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A complement to the theory of  $G$ -CW complexes

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Let  $G$  be a topological group. By a  $G$ -space  $X$  we mean a topological space  $X$  together with a continuous  $G$ -action on  $X$ . If a  $G$ -equivariant map  $f: X \rightarrow Y$  between  $G$ -spaces induces isomorphisms  $f_*: \pi_n(X^H, x) \rightarrow \pi_n(Y^H, f(x))$  for every  $n \geq 0$ , every (closed) subgroup  $H$  of  $G$  and  $x \in X^H = \{x \in X; gx = x \text{ for every } g \in H\}$ , then  $f$  is called a weak  $G$ -homotopy equivalence.

We have defined the notion of (Hausdorff)  $G$ -CW complexes and have shown that any weak  $G$ -homotopy equivalence between  $G$ -CW complexes is a  $G$ -homotopy equivalence. The purpose of this talk is to show a canonical construction of a pair of a  $G$ -CW complex  $K_X$  and a weak  $G$ -homotopy equivalence  $p: K_X \rightarrow X$  for a (Hausdorff)  $G$ -space  $X$  [Theorem 1]. As an application we prove that the singular  $G$ -(co)homology theory on  $X$  defined by Illman coincides with the cellular  $G$ -(co)homology theory on  $K_X$  [Theorem 2].

The construction of  $K_X$  is as follows. Take the geometric realization  $|S(X)|$  of the singular complex of  $X$ . The underlying discrete group of  $G$  operates on  $|S(X)|$ . Define  $|S_G(X)|$  to be the  $G$ -space with the strongest topology so that the  $G$ -action be

continuous. Since  $\pi_n(|S_G(X)|^H) \rightarrow \pi_n(X^H)$  are onto homomorphisms, we have only to kill the kernels by attaching many many  $G$ -cells.

Remark. We need to define the notion of a non-Hausdorff  $G$ -CW complex when  $X$  is not a Hausdorff space.

Question. Is the induced map  $\rho/G : K_X/G \rightarrow X/G$  a weak homotopy equivalence for our  $(K_X, \rho)$ ?

#### References

- S. Illman : Equivariant singular homology and cohomology I,  
Memoires Amer. Math. Soc. no.156 (1975).
- T. Matumoto : Equivariant  $K$ -theory and Fredholm operators,  
J. Fac. Sci. Univ. Tokyo Sect.I 18 (1971), 109-125.
- \_\_\_\_\_ : On  $G$ -CW complexes and a theorem of J.H.C.Whitehead,  
Ibid 18 (1971), 363-374.
- \_\_\_\_\_ : Equivariant cohomology theories on  $G$ -CW complexes,  
Osaka J. Math. 10 (1973), 51-68.