

The Prime Graph of a Sporadic Simple Group

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1. Introduction

Let G be a finite group and S a sporadic simple group. We denote by $\pi(G)$ the set of all primes dividing the order of G . The prime graph $\Gamma(G)$ of G is defined in the usual way connecting p and q in $\pi(G)$ when there is an element of order pq in G . The main purpose of this paper is to determine finite group G satisfying $\Gamma(G) = \Gamma(S)$. G. Chen characterized S by $\Gamma(S)$ and $|S|$. Let $\pi_e(G)$ be the set of orders of all elements in G . W. Shi proved that G satisfying $\pi_e(G) = \pi_e(S)$ is isomorphic to S , except for J_2, Co_1 . Our main theorem generalizes their results. Moreover, we prove that a simple group G satisfying $|G| = |S|$ is isomorphic to S as a corollary of our main theorem.

2. Theorem

Let S be a sporadic simple group. Suppose that G is a finite group satisfying $\Gamma(G) = \Gamma(S)$.

- (1) If S is one of $J_1, M_{22}, M_{23}, M_{24}, Co_2$, then G is isomorphic to S .
- (2) If S is M_{11} , then $G \simeq M_{11}$ or $L_2(11)$.
- (3) If S is one of $J_3, J_4, Suz, O'N, Ly, Fi_{23}, Fi'_{24}, M, BM, Th, Ru, Co_1$, then $G/O_\pi(G) \simeq S$ where π is a subset of the numbers in the 2nd column of the following.

S	π
J_3, Suz	2, 3, 5
J_4	2, 3, 11
Ly, Fi'_{24}, Th, Co_1	2, 3
M	3
$O'N, Fi_{23}, BM, Ru$	2

- (4) If S is $HS, He, M^cL, Co_3, Fi_{22}$ or HN , then $G/O_\pi(G)$ is one of the groups in the 2nd column where π is a subset of the numbers in the 3rd column of the following.

S	$G/O_\pi(G)$	π
HS	$HS, U_6(2)$	2, 3, 5
He	$L_2(16), L_2(16).2,$ $L_2(16).4, O_8^-(2),$ $O_8^-(2).2, S_8(2), He \text{ or } He.2$	2, 3, 5, 7
M^cL	$M_{22}, M_{22}.2, HS, HS.2, U_6(2), U_6(2).2, M^cL$	2, 3, 5
Co_3	M_{24}, Co_3	2
Fi_{22}	$Suz, Suz.2, Fi_{22}, Fi_{22}.2$	2, 3, 5
HN	$HN, HN.2$	2, 3, 5, 7

(5) If S is M_{12} , then one of the following holds : (a) $G \simeq 11^{2n} : SL_2(5)$ for any $n \in \mathbf{Z}$, $G \simeq 11^{2n} : SL_2(5).2$ for any $2 \leq n \in \mathbf{Z}$, (b) $G/O_\pi(G) \simeq L_2(11), L_2(11).2, M_{11}, M_{12}$ or $M_{12}.2$ where $\pi \subseteq \{2, 5\}$.

(6) If S is J_2 , then one of the following holds : (a) G is solvable and G is a Frobenius group or a 2-Frobenius group, (b) $G/O_\pi(G) \simeq J_2, L_3(4), L_3(4).2_1, L_3(4).2_3, S_6(2), O_8^+(2), U_3(5), U_3(5).2, U_4(3), U_4(3).2_2, U_4(3).2_3, A_7, A_7.2, A_8, A_8.2, A_9, L_2(7), L_2(7).2, L_2(8), L_2(8).3, U_3(3)$ or $U_3(3).2$ where $\pi \subseteq \{2, 3, 5\}$.

参考文献

- [1] M. HAGIE, The prime graph of a sporadic simple group, Communications in algebra, accepted.