$ onto the yz plane, respectively. Richards et al. (2008)'s results indicate that $\mathbf{M}_{yz,t}$ is almost perpendicular to $\mathbf{U}_{yz}$ for all the measured conditions, as shown in Figure 17. We have assumed in Section 6.2 that the rotational moment $\mathbf{M}_r$ has a negligible influence on the direction of $\mathbf{M}_{inst}$. Therefore, we consider that $\mathbf{M}_{yz,inst}$ ($=\mathbf{M}_{yz,t} + \mathbf{M}_{yz,r}$) have the same direction as $\mathbf{M}_{yz,t}$ in the following discussion. Then the direction of $\mathbf{M}_{yz,inst}$ of the experiment cases can be calculated using Richards et al. (2008)'s experimental results about $\mathbf{M}_{yz,t}$.

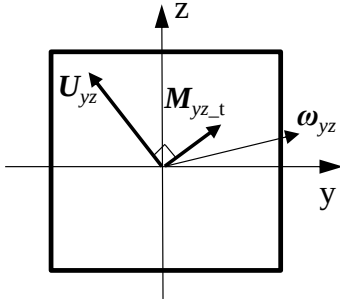


Figure 17. The direction of translational moment $\mathbf{M}_{yz,t}$. $\mathbf{M}_{yz,t}$ is perpendicular to $\mathbf{U}_{yz}$.

It can be inferred that the projected vectors of the moment term $\mathbf{w}_M$ onto the yz plane, denoted as $\mathbf{w}_{yz,M}$, has the effect of making the direction of $\boldsymbol{\omega}_{yz}$ tend to come close to the direction of $\mathbf{M}_{yz,inst}$ (see Appendix B). We consider that M_x as zero, so $\mathbf{M}_{yz,inst}$ is equal to $\mathbf{M}_{inst}$. On the other hand, $\mathbf{M}_{YZ,inst}$ is the projected vector of $\mathbf{M}_{inst}$ onto the YZ plane of the translating reference frame. Because the direction of $\boldsymbol{\omega}_{yz}$ tends to come close to the direction of $\mathbf{M}_{yz}$ in the body reference frame, $\boldsymbol{\omega}_{YZ}$ tends to come close to the direction of $\mathbf{M}_{YZ,inst}$ in the translating reference frame (see Appendix B).

Similar to the definition of α_A , we define α_M as the angle between the $\mathbf{M}_{YZ,inst}$ and the Y direction, as shown in Figure 18. We also define α'_A as the angle that makes $|\alpha_A - \alpha'_A|$ equal to 180° . Then it can be inferred that for $-90^\circ < \alpha_A < 0^\circ$, α_A tends to increase if $\alpha_A < \alpha_M < \alpha'_A$, while α_A tends to decrease if $\alpha_M < \alpha_A$ or $\alpha_M > \alpha'_A$ (see Appendix B). We refer to α_M being in the increase interval when $\alpha_A < \alpha_M < \alpha'_A$ and in the decrease interval when $\alpha_M < \alpha_A$ or $\alpha_M > \alpha'_A$.

Figure 19 shows the values of α_M , α_A and α_A' of two cases, $(75^\circ, 60^\circ, 45^\circ)$ and $(75^\circ, 60^\circ, 0^\circ)$, which represent the two types of time series of α_A shown in Section 6.1. In these figures, the abrupt changes of α_M occur when β_w is near 0° or 90° or 180° . In Figure 19(a), α_M alternates between the increase interval and decrease interval. This feature makes α_A in Figure 19(a) not change significantly over time. In Figure 19(b), α_M lies in the decrease interval in most of the time. Therefore, α_A tends to decrease over time.

In the above discussion, based on the assumption that the rotational moment $\mathbf{M}_r$ has a negligible influence on the direction of instantaneous moment $\mathbf{M}_{\text{inst}}$ in the 6-DOF motion, we have shown that for different initial conditions, the aerodynamic moment results in two types of time series of α_A of tumbling pattern cases. This feature of rotational moment $\mathbf{M}_r$ may be helpful for modelling of the aerodynamics of the 6-DOF motion of square plates.

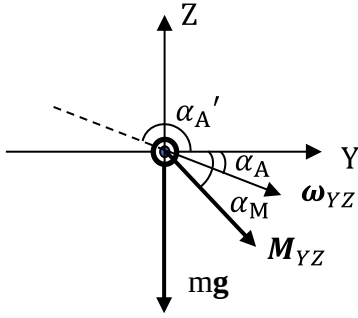


Figure 18. Definition of α_M .

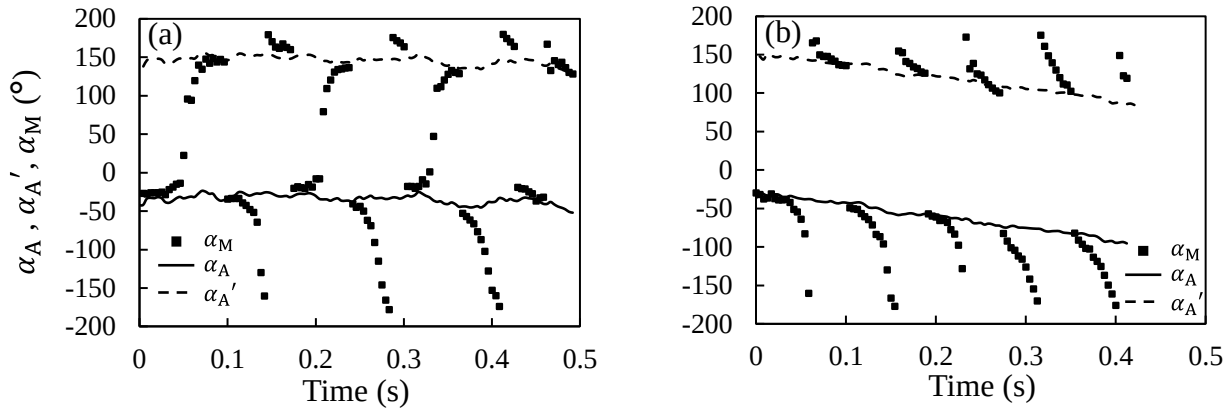


Figure 19. Time series of α_A , α_A' and α_M . (a) case $(75^\circ, 60^\circ, 45^\circ)$, (b) case $(75^\circ, 60^\circ, 0^\circ)$.

7. Conclusions

We performed free flight experiments of a square plate in a uniform flow.

Based on the experiments, we investigated the characteristics of the rotational motion of the plates. According to the rotational characteristics, the plate trajectories can be categorized into four flight patterns: flutter, tumbling, transition and quasi-translation.

We proposed two basic hypotheses on the rotational force and verified them by analysing the experiment results. The first one is that the magnitude of the rotational force component $\mathbf{F}_n$ (see

Figure 10) increases as the angular velocity increases. The second one is that the direction of $\mathbf{F}_n$ may be determined by Equation (10), i.e., its direction is dependent on the relative wind velocity and the angular velocity.

We also proposed a hypothesis on the rotational moments, i.e., the rotational moment $\mathbf{M}_r$ has a negligible influence on the direction of instantaneous moment $\mathbf{M}_{inst}$ in the 6-DOF motion. This hypothesis was supported by the experiment results.

The above hypotheses about the rotational force and moment may be used to simplify the modelling of the aerodynamics of the 6-DOF motion of square plates. These hypotheses in this study are proposed based on the observations of the actual flight motion, so they can capture the general characteristics of the aerodynamics of square plates in flight. Here we offer possible ideas for such a modelling as follows. The aerodynamic force and moment are divided into a translational part and a rotational part, respectively. For the translational part, the translational force and moment coefficients obtained in the previous experiments are used (for example, Richards et al., 2008). For the rotational part, the rotational force is decomposed into the forces $\mathbf{F}_n$ and $\mathbf{F}_p$, as shown in Section 5.1. The direction of the force $\mathbf{F}_n$ is calculated by Equation (10), and the magnitude of $\mathbf{F}_n$ is calculated using the experimental data of the 3-DOF motion (Tachikawa, 1983). The rotational moment $\mathbf{M}_r$ should be modelled in a way that it has a negligible influence on the direction of $\mathbf{M}_{inst}$, and a possible idea is that $\mathbf{M}_r$ has the same direction of the translational moment $\mathbf{M}_t$.

The conclusions obtained in this paper are considered not limited to the size of the plate in the experiments. Reynolds number may vary with changes in the plate size. However, Reynolds number usually does not have a great influence on the aerodynamics of a bluff body with sharp edges as long as Reynolds number is relatively high (usually greater than 10^4 to 10^5). However, the generality of the conclusions obtained in this paper may need validation in further experiments.

In this study, we did not extract the force and moment from the experiment data because of the limitation of accuracy, so the proposed hypotheses about the aerodynamics were verified on the analysis of the flight characteristics (trajectory, velocity, orientation, and angular velocity). To further verify these proposed hypotheses, we need to improve the measurement accuracy of the motion capture or use other methods that can directly measure the forces.

Acknowledgement

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APPENDIX A

In the body's principal reference frame, the angular momentum $\mathbf{L}$ is

$$\mathbf{L} = \mathbf{I}\boldsymbol{\omega} = (I_x\omega_x, I_y\omega_y, I_z\omega_z) \quad (\text{A.1})$$

For a square plate, the kinematic term $\mathbf{w}_k$ is

$$\mathbf{w}_k = (0, (I_z - I_x)\omega_z\omega_x/I_y, (I_x - I_y)\omega_x\omega_y/I_z) \quad (\text{A.2})$$

Then

$$\mathbf{w}_k \cdot \mathbf{L} = 0 \quad (\text{A.3})$$

$$\mathbf{w}_k \cdot \boldsymbol{\omega} = 0 \quad (\text{A.4})$$

where $I_y = I_z$ is used. Therefore, $\mathbf{w}_k$ is perpendicular to $\mathbf{L}$ and $\boldsymbol{\omega}$. This also means that $\mathbf{w}_k$ is perpendicular to the plane that contains $\mathbf{L}$ and $\boldsymbol{\omega}$ (see Figure A.1).

if no aerodynamic moment is applied to the square plate, the time derivative of $\boldsymbol{\omega}$ is equal to $\mathbf{w}_k$ according to Equation (14). This means the velocity of point B is always perpendicular to line AB. Then the trajectory of point B is a circle. Therefore, $\mathbf{w}_k$ would make $\boldsymbol{\omega}$ rotate about $\mathbf{L}$.

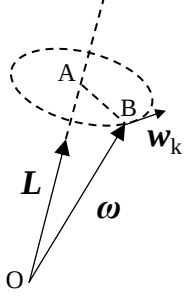


Figure A.1. Direction of $\mathbf{w}_k$. Point O is the mass center. OA is in the extended line of $\mathbf{L}$. Point B is the end of $\boldsymbol{\omega}$. OA is perpendicular to AB. $\mathbf{w}_k$ is perpendicular to the plane OAB. The trajectory of point B is a circle if aerodynamic moment is 0.

APPENDIX B

If we only consider the effect of aerodynamic moment $\mathbf{w}_{yz_M}$, the second and third Equation of Equation (14) becomes

$$\begin{aligned}\frac{d\omega_y}{dt} &= \frac{M_y}{I_y} \\ \frac{d\omega_z}{dt} &= \frac{M_z}{I_z}\end{aligned}\tag{B.1}$$

Since $I_y = I_z$, we can obtain

$$\frac{d\omega_y}{d\omega_z} = \frac{d\omega_y}{dt} \frac{dt}{d\omega_z} = \frac{M_y}{I_y} \frac{I_z}{M_z} = \frac{M_y}{M_z}\tag{B.2}$$

At time t , the angular velocity is $\boldsymbol{\omega}_{yz}$, as shown in Figure B.1. After a small enough increment of time Δt , the ratio of the increment $\Delta\omega_y/\Delta\omega_z$ is approximately equal to $d\omega_y/d\omega_z$ as well as M_y/M_z . Therefore, $\Delta\boldsymbol{\omega}_{yz}(0, \Delta\omega_y, \Delta\omega_z)$, the increment of $\boldsymbol{\omega}_{yz}$ has the same direction of $\mathbf{M}_{yz}$. At time $t + \Delta t$, the angular velocity becomes $\boldsymbol{\omega}_{yz} + \Delta\boldsymbol{\omega}_{yz}$ and closer to $\mathbf{M}_{yz}$. Therefore, $\mathbf{w}_{yz_M}$ has the effect of making the direction of $\boldsymbol{\omega}_{yz}$ tend to come close to the direction of $\mathbf{M}_{yz}$.

Next, we change the view from the body's principal reference frame to the translating reference frame, which is nonrotating. We consider that M_x as zero, so $\mathbf{M}_{yz}$ is equal to $\mathbf{M}$. We also consider that ω_x as zero, so $\boldsymbol{\omega}_{yz}$ is equal to $\boldsymbol{\omega}$. $\mathbf{M}_{YZ}$ and $\boldsymbol{\omega}_{YZ}$ are the projected vectors of $\mathbf{M}$ and $\boldsymbol{\omega}$ onto the YZ plane of the translating reference frame, respectively, as shown in Figure B.2. Because $\Delta\boldsymbol{\omega}_{yz}$ has the same direction of $\mathbf{M}_{yz}$, $\Delta\boldsymbol{\omega}_{YZ}$ also has the same direction of $\mathbf{M}_{YZ}$. Therefore, the direction of $\boldsymbol{\omega}_{YZ}$ tends to come close to that of $\mathbf{M}_{YZ}$. We introduce the vector $\boldsymbol{\omega}_{YZ}'$ which has the opposite direction of $\boldsymbol{\omega}_{YZ}$ in the YZ plane. Like the definition of α_A , we define the angle α_A' as the angle between the $\boldsymbol{\omega}_{YZ}$ and Y direction. Similarly, α_M is the angle between the $\mathbf{M}_{YZ}$ and Y direction. As shown in Figure B.2, $\boldsymbol{\omega}_{YZ}$ and $\boldsymbol{\omega}_{YZ}'$ divide

1 the YZ plane into two parts. ω_{YZ} tends to rotate towards the part that contains M_{YZ} . That is to
 2 say, for $-90^\circ < \alpha_A < 0^\circ$, α_A tends to increase if $\alpha_A < \alpha_M < \alpha_A'$, while α_A tends to decrease
 3 if $\alpha_M < \alpha_A$ or $\alpha_M > \alpha_A'$.
 4

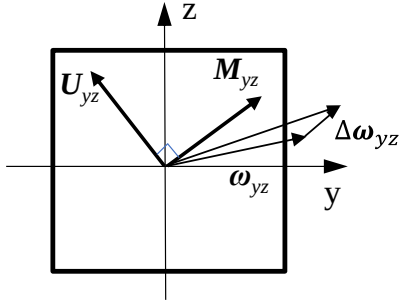


Figure B.1. Tendency of change of ω_{yz} . The direction of ω_{yz} tends to come close to the direction of M_{yz} .

5

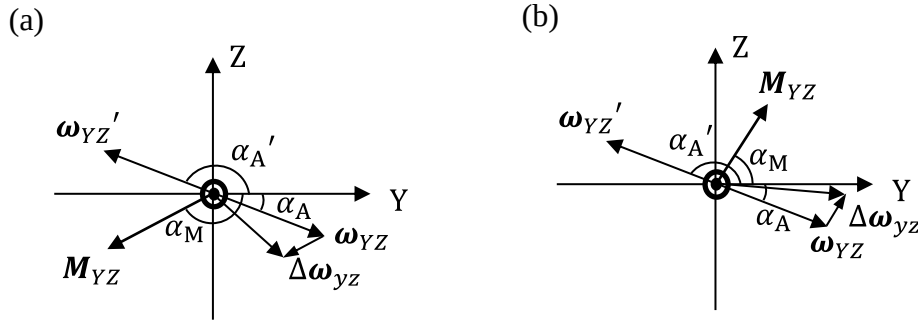


Figure B.2. Tendency of change of α_A . $\Delta\omega_{YZ}$ has the same direction of M_{YZ} . (a) α_A tends to decrease when $\alpha_M < \alpha_A$ or $\alpha_M > \alpha_A'$. (b) α_A tends to increase when $\alpha_A < \alpha_M < \alpha_A'$.

6