

Note on central extensions and Leopoldt's conjecture

金沢大理 古田孝臣 (Yoshiomi Furuta)

Introduction

Let k be an algebraic number field of finite degree, and ℓ be a prime number. Throughout this paper, we always assume

$$(*) \quad \sqrt{-1} \in k \quad \text{when } \ell = 2.$$

We denote by $G(K/k)$ the Galois group of a Galois extension K/k . Denote by $k^{(\ell)}$ the maximal ℓ -extension of k unramified outside ℓ . Then it is well-known that Leopoldt's conjecture for k and ℓ is equivalent to $H^{-3}(G(k^{(\ell)}/k), \mathbb{Z}) = 0$. This is connected with a certain problem of central extensions through the relationship between the structure of the Galois group of a central extension and (the dual of) Schur's multiplier $H^{-3}(G, \mathbb{Z})$ (Theorem 4). The problem is reduced in Section 3 to a simpler case (Theorem 8).

§1. Leopoldt's conjecture and abundant central extensions.

For any pro-finite group G and a natural number n , the cohomology group $H^{-n}(G, \mathbb{Z})$ of minus dimension is defined by

$$H^{-n}(G, \mathbb{Z}) \simeq \varprojlim H^{-n}(G/U_\lambda, \mathbb{Z}),$$

where U_λ runs over open subgroups of G of finite index, and $\varprojlim$ is of the deflation map. Then we have $H^{-n}(G, \mathbb{Z}) \simeq H^n(G, \mathbb{Z})^\wedge$. Hence $H^{-3}(G, \mathbb{Z}) \simeq H^3(G, \mathbb{Z})^\wedge \simeq H^2(G, \mathbb{Q}/\mathbb{Z})^\wedge$, which is called (the dual of) Schur's multiplier of G .

For a tower of Galois extensions $M \supset K \supset k$, denote by $K_{M/k}^*$ the genus field of K/k in M , which is by definition, the composite of K and the maximal abelian extension of k in M . Denote by $\hat{K}_{M/k}$ the maximal central extension of K/k in M , namely the maximal extension in M whose Galois group over K is contained in the center of the Galois group over k . Then we have the following theorem (Cf. Heider [3, §2], Furuta [1, Theorem 5]).

THEOREM 1.

$$G(\hat{K}_{M/k}/K_{M/k}^*) = \frac{H^{-3}(G(K/k), \mathbb{Z})}{\text{Def}_{G(M/k) \rightarrow G(K/k)} H^{-3}(G(M/k), \mathbb{Z})}.$$

We call M *abundant* for K/k when $G(\hat{K}_{M/k}/K_{M/k}^*) \simeq H^{-3}(G(K/k), \mathbb{Z})$, namely $\text{Def}_{G(M/k) \rightarrow G(K/k)} H^{-3}(G(M/k), \mathbb{Z}) = 0$.

For a Galois extension M/k , it follows from Theorem 1 and the definition of cohomology groups of pro-finite groups that $H^{-3}(G(M/k), \mathbb{Z}) = 0$ if and only if M is abundant for any finite Galois extension K over k contained in M .

Now denote by $\bar{k}$ the algebraic closure of k . Then it is well-known that

$$(1.1) \quad H^{-3}(G(\bar{k}/k), \mathbb{Z}) = 0.$$

(Cf. Serre [9, Theorem 4], Heider [3, §5], Yamashita [10, Theorem 3], Miyake [8]). Hence $\bar{k}$ is abundant for any Galois extension K/k , and we have

$$\text{THEOREM 2.} \quad G(\hat{K}_{\bar{k}/k}/K^*_{\bar{k}/k}) \simeq H^{-3}(G(K/k), \mathbb{Z}).$$

Now we are interested how small the abundant extension for K/k , whose existence is guaranteed as above, can be chosen, and especially in the following problem:

PROBLEM *For any Galois extension K/k , does there exist an abundant extension M for K/k such that only prime divisors ramified in K/k are ramified in M/K ?*

The above problem is closely related to Leopoldt's conjecture e.g. as follows.

We assume always (*) as in Introduction, and denote by $k^{(\ell)}$ the maximal ℓ -extension unramified outside ℓ . Then the

following theorem is well-known (Cf. Heider [4, Satz 6, Bemerkung], Heider [5, Satz 11], Iwasawa [6], Kuz'min [7, Theorem 7.2])).

THEOREM 3. *Under the assumption (*), Leopoldt's conjecture for ℓ is true for k if and only if*

$$H^{-3}(G(k^{(\ell)}/k), \mathbb{Z}) = 0.$$

By the remark after Theorem 1, we have

THEOREM 4. *Under the assumption (*), Leopoldt's conjecture for ℓ is true for k if and only if $k^{(\ell)}$ is abundant for any Galois extension of finite degree over k contained in $k^{(\ell)}$.*

§2. Central extensions for a sequence of fields.

Let K/k be an ℓ -extension of finite degree. Then there is a sequence of extensions $k = K_0 \subset K_1 \subset \dots \subset K_t = K$ such that K_{i+1}/K_i is cyclic of degree ℓ and each K_i is normal over k . Denote by $\hat{K}_i$ the maximal central extension of K_i/k in M . At first we reduce the structure of $G(\hat{K}_{M/k}/K_{M/k}^*)$ to that of $G(\hat{K}_{i+1}/\hat{K}_i)$.

Let $M \supset L \supset K \supset k$ be a tower of Galois extensions over

k , and assume that M is abelian over K . Put $G = G(L/k)$, $H = G(K/k)$, $A = G(M/K)$ and $B = G(M/L)$. Then A and B are H -module and G -module respectively by means of conjugation. Let I_G and I_H be the augmentation ideals of the group rings $\mathbb{Z}[G]$ and $\mathbb{Z}[H]$ respectively. Denote by $L^{(i)}$ and $K^{(i)}$ be the extensions of L and K in M corresponding to $I_G^i B$ and $I_H^i A$ respectively. Note that $L^{(1)} = \hat{L}_{M/k}$ and $K^{(1)} = \hat{K}_{M/k}$.

Let $H_0 = H/[H, H]$, where $[H, H]$ is the commutator subgroup of H . For $\tau \in H$ and $a \in A$, denote by $\bar{\tau}$ and $\bar{a}$ the class of H_0 and A/B which contain τ and a respectively. Set

$$R(H, A, B) = \langle \prod (\bar{\tau} \otimes \bar{a}_{\bar{\tau}}) \in H_0 \otimes (A/B) ; \prod a_{\bar{\tau}}^{\tau-1} = 1 \rangle$$

where $\otimes$ stands for the tensor product over $\mathbb{Z}$ by means of the exponential map.

Theorem 5. *Notation being as above, we assume that $G(L/K)$ is contained in the center of $G(K/k)$. Then we have*

- $$(1) \quad K^{(i+1)} \supset L^{(i)} \supset K^{(i)}$$
- $$(2) \quad G(\hat{L}_{M/k} / \hat{K}_{M/k}) \simeq \frac{H_0 \otimes (A/B)}{R(H, A, B)}$$

Proof. (1) Put $G_1 = G(L/K)$. Then $H = G/G_1$. Since A is abelian, we have $b^{g_1} = b$ for $b \in B$ and $g_1 \in G_1$. Hence we can treat B as H -module, and we have $I_G^i B = I_H^i B$. Then

$I_H^i A \supset I_H^i B = I_G^i B$. Hence $K^{(i)} \subset L^{(i)}$. Moreover we have $\hat{K}_{M/k} \supset \hat{K}_{L/k} = L$ by assumption. Hence $I_H A \subset B$. This implies $I_H^{i+1} A \subset I_H^i B = I_G^i B$, which means $K^{(i+1)} \supset L^{(i)}$.

(2) For $a \in A$, let $\tilde{a}$ be the class of $A \bmod I_H A$ which contains a . Let ϕ be a homomorphism of $H_0 \otimes (A/I_H A)$ to $I_H A/I_H^2 A$ defined by

$$\phi(\bar{\tau} \otimes \tilde{a}) = a^{\tau-1} \bmod I_H^2 A$$

for $\tau \in H$ and $a \in A$. Then ϕ is well-defined and surjective by Furuta and Yamashita [2, Lemma 2]. Moreover [2, Theorem] implies

$$(2.1) \quad \text{Ker } \phi = \langle \prod (\bar{\tau} \otimes \tilde{a}_{\bar{\tau}}) \in H_0 \otimes A/I_H A ; \prod a_{\bar{\tau}}^{\tau-1} = 1 \rangle.$$

Hence

$$G(K_{M/k}^{(2)}/\hat{K}_{M/k}) \simeq I_H A/I_H^2 A \simeq \frac{H_0 \otimes (A/I_H A)}{\text{Ker } \phi}$$

and

$$G(K_{M/k}^{(2)}/\hat{L}_{M/k}) \simeq I_H B/I_H^2 A \simeq \frac{H_0 \otimes (B/I_H A)}{(\text{Ker } \phi) \cap (H_0 \otimes (B/I_H A))}.$$

A canonical exact sequence $0 \rightarrow B/I_H A \xrightarrow{i} A/I_H A \xrightarrow{\pi} A/B \rightarrow 0$ implies an exact sequence

$$H_0 \otimes (B/I_H A) \xrightarrow{\text{id} \otimes i} H_0 \otimes (A/I_H A) \xrightarrow{\text{id} \otimes \pi} H_0 \otimes (A/B) \longrightarrow 0.$$

Put $\kappa = \text{id} \otimes \pi$ and let ϕ' be the restriction of ϕ to $H_0 \otimes (B/I_H A)$. Then we have the following diagram of exact sequences

$$\begin{array}{ccccccc}
& & & & 0 & & \\
& & & & \downarrow & & \\
0 & \longrightarrow & \text{Ker } \phi' & \longrightarrow & H_0 \otimes (B/I_H A) & \xrightarrow{\phi'} & I_H B / I_H^2 A \longrightarrow 0 \\
& & & & \downarrow i' & & \downarrow i'' \\
0 & \longrightarrow & \text{Ker } \phi & \longrightarrow & H_0 \otimes (A/I_H A) & \xrightarrow{\phi} & I_H A / I_H^2 A \longrightarrow 0 \\
& & & & \downarrow \kappa & & \downarrow j \\
& & & & H_0 \otimes (A/B) & & I_H A / I_H B \\
& & & & \downarrow & & \downarrow \\
& & & & 0 & & 0
\end{array}$$

We define a homomorphism $\varphi_{A/B}$ of $H_0 \otimes (A/B)$ to $I_H A / I_H B$ by $\varphi_{A/B} \circ \kappa = j \circ \phi$. Then $\varphi_{A/B}$ is well-defined and surjective.

Moreover we have $\text{Ker } \varphi_{A/B} = \kappa(\text{Ker}(j \circ \phi)) = \kappa \phi^{-1}(\text{Im } i'') = \kappa((\text{Ker } \phi)(\text{Im } i')) = \kappa(\text{Ker } \phi)$. Therefor it follows from (2.1) and the definition of κ that $\text{Ker } \varphi_{A/B} = R(H, A, B)$. Hence

$G(\hat{L}_{M/k} / \hat{K}_{M/k}) \simeq I_H A / I_H B \simeq \frac{H_0 \otimes (A/B)}{\text{Ker } \varphi_{A/B}} = \frac{H_0 \otimes (A/B)}{R(H, A, B)}$, which is to be proved.

Let $M \supset L \supset K \supset k$ be a tower of Galois extensions over k , and assume that $G(L/K)$ is contained in the center of $G(L/k)$. Let K' be the maximal abelian extension of k contained in K , and M' be the maximal abelian extension of K in M . For $\tau \in G(K/k)$ and $a \in G(M'/K)$, let $\bar{\tau}$ and $\bar{a}$ be elements of $G(K'/k)$ and $G(L/K)$ whose extensions are τ and a respectively. Set

$$(2.2) \quad R(M, L, K, k) = \langle \prod (\bar{\tau} \otimes \bar{a}_{\bar{\tau}}) \in G(K'/k) \otimes G(L/K); \prod a_{\bar{\tau}}^{\tau-1} = 1, \tau \in G(K/k), a_{\bar{\tau}} \in G(M'/K) \rangle.$$

Then we have

THEOREM 6. Let $M \supset L \supset K \supset k$ be a tower of Galois extensions over k . Assume that $G(L/K)$ is cyclic and contained in the center of $G(L/k)$. Then

$$G(\hat{L}_{M/k}/\hat{K}_{M/k}) \cong \frac{G(K'/k) \otimes G(L/K)}{R(M,L,K,k)}.$$

Proof. We apply Theorem 5 by setting $A = G(M'/K)$, $B = G(M'/L)$, $G = G(L/k)$ and $H = G(K/k)$. Then $R(M,L,K,k) = R(H,A,B)$ and $\hat{K}_{M/k} = \hat{K}_{M'/k}$ since $\hat{K}_{M/k}$ is abelian over K . Moreover $\hat{L}_{M/k}/K$ is abelian, for obviously $G(\hat{L}_{M/k}/L)$ is contained in the center of $G(\hat{L}_{M/k}/K)$ and L/K is cyclic. Hence $\hat{L}_{M/k} = \hat{L}_{M'/k}$. Thus Theorem 5 implies the theorem.

Let $M \supset L \supset K \supset k_1 \supset k$ be a tower of Galois extensions over k , and assume that $G(L/K)$ is cyclic and contained in the center of $G(L/k)$. For $\tau_1 \in G(K/k_1)$, let $\bar{\tau}_1$ be the class of $G(K/k_1)$ mod. $[G(K/k_1), G(K/k_1)]$ which contains τ_1 , and let $\bar{\tau}_1$ be the class of $G(K/k)$ mod. $[G(K/k), G(K/k)]$ which contains τ_1 . We define a homomorphism $\lambda_{k_1 \rightarrow k}$ of $R(M,L,K,k_1)$ to $R(M,L,K,k)$ by

$$\lambda_{k_1 \rightarrow k}(\bar{\tau}_1 \otimes \bar{a}) = \bar{\tau}_1 \otimes \bar{a},$$

where $a \in G(M'/K) = G(M/K)/[G(M/K), G(M,K)]$ and $\bar{a}$ is an element of $G(L/K)$ whose extension to M' is equal to a , M' being as above the maximal abelian extension of K in M . Then

since L/K is cyclic, we have

$$(2.2') \quad R(M, L, K, k) = \langle \bar{\tau} \otimes \bar{a} \in G(K'/k) \otimes G(L/K) ; a^{\tau-1} = 1 \\ \tau \in G(K/k), a \in G(M'/K) \rangle.$$

This implies immediately

Theorem 7. *Let $M \supset L \supset K \supset k$ be a tower of Galois extensions over k , and assume that $G(L/K)$ is cyclic and contained in the center of $G(L/k)$. For $\tau \in G(K/k)$, denote by K_τ the intermediate field of K/k invariant by τ . Then we have*

$$R(M, L, K, k) = \prod_{\tau \in G(K/k)} \lambda_{K_\tau \rightarrow k} R(M, L, K, K_\tau).$$

In the product, it is enough that τ runs only over representatives of $G(K/k) \bmod. [G(K/k), G(K, k)]$.

§3. Reduction to extensions of type (l, l) .

In order to prove Theorem 8, we prepare the following lemma.

LEMMA. *Let K_1/k be a cyclic extension of degree l . Let K_2/k be a cyclic extension of degree a power of l , and F be the extension of k of degree l contained in K_2 . Put $L = K_1 K_2$, and suppose $K_1 \cap K_2 = k$. If M is a Galois extension of k which contains L and abundant for $K_1 F/k$, then M is also abundant for L/k .*

Proof. Put $L_1 = K_1 F$. Let $\hat{L}_1$ and L_1^* be the maximal central and the genus field for L_1/k in M respectively. Since L_1/k is of type (ℓ, ℓ) , the order of $H^{-3}(G(L_1/k), \mathbb{Z})$ is equal to ℓ . Hence $(\hat{L}_1 : L_1^*) = \ell$. Moreover L_1^* is the genus field for L/k in M , since the both L_1 and L are abelian over k . By the definition of $\hat{L}_1$, it is clear that $G(\hat{L}_1/L_1)$ and hence $G(\hat{L}_1/L)$ is contained in the center of $G(\hat{L}_1/k)$. Moreover $\hat{L}_1$ is non-abelian over k . Hence $\hat{L}_1$ is a non-abelian central extension for L/k . Since $H^{-3}(G(L/k), \mathbb{Z})$ is of order ℓ , the extension $\hat{L}_1$ must be the maximal central extension for L/k in M . This means that M is abundant for L/k .

THEOREM 8. *Let ℓ be a rational prime, and k_0 be an algebraic number field of finite degree satisfying (*). then Leopoldt's conjecture is true for any algebraic number field k of finite degree which contains k_0 and for ℓ if and only if the following condition (#) is satisfied:*

(#) *Let k be any algebraic number field of finite degree which contains k_0 . Let L be any abelian extension of k which is of type (ℓ, ℓ) and unramified outside ℓ . Then there always exists an abundant extension M for L/k such that M is also unramified outside ℓ .*

Proof. The necessity of (#) is trivial by Theorem 4.

Thus we prove the sufficiency. Let $k^{(\ell)}$ be as above the maximal ℓ -extension of k unramified outside ℓ , and K be a Galois extension of finite degree over k which is contained in $k^{(\ell)}$. To prove the theorem it is enough by Theorem 4 to show that $k^{(\ell)}$ is abundant for K/k . Since K/k is an ℓ -extension, there is a sequence $k = K_0 \subset K_1 \subset \dots \subset K_t = K$ such that each K_i is normal over k , $(K_i : K_{i-1}) = \ell$, and $G(K_i/K_{i-1})$ is contained in the center of $G(K_i/k)$. Let $\bar{k}$ be the algebraic closure of k , and let R be as in (2.2).

We prove first the following equality

$$(2.3) \quad R(k^{(\ell)}, K_i, K_{i-1}, k) = R(\bar{k}, K_i, K_{i-1}, k)$$

for $i = 2, 3, \dots, t$.

For $\tau \in G(K_{i-1}/k)$, let K_τ be the intermediate field of K_{i-1}/k which is invariant by τ . Let M be any one of $k^{(\ell)}$ or $\bar{k}$. Then the maximal central extension $\hat{K}_{i-1, M/K_\tau}$ of K_{i-1}/K_τ in M is abelian over K_τ . Because $G(\hat{K}_{i-1, M/K_\tau}/K_{i-1})$ is contained in the center of $G(\hat{K}_{i-1, M/K_\tau}/K_\tau)$ and K_{i-1}/K_τ is cyclic. In the same manner, K_i is abelian over K_τ . Therefore $K_{i, M/K_\tau}^*$ and $\hat{K}_{i-1, M/K_\tau}$ are both the maximal abelian extension over K_τ contained in M . Hence $K_{i, M/K_\tau}^* = \hat{K}_{i-1, M/K_\tau}$. Then Theorem 6 implies

$$G(\hat{K}_{i, M/K_\tau}/K_{i, M/K_\tau}^*) = G(\hat{K}_{i, M/K_\tau}/\hat{K}_{i-1, M/K_\tau})$$

$$= \frac{G(K_{i-1}/K_\tau) \otimes G(K_i/K_{i-1})}{R(M, K_i, K_{i-1}, K_\tau)}$$

On the other hand Theorem 7 implies

$$R(M, K_i, K_{i-1}, k) = \prod_{\tau \in G(K_{i-1}/k)} \lambda_{K_\tau \rightarrow k} R(M, K_i, K_{i-1}, K_\tau).$$

Therefor in order to prove (2.3) it is enough to show

$$(2.4) \quad G(\hat{K}_{i,k^{(\ell)}}/K_\tau / \hat{K}_{i,k^{(\ell)}}^*/K_\tau) \simeq G(\hat{K}_{i,\bar{k}}/K_\tau / \hat{K}_{i,\bar{k}}^*/K_\tau).$$

Now if K_i is cyclic over K_τ , then $\hat{K}_{i,M/K_\tau} = K_{i,M/K_\tau}^*$ in both cases $M = k^{(\ell)}$ and $M = \bar{k}$. Hence (2.4) is trivial. Suppose that K_i is non-cyclic over K_τ . Then there is an intermediate field F_1 of K_i/K_τ such that $(F_1 : K_\tau) = \ell$, $K_i = F_1 K_{i-1}$ and $F_1 \cap K_{i-1} = K_\tau$. Let F_2 be the intermediate field of K_{i-1}/K_τ which is of degree ℓ over K_τ . Then $F_1 F_2$ is of type (ℓ, ℓ) over K_τ . Since $k^{(\ell)}$ is the maximal ℓ -extension over K_τ , $k^{(\ell)}$ is abundant for $F_1 F_2 / K_\tau$ by the assumption (#) of the theorem. Similarly, $\bar{k}$ is also abundant for $F_1 F_2 / K_\tau$ by (1.1). Therefor it follows from Lemma that both $k^{(\ell)}$ and $\bar{k}$ are abundant for K_i / K_τ , which implies (2.4) and hence (2.3).

Next we prove that (2.3) implies Leopoldt's conjecture to be true for k and ℓ . Now it follows from (2.3) and Theorem 6 that

$$G(\hat{K}_{i,k^{(\ell)}}/k / \hat{K}_{i-1,k^{(\ell)}}/k) \simeq G(\hat{K}_{i,\bar{k}}/k / \hat{K}_{i-1,\bar{k}}/k)$$

for $i = 2, \dots, t$. Hence we have

$$\prod_{i=2}^t (\hat{K}_{i,k^{(\ell)}}/k : \hat{K}_{i-1,k^{(\ell)}}/k) = \prod_{i=2}^t (\hat{K}_{i,\bar{k}}/k : \hat{K}_{i-1,\bar{k}}/k),$$

which implies

$$(2.5) \quad (\hat{K}_{k^{(\ell)}/k} : \hat{K}_{1,k^{(\ell)}/k}) = (\hat{K}_{\bar{k}/k} : \hat{K}_{1,\bar{k}/k}).$$

Let A_k and $A_k^{(\ell)}$ be the maximal abelian extension of k in $\bar{k}$ and in $k^{(\ell)}$ respectively. Then we have $\hat{K}_{1,k^{(\ell)}/k} =$

$K_{1,k^{(\ell)}/k}^* = A_k^{(\ell)}$ and $\hat{K}_{1,\bar{k}/k} = K_{1,\bar{k}/k}^* = A_k$, for K_1 is cyclic over k . Hence (2.5) implies

$$(2.6) \quad (\hat{K}_{k^{(\ell)}/k} : A_k^{(\ell)}) = (\hat{K}_{\bar{k}/k} : A_k).$$

Let K' be the maximal abelian extension over k contained in K . Then since $K_{k^{(\ell)}/k}^* = KA_k^{(\ell)}$ and $K_{\bar{k}/k} = KA_k$, we have

$G(K_{k^{(\ell)}/k}^*/A_k^{(\ell)}) \simeq G(K/K')$ and $G(K_{\bar{k}/k}^*/A_k) \simeq G(K/K')$. Hence

(2.6) implies

$$(\hat{K}_{k^{(\ell)}/k} : K_{k^{(\ell)}/k}^*) = (K_{\bar{k}/k} : K_{\bar{k}/k}^*).$$

The right hand side is equal to the order of $H^{-3}(G(K/k), \mathbb{Z})$ since $\bar{k}$ is abundant for K/k . This implies by Theorem 1 that $k^{(\ell)}$ is also abundant for K/k , which is to be proved.

Remark. When L/k is abelian of type (ℓ, ℓ) , Schur's multiplier $H^{-3}(G(L/k), \mathbb{Z})$ is cyclic of order ℓ . Therefore the existence of an abundant extension M for L/k in (#) is equivalent to the existence of a Galois extension $\hat{L}$ and an abelian extension L^* over k satisfying the following condition:

(i) $\hat{L}$ is non-abelian over k , $\hat{L} \supset L^*$, and $(\hat{L} ; L^*) = \ell$,

- (ii) $G(\hat{L}/L)$ is contained in the center of $G(\hat{L}/k)$,
- (iii) $\hat{L}/L$ is unramified outside $\mathfrak{l}$.

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Department of mathematics

Kanazawa University

Kanazawa 920

Japan