

Gray-box Model-based Predictive Control of Czochralski Process^{*}

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Abstract

The present study proposes a gray-box (GB) model-based predictive control method to produce high-quality 300 mm silicon ingots in the commercial Czochralski (CZ) process. The GB model consists of an energy transfer, hydrodynamic, and geometrical model and a statistical model, predicts three controlled variables, i.e., crystal radius, growth rate, and melt position, and represents the time-varying and nonlinear characteristics of the CZ process. Solving an optimization problem with the GB model requires heavy computational load; therefore, the proposed method derives the prediction model by successive linearization of the GB model to compute optimal manipulated variables in several seconds. The proposed method was compared with the conventional method using PID controllers in disturbance rejection performance through control simulations. The results have demonstrated that the integral absolute error (IAE) of the proposed method was reduced by 60% on average and 89% at maximum even when a plant-model mismatch exists.

Keywords: A1. Gray-box model, A1. Model predictive control (MPC), A1.

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1. Introduction

Silicon wafers are raw materials for semiconductors and vital components of most electronics goods, including computers, telecommunication products, and consumer electronics. SEMI Silicon Manufacturers' Group (SMG) reported
5 that the shipment of silicon materials for semiconductor applications in 2020 is 1.3 times larger than in 2010 and 1.2 times larger than in 2015 [1].

The Czochralski (CZ) process produced more than 95% of single-crystal silicon ingots with a diameter of 300 mm in 2009 [2]. Fig. 1 shows a schematic diagram of the CZ process. Polycrystalline silicon is charged into a crucible
10 and melted. Then, a seed crystal is attached to the surface of the melt. The crucible and the crystal are rotated and lifted, resulting in crystal growth at the solid-liquid interface, and finally, a single-crystal silicon ingot is obtained. This study focuses on the ingot body production process, in which it is essential to control the crystal radius, the growth rate, and the melt position to produce
15 high-quality products.

The block flow diagram of the current industrial CZ process is represented in Fig. 2, where subscripts set and profile mean a set-point and profile. The

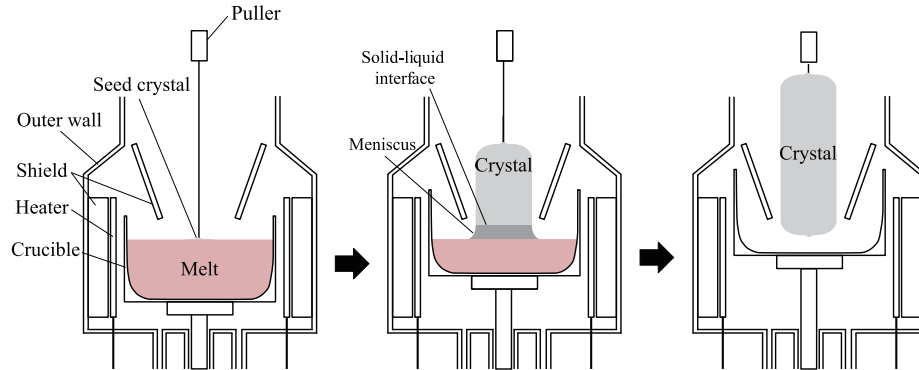


Fig. 1: Schematic diagram of the Czochralski process [3].

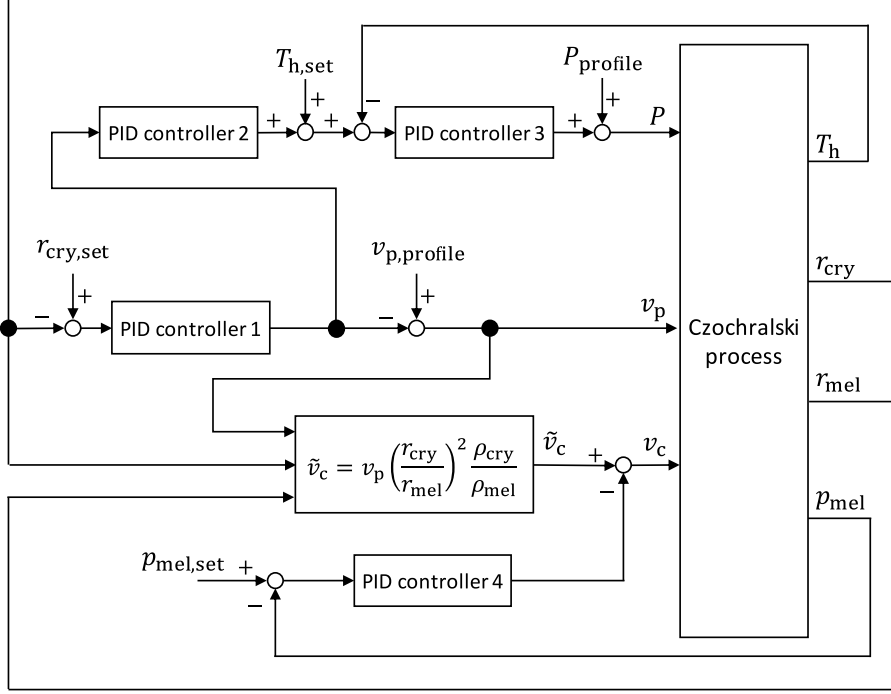


Fig. 2: Block flow diagram of the industrial Czochralski process. Manipulated variables are P , v_p , and v_c . Controlled variables are the heater temperature T_h , the crystal radius r_{cry} , and the melt position p_{mel} . Subscripts set and profile denote a set-point and a profile. r_{mel} , ρ_{cry} , and ρ_{mel} denote the radius of the melt surface, the crystal density, and the melt density, respectively.

crystal radius r_{cry} , the heater temperature T_h , and the melt position p_{mel} are controlled by manipulating the crystal pulling rate v_p , the crucible rise rate v_c ,
 20 and the heater input power P . p_{mel} is a distance between the melt surface and the bottom of the shield above the melt. The crystal growth rate v_g cannot be measured in real time; thus, v_p is manipulated to track the predetermined value $v_{p,\text{profile}}$ in order to control v_g . The melt surface radius r_{mel} changes as the crystal grows since the crucible is bowl-shaped. ρ_{cry} and ρ_{mel} are the density of
 25 the crystal and melt.

In the CZ process, the input-output relationship changes due to crystal growth. Besides, the radiative heat transfer has a nonlinear effect on the con-

trolled variables (CVs). The application of 300 mm silicon wafers to silicon power devices has recently been promoted in Europe; thus, it is required to develop 300 mm silicon wafers with higher quality at a lower cost [4]. This paper proposes a model predictive control (MPC) system to improve the control performance of the conventional PID control system.

Table 1 summarizes previous research on MPC of the CZ process. Irizarry-Rivera and Seider [5, 6] developed two model predictive controllers: one controls r_{cry} by manipulating v_{p} , and the other controls v_{p} by manipulating P . Abdollahi *et al.* [7] derived the optimal temperature trajectory using quadratic programming and then implemented an MPC strategy to track the optimal temperature. Rahmanpour *et al.* [8] developed a control strategy with two model predictive controllers to control r_{cry} and melt temperature T_{mel} . Liu *et al.* [9] proposed a control structure where r_{cry} is controlled by manipulating T_{h} with constant v_{p} . Zhang *et al.* [10] and Lee *et al.* [11] used MPC to determine a feedforward trajectory of temperature in the traditional control structure.

As shown in Table 1, two of seven studies focused on the commercial CZ process producing 300 mm silicon ingots. Although control of r_{cry} , v_{g} , and p_{mel}

Table 1: Research on MPC of the CZ process.

Reference	Controlled variable	Manipulated variable	Crystal diameter [mm]
[5, 6]	r_{cry}	v_{p}	40
	v_{p}	P	
[7]	$r_{\text{cry}}, T_{\text{h}}$	P, v_{p}	50
[8]	$r_{\text{cry}}, T_{\text{mel}}$	P, v_{p}	160
[9]	$r_{\text{cry}}, T_{\text{h}}$	T_{h}	208
[10]	$r_{\text{cry}}, T_{\text{h}}, v_{\text{p}}$	P, v_{p}	300
[11]	$r_{\text{cry}}, T_{\text{h}}, v_{\text{p}}$	P, v_{p}	300
Present study	$r_{\text{cry}}, v_{\text{g}}, p_{\text{mel}}$	$P, v_{\text{p}}, v_{\text{c}}$	300

45 is required to manufacture high-quality products, p_{mel} is not used as the CV in
 the previous studies. Besides, in four of seven previous studies, v_p is controlled
 instead of v_g . The reason would be there had been no model predicting the three
 variables (r_{cry} , v_g , and p_{mel}) with high accuracy. Zheng *et al.* [3] developed a
 first-principle model to predict r_{cry} and v_g from P , v_p , and v_c in the process
 50 producing 300 mm silicon ingots. Kato *et al.* [12] improved the first-principle
 model and built a gray-box (GB) model to predict all CVs used in the industrial
 process. The present study proposes an MPC method based on the GB model.
 Although the GB model can accurately predict the CVs, solving an optimization
 problem using the GB model requires a high computational cost. The proposed
 55 method derives a prediction model by successive linearization of the GB model
 to reduce the cost. Our contributions are summarised as follows:

- Develop an MPC method based on the gray-box model predicting three
CVs with high accuracy.
- Improve disturbance rejection performance than the conventional control
 60 method even when a plant-model mismatch exists.

2. Gray-box model of CZ process

Fig. 3 shows the structure of the GB model [12]. The GB model consists
 of two models: a statistical model and an energy transfer, hydrodynamic and
 geometrical (EHG) model. In the EHG model, r_{cry} , v_g , and T_h are calculated
 65 from v_p , v_c , P , and G_{cry} . G_{cry} is the axial thermal gradient of the crystal at

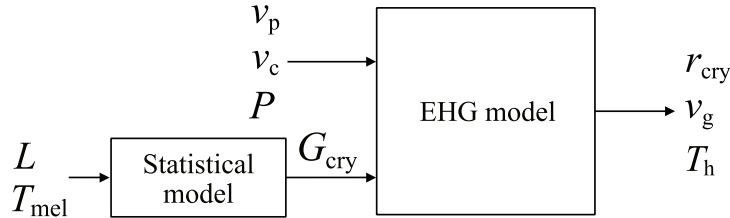


Fig. 3: Gray-box model of the CZ process.

the solid-liquid interface. The statistical model predicts G_{cry} using the crystal length L and T_{mel} .

As shown in Fig. 4, the CZ process is assumed to consist of eight components. The shield, the heater, and the crucible are divided into three parts by the melt surface and crucible bottom positions for precisely calculating the energy balance. The energy balance of each section is calculated from radiative and conductive heat transfer rates and P . $Q_{i,j}$ and $Q_{i,j}^*$ are the radiative and conductive heat transfer rates, respectively. Details of the energy balance are described by Zheng *et al.* [3].

v_g can be expressed as follows:

$$v_g = \frac{k_{\text{men}} G_{\text{men}} - k_{\text{cry}} G_{\text{cry}}}{\rho_{\text{cry}} \Delta H_f}, \quad (1)$$

$$G_{\text{men}} = -\frac{T_{\text{mel}} - T^*}{h_{\text{men}}}, \quad (2)$$

where k_{men} and k_{cry} are the heat conductivity of the meniscus section and the crystal. G_{men} is the axial thermal gradient of the meniscus section at the solid-liquid interface, ΔH_f is the specific latent heat of fusion for silicon, and h_{men}

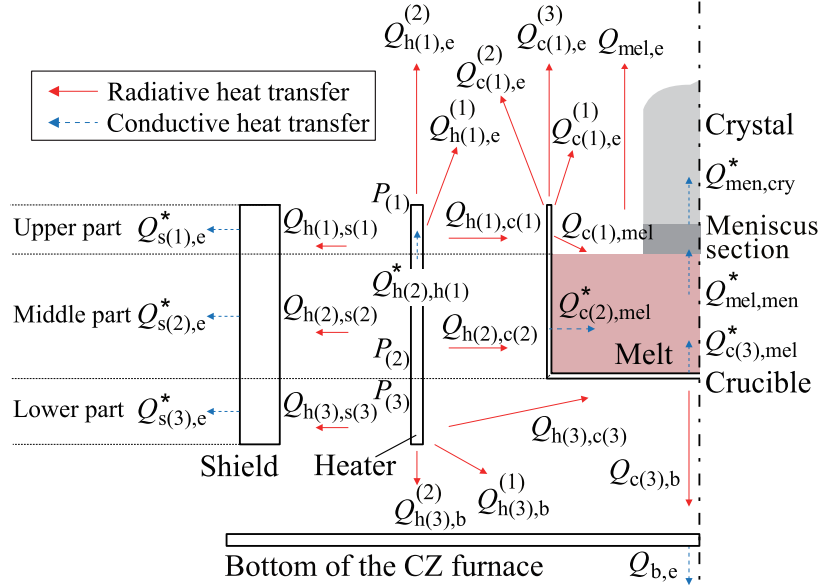


Fig. 4: Heat transfer considered in the EHG model [3].

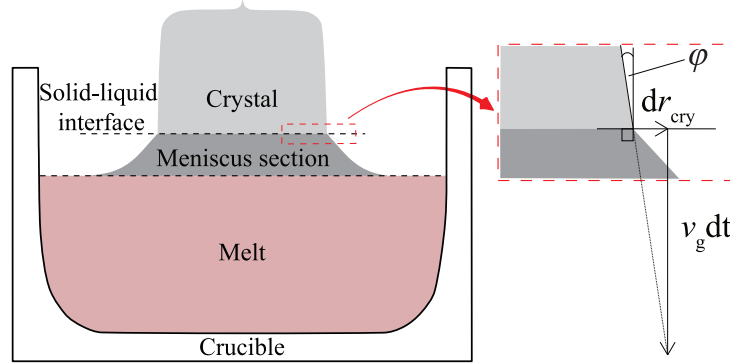


Fig. 5: Geometrical relationship around the solid-liquid interface [3].

is the meniscus height. T^* is the crystallization point of silicon equal to the temperature at the solid-liquid interface.

As shown in Fig. 5, r_{cry} is expressed using v_g and the crystal slope angle φ [13] as follows:

$$\frac{dr_{\text{cry}}}{dt} = v_g \tan \varphi. \quad (3)$$

From the mass balance at the meniscus section, φ is expressed as follows:

$$\frac{d\varphi}{dt} = \frac{v_p - v_c - \psi_1(r_{\text{cry}}, \varphi)v_g}{\psi_2(r_{\text{cry}}, \varphi)}, \quad (4)$$

where ψ_1 and ψ_2 are functions of r_{cry} and φ . Details of ψ_1 and ψ_2 are described by Winkler *et al.* [14].

The GB model uses a statistical model that predicts G_{cry} from L and T_{mel} to improve the prediction accuracy of the CVs. Before constructing the statistical model, training data is generated using the EHG model because G_{cry} is unmeasurable. The statistical model is then constructed by Gaussian Process Regression [15]. Details of the statistical model are described by Kato *et al.* [12].

The GB model requires initial values of the temperatures at the upper and middle parts of the heater $T_{h(1)}$ and $T_{h(2)}$, T_{mel} , and φ . Although these initial values affect the prediction accuracy of the CVs, they are not measured in the industrial CZ process. The CVs can be predicted with high accuracy by calculating the initial values using the method proposed by Kato *et al.* [12]. This method is used in the present work.

This study proposes a model-based predictive control method with higher control performance than the conventional method using PID controllers; thus, we need a model that can accurately predict the CVs of the industrial CZ process. The GB model satisfies this requirement and does not require further improvement in prediction accuracy for the purpose.

3. Proposed control method

The CVs and the manipulated variables (MVs) are defined as follows:

$$\mathbf{y}(t) = [r_{\text{cry}}(t), v_{\text{g}}(t), p_{\text{mel}}(t)]^{\text{T}}, \quad (5)$$

$$\mathbf{u}(t) = [P(t), v_{\text{p}}(t), v_{\text{c}}(t)]^{\text{T}}. \quad (6)$$

Here, the number of the MVs and the CVs n_u and n_y are three. MPC requires predictions of $\mathbf{y}(t)$ over the prediction horizon ($t = t_0 + 1, \dots, t_0 + H_{\text{p}}$) at each sampling instant t_0 . The proposed method computes the output predictions $\hat{\mathbf{y}}$ expressed as

$$\hat{\mathbf{y}}(t) = \hat{\mathbf{y}}_{\text{free}}(t) + \hat{\mathbf{y}}_{\text{forced}}(t), \quad (7)$$

where $\hat{\mathbf{y}}_{\text{free}}$ is a free response, which is an output of the GB model without change in input variables ($\Delta \mathbf{u}(t) = \mathbf{u}(t) - \mathbf{u}(t-1) = 0$) over the prediction horizon. $\hat{\mathbf{y}}_{\text{forced}}$ is a forced response, which depends on $\Delta \mathbf{u}(t)$ over the prediction horizon. Successive linearization of the GB model is applied and a state-space model computing $\hat{\mathbf{y}}_{\text{forced}}$ is derived every linearization interval Δt_1 to solve the optimization problem in several seconds. The next MVs $\mathbf{u}(t_0)$ are determined by solving the optimization problem in 3.2 for each control interval Δt_{c} .

3.1. Successive linearization

It is challenging to linearize the GB model analytically because the GB model has several nonlinear algebraic equations, including a term of radiation heat transfer proportional to the fourth power of the component's temperature. Based on the method developed by Megías *et al.* [16], the GB model is linearized in the following steps:

1. Calculate a free response $\hat{\mathbf{y}}_{\text{free}}(t)$ over the prediction horizon by the GB model with $\mathbf{u}(t) = \mathbf{u}(t_0 - 1)$.
2. For $i = 1, 2$, and 3 , calculate $\hat{\mathbf{y}}_{u_i}(t)$, which is an output of the GB model over the prediction horizon with

$$u_j(t) = \begin{cases} u_j(t_0 - 1) + \Delta u_{j,\max} & (j = i), \\ u_j(t_0 - 1) & (j \neq i), \end{cases} \quad (8)$$

where u_j is the j th variable in (6), and $\Delta u_{j,\max}$ is the maximum absolute change of u_j in one-batch operation data obtained from the industrial process.

3. For $i = 1, 2$, and 3 , calculate difference between the responses $\hat{\mathbf{y}}_{u_i}(t)$ and $\hat{\mathbf{y}}_{\text{free}}(t)$ given by

$$\mathbf{Y}_{u_i} = \begin{bmatrix} (\hat{\mathbf{y}}_{u_i}(t_0 + 1) - \hat{\mathbf{y}}_{\text{free}}(t_0 + 1))^{\top} \\ (\hat{\mathbf{y}}_{u_i}(t_0 + 2) - \hat{\mathbf{y}}_{\text{free}}(t_0 + 2))^{\top} \\ \vdots \\ (\hat{\mathbf{y}}_{u_i}(t_0 + H_p) - \hat{\mathbf{y}}_{\text{free}}(t_0 + H_p))^{\top} \end{bmatrix}. \quad (9)$$

4. Derive a state-space model

$$\hat{\mathbf{y}}_{\text{forced}}(t) = \mathbf{C}\mathbf{x}(t), \quad (10)$$

$$\mathbf{x}(t + 1) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\Delta\mathbf{u}(t), \quad (11)$$

where $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ are matrices calculated from $\mathbf{Y}_{u_1}$, $\mathbf{Y}_{u_2}$, and $\mathbf{Y}_{u_3}$, and $\mathbf{x}$ is a state vector.

Details of the derivation of the state-space model are described by Jimoh and Dan'Isa [17].

3.2. Optimization problem of model predictive control

For each control interval, the optimal MVs $\mathbf{u}_{\text{opt}}(t)$ over the control horizon ($t = t_0, \dots, t_0 + H_c - 1$) are obtained by solving the following optimization

problem:

$$\begin{aligned} \min_{\mathbf{u}(t) \ (t=t_0, \dots, t_0+H_c-1)} \quad & \sum_{t=t_0+1}^{t_0+H_p} \mathbf{e}(t)^\top \mathbf{Q} \mathbf{e}(t) \\ & + \sum_{t=t_0}^{t_0+H_c-1} \Delta \mathbf{u}(t)^\top \mathbf{R} \Delta \mathbf{u}(t), \end{aligned} \quad (12)$$

$$\text{s.t.} \quad \mathbf{e}(t) = \mathbf{y}_{\text{set}}(t) - \hat{\mathbf{y}}(t), \quad (13)$$

$$\hat{\mathbf{y}}(t) = \hat{\mathbf{y}}_{\text{free}}(t) + \hat{\mathbf{y}}_{\text{forced}}(t), \quad (14)$$

$$\hat{\mathbf{y}}_{\text{forced}}(t) = \mathbf{C} \mathbf{x}(t), \quad (15)$$

$$\mathbf{x}(t+1) = \mathbf{A} \mathbf{x}(t) + \mathbf{B} \Delta \mathbf{u}(t), \quad (16)$$

$$\mathbf{x}(t_0) = \mathbf{0}, \quad (17)$$

$$\Delta \mathbf{u}_{\min} \leq \Delta \mathbf{u}(t) \leq \Delta \mathbf{u}_{\max}, \quad (18)$$

$$\mathbf{u}(t) \geq \mathbf{0}, \quad (19)$$

$$\Delta \mathbf{u}(t) = \mathbf{0} \quad (t = t_0 + H_c, \dots, t_0 + H_p - 1), \quad (20)$$

where $\mathbf{Q}$ and $\mathbf{R}$ are diagonal weighting matrices. H_c and H_p are the lengths of the control horizon and the prediction horizon. $\Delta \mathbf{u}_{\max}$ is a vector whose i th element is $\Delta u_{i,\max}$, and $\Delta \mathbf{u}_{\min}$ is a vector whose i th element is $-\Delta u_{i,\max}$. The

next MVs are the optimal MVs at $t = t_0$: $\mathbf{u}(t_0) = \mathbf{u}_{\text{opt}}(t_0)$.

4. Simulation condition

We evaluate the set-point trajectory and disturbance rejection performance of the conventional and proposed methods through control simulations in the following procedure:

1. Control the process by the conventional method until relative control deviations of all CVs to $\epsilon_{y_{i,\max}}$ are smaller than 0.01 during 10 minutes.
2. Switch the control method to the proposed one when evaluating the performance of the proposed method.
3. Control the process for 30 minutes without disturbance.

- 135 4. After the 30 minutes, inject a constant disturbance during a minute into one of the MVs or the CVs except for v_g . The magnitude of the disturbance to the MV and the CV is $\Delta u_{i,\max}$ and $\epsilon_{y_i,\max}$, respectively.
5. Continue the simulation for the next 120 minutes.

$\epsilon_{y_i,\max}$ is the maximum absolute control deviation of the i th CV y_i in one-batch industrial process data.

The dynamic characteristics of the CZ process change as the crystal grows; thus, we compare the performance at the early, middle, and late stages. Each of them corresponds to the stage where the crystal length is 0–33%, 33%–66%, and 66%–100% of the product’s crystal length.

145 We use the GB model as the control target. In the proposed method, the control simulations with or without a plant-model mismatch are conducted. In cases with a plant-model mismatch, two types of mismatches are used. In the control target, the overall heat transfer coefficient of the heater U_h is 490 W/(m²K) and the melt emissiity ϵ_{mel} is 0.25. In the prediction model, U_h is changed to
 150 $490 \times 1.1 = 539$ W/(m²K) or ϵ_{mel} is changed to $0.25 \times 0.95 = 0.2375$.

In the conventional method, the PID controllers are designed using the improved, simplified internal model control (SIMC) tuning rules [18]. The control interval of the conventional method is 1 s. In the proposed method, $\Delta t_c = \Delta t_l = 60$ s, $H_p = 60$, and $H_c = 10$. The weighting matrices are as follows:

$$\mathbf{Q} = \text{diag} \left(\frac{100}{\epsilon_{y_1,\max}}, \frac{100}{\epsilon_{y_2,\max}}, \frac{100}{\epsilon_{y_3,\max}} \right), \quad (21)$$

$$\mathbf{R} = \text{diag} \left(\frac{1}{\Delta u_{1,\max}}, \frac{1}{\Delta u_{2,\max}}, \frac{1}{\Delta u_{3,\max}} \right). \quad (22)$$

H_p , H_c , $\mathbf{Q}$, and $\mathbf{R}$ were determined to minimize the sum of the integral absolute errors (IAEs) of the three CVs through prior several control simulations. The differences between the control simulations and the real control are summarized in Table 2.

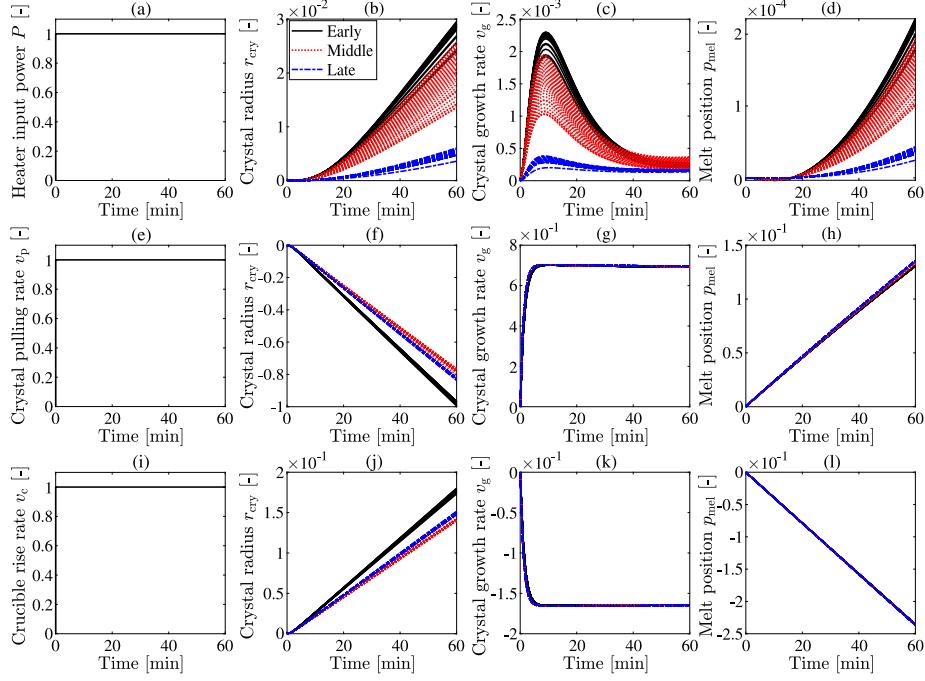


Fig. 6: Responses used for linearization. Responses with steps of (a) P , (b) v_p , and (c) v_c are shown in (b)–(d), (f)–(h), (j)–(l), respectively. 30 responses are plotted at each stage.

5. Results

5.1. Linearization of gray-box model

Fig. 6 shows $\mathbf{Y}_{u_i}$ used in procedure 3 in section 3.1 during control simulations without a plant-model mismatch. The values of u_i and y_i are nondimensional-

Table 2: Differences between the control simulations and the real control.

	Control simulations		Real control
	MPC	PID	PID
Controlled process	Gray-box model		Real process
Control interval	60 sec	1 sec	—
Tuning method	Minimizing sum of IAEs of controlled variables	Improved SIMC tuning rules	—
Controlled variables	$r_{\text{cry}}, v_g, p_{\text{mel}}$		$r_{\text{cry}}, p_{\text{mel}}, T_h$

ized by scaling factors $\Delta u_{i,\max}$ and $\epsilon_{y_i,\max}$, respectively. Note that the values
 160 of the scaling factors affect the changes in the CVs.

The input-output relationships between P and the CVs (b–d), v_p and r_{cry} (f), and v_c and r_{cry} (j) change with time more significantly than the others. The change in r_{cry} 60 minutes after the step change in v_p and v_c at the early stage is about 20% larger than that at the other stages. This is because v_g at the
 165 early stage is smaller than that at the other stages; when v_g becomes small, the change in φ becomes large, and consequently the change in r_{cry} becomes large as shown in Eqs. (3) and (4). Successive linearization is required to represent these time-varying input-output relationships in a prediction model.

The changes in the CVs 60 minutes after the step change in v_p and v_c are 5
 170 (b and j) to 1000 (d and l) times larger than those after the step change in P . This result is qualitatively consistent with the fact that v_p and v_c are mainly used to control r_{cry} in the industrial CZ process. Since v_c highly affects the CVs, it should be essential to manipulate v_c to achieve high control performance.

5.2. Control simulation

The IAEs of the conventional method (PID) and the proposed method
 175 (MPC) are shown in Tables 3–5, where the IAE of y_i is nondimensionalized by $\epsilon_{y_i,\max}$. In the cases without a plant-model mismatch, the sum of IAEs of MPC is 11–87% of that of PID. In the cases with a plant-model mismatch, the sum of IAEs of MPC is 11–98% of that of PID. MPC achieved better dis-
 180 turbance rejection performance than PID even when a 5% or 10% plant-model mismatch exists. Quad-Core Intel Core i5 CPU (2.3 GHz) was used, and it took less than 30 seconds to compute MVs at each control interval (60 seconds).

The control simulation results in cases 19–21 are shown in Fig. 7. Each vertical axis of the MVs and the CVs is nondimensionalized by $\Delta u_{i,\max}$ and
 185 $\epsilon_{y_i,\max}$, respectively, and scaled so that each initial value in the conventional method is zero. Even with a 10% plant-model mismatch, MPC reduced the control deviation of r_{cry} and p_{mel} much faster than PID.

Table 3: IAEs of controlled variables in the presence of disturbance in early stage.

Case	VWD ^a	CPP ^b	r_{cry}		v_g		p_{mel}		Sum	
			MPC	PID	MPC	PID	MPC	PID	MPC	PID
1	P	-	0.10		1.1		0.020		1.2	
2		U_h	0.12	1.3	1.2	3.0	0.020	0.52	1.3	4.8
3		ε_{mel}	0.11		1.1		0.020		1.2	
4	v_p	-	0.57		3.6		0.056		4.2	
5		U_h	0.56	2.1	3.5	4.5	0.057	0.76	4.1	7.3
6		ε_{mel}	0.58		3.6		0.054		4.2	
7	v_c	-	0.33		2.9		0.17		3.4	
8		U_h	0.35	1.5	2.9	3.1	0.18	1.9	3.5	6.6
9		ε_{mel}	0.33		2.8		0.17		3.3	
10	r_{cry}	-	0.94		4.9		0.27		6.1	
11		U_h	0.90	2.4	4.8	4.1	0.27	0.53	5.9	7.1
12		ε_{mel}	1.1		5.5		0.27		6.9	
13	p_{mel}	-	0.52		3.0		0.23		3.8	
14		U_h	0.52	1.3	3.1	3.0	0.23	1.7	3.9	6.0
15		ε_{mel}	0.53		3.1		0.23		3.8	

^a Variable with disturbance

^b Changed physical property

6. Conclusion

The present study proposed a model predictive control method based on successive linearization of the gray-box model of the Czochralski process. The proposed method's control performance was compared with that of the conventional method using PID controllers through control simulations. The results demonstrated that the proposed method provided a higher disturbance rejection performance than the conventional method in the cases with or without a 5% or 10% plant-model mismatch. The sum of integral absolute errors of the controlled variables of the proposed method was 11–98% of that of the conventional method. Besides, the proposed method was able to calculate the MVs in a sufficiently shorter time than the control interval.

Table 4: IAEs of controlled variables in the presence of disturbance in middle stage.

Case	VWD ^a	CPP ^b	r_{cry}		v_g		p_{mel}		Sum	
			MPC	PID	MPC	PID	MPC	PID	MPC	PID
16	P	-	0.23		1.5		0.0065		1.7	
17		U_h	0.20	5.7	1.4	4.9	0.0069	0.030	1.6	11
18		ε_{mel}	0.30		1.6		0.0054		1.9	
19	v_p	-	0.73		3.9		0.047		4.7	
20		U_h	0.70	6.1	3.9	6.6	0.048	0.53	4.7	13
21		ε_{mel}	0.82		3.9		0.046		4.8	
22	v_c	-	0.46		3.2		0.16		3.8	
23		U_h	0.44	5.8	3.2	5.0	0.16	1.9	3.8	13
24		ε_{mel}	0.55		3.2		0.16		3.9	
25	r_{cry}	-	1.7		7.1		0.26		9.1	
26		U_h	1.7	6.8	7.1	6.3	0.28	0.051	9.0	13
27		ε_{mel}	1.6		6.3		0.22		8.1	
28	p_{mel}	-	0.54		3.4		0.22		4.2	
29		U_h	0.52	5.7	3.3	4.9	0.22	1.1	4.1	12
30		ε_{mel}	0.60		3.5		0.22		4.3	

^a Variable with disturbance

^b Changed physical property

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Table 5: IAEs of controlled variables in the presence of disturbance in late stage.

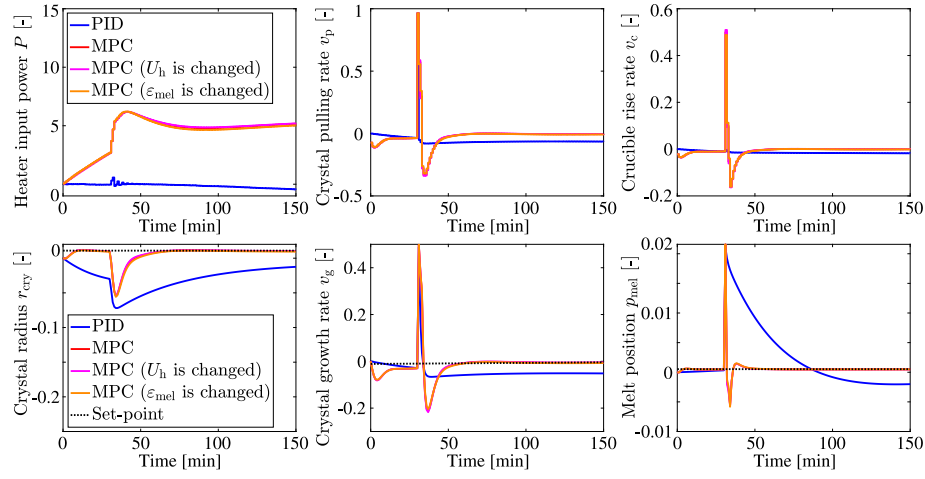
Case	VWD ^a	CPP ^b	r_{cry}		v_g		p_{mel}		Sum	
			MPC	PID	MPC	PID	MPC	PID	MPC	PID
31	P	-	0.26		1.7		0.014		2.0	
32		U_h	0.25	3.5	1.7	2.0	0.014	13	2.0	18
33		ε_{mel}	0.26		1.7		0.014		2.0	
34	v_p	-	0.68		4.1		0.048		4.8	
35		U_h	0.66	3.9	4.0	2.4	0.047	13	4.8	19
36		ε_{mel}	0.69		4.1		0.048		4.8	
37	v_c	-	0.45		3.4		0.16		4.0	
38		U_h	0.44	3.4	3.4	2.9	0.16	15	4.0	21
39		ε_{mel}	0.46		3.4		0.16		4.0	
40	r_{cry}	-	1.4		7.0		0.30		8.7	
41		U_h	1.3	3.3	6.5	2.3	0.27	13	8.0	18
42		ε_{mel}	1.4		6.8		0.26		8.4	
43	p_{mel}	-	0.63		3.8		0.23		4.6	
44		U_h	0.61	3.5	3.8	2.4	0.23	14	4.6	20
45		ε_{mel}	0.63		3.8		0.23		4.6	

^a Variable with disturbance

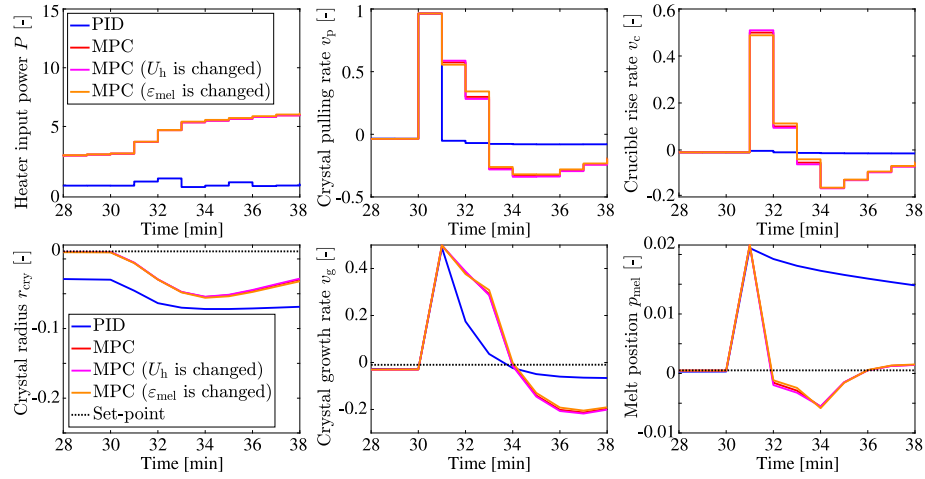
^b Changed physical property

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(a) Result for 150 minutes.



(b) Result from time = 28 to 38 min.

Fig. 7: Control simulation results in cases 19–21. The disturbance of the crystal pulling rate v_p was injected from time = 30 to 31 min.

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