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Inventory Effects of Two Risk-Averse Market Makers

Keiichi Tanaka[1]

[1] *Faculty of Economics, Kyoto University. E-mail: tanakak@econ.kyoto-u.ac.jp*

The inventory positions of two risk-averse market makers are introduced into a Kyle (1985) type batch trading model and the effects analyzed. An equilibrium is defined including participation constraints and incentive compatibility and is characterized as a γ-coalitional equilibrium. At equilibrium, the two market makers share the risk of clearing orders so that the aggregate pricing schedule becomes linear. The pricing schedule coincides with the regret-free pricing of a fictitious risk-averse market maker. The inventories shift the pricing schedule vertically, and the coalition parameter changes the slope. Orders by the informed trader are then found to depend on inventories.

Keywords: market microstructure, inventory, risk-averse market maker, coalition

JEL Classification Numbers: D81, G12, G14

1. Introduction

It is intuitive that a risk-averse market maker with a long position in a risky asset in order to reduce their position will quote *good ask prices* and *bad bid prices* to traders. A good price means an aggressive price relative to the reference price that would be quoted under different conditions, e.g., a low ask price and a high bid price. Any impact of a market maker's inventory position on the quoted price of a security is called the inventory effect. In the inventory effect literature, Stoll (1978) shows that in the absence of asymmetric information, the inventory position of the risk-averse market maker changes the bid and ask price in a similar fashion, but does not affect the bid-ask spread.

When considering the behavior of a risk-averse market maker, the degree of risk aversion[1)] and the inventory position are important factors for a utility-based pricing rule. The regret-free pricing of a market maker is a pricing rule whereby for each quantity of order, the market maker is indifferent as to whether or not to

[1)] The definition of risk aversion used in this paper is the Arrow-Pratt measure of absolute risk aversion.

execute the order using the pricing schedule. The inventory effect may be evident since a risk-averse market maker will take the inventory position into account when determining the regret-free pricing, whereas a risk-neutral market maker will not. However, it is unresolved whether the *assumption* of regret-free pricing, which many models employ, is economically reasonable. Furthermore, regret-free pricing is not a unique feasible pricing rule by a market maker without further constraints.

The aim of this paper is to present several inventory effects for which a coalition of two risk-averse market makers is essential, given the presence of asymmetric information. Our analysis basically follows the one-period models of Kyle (1985) and Subrahmanyam (1991). We focus on a linear equilibrium led by an implicit coalition of two risk-averse market makers, rather than Bertrand competition between them, and *derive* the equilibrium pricing schedules, each of which coincides with the regret-free pricing schedule of a risk-averse market maker who may be fictitious. Our results include an “indirect” inventory effect due to the coalition, and different behavior for the informed trader, as well as some further implications of the pricing and the inventory positions obtained in other models such as those of Stoll (1978) and Subrahmanyam (1991). Our research is a primary, but necessary, step when building multi-period trading models with risk-averse market makers, because an inventory position is a state variable to describe the behavior of participants. As part of this application, Subrahmanyam’s (1991) assumptions and discussion are justified.

Three basic frameworks are used to study issues of market microstructure: inventory models (Stoll, 1978), sequential trading models (Glosten and Milgrom, 1985), and batch trading models (Kyle, 1985). One of the main distinctive features among these is the existence of information asymmetry between the market maker and traders. However, only inventory models consider the effects of a market maker’s inventory position on pricing. To start with, Stoll (1978) analyzes the pricing strategy of a risk-averse market maker who quotes bid and ask prices for a trader in the absence of asymmetric information. The pricing strategy is assumed to follow regret-free pricing. He shows that the bid-ask spread depends on the trading size, the degree of risk aversion, and the variance of the security price. The bid-ask spread due to the risk aversion of the market maker is compensation paid to the market maker by the trader. The inventory position affects both bid and ask prices in the same manner, so that the bid-ask spread is independent of the inventory position. As an alternative, Glosten and Milgrom (1985) consider a risk-neutral market maker facing asymmetric information with informed traders in a sequential (multi-period) trading model. Those authors show that the bid-ask spread reflects the asymmetric information between the market maker and the informed trader. Lastly, Kyle (1985) assumes competitive risk-neutral market makers who set trading prices with zero expected profits as a result of the competition and risk neutrality. However, inventory positions have not been studied in successive models because of the setting of risk-neutral market makers who do not take the inventory positions into account when pricing. That said, Subrahmanyam (1991) studies a risk-averse market maker who applies regret-free pricing in a one-period setting, but does not consider the inventory position.

When two market makers have different risk characteristics, they will not quote the same pricing schedule. We define the aggregate pricing schedule as a function of the better price of quotes by two market makers for each quantity of order. In our model, the assumption of regret-free pricing found in the literature is replaced by two conditions: participation constraint and incentive compatibility. The risk-averse market makers' motivation to quote for traders is to improve their utility from the reservation utility with no trades by reducing their inventory position. The participation constraint defines the regret-free pricing of each market maker for a given linear trading strategy of the informed trader. Incentive compatibility is also imposed so as to rule out prices which both market makers can quote. It means that for each order quantity, the potential winning market maker should offer a better price than the regret-free price of the losing market maker. We focus on equilibria under which the aggregate pricing schedule is linear and laid in the area surrounded by the regret-free pricing schedules of the two market makers by these constraints. Although there exist multiple equilibria, each equilibrium is characterized by a parameter γ, and we call the equilibrium a γ-coalitional equilibrium. The number of γ shows how to divide a rent, as measured by the difference between the regret-free prices of two market makers. Under Bertrand competition, and without coalition, the aggregate pricing schedule would not be linear due to the inventory positions. Therefore the coalition is formed so that an aggregate pricing schedule becomes strictly linear, and it is useful to handle some nonlinear issues associated with risk aversion. A γ-coalitional equilibrium is actually a broader concept than Bayesian equilibrium due to this incentive compatibility. To the best of our knowledge, the notion of γ-coalitional equilibrium is new to the literature.

Inventory positions, the degree of risk aversion, and a coalition parameter γ are thereby important factors for the determination of equilibrium. Our results on inventory effects are then summarized as follows. (*i*) An inventory position averaged by some weight shifts the equilibrium pricing schedule vertically. (*ii*) An averaged degree of risk aversion of market makers changes the slope of the pricing schedule. (*iii*) Two market makers coexist and share the risk of clearing orders: a relatively long market maker takes purchase orders and a relatively short market maker takes sale orders so as to reduce each inventory position towards zero. (*iv*) The equilibrium trading strategy of the informed trader is different from that of one without inventory. In some cases the sign of the order is opposite. The weight parameters for average numbers in (*i*) and (*ii*) above depend on the coalition parameter, the degree of risk aversion of the market makers, and their inventory positions. Importantly, results (*i*) and (*ii*) are consistent with Stoll (1978). Although Stoll (1978) shows that the bid-ask spread is independent of the inventory position as discussed earlier, our conclusion (*ii*) implies that Kyle's λ also may be affected by inventory positions via the endogenous derivation of the coalition parameter. The coexistence and risk sharing of market makers in (*iii*) is new to the literature, although it appears that it occurs in practice. (*iv*) is another new insight.

The remainder of this paper is organized as follows. Section 2 describes the model and derives the equilibrium. Section 3 develops the endogenous derivation of a coalition parameter γ (and allocation of inventory positions). Section 4 provides some concluding remarks, and the mathematical proofs are given in the Appendix.

2. Model

2.1. Setup

We model a stock market in a one-period setting by following Kyle (1985) in several respects. In a given trading place, a stock is assumed to be traded by three types of traders: an informed trader, liquidity traders, and two market makers. The informed trader is risk-neutral and has private information about the liquidation value of the stock at the end of the period. For liquidity traders, aggregate demand is exogenously determined. The two market makers A and B are risk-averse and each of them possesses an inventory position in the stock. A pricing schedule is a function on $\mathbb{R}$ that maps an order quantity to the clearing price. The role of market makers includes determining their own pricing schedule, clearing orders submitted by both the informed trader and the liquidity traders, and taking the net position of the transactions as the result of order clearing.

We denote by $\tilde{v}$ the ex-post liquidation value of the stock, and by $\tilde{u}$ the quantity demanded by the liquidity traders. The liquidation value $\tilde{v}$ and the demand $\tilde{u}$ are exogenously given and follow independent normal distributions with mean and variance of (p_0, Σ_0) and $(0, \sigma_u^2)$, respectively.

Trading activity consists of several steps. First, before trading, each market maker i ($i = A, B$) is endowed with stock of z_i shares and money of $-p_0 z_i$. Each market maker determines his or her own pricing schedule P^i ($i = A, B$) and shows it to the other market maker. Then the aggregate pricing schedule P^M, defined by[2)]

$$P^M(y) = \begin{cases} \min(P^A(y), P^B(y)) & \text{for } y > 0, \\ \max(P^A(y), P^B(y)) & \text{for } y < 0, \end{cases} \tag{1}$$

is announced to both the informed trader and the liquidity traders. By definition, the aggregate pricing schedule P^M gives the best price of the two market makers to the traders. After the random variables $\tilde{v}$ and $\tilde{u}$ are realized and observed by the informed trader and the liquidity traders, respectively, each of the traders submits an order to the market makers. The quantity $\tilde{x}$ of order by the informed trader is calculated on the basis of the observation of $\tilde{v}$ and denoted by $\tilde{x} = X(\tilde{v})$. The liquidity traders submit $\tilde{u}$ itself as the aggregate order. Then the market makers

[2)] We apply this formula because traders will know the better price at equilibrium. One could define the aggregate pricing schedule by a different formula such as the favor price for market makers rather than traders

$$P^M(y) = \begin{cases} \max(P^A(y), P^B(y)) & \text{for } y > 0, \\ \min(P^A(y), P^B(y)) & \text{for } y < 0. \end{cases}$$

However, this definition would yield no existence of a linear equilibrium.

observe the total demand $\tilde{x}+\tilde{u}$. The trading price is determined as $\tilde{p} = P^M(\tilde{x}+\tilde{u})$ from the announced aggregate pricing schedule, and either of the market makers clears the submitted orders at the trading price. Finally, at the end of the period all shares of the stock are liquidated at $\tilde{v}$.

Strategies to be considered are X, P^A, and P^B. The demand $\tilde{u}$ by the liquidity traders is a risk for the informed trader to determine strategy X. Similarly, the liquidation value $\tilde{v}$ is a risk for the two market makers when determining strategies P^A and P^B.

We make several standing assumptions in this paper.

Assumption 1. (i) The utility function of market maker i ($i = A, B$) is a constant absolute risk aversion (CARA) on wealth W with the degree of risk aversion $R_i > 0$

$$u_i(W) = -\exp(-R_i W), \quad i = A, B. \tag{2}$$

(ii) The aggregate pricing schedule P^M is required to be linear for the traders' order submission.
(iii) Trades are not split between the market makers, so that either of the market makers face traders in transaction. Market maker B wins the trade y if $P^A(y) = P^B(y)$.
(iv) All participants know the inventory positions held by each market maker.

Assumption 1 (i) makes our analysis easy, so that the comparison of the expected utility is equivalent to the comparison of the mean-variance utility $E(W) - R_i Var(W)/2$ when W follows a normal distribution. Assumption (ii) is the most important to encourage market makers to form an implicit coalition since there will be no utility gain when P^M is not linear. It may be justified if our model is extended to a multi-period setting because a linear pricing schedule is necessary to rule out manipulation (Huberman and Stanzl, 2004). The assumptions of (ii) and the risk-neutral informed trader imply that the informed trader takes a linear trading strategy $X(v) = \alpha + \beta v$, which is a usual assumption of Kyle (1985) and the successive batch trading models. That is why we can concentrate on the linear trading strategies of the informed trader. Later, these linear conditions give us the benefit of tractability in equilibrium. Assumption (iv) can be justified by observing the trading history of the stock since it is issued[3].

A pricing schedule is neither necessarily continuous nor linear on the whole line, although it may be locally linear.

Definition 1. (i) A piecewise linear pricing schedule is a function $P : \mathbb{R} \to \mathbb{R}$ such that there exists a sequence $\{y_j\}_{j\in\mathbb{Z}}$ in $\mathbb{R}$ such that

$$\cdots < y_{-j} < y_{-(j-1)} < \cdots < y_{-1} < y_0 = 0 < y_1 < \cdots < y_{j-1} < y_j < \cdots ,$$

[3] Treating the inventory as a private information to the market maker is an interesting issue but it is beyond the scope of our research. I thank to an anonymous referee who points out this issue.

P is a linear function $P(y) = a_j + b_j y$ on each interval (y_j, y_{j+1}), and for each j it holds that either $P(y_j) = a_j + b_j y_j$ or $P(y_j) = a_{j-1} + b_{j-1} y_j$.
(ii) A piecewise linear pricing schedule is called a linear pricing schedule if it is a linear function on $\mathbb{R}$.

2.2. Definition of equilibrium

When two market makers have different risk characteristics, they will not generally quote the same pricing schedule. The regret-free pricing of a market maker is a pricing rule under which for each quantity of order, the market maker is indifferent as to whether or not to execute the order at the pricing schedule[4)]. The degree of risk aversion and the inventory position are parameters of the regret-free pricing.

In our model, the assumption of regret-free pricing in the literature is replaced by two conditions: participation constraint and incentive compatibility. The risk-averse market makers' motivation to quote for traders is to improve their utility from the reservation utility with no trades by reducing their inventory position. The participation constraint defines the regret-free pricing of each market maker for a given linear trading strategy of the informed trader. Incentive compatibility is also imposed so as to rule out prices which both market makers can quote. It means that for each order quantity, the potential winning market maker should offer a better price than the regret-free price of the losing market maker.

First we consider the participation constraint. If a market maker i actually clears the orders for y at $P^i(y)$, then the expected utility $U_i(y)$ from the wealth after the liquidation is given by

$$U_i(y) \quad = \quad E[u_i((\tilde{v} - p_0)z_i - (\tilde{v} - P^i(y))y) | \tilde{x} + \tilde{u} = y]. \tag{3}$$

The reservation utility U_i^r is the expected utility of no trades given by

$$U_i^r(y) \quad = \quad E[u_i((\tilde{v} - p_0)z_i) | \tilde{x} + \tilde{u} = y]. \tag{4}$$

In order for each market maker to have the motivation to carry out market making for traders, the pricing schedule should satisfy the participation constraint $U_i(y) \geq U_i^r(y)$ for all order $y \in \mathbb{R}$. The following lemma shows that the participation constraint yields the regret-free pricing schedule P^{i*} that is a linear function of the quantity of order.

Lemma 1. *Fix i ($i = A$ or B) and suppose that the informed trader takes a linear trading strategy $X(v) = \alpha + \beta v$. Then, a pricing schedule P^i satisfies the participation constraint $U_i(y) \geq U_i^r(y)$ for all $y \in \mathbb{R}$ if and only if it satisfies*

$$P^i(y) \geq P^{i*}(y; X) \text{ for } y > 0, \quad P^i(y) \leq P^{i*}(y; X) \text{ for } y < 0, \tag{5}$$

where

$$P^{i*}(y; X) = \mu_0(\alpha, \beta) + \lambda_0(\beta) y + (\lambda_i(\beta) - \lambda_0(\beta))(y - 2z_i), \tag{6}$$

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4) It is equivalent to the zero expected profit pricing if the market maker is risk neutral.

and

$$\begin{aligned}
\mu_0(\alpha,\beta) &= p_0 - \frac{\beta\Sigma_0}{\beta^2\Sigma_0 + \sigma_u^2}(\alpha + \beta p_0), \\
\lambda_0(\beta) &= \frac{\beta\Sigma_0}{\beta^2\Sigma_0 + \sigma_u^2}, \\
\lambda_i(\beta) &= \lambda_0(\beta) + \frac{\Sigma_0\sigma_u^2}{\beta^2\Sigma_0 + \sigma_u^2}\frac{R_i}{2}.
\end{aligned} \tag{7}$$

Proof. See Appendix 1. □

Note that Equation (5) is defined for all y but $y = 0$. Lemma 1 argues about the participation constraint of a market maker and thus any price does not matter for him/her for $y = 0$.

We call $P^{i*}(\cdot; X)$ the regret-free pricing schedule of i for the informed trader's linear trading strategy $X(v) = \alpha + \beta v$. We may omit the dependence on the trading strategy X of the informed trader in this paper as $P^{i*}(\cdot) = P^{i*}(\cdot; X)$ when there is no confusion. The degree of risk aversion R_i affects the slope λ_i of P^{i*} and the inventory position shifts the level of P^{i*} vertically, e.g., P-intercept $\mu_0(\alpha,\beta) - 2(\lambda_i(\beta) - \lambda_0(\beta))z_i$.

Due to differences in these characteristics, each market maker has a comparative advantage for some orders. As a result, for a given linear trading strategy of the informed trader, there will be three areas in the order-price plane by the number (zero, one, two) of market makers whose utility is improved by the transaction. The signs of the utility gains for two market makers are shown in Figure 1. For example $(-, +)$ means that in the area, by clearing the order for the quantity at the clearing price, market maker A cannot improve expected utility from the reservation utility with no trades, while market maker B can improve expected utility. Therefore, as a condition for equilibrium we impose incentive compatibility so as to rule out prices that both market makers can quote, i.e., the area of $(+, +)$ in Figure 1[5]. It means that for each order quantity, the potential winning market maker should offer a better price than the regret-free price of the losing market maker.

Since we shall focus on a linear trading strategy and a linear aggregate pricing schedule as mentioned before, we come up with the following definition of a linear equilibrium.

Definition 2. A linear equilibrium is a triple (X, P^A, P^B) of linear trading strategy X and piecewise linear pricing schedules P^A and P^B satisfying the following four conditions:
(i) (Linear aggregate pricing schedule)
The aggregate pricing schedule P^M defined by equation (1) of the pair of piecewise linear pricing schedules (P^A, P^B) is a linear pricing schedule.

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[5] Participation constraints rule out the area of $(-, -)$.

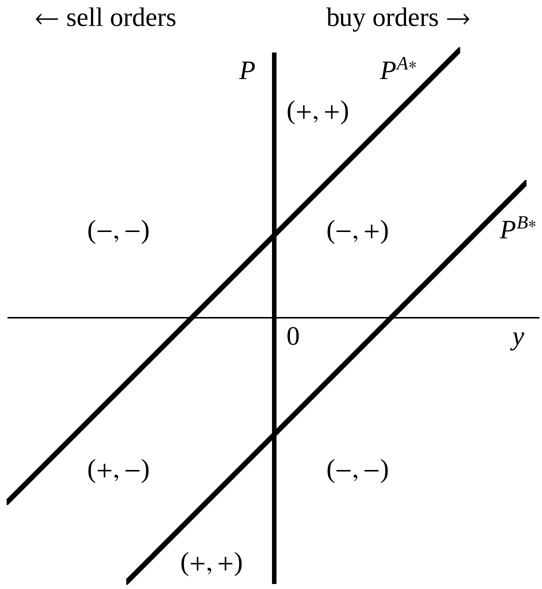

Figure 1 The regret-free pricing schedules of A and B ($z_A < z_B$) and the utility gains ($U_A - U_A^r, U_B - U_B^r$) of the market makers.

(ii) (Profit maximization of the informed trader)
Given linear aggregate pricing schedule P^M and for all $v \in \mathbb{R}$, X is a linear trading strategy and satisfies

$$E[\tilde{\pi}_I(X, P^M)|\tilde{v} = v] \geq E[\tilde{\pi}_I(X', P^M)|\tilde{v} = v] \tag{8}$$

for any linear trading strategy X' where $\tilde{\pi}_I(X, P^M)$ is the trading profit of the informed trader $\tilde{\pi}_I(X, P^M) = (\tilde{v} - P^M(X(\tilde{v}) + \tilde{u}))X(\tilde{v})$.
(iii) (Participation constraint of market makers)
Given linear trading strategy X, piecewise linear pricing schedules P^A and P^B satisfy the participation constraint:

$$\left(P^i(y) - P^{i*}(y; X)\right)y \geq 0, \quad \forall y \in \mathbb{R}, \quad i = A, B. \tag{9}$$

(iv) (Incentive compatibility of winning market maker)
Given linear trading strategy X, for each $y \in \mathbb{R}$, piecewise linear pricing schedules P^A and P^B satisfy the incentive compatibility condition:

$$\begin{aligned}&\text{For } \forall y \in \mathbb{R} \text{ and } i, j \in \{A, B\}, \text{ if } P^M(y) = P^i(y) \text{ for some } i, \text{ then}\\ &P^j(y) = P^{j*}(y; X) \text{ and } \left(P^i(y) - P^j(y; X)\right)y \leq 0 \text{ for } j \neq i.\end{aligned} \tag{10}$$

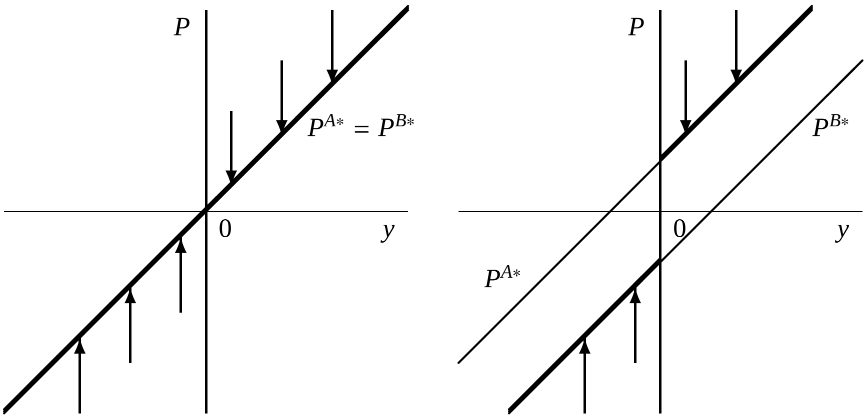

Figure 2 Bertrand competition without inventory (left) and with inventory (right).

In the above definition, the participation constraint condition (*iii*) requires that each price $P^i(y)$ of market maker i at order y should be more conservative (or worse) than the regret-free price $P^i(y; X)$ for $i = A, B$. The incentive compatibility condition (*iv*) tells us that the winning price $P^i(y)$ of the winning market maker i at order y should be more aggressive (or better) than the price $P^j(y; X)$ of the loser j who quotes the regret-free price $P^{j*}(y; X)$. If the characteristics of the two market makers are exactly the same, then conditions (*iii*) and (*iv*) imply that an equilibrium pricing schedule coincides with the regret-free pricing of the market maker as assumed in Subrahmanyam (1991). One might think that the condition (*i*) is the same as Assumption 1 (*ii*). The difference is that Assumption 1 (*ii*) describes a condition under which neither of the market makers improves their expected utility from the reservation utility, while Definition 2 (*i*) states that P^M is actually linear.

If the two market makers are assumed to be in Bertrand competition at each order quantity when a trading strategy X is given, the resulting pricing schedule is not linear (See Figure 2). Local competition does not lead to a good global form in the presence of risk aversion and inventory in general.

As long as the regret-free pricing schedules between market makers are distinct, multiple linear equilibria may exist, dependent on the "bargaining power" between the two market makers. For each order y, the difference of the regret-free pricing schedules $|P^{A*}(y; X) - P^{B*}(y; X)|$ can be regarded as the rent to be shared by the two market makers and the informed trader. Such bargaining power between the two market makers is represented by γ as follows.

Two conditions (*iii*) and (*iv*) of Definition 2 imply that the aggregate pricing schedule P^M must be laid in the area surrounded by the regret-free pricing schedules P^{A*} and P^{B*}

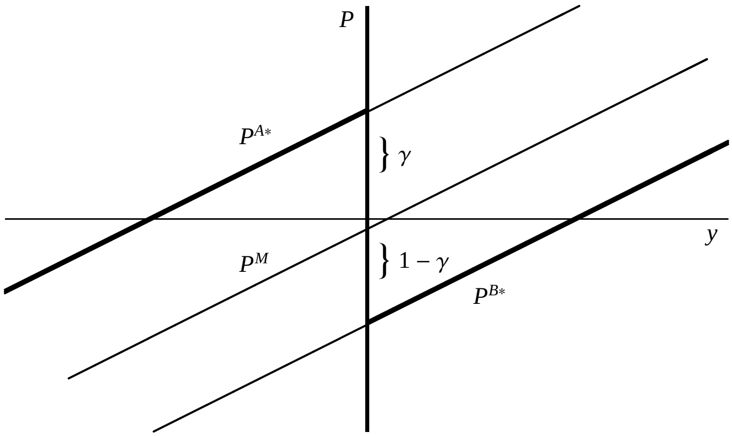

Figure 3 γ-coalitional pricing schedule ($z_A < z_B$).

$$\min\{P^{A*}(y;X), P^{B*}(y;X)\} \leq P^M(y) \leq \max\{P^{A*}(y;X), P^{B*}(y;X)\}, \quad \forall y \in \mathbb{R}. \tag{11}$$

By inequality (11) and the fact that all graphs of P^M, P^{A*}, and P^{B*} are linear lines on a plane[6)], any equilibrium aggregate pricing schedule P^M is characterized by some constant $\gamma \in [0,1]$ satisfying

$$P^M(y) = (1-\gamma)P^{A*}(y;X) + \gamma P^{B*}(y;X), \quad \forall y \in \mathbb{R}. \tag{12}$$

Formally such an equilibrium is defined as a γ-coalitional equilibrium as below. This definition is well-defined since we can find the corresponding γ for any linear equilibrium.

Definition 3. For a given constant $\gamma \in [0,1]$, a linear equilibrium (X, P^A, P^B) is said to be a γ-coalitional equilibrium if it satisfies for all $y \in \mathbb{R}$

$$\begin{aligned} if P^A(y) = P^M(y), &\quad \text{then} |P^A(y) - P^{A*}(y;X)| = \gamma |P^{A*}(y;X) - P^{B*}(y;X)|, \\ if P^B(y) = P^M(y), &\quad \text{then} |P^B(y) - P^{B*}(y;X)| = (1-\gamma) |P^{A*}(y;X) - P^{B*}(y;X)|. \end{aligned}$$

In other words, at an equilibrium the two market makers agree that A reserves γ of the surplus $|P^{A*}(y;X) - P^{B*}(y;X)|$ if A wins, and B reserves $1-\gamma$ of the surplus if B wins as indicated in Figure 3. The closer γ is to 1, the higher the expected utility of A from the transaction, and vice versa.

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6) The regret-free pricing schedule P^{i*} is not to be confused with a pricing schedule P^i that is actually quoted. Note that P^{i*} is linear by Lemma 1 while P^i is piecewise linear by conditions of Definition 2 at equilibrium.

2.3. Analysis of equilibrium

We derive a γ-coalitional equilibrium for a given γ in this subsection. Some endogenous derivations of γ are shown in the next section. Once the trading strategy X of the informed trader and a coalition parameter γ are specified, it is easy to calculate the aggregate pricing schedule P^M thanks to equation (12). Then the pricing schedules of the winning market maker and the loser can be obtained by incentive compatibility. We show the unique existence of a γ-coalitional equilibrium where the forms of parameters are similar to the results of Kyle (1985) with modifications due to risk aversion and the inventory position. These results are obtained in a straightforward manner from the equilibrium definition. Since the slope of the regret-free pricing schedule depends on the degree of risk aversion, it is convenient to study equilibrium by considering whether the regret-free pricing schedules are in parallel with each other.

2.3.1. Case of same risk aversion

We start the investigation by looking at an easier case where the regret-free pricing schedules of the two market makers are in parallel with each other, or equivalently, a case of market makers having the same degree of risk aversion.

Theorem 1. *Suppose that $R_A = R_B = R$. Then there exists a γ-coalitional equilibrium uniquely as follows:*

$$X(v) = \beta(v - \mu), \quad P^M(y) = \mu + \lambda y, \tag{13}$$

and if $z_A \geq z_B$, then

$$P^A(y) = \begin{cases} \mu + \lambda y & \text{for } y \geq 0 \\ P^{A*}(y) & \text{for } y < 0, \end{cases} \qquad P^B(y) = \begin{cases} P^{B*}(y) & \text{for } y \geq 0 \\ \mu + \lambda y & \text{for } y < 0. \end{cases} \tag{14}$$

If $z_A < z_B$, then

$$P^A(y) = \begin{cases} P^{A*}(y) & \text{for } y \geq 0 \\ \mu + \lambda y & \text{for } y < 0, \end{cases} \qquad P^B(y) = \begin{cases} \mu + \lambda y & \text{for } y \geq 0 \\ P^{B*}(y) & \text{for } y < 0, \end{cases} \tag{15}$$

where

$$\begin{aligned} \mu &= p_0 - \Sigma_0 R((1-\gamma)z_A + \gamma z_B), \\ \lambda &= \frac{\Sigma_0}{4}\Big(R + \sqrt{4(\Sigma_0\sigma_u^2)^{-1} + R^2}\Big), \quad \beta = \frac{\sigma_u^2}{2}\Big(\sqrt{4(\Sigma_0\sigma_u^2)^{-1} + R^2} - R\Big), \end{aligned} \tag{16}$$

and

$$P^{A*}(y) = \mu + \lambda y + \frac{\Sigma_0 R}{2 - \beta\Sigma_0 R}\gamma(z_B - z_A), \tag{17}$$

$$P^{B*}(y) = \mu + \lambda y - \frac{\Sigma_0 R}{2 - \beta\Sigma_0 R}(1-\gamma)(z_B - z_A). \tag{18}$$

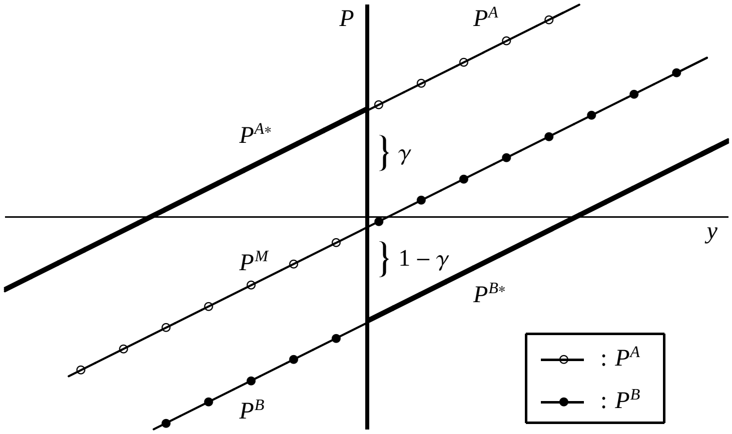

Figure 4 γ-coalitional equilibrium pricing schedule ($z_A < z_B$).

Proof. See Appendix 2. □

Figure 4 illustrates the equilibrium pricing schedule for the case of $z_A < z_B$. Market maker A quotes the piecewise linear pricing schedule P^A which is shown as lines with circles (○), and B quotes P^B with bullets (•). Although each market maker gives a piecewise linear pricing schedule, the aggregate pricing schedule is linear and the market makers share the risk of transaction with traders depending on their inventory positions and degree of risk aversion. These properties are later discussed in more detail.

2.3.2. Case of different risk aversion

Next we proceed to the case of $R_A \neq R_B$. Since the aggregate pricing schedule P^M is laid in the area surrounded by the regret-free pricing schedules P^{A*} and P^{B*}, P^M must go through the intersection of two lines of P^{A*} and P^{B*} if any. This is indeed the case when $R_A \neq R_B$. The generality is not lost by assuming $R_A < R_B$.

Given a linear trading strategy $X(v) = \alpha + \beta v$ of the informed trader, let $(y^*, p^*(\alpha, \beta))$ be the intersection of two lines of P^{A*} and P^{B*} on the (y, p) plane, given by

$$y^* = \frac{2(R_B z_B - R_A z_A)}{R_B - R_A}, \tag{19}$$

$$p^*(\alpha,\beta) = \mu_0(\alpha,\beta) + \lambda_0(\beta)y^* + \frac{\Sigma_0\sigma_u^2}{\beta^2\Sigma_0 + \sigma_u^2}\frac{R_A R_B(z_B - z_A)}{R_B - R_A}. \tag{20}$$

Note that y^* is determined by characteristics of market makers and does not depend on the informed trader's trading strategy. Then y^* is another turning point in the quantity of order that each market maker takes into consideration for the pricing schedule.

Theorem 2. *Suppose that $R_A < R_B$. Then there exists a γ-coalitional equilibrium uniquely as follows:*

$$X(v) = \beta(v-\mu), \quad P^M(y) = \mu + \lambda y, \tag{21}$$

and if $y^* = \frac{2(R_B z_B - R_A z_A)}{R_B - R_A} \geq 0$, *then*

$$P^A(y) = \begin{cases} \mu + \lambda y & \text{for } y \geq y^* \text{ or } y \leq 0 \\ P^{A*}(y) & \text{for } 0 < y < y^*, \end{cases} \qquad P^B(y) = \begin{cases} P^{B*}(y) & \text{for } y \geq y^* \text{ or } y \leq 0 \\ \mu + \lambda y & \text{for } 0 < y < y^*. \end{cases} \tag{22}$$

If $y^* < 0$,

$$P^A(y) = \begin{cases} \mu + \lambda y & \text{for } y \leq y^* \text{ or } y \geq 0 \\ P^{A*}(y) & \text{for } y^* < y < 0, \end{cases} \qquad P^B(y) = \begin{cases} P^{B*}(y) & \text{for } y \leq y^* \text{ or } y \geq 0 \\ \mu + \lambda y & \text{for } y^* < y < 0, \end{cases} \tag{23}$$

where

$$\begin{aligned} \mu &= p_0 - \Sigma_0\Big[(1-\gamma)R_A z_A + \gamma R_B z_B\Big], \quad R_\gamma = (1-\gamma)R_A + \gamma R_B, \\ \lambda &= \frac{\Sigma_0}{4}\Big(R_\gamma + \sqrt{4(\Sigma_0\sigma_u^2)^{-1} + R_\gamma{}^2}\Big), \quad \beta = \frac{\sigma_u^2}{2}\Big(\sqrt{4(\Sigma_0\sigma_u^2)^{-1} + R_\gamma{}^2} - R_\gamma\Big), \end{aligned} \tag{24}$$

and

$$P^{A*}(y) = \mu + \lambda y - \frac{\Sigma_0}{2 - \beta\Sigma_0 R_\gamma}\frac{\gamma(R_B - R_A)}{2}(y - y^*), \tag{25}$$

$$P^{B*}(y) = \mu + \lambda y + \frac{\Sigma_0}{2 - \beta\Sigma_0 R_\gamma}\frac{(1-\gamma)(R_B - R_A)}{2}(y - y^*). \tag{26}$$

Proof. See Appendix 3. □

Figure 5 illustrates the equilibrium pricing schedules where the less risk-averse market maker A wins sufficiently large order quantities at equilibrium. The more risk-averse market maker B wins only small transactions of either $[0, y^*]$ or $[y^*, 0]$, and all other orders are executed by A since A is less risk averse than B so that λ_A is smaller than λ_B. By letting $R_B \to R_A$, we see $y^* \to \pm\infty$ and the above results converge to the results of the same risk parameter as shown in the previous subsection. In that sense, Theorem 1 is a corollary of Theorem 2.

2.3.3. Discussion

We discuss some properties of the equilibrium. As shown in Theorems 1 and 2 (also Figure 4 and 5), the two market makers coexist and share the risk of trading; when $z_A < z_B$, A shows better bid prices than B and wins sale transactions from traders, while B shows better ask prices than A and wins purchase transactions.

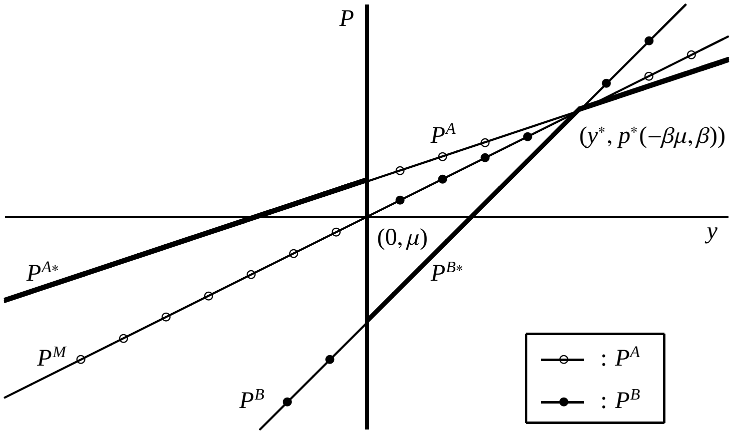

Figure 5 γ-coalitional equilibrium pricing schedule ($y^* > 0$).

The reason is the comparative advantage in the inventory position due to risk aversion, which motivates each market maker to reduce their position. This aggregate behavior is represented by the aggregate pricing schedule P^M. It is easy to see that the aggregate pricing schedule is the same as the regret-free pricing schedule of a fictitious market maker with degree of risk aversion $R_\gamma = (1-\gamma)R_A + \gamma R_B$ and the inventory position of

$$\frac{(1-\gamma)R_A z_A + \gamma R_B z_B}{(1-\gamma)R_A + \gamma R_B}. \tag{27}$$

As a direct inventory effect, μ is different from p_0 by the inventory position of the fictitious market maker. Namely $\mu = P^M(0)$ is a biased estimator of $\tilde{v}$ due to the inventory [7]

$$\mu = p_0 - \Sigma_0[(1-\gamma)R_A z_A + \gamma R_B z_B]. \tag{28}$$

On the other hand λ and β depend on the degree of risk aversion R_γ of the fictitious market maker[8]

$$\lambda = \frac{\Sigma_0}{4}\left(R_\gamma + \sqrt{4(\Sigma_0\sigma_u^2)^{-1} + {R_\gamma}^2}\right). \tag{29}$$

The more risk averse is the market maker, the more sensitive the aggregate pricing schedule is to the quantity of order. Our formula of λ is consistent with the result of Subrahmanyam (1991) (formula (16) in Proposition 4).

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7) However, such behavior may be ruled out if we extend to the multi-period setting and consider the conditions necessary to rule out manipulation, such as $\mu = p_0$ for each period. That topic is beyond the scope of this paper.
8) In the case of the same degree of risk aversion of the two market makers, γ does not matter for λ.

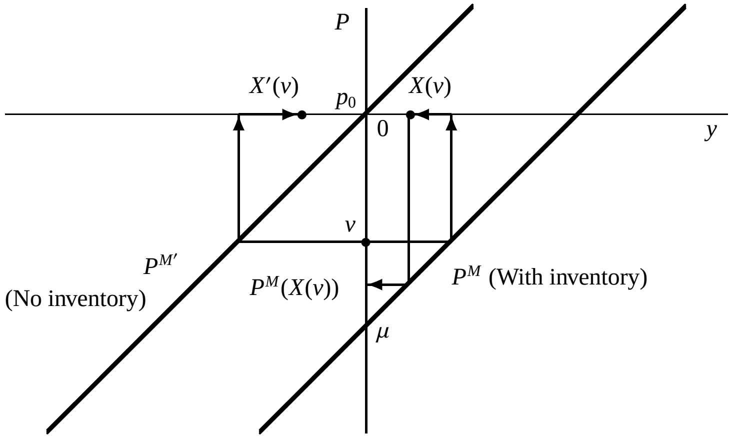

Figure 6 Behavior of informed trader.

So far the inventory positions of market makers affect μ only on the aggregate pricing schedule at equilibrium. It appears that λ is independent of the inventories of the market makers. The weighted inventory position changes the aggregate pricing schedule vertically, and the weighted degree of risk aversion determines the slope. In a nutshell, similar inventory effects to those given by Stoll (1978) are seen so long as γ is treated as exogenously given. If a coalition parameter γ is determined endogenously depending on the inventory position of the market makers, there is an inventory effect on the equilibrium price via the average degree of risk aversion R_γ. Such a phenomenon can be denoted as an indirect effect of inventory through the coalition and is new to the literature. In the next section we examine several cases of endogenous derivation.

Another new inventory effect is seen in the trading strategy of the informed trader due to the fact that μ differs from p_0. If the informed trader observes v, he or she knows that their expected profit will be positive at equilibrium when their order quantity is between 0 and $2X(v)$ and the profit attains the maximum at $X(v)$, since the profit function is quadratic in the informed trader's order. The sign of their order $X(v)$ depends on μ, not p_0; they wish to buy if $v > \mu$ while they wish to sell if $v < \mu$. Thus, if $\mu < v < p_0$ (or $p_0 < v < \mu$), the sign of the order is opposite to the case of no inventory. With the inventories he or she will buy (or sell if $p_0 < v < \mu$) while he or she will sell (or buy if $p_0 < v < \mu$) without inventory. In Figure 6, P^M and X denotes strategies with the inventories and $P^{M'}$ and X' denotes those for the case of $\mu = p_0$ or no inventory. These decisions make sense because with the inventory the expected trading (purchase) price $E[P^M(X(v) + \tilde{u})|\tilde{v} = v] = P^M(X(v))$ is cheaper than the liquidation value v. Without the inventory, the expected trading price is more expensive than the liquidation value and hence the informed trader will sell.

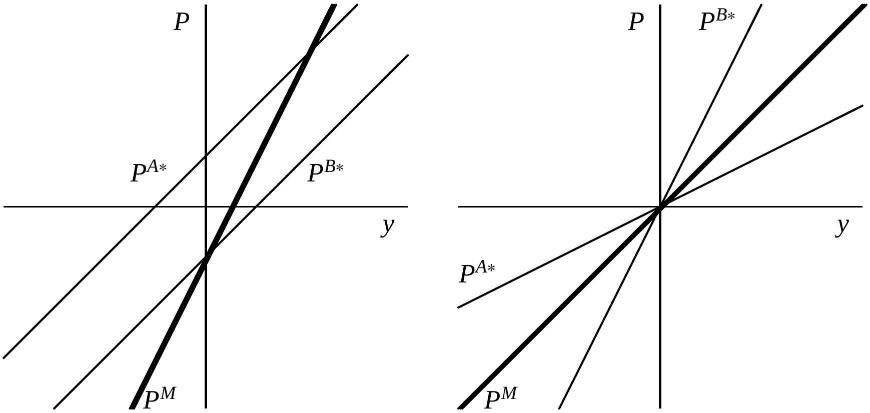

Figure 7 (left) In case of $R_A z_A \neq R_B z_B$, there exists a linear Bayesian equilibrium that is not a γ-coalitional equilibrium. (right) In the case of $R_A z_A = R_B z_B$, a linear Bayesian equilibrium is always a γ-coalitional equilibrium.

The reciprocal of the conditional variance of $\tilde{v}$, given a trading price $\tilde{p}$, measures the price efficiency because the price partially reveals private information. The price efficiency at the γ-coalitional equilibrium is calculated as

$$(Var[\tilde{v}|\tilde{p}])^{-1} = (Var^y[\tilde{v}])^{-1} = \frac{2}{\Sigma_0} - \beta R_\gamma. \tag{30}$$

That implies that the price under the γ-coalition of risk-averse market makers is less efficient than the case of risk-neutral market makers due to risk aversion. The difference is measured by the average degree of risk aversion R_γ, not the inventory position.

Finally, let us revisit the definition of a linear equilibrium. One may think that the definition of a linear equilibrium is a weak notion. However, it is actually stronger than a linear Bayesian equilibrium under Assumption 1 in the following sense. A linear Bayesian equilibrium is defined by a triple (X, P^A, P^B) of a linear strategy and piecewise linear pricing schedules satisfying, for each $v, y \in \mathbb{R}$,

$$E[\tilde{\pi}_I(X, P^A, P^B)|\tilde{v} = v] \geq E[\tilde{\pi}_I(X', P^A, P^B)|\tilde{v} = v] \tag{31}$$

$$E[u_A(W_A(P^A, P^B, y))|X(\tilde{v}) + \tilde{u} = y] \geq E[u_A(W_A(P^{A'}, P^B, y))|X(\tilde{v}) + \tilde{u} = y] \tag{32}$$

$$E[u_B(W_B(P^A, P^B, y))|X(\tilde{v}) + \tilde{u} = y] \geq E[u_B(W_B(P^A, P^{B'}, y))|X(\tilde{v}) + \tilde{u} = y] \tag{33}$$

for all X': linear trading strategy, and $P^{A'}, P^{B'}$: piecewise linear pricing schedules, where we denote by $W_i(P^A, P^B, y)$ the wealth of market maker i at the end of the trading period when the order y is submitted and market makers obey pricing schedules P^A and P^B. If a triple (X, P^A, P^B) is a linear equilibrium, then it constitutes a linear Bayesian equilibrium. The reverse does not hold in general due to the incentive compatibility condition (*iv*) (see Figure 7).

3. Endogenous derivation of a coalition parameter

By changing γ we obtain a family of γ-coalitional equilibria. In this section, we present several ways to choose an equilibrium, i.e., endogenously derive γ by an appropriate optimality condition. The first method is where two market makers are employees of a monopolistic market-making firm that orders the employee γ to maximize total expected profit. The second one is where market makers cooperate so as to minimize the maximized expected profit of the informed trader. The third one is to use a Nash bargaining solution between two market makers.

We denote by $\tilde{\pi}_M$, $\tilde{\pi}_I$, and $\tilde{\pi}_U$ the total profit of the market makers, the informed trader, and the uninformed trader, respectively. The unconditional expected profits of the participants at a γ-coalitional equilibrium are given by

$$E[\tilde{\pi}_M] = E[(\tilde{v} - P^M(\tilde{y}))(-\tilde{y}) + (\tilde{v} - p_0)z] = -\frac{1}{2}\beta(\Sigma_0 + (\mu - p_0)^2) + \lambda\sigma_u^2, \tag{34}$$

$$E[\tilde{\pi}_I] = E[(\tilde{v} - P^M(\tilde{y}))\beta(\tilde{v} - \mu)] = \frac{1}{2}\beta(\Sigma_0 + (\mu - p_0)^2), \tag{35}$$

$$E[\tilde{\pi}_U] = E[(\tilde{v} - P^M(\tilde{y}))\tilde{u}] = -\lambda\sigma_u^2. \tag{36}$$

If γ and inventory positions are control variables, then maximizing $E[\tilde{\pi}_M]$ is equivalent to minimizing $E[\tilde{\pi}_I]$. If only γ is controlled, then "optimal" γ may depend on inventory positions and the relationship between market makers.

3.1. Order by a risk-neutral market-making firm

In this subsection, we assume that two individual risk-averse market makers are employed by a firm that is a risk-neutral monopolist in the market holding their own inventory position z. The firm can allocate inventory positions to these market makers and order them to form a coalition with specific γ so that the expected firm's profit from their activities is maximized. Our aim here is to derive the optimal γ and allocations of the inventory positions z_A and z_B with $z = z_A + z_B$.

It is sufficient for the firm's optimality that λ is maximized and $\mu = p_0$ since $\beta = 1/(2\lambda)$ is minimized when λ is maximized if γ and each inventory position are controlled.

Theorem 3. *If $R_A < R_B$, then the optimal selection of γ and inventory allocations by the firm among γ-coalitional equilibrium are given by*

$$\gamma = 1, \quad z_A = z, \quad z_B = 0. \tag{37}$$

If $R_A = R_B$, then the optimal selections are given by

$$\forall\gamma \in [0, 1]\,(\gamma \neq \frac{1}{2}), \quad z_A = \frac{-\gamma}{1 - 2\gamma}z, \quad z_B = \frac{1 - \gamma}{1 - 2\gamma}z. \tag{38}$$

In both cases the aggregate pricing schedule yields $\mu = p_0$ for the zero order $y = 0$ and the maximized expected firm's profit is positive $E[\tilde{\pi}_M] = \frac{1}{2}\Sigma_0\sigma_u^2 R_\gamma > 0$.

When $\gamma = 1$ is optimal in Theorem 3, the aggregate pricing schedule coincides with the regret-free pricing schedule of the more risk-averse market maker B with no inventory position

$$P^M(y) \quad = \quad P^{B*}(y) = p_0 + \lambda y, \quad \text{where} \quad \lambda = \frac{\Sigma_0}{4}\Big(R_B + \sqrt{4(\Sigma_0\sigma_u^2)^{-1} + {R_B}^2}\Big). \tag{39}$$

This result is exactly the same situation assumed by Subrahmanyam (1991), i.e., that the risk-averse market maker sets prices as the regret-free price.

The risk-neutral firm can obtain a positive expected profit because it is a monopoly. The firm behaves as if it were risk-averse market maker B, that is, the most conservative behavior obtainable as an individual market maker, while the expected utility is calculated as that of a risk-neutral market maker, that is, the most aggressive.

3.2. Minimizing the maximized expected profit of the informed trader

The market makers [9] may be able to choose parameters such as γ and inventory positions so as to minimize the maximized expected profit of the informed trader.

If they can choose γ and reallocate inventory positions between them, the optimal choices are the same as in Theorem 3 since minimizing the expected profit of the informed trader is one of the sufficient conditions for maximizing the total expected profit of market makers.

What about if they can control γ only without changing their inventories? If the degree of risk aversion is the same $R_A = R_B$, λ does not depend on γ as shown in Theorem 1 and the expected profit of liquidity traders is fixed irrespective of γ and inventory positions. Therefore, minimizing the expected profit of the informed trader is equivalent to maximizing the total expected profit of market makers, and arguments in Theorem 3 can be applied to this case.

If $R_A = R_B$ and $z_A = z_B$, any $\gamma \in [0, 1]$ is optimal. If $R_A = R_B$ and $z_A < z_B$, then we obtain the optimal γ as

$$\gamma \quad = \quad \min\Big(\max\Big(\frac{z_A}{z_A - z_B}, 0\Big), 1\Big) = \begin{cases} 1 & \text{if } z_A < z_B < 0, \\ \dfrac{z_A}{z_A - z_B} & \text{if } z_A < 0 < z_B, \\ 0 & \text{if } 0 < z_A < z_B. \end{cases} \tag{40}$$

If $R_A < R_B$ and $R_A z_A < R_B z_B \leq 0$, then it is easy to see that the optimal γ is one. For other cases, the optimal γ is not obtained analytically in a closed form.

3.3. Nash bargaining solution

It is a natural extension to understand the choice of γ as a Nash bargaining problem between market makers by taking inventory positions as given. Higher γ improves the expected utility of A and lowers one of B. If they do not agree with

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[9] In this subsection and the next, the market makers are general decision makers, unlike the setting of the previous subsection.

γ, both of them fail to improve their utility from the reservation level. Bargaining power is reflected in winning orders that are determined by their risk aversion and inventory position, independent of the choice of γ.

For the case of $R_A = R_B$, it can be shown that $\gamma = \frac{1}{2}$ is the Nash bargaining solution between two market makers (see Appendix 4 for the derivation). The two market makers possess the same bargaining power since the wining area is symmetrical given the same degree of risk aversion. For the case of $R_A < R_B$, it is not clear whether there exists a Nash bargaining solution. Our conjecture, however, is that a solution γ, if any, will be very close to one because the bargaining power of market maker B is weak due to the restricted winning order area of B, compared with that of A.

4. Concluding remarks

We present a model of two risk-averse market makers to derive regret-free pricing, rather than assume such pricing. In a coalition, the two market makers make use of their comparative advantage in inventory and share the risk of transactions with traders. The informed trader employs a different trading strategy from one without the market makers' inventory positions.

We see that there are two inventory effects on the equilibrium pricing schedule. The first is a direct effect where the inventory positions of the market makers vertically shift the equilibrium pricing schedule. The second is an indirect effect where the inventory position of the market makers may change the slope λ of the pricing schedule through the coalition parameter γ, depending on the relationship between the market makers. The slope $\lambda = \lambda_0 + (\lambda - \lambda_0)$ of the risk-averse market maker can be interpreted as representing two parts of the bid-ask spread. The λ_0, that is, the Kyle's λ of a risk-neutral market maker, represents that part of the bid-ask spread due to asymmetric information existing between the market maker and the informed trader on liquidation value. The difference between λ and λ_0 represents the bid-ask spread as the compensation for the risk-averse market maker to bear the risk of the order. A coalition that may depend on the inventories of the market makers changes the compensation required by the market maker due to the averaged inventory position. This insight into an indirect inventory effect is new to the literature.

There are several possible extensions of this paper. One extension to multi-period and continuous time settings would be possible following Kyle (1985) and Back (1992). The decision of the informed trader is then made backwards, as is that of the market makers, although inventory positions as state variables are determined forwards. The equilibrium price is then part of the forward-backward equation. A second extension is the study of n market makers ($n \geq 3$). The essence of a γ-coalition is a coalition between the best market maker and the second-best market maker for each order quantity. Therefore, there may not exist a coalitional equilibrium for $n \geq 3$ in general, and a modification of the incentive compatibility condition will be required for large n as in linear Bayesian equilibria.

Appendix

A.1. Proof of Lemma 1

Fix y and a market maker i. In this proof we denote the conditional expectation $E[\cdot|\alpha + \beta\tilde{v} + \tilde{u} = y]$ by $E^y[\cdot]$ and the conditional variance by $Var^y[\cdot]$ similarly. Suppose the market maker wins the order and buys $-y$ from traders at $P^i(y)$. Then the wealth of i after the stock position is cleared at $\tilde{v}$ will be $(\tilde{v}-p_0)z_i-(\tilde{v}-P^M(y))y$, and the expected utility is given by

$$\begin{aligned}U_i(y) &= E^y[u_i(-p_0z_i + P^i(y)y + \tilde{v}(z_i - y))] \\ &= u_i(E^y[-p_0z_i + P^i(y)y + \tilde{v}(z_i - y)] - \frac{R_i}{2}Var^y[-p_0z_i + P^i(y)y + \tilde{v}(z_i - y)])\end{aligned}$$

since $u_i(W) = -\exp(-R_iW)$. On the other hand, the reservation utility is

$$U_i^r(y) = u_i(-p_0z_i + E^y[\tilde{v}]z_i - \frac{R_iz_i^2}{2}Var^y[\tilde{v}]).$$

Then the participation constraint $U_i(y) \geq U_i^r(y)$ is equivalent to

$$E^y[P^i(y)y + \tilde{v}(z_i - y)] - \frac{R_i}{2}Var^y[\tilde{v}(z_i - y)] \geq E^y[\tilde{v}]z_i - \frac{R_i}{2}Var^y[\tilde{v}z_i].$$

Since $\alpha + \beta\tilde{v} + \tilde{u}$ is normally distributed, by the theorem on normal correlation we have

$$E^y[\tilde{v}] = p_0 + \frac{\beta\Sigma_0}{\beta^2\Sigma_0 + \sigma_u^2}(y - \alpha - \beta p_0), \quad Var^y[\tilde{v}] = \frac{\Sigma_0\sigma_u^2}{\beta^2\Sigma_0 + \sigma_u^2}.$$

The participation constraint is further reduced to an inequality

$$\Big(P^i(y) - [p_0 + \frac{\beta\Sigma_0}{\beta^2\Sigma_0 + \sigma_u^2}(y - \alpha - \beta p_0)) + \frac{\Sigma_0\sigma_u^2}{\beta^2\Sigma_0 + \sigma_u^2}\frac{R_i}{2}(y - 2z_i)]\Big)y \geq 0.$$

A.2. Proof of Theorem 1

We only consider the case of $z_A \geq z_B$ in this proof. The opposite case can be verified in the same manner.

Suppose that $X(v) = \alpha + \beta v$ and $P^M(y) = \mu + \lambda y$. First, let us consider the informed trader's expected profit maximization problem. Given P^M, the expected profit of the informed trader is given by

$$E[\tilde{\pi}(X, P^M)|\tilde{v} = v] = E[(\tilde{v} - P^M(X + \tilde{u}))X|\tilde{v} = v] = (v - \mu)X - \lambda X^2$$

which is maximized at $X = (v - \mu)/(2\lambda)$ so that

$$\alpha = \frac{-\mu}{2\lambda}, \quad \beta = \frac{1}{2\lambda}. \tag{41}$$

Next we consider conditions for market makers' pricing schedules. By Lemma 1, market makers can afford to show

$$\begin{aligned} P^{A*}(y;X) &= \mu_0(\alpha,\beta) + \lambda_0(\beta)y + (\lambda_A(\beta) - \lambda_0(\beta))(y - 2z_A), \\ P^{B*}(y;X) &= \mu_0(\alpha,\beta) + \lambda_0(\beta)y + (\lambda_B(\beta) - \lambda_0(\beta))(y - 2z_B), \end{aligned}$$

or worse price schedules. With equation (12) we have equations

$$\lambda = \lambda_A(\beta) = \lambda_B(\beta), \tag{42}$$

$$\mu = \mu_0(\alpha,\beta) - (1-\gamma)(\lambda_A(\beta) - \lambda_0(\beta))2z_A - \gamma(\lambda_B(\beta) - \lambda_0(\beta))2z_B. \tag{43}$$

By solving equations (41) – (43), we have

$$\begin{aligned} \lambda &= \frac{\Sigma_0}{4}\Big(R + \sqrt{4(\Sigma_0\sigma_u^2)^{-1} + R^2}\Big), \quad \beta = \frac{\sigma_u^2}{2}\Big(\sqrt{4(\Sigma_0\sigma_u^2)^{-1} + R^2} - R\Big), \\ \mu &= p_0 - \Sigma_0 R((1-\gamma)z_A + \gamma z_B), \quad \alpha = -\beta\mu. \end{aligned}$$

Note that β satisfies a quadratic equation

$$\beta^2\Sigma_0 + \beta\Sigma_0\sigma_u^2 R - \sigma_u^2 = 0. \tag{44}$$

By using this equation, it is easy to show that

$$\begin{aligned} \lambda_0(\beta) &= \frac{\beta\Sigma_0/\sigma_u^2}{2 - \beta\Sigma_0 R}, \\ 2(\lambda_A(\beta) - \lambda_0(\beta)) &= \frac{\Sigma_0 R}{2 - \beta\Sigma_0 R}, \\ \mu_0(\alpha,\beta) &= p_0 - \frac{\beta\Sigma_0/\sigma_u^2}{2 - \beta\Sigma_0 R}\beta\Sigma_0 R((1-\gamma)z_A + \gamma z_B), \end{aligned}$$

and

$$\begin{aligned} P^{A*}(y) &= \mu - \frac{\Sigma_0 R}{2 - \beta\Sigma_0 R}\gamma(z_A - z_B) + \lambda y, \\ P^{B*}(y) &= \mu + \frac{\Sigma_0 R}{2 - \beta\Sigma_0 R}(1-\gamma)(z_A - z_B) + \lambda y. \end{aligned}$$

A.3. Proof of Theorem 2

The procedure is in a fashion parallel to the proof of Theorem 1. Suppose that $X(v) = \alpha + \beta v$ and $P^M(y) = \mu + \lambda y$. The FOC of the maximization problem of the informed trader gives

$$\alpha = \frac{-\mu}{2\lambda}, \quad \beta = \frac{1}{2\lambda}.$$

By the definition of the γ-coalitional equilibrium, P^M is a line passing through $(y^*, p^*(\alpha,\beta))$ satisfying equation (7), which implies

$$\begin{aligned} P^M(y) &= p^* + \lambda(y - y^*), \\ \lambda &= (1-\gamma)\lambda_A(\beta) + \gamma\lambda_B(\beta), \\ \mu &= p^* - \lambda y^*. \end{aligned}$$

Then by solving these equations we have

$$\begin{aligned} R_\gamma &= (1-\gamma)R_A + \gamma R_B, \\ \lambda &= \frac{\Sigma_0}{4}\Big(R_\gamma + \sqrt{4(\Sigma_0\sigma_u^2)^{-1} + R_\gamma{}^2}\Big), \\ \beta &= \frac{\sigma_u^2}{2}\Big(\sqrt{4(\Sigma_0\sigma_u^2)^{-1} + R_\gamma{}^2} - R_\gamma\Big), \end{aligned}$$

where R_γ replaces R in Theorem 1. In this case, β satisfies

$$\beta^2\Sigma_0 + \beta\Sigma_0\sigma_u^2 R_\gamma - \sigma_u^2 = 0. \tag{45}$$

Furthermore, one can obtain

$$\begin{aligned} \mu &= p_0 - \Sigma_0\Big[(1-\gamma)R_A z_A + \gamma R_B z_B\Big], \\ p^* &= p_0 + \frac{\Sigma_0/\sigma_u^2}{R_B - R_A}\Big(\beta(R_B z_B - R_A z_A) + R_A R_B(z_B - z_A)\sigma_u^2\Big), \\ P^{A*}(y) &= \mu + \lambda y + (\lambda_A(\beta) - \lambda)(y - y^*), \\ P^{B*}(y) &= \mu + \lambda y - (\lambda_B(\beta) - \lambda)(y - y^*), \end{aligned}$$

by noticing

$$\begin{aligned} \lambda_A(\beta) - \lambda &= -\frac{\Sigma_0}{2 - \beta\Sigma_0 R_\gamma}\frac{\gamma(R_B - R_A)}{2}, \\ \lambda_B(\beta) - \lambda &= \frac{\Sigma_0}{2 - \beta\Sigma_0 R_\gamma}\frac{(1-\gamma)(R_B - R_A)}{2}. \end{aligned}$$

A.4. Nash bargaining solution when $R_A = R_B$

We denote by $P^{A,\gamma}$ and $P^{B,\gamma}$ the pricing schedule of market makers A and B at the γ-coalitional equilibrium. Since the wealth of market makers after the liquidation is given by

$$\begin{aligned} W_A(P^{A,\gamma}, P^{B,\gamma}, y) &= -p_0 z_A + (\mu + \lambda y)y + \tilde{v}(z_A - y), \\ W_B(P^{A,\gamma}, P^{B,\gamma}, y) &= -p_0 z_B + (\mu + \lambda y)y + \tilde{v}(z_B - y), \end{aligned}$$

the certainty equivalence of the wealth $w_A(\gamma, y)$ and $w_B(\gamma, y)$ defined by

$$\begin{aligned} E^y[u_A(W_A(P^{A,\gamma}, P^{B,\gamma}, y))] &= u_A(w_A(\gamma, y)), \\ E^y[u_B(W_B(P^{A,\gamma}, P^{B,\gamma}, y))] &= u_B(w_B(\gamma, y)), \end{aligned}$$

are given as follows,

$$
\begin{aligned}
w_A(\gamma, y) &= \frac{y}{\beta^2\Sigma_0 + \sigma_u^2}\Big(\sigma_u^2\Sigma_0 R\gamma(z_A - z_B) + z_A\Sigma_0\beta\Big) \\
&\quad + \frac{z_A\Sigma_0 R}{\beta^2\Sigma_0 + \sigma_u^2}\Big(-\beta^2\Sigma_0((1-\gamma)z_A + \gamma z_B) - \frac{1}{2}\sigma_u^2 z_A\Big), \\
w_B(\gamma, y) &= \frac{y}{\beta^2\Sigma_0 + \sigma_u^2}\Big(-\sigma_u^2\Sigma_0 R(1-\gamma)(z_A - z_B) + z_B\Sigma_0\beta\Big) \\
&\quad + \frac{z_B\Sigma_0 R}{\beta^2\Sigma_0 + \sigma_u^2}\Big(-\beta^2\Sigma_0((1-\gamma)z_A + \gamma z_B) - \frac{1}{2}\sigma_u^2 z_B\Big).
\end{aligned}
$$

After a tedious calculation, the unconditional expected utility gains are calculated as

$$
\begin{aligned}
U_A(\gamma) &= E\Big[E^y[u_A(w_A(\gamma, y))] - E^y[u_A(w_A(\gamma, y) - \frac{\Sigma_0 R}{2 - \beta\Sigma_0 R}\gamma(z_A - z_B))]|y > 0\Big] \\
&= \Big[1 - \exp\Big(\frac{\Sigma_0 R^2}{2 - \beta\Sigma_0 R}\gamma(z_A - z_B)\Big)\Big]I_A(\gamma), \\
U_B(\gamma) &= \Big[1 - \exp\Big(\frac{\Sigma_0 R^2}{2 - \beta\Sigma_0 R}(1-\gamma)(z_A - z_B)\Big)\Big]I_B(\gamma),
\end{aligned}
$$

where

$$
\begin{aligned}
g(\gamma, y) &= \frac{1}{\sqrt{2\pi(\beta^2\Sigma_0 + \sigma_u^2)}}\exp\Big(-\frac{(y - \beta(p_0 - \mu))^2}{2(\beta^2\Sigma_0 + \sigma_u^2)}\Big), \\
I_A(\gamma) &= \int_0^\infty u_A(w_A(\gamma, y))g(\gamma, y)dy \\
&= \exp\Big[\frac{R^3\Sigma_0^2\sigma_u^2(\beta + \frac{1}{2}\sigma_u^2 R)}{\beta^2\Sigma_0 + \sigma_u^2}\gamma^2(z_A - z_B)^2 \\
&\quad + \frac{1}{2}R^2\Sigma_0 z_A^2\Big]N\Big(\frac{-R\Sigma_0(\beta + \sigma_u^2 R)}{\sqrt{\beta^2\Sigma_0 + \sigma_u^2}}\gamma(z_A - z_B)\Big), \\
I_B(\gamma) &= \int_{-\infty}^0 u_B(w_B(\gamma, y))g(\gamma, y)dy \\
&= \exp\Big[\frac{R^3\Sigma_0^2\sigma_u^2(\beta + \frac{1}{2}\sigma_u^2 R)}{\beta^2\Sigma_0 + \sigma_u^2}(1-\gamma)^2(z_A - z_B)^2 + \frac{1}{2}R^2\Sigma_0 z_B^2\Big] \\
&\quad \times N\Big(\frac{-R\Sigma_0(\beta + \sigma_u^2 R)}{\sqrt{\beta^2\Sigma_0 + \sigma_u^2}}(1-\gamma)(z_A - z_B)\Big).
\end{aligned}
$$

It is straightforward to see that the Nash product $U_A(\gamma)U_B(\gamma)$ is maximized when $\gamma = \frac{1}{2}$.

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