

Omnidirectional Mobile Mechanisms
and Integrated Motor Mechanisms
for Wheeled Locomotion Devices

Tatsuro Terakawa

2019

Omnidirectional Mobile Mechanisms and Integrated Motor Mechanisms for Wheeled Locomotion Devices

Tatsuro Terakawa

Department of Mechanical Engineering and Science
Graduate School of Engineering, Kyoto University

A thesis submitted for the degree of
Doctor of Engineering

2019

Acknowledgment

First of all, I would like to express the greatest appreciation to my supervisor, Prof. Masaharu Komori, for his guiding me all the way through my research life. Without his persistent help, precise advice, and excellent creativity, this thesis would not have been possible. He has supported me not only in carrying out the research but also in cultivating the mindset of a researcher since I was in bachelor.

My heartfelt appreciation goes to Prof. Fumitoshi Matsuno and Prof. Atsushi Matsubara for their time and meticulous comments during my thesis review. Besides, Prof. Fumitoshi Matsuno took very good care of me in the class of FBL/PBL. Prof. Atsushi Matsubara gave me opportunities to interact with researchers and students in foreign universities. These experiences broaden my horizons.

I am deeply grateful to Kyoto University Design School (KDnS) and its members. I would like to offer my special thanks to Prof. Takashi Kusumi for his ready consent to participate in the examination on my program completion. I experienced his leading class of Open Innovation Practice Implementation, which brought me perspectives useful for my interests. Prof. Toru Ishida and Prof. Tetsuo Sawaragi gave me generous supports and warm encouragement in the KDnS life. I received enormous teachings, ideas, skills, and mindsets from Prof. Kumiyo Nakakoji through various opportunities including the classes, practices, and off-campus learning. Thanks to Dr. Yusuke Kita, I came across with *Discussions on Studies of Design*, which was the ground of my activities in KDnS. I would like to show my appreciation to other faculties, staffs, students, and the people related to this program as well.

I owe a very important debt to the coauthors of my research papers. Prof. Kenji Fujimoto gave insightful comments and suggestions on the control theory. Mr. Mitsuru Sakamoto and Mr. Yuji Kawato, Issi Corporation, made an enormous contribution to designing and developing the experimental devices. I received diversified support in carrying on analyses and experiments from Mr. Yuya Morita, Mr. Yugo Nishida, Mr. Kippei Matsuda, Mr. Yuuta Yamaguchi, Mr. Shinji Mikami, Mr. Shouta Tamura, and Mr. Ryo Kataoka.

I thank all members of our laboratory. Assist. Prof. Tetsuya Nonaka assisted me in constructing the research environment. The student members have made my laboratory life fun and stimulating.

I am particularly grateful for exciting and precious discussions given by the members of ARTIFACT Kenkyu-kai, Mr. Yuki Okura, Mr. Yudai Kishimoto, and Ms. Miduho Morita, and the other participants.

I am very much obliged to the Japan Society for the Promotion of Science (JSPS) for the financial support as Research Fellowships for Young Scientists (DC2). Our work was partly supported by Grants-in-Aid for Scientific Research (JSPS Kakenhi Grant Number JP18J13377 and JP25289023) and the Research Expense Allowance Collaborative Graduate Program in Design, Kyoto University.

Finally, I express my greatest gratitude to my family for their fulfilling supports for my physical and mental wellness, which have brought me to what I am.

Contents

Chapter 1	General Introduction	1
1.1	Locomotion Mechanisms	1
1.2	Conventional Omnidirectional Wheeled Mechanisms	2
1.3	Motors	3
1.4	Aims and Approach	4
1.5	Thesis Overview	4
Chapter 2	Wheeled Mechanism Analysis and a Novel Omnidirectional Mechanism ‘Slidable Wheel’	7
2.1	Wheeled Mechanisms	7
2.2	Analysis and Synthesis of Wheeled Mechanisms	9
2.2.1	Kinematic analysis of Wheeled Mechanisms	9
2.2.2	Synthesis of a Slidable Wheel	11
2.2.3	Comparisons of Wheeled Mechanisms	12
2.3	Kinematic Analysis of Omnidirectional Mobile Robot Using Slidable Wheels	14
2.3.1	Slidable Wheeled Omnidirectional Mobile Robot (SWOM)	14
2.3.2	Kinematics of SWOM	16
2.3.3	Reachable Region under Fixed Steering Angles	18
2.4	Example of Unlimited Travel: Unlimited Translation	20
2.4.1	Fundamental Traveling Strategy	20
2.4.2	Analysis and Simulation	21
2.4.3	Singular Configuration in Unlimited Translation	24
2.5	Experiments	25
2.5.1	SWOM Prototype	25
2.5.2	Experiments of Fundamental Movements	26
2.5.3	Experiments of Omnidirectional Movements	27
2.6	Summary	31
Chapter 3	Feedback Control of an Omnidirectional Mobile Robot with Slidable Wheels	33
3.1	Introduction	33
3.2	Design of control system for SWOM	33
3.2.1	Kinematic Model	33
3.2.2	Feedback Linearization	34
3.2.3	Singular Configuration Avoidance	36

3.3	Simulation and Experiment	38
3.3.1	Simulation	38
3.3.2	Experiment	39
3.3.3	Discussions	42
3.4	Summary	44
Chapter 4	Active Omni Wheel Achieving Seamless Periphery and Simple Structure	45
4.1	Introduction	45
4.2	Proposition of New AOW	46
4.2.1	Conventional AOW	46
4.2.2	Structure and Motion of New AOW	48
4.3	Analysis of New AOW	49
4.3.1	Geometry	49
4.3.2	Kinematics	52
4.3.3	Acceleration Limit	53
4.4	Two-wheel-drive Vehicle Using AOW	55
4.5	Development and Experiments	58
4.5.1	Design and Development of Prototype	58
4.5.2	Experiments	60
4.5.3	Comparison of Conventional and New AOWs	62
4.6	Summary	64
Chapter 5	Transmission Mechanism to Drive Active Omni Wheels with Fewer Motors	65
5.1	Introduction	65
5.2	Transmission Mechanism for Fewer Motors	65
5.2.1	Basic Concept	65
5.2.2	Kinematics and Constraints	67
5.3	Synthesis of Two-wheel-drive Vehicles with Fewer Motors	69
5.3.1	Vehicle with One Motor and Two AOWs	69
5.3.2	Vehicle with Two Motors and Two AOWs	70
5.3.3	Vehicle with Two Motors, One AOW, and One Omni Wheel	73
5.4	Experiments	75
5.4.1	Developed Two-wheel-drive Vehicle	75
5.4.2	Experiment on the Transmission Mechanism	77
5.4.3	Experiment on the Vehicle Motion	78
5.5	Summary	80
Chapter 6	Reducer-Integrated Motor Using Strain Wave Gearing System	83
6.1	Introduction	83
6.2	Reducer-Integrated Motor	84

6.2.1	Principle of a Strain Wave Gearing System	84
6.2.2	Proposed Reducer-Integrated Motor	85
6.3	Tooth Tip Interference Analysis	87
6.3.1	Problem Definition	87
6.3.2	Numerical Analysis	89
6.3.3	Structural Condition for Actuators	92
6.4	Input-Output Relation Analysis	93
6.5	Experiments	96
6.5.1	Proposed Motor Prototype	96
6.5.2	Experiments	98
6.5.3	Discussion	100
6.6	Summary	101
Chapter 7	Reducer-Integrated Motor Using Inscribed Planetary Gear Mechanism	103
7.1	Introduction	103
7.2	New Reducer-Integrated Motor	104
7.2.1	Principle of an Inscribed Planetary Gear	104
7.2.2	Proposed New Reducer-Integrated Motor	104
7.3	Conditions for Engagement of Gear Pairs	106
7.3.1	Engagement Equations for Two Kinds of External and Internal Gear Pairs	107
7.3.2	Conditions for Tooth Engagement of Each Gear Pair	108
7.4	Design Method for the Reducer Part	111
7.4.1	Design Policy	111
7.4.2	Strength Analysis	113
7.4.3	Design Algorithm	116
7.5	Experiments	117
7.5.1	Proposed Motor Prototype	118
7.5.2	Experimental Results and Discussion	119
7.5.3	Evaluation of the Load-Supporting Capacity	120
7.6	Summary	121
Chapter 8	Conclusions and Future Work	123
8.1	Conclusions	123
8.2	Future Work	125
	References	127
	List of Publications	133

Chapter 1

General Introduction

1.1 Locomotion Mechanisms

Locomotion devices such as vehicles and mobile robots are now widely used to transport people and goods efficiently. There exist various locomotion mechanisms to meet various applications. The wheeled mechanism is the most popular one, which participates in the fields of not only industry and logistics [1]–[3] but also other services. Wheeled robots take over hard work in agriculture and forestry [4]–[6] and dangerous tasks in a harsh environment with height [7], chemicals [8], or outer space [9]. These days, it is also expected to support people in welfare and nursing care [10]–[12] and even at home [13]–[15].

Another well-known style of locomotion is a legged mechanism similar to humans, animals, and insects. Step-by-step walking enables the robots to travel over uneven terrain [16], [17]. This mechanism brings harmonization with a human environment, for example, in going up stairs and climbing up ladders [18], [19]. In vehicles and mobile robots, the tracked mechanism is also often used. Tracks, or crawlers, has such a large ground contacting surface that can run through rough terrain [20], [21]. Its excellent traveling performance is useful for rescue [22]–[24]. Recently, slithering locomotion of snakes has been studied and several snake-like robots have been developed. Their slender forms are very suitable for in-pipe inspection [25], [26]. They are engaged in urban search in a disaster situation [27], [28]. The hybrid mechanisms have also been proposed: wheel-leg [29], [30], track-wheel [31], leg-track [32], and leg-track-wheel [33].

This research focuses on the wheeled mechanism. Compared to other mechanisms, wheels have advantages in high energy efficiency and high-speed runnability thanks to their simple structure and motion [34]. High stability is also achieved just by using three or more wheels. Furthermore, control complexity is low in general usage. These characteristics will meet people requirements in many situations, which may result in the extremely widespread use of the wheeled mechanisms as described above.

In the wheel mechanisms, however, there exists a disadvantage in mobility when compared to other mechanisms. Regarding the legged locomotion, the robots are basically able to move in any direction immediately. On the other hand, the wheeled vehicles cannot move sideways unless the

wheels slip; they need a switchback or a turnabout, which costs extra time and space. This problem disrupts efficient motion in places with a limited space or many obstacles.

1.2 Conventional Omnidirectional Wheeled Mechanisms

To solve the problem in the limited mobility of the wheeled vehicles, omnidirectional wheeled mechanisms, which enable the vehicle to move in any direction at any time, have been proposed. A primary example is omni wheels [Fig. 1.1(a)]. The omni wheel has a disk shape like a normal wheel but has outer rollers around the circumference that rotate passively. When the main body of the wheel is driven by a motor, the omni wheel can move actively in one direction. When the omni wheel receives a side force, it moves passively in another direction because of the passive rotation of the outer rollers. By using three or more omni wheels, an omnidirectional vehicle can be constructed [35]–[38].

As another example, omnidirectional mobile robots using spherical wheels [Fig. 1.1(b)] have been proposed [39]–[42]. The spherical wheel has a ball whose radius is constant in all directions and rotates it in the desired direction by using multiple rollers driven by motors. Then, the wheel can move actively in an arbitrary direction.

Caster wheels that move passively are often used in office chairs or handcarts. By driving them actively by motors, omnidirectional mobile robots are realized [43]–[46]. The caster wheel [Fig. 1.1(c)] has two rotation axes, i.e., an axle of the wheel and a steering axis, with an offset between them. When the wheel axle is driven, the caster wheel moves in one direction. When the steering axis is driven, the caster wheel moves sideways with rotation. By using two or more caster wheels and controlling the rotation of these axes appropriately, the robot can move in a desired direction at any time.

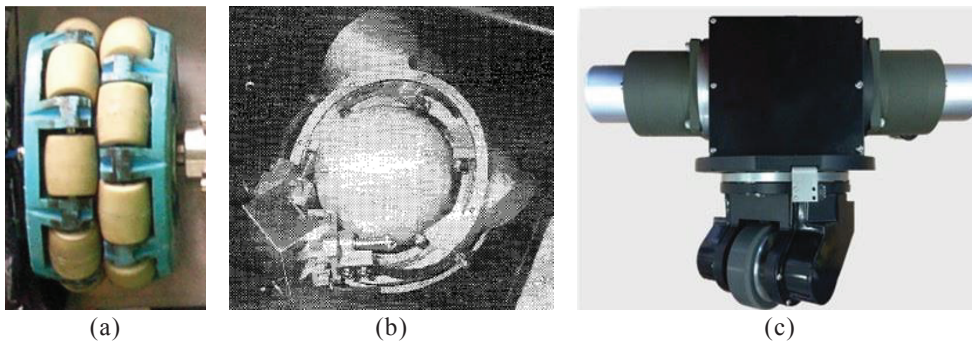


Fig. 1.1

Examples of omnidirectional wheeled mechanisms: (a) Omni wheel [37], (b) Spherical wheel [42], and (c) Caster wheel [44].

These omnidirectional wheeled mechanisms help locomotion devices to acquire omnidirectional mobility equivalent to legged mechanisms. However, some troubles arise resulting from their structure. For example, the complicated structure of the omni wheels leads to low load capacity and vertical vibration. The redundant motors of the spherical wheels and caster wheels cause excessive constraints requiring accurate motor control, otherwise, wheel slippage or undesirable force will occur.

1.3 Motors

Motors play important roles in the performance of wheeled locomotion devices. This is also true in various robots and machines. The most popular type of motor is the electromagnetic motor combined with a reducer, which can realize high-precision positioning and large torque output. However, the conventional reducers that utilize spur gear pairs or planetary gear mechanisms tend to be large, because they need multistage gear trains to gain high reduction ratios. This makes the entire motor system large, and consequently, the robots and machines using motors large. In the wheeled vehicles, this problem leads to reducing movement efficiency and increasing the risk of collisions.

To address this issue, compact reducer mechanisms, which realize high reduction ratio in a single stage, have been developed. For example, a strain wave gearing system [47] [Fig. 1.2(a)] and an inscribed planetary gear, or a KHV planetary gear, mechanism [48]–[51] [Fig. 1.2(b)] achieve downsizing by using the principle of differential gears. These reducers have an external and internal gear pair with a small difference in teeth number and output the relative rotation between them. The strain wave gearing system cause the relative rotation by rotating an oval cam inside the external gear and deforming the elastic external gear, while the inscribed planetary gear does it by revolving either of the external or internal gear. Besides these mechanisms, other reducers, including a ball reducer [52] and a cycloidal reducer [53], are proposed.

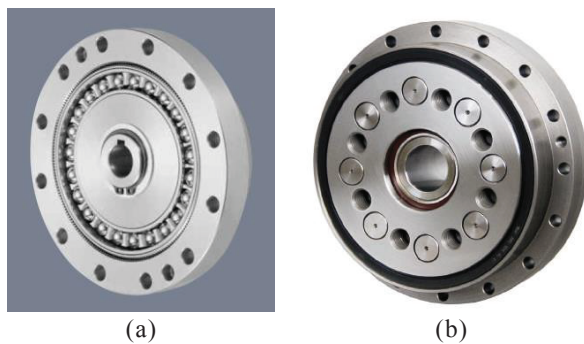


Fig. 1.2
Examples of compact reducer mechanisms: (a) Strain wave gearing system [74] (b) Inscribed planetary gear mechanism [85].

Indeed these reducers have a great effect on the downsizing of a reducer mechanism itself. Considering the whole motor system, however, its effect is limited because an electromagnetic motor is typically as large as the conventional reducers using multistage spur gears. Therefore, a progressive solution is still required.

1.4 Aims and Approach

This research aims to solve the abovementioned problems of wheeled mechanisms and motors. Through analyses, syntheses, and experiments, novel omnidirectional mobile mechanisms and integrated motor mechanisms are proposed based on the theory of mechanisms and kinematics of machinery.

Regarding the omnidirectional mobile mechanisms, this research offers two types of solutions. One approach is the proposition of a novel wheeled mechanism using a normal wheel. Here, an omnidirectional wheeled mechanism with a simple structure and non-excessive constraints is synthesized through the analysis of the existing wheeled mechanisms. Another approach develops new technologies for the wheeled mechanism with outer rollers to solve its concerns. The wheeled mechanism is made advanced in its structure for the sake of low vibration and in the transmission mechanism to reduce the number of motors. With respect to the motor mechanisms, the motor system miniaturization is achieved by integrating the motor and reducer with the aid of differential gears and simultaneous tooth engagement. The effectiveness of each of the proposed mechanisms is evaluated through experiments.

1.5 Thesis Overview

The thesis consists of eight chapters.

Chapter 1 introduces a background of this research and points out problems in the current situation for wheeled locomotion devices. The purpose and strategy of the thesis are described.

Chapter 2 starts with the analysis of every kind of wheeled mechanisms, and then, the generalized kinematic conditions for omnidirectional movement are extracted. By using the conditions, a novel wheeled mechanism is synthesized and an omnidirectional mobile robot with it is proposed. Through the analysis and experiments, the effectiveness of the proposed mechanism is verified.

Chapter 3 is for designing a controller to stabilize the mobile robot proposed in Chapter 2. Considering the conditions, feedback linearization based on the kinematics is applied. Simulations and experiments have been carried out.

Chapter 4 presents another omnidirectional wheeled mechanism by improving the mechanism proposed in the past to overcome the structural problems in vibration and the number of parts. How the proposed wheel and the vehicle using it should be designed is discussed with developing the prototype.

Chapter 5 deals with the transmission mechanism for one omnidirectional wheeled mechanism. Based on the characteristics of the wheeled mechanism, the transmission mechanism with switchable transmission paths is synthesized. The prototype vehicle can move in the longitudinal and lateral directions and turn with a smaller number of motors than usual.

Chapter 6 includes the proposition of a novel reducer-integrated motor using the principle of a strain wave gearing system. The prototype is developed based on the condition for avoiding tooth tips interference and experimental tests are conducted to compare several movement patterns.

Chapter 7 provides another reducer-integrated motor to make the design flexibility higher than the motor proposed in Chapter 6. The analysis clarified the design conditions and design algorithm. The experiment verifies its effectiveness.

Chapter 8 is a summarization of the conclusions in each chapter and discussions on the perspective in future work.

Chapter 2

Wheeled Mechanism Analysis and a Novel Omnidirectional Mechanism ‘Slidable Wheel’

2.1 Wheeled Mechanisms

Many kinds of wheeled mechanisms have been developed and used. Here, each of them is explained in detail.

Fixed wheel [Fig. 2.1(a)] is the most standard type of wheel that can rotate around the axle and move actively in one direction. It can turn around the ground contact point passively. The structure is simple and small. However, its mobility is limited because the wheel cannot move sideways.

Two fixed wheels cause constraint on the vehicle in general, so they should be arranged in parallel.

Orientable wheel [Fig. 2.1(b)] has another rotation axis to change its direction actively. This function enables the wheel to move in an arbitrary direction. Note that the direction that the wheel can move immediately is always limited, so that omnidirectional movement is not achieved.

Omni wheel [35]–[38] [Fig. 2.1(c)] brings omnidirectional mobility to the mobile robot thanks to the outer rollers that can rotate freely. Its structure is so complicated that there are concerns about low load capacity and mechanical vibration. The complexity of the control method seems as low as that of a fixed wheel. Even three omni wheels never cause overconstraint situation, unlike in the

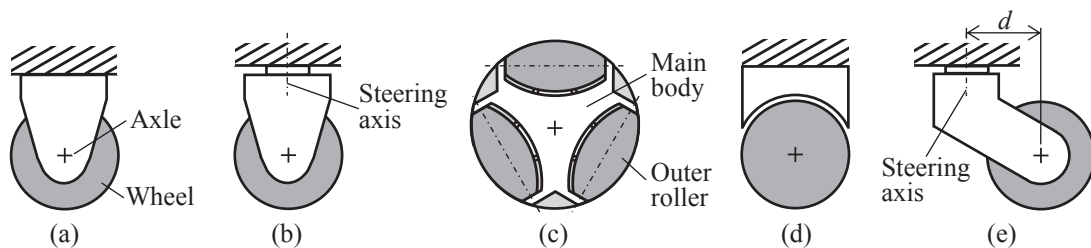


Fig. 2.1

Varieties of wheel mechanisms. (a) Fixed wheel, where only the wheel itself can rotate around its axle. (b) Orientable wheel, where the wheel can both rotate and turn around the steering axis passing through the ground contact point. (c) Omni wheel, which can move in a direction perpendicular to the wheel plane by using free rollers along the circumference. (d) Spherical wheel, which has a ball-shaped wheel that is rotatable in any direction. (e) Caster wheel, which has an offset between the axle of the wheel and the steering axis.

fixed wheels. Singular configuration, where the robot motion becomes uncontrollable, can occur depending on the wheel arrangement, e.g., when three or more omni wheels are arranged in parallel, but is avoidable. The two-layered structure is often used to compensate the gaps in between the outer rollers. The radius of the outer rollers should be smaller than that of the wheel main body.

Mecanum wheel [54]–[57] has a structure similar to that of the omni wheel, but the axes of the outer rollers are tilted away from the wheel plane. This structure expands the flexibility for arranging the wheels because the wheels can put in parallel by adjusting the tilting angles of the outer rollers.

Orthogonal wheel [58]–[61] utilizes two parallel sphere-like-shaped rollers rotating actively in one direction and passively in the perpendicular direction. Its movement principle is similar to those of the omni wheels. The number of parts can be kept smaller but the size becomes larger compared to the omni wheel.

Active Omni Wheel (AOW) [62] has almost the same appearance as the omni wheels, but both of the wheel main body and outer rollers can be rotated actively, so that passive joints are eliminated. The two degree-of-freedom (DOF) movement is easily decoupled in the motion control. Two AOWs cause motor-redundancy and then an overconstraint-like situation occurs where the motors have to drive in harmony with high accuracy.

Spherical wheel [39]–[42] [Fig. 2.1(d)] usually provide two DOF movement as well as the AOW. Its isotropy in structure leads to high surmountability in every direction, but also to large occupation volume. The rollers driving a ball wheel bring technical challenge because the roller should transmit its rotation in one direction but must not in another direction. A spherical wheel with one active DOF and one passive DOF has been also proposed [63], which needs to handle singularity.

Caster wheel [43]–[46] [Fig. 2.1(e)] also has basically two active DOFs because a caster wheel with one active DOF causes the unacceptable singular condition [64]. Both structural simplicity and superb mobility are established because the omnidirectional movement is achieved with normal wheels. Controlling the caster wheel is relatively difficult, and moreover, overconstraint occurs in two or more two-DOF caster wheels.

To make the robot attitude stable, three or more wheels are required in principle. With respect to climbing steps or running through uneven terrain, it is preferable that all the wheels are actively driven. Following the requirement of using three active wheels without overconstraint, the omni, Mecanum, orthogonal, and spherical wheels with one active DOF become candidates. However, it can be difficult for these wheeled mechanisms to move precisely because resistance as a result of friction and viscosity that change depending on the conditions disturbs the passive rotation of the rollers in the wheels. Furthermore, these mechanisms are relatively complicated in structure.

Another way to avoid the constraint in motor redundancy is to share the motors among the wheels with two active DOFs (e.g., in spherical wheels [65] and in caster wheels [66], [67]). However, it is accompanied by complicated transmission mechanisms and limited design flexibility.

According to the aforementioned concerns, the wheeled mechanism with high mobility, simple structure, active drive, and fewer constraints will be useful. Based on this situation, the analysis and synthesis of wheeled mechanisms are carried out to propose a novel mechanism.

2.2 Analysis and Synthesis of Wheeled Mechanisms

2.2.1 Kinematic analysis of Wheeled Mechanisms

First, the existing wheeled mechanisms are analyzed. In order to discuss the kinematics common to all types of wheels, including the fixed wheel, orientable wheel, omni wheel, spherical wheel, and caster wheel, the motion of a mobile robot with each wheel type is considered. As shown in Fig. 2.2(a), the global reference frame $O-X_I Y_I$ is fixed on the floor. The pose of the local reference frame $P-X_R Y_R$ in the global frame is represented by a vector ξ , as follows.

$$\xi = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \quad (2.1)$$

In the local reference frame, as shown in Fig. 2.2(b), the wheel of the mobile robot is located at a distance l from the origin P and at an angle α from X_R . The axis of the wheel is tilted at an angle β from a line passing through P and the center of the wheel. Parameters $\dot{g}$ and $\dot{f}$ denote the velocity of the wheel in the direction parallel to the wheel plane and the direction perpendicular to it, respectively. Then, the kinematic equations for these directions are represented as follows:

$$[\sin(\alpha + \beta) \quad -\cos(\alpha + \beta) \quad -l \cos \beta] R(\theta) \dot{\xi} + \dot{g} = 0 \quad (2.2)$$

$$[\cos(\alpha + \beta) \quad \sin(\alpha + \beta) \quad l \sin \beta] R(\theta) \dot{\xi} - \dot{f} = 0 \quad (2.3)$$

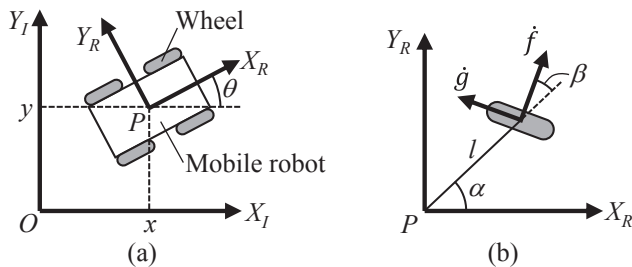


Fig. 2.2

Reference frames for kinematic analysis of a mobile robot. (a) Position and orientation of the mobile robot in the global reference frame. (b) Position and orientation of the wheel in the local reference frame.

where the rotation matrix $R(\theta)$ transforms the vector in the global reference frame to the vector in the local frame.

$$R(\theta) = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2.4)$$

When the wheel is a fixed wheel [Fig. 2.1(a)] or an orientable wheel [Fig. 2.1(b)] with radius r and rotation angle φ , the kinematic equations of Eqs. (2.2) and (2.3) are given as follows:

$$[\sin(\alpha + \beta) \quad -\cos(\alpha + \beta) \quad -l \cos \beta] R(\theta) \dot{\xi} + r \dot{\varphi} = 0 \quad (2.5)$$

$$[\cos(\alpha + \beta) \quad \sin(\alpha + \beta) \quad l \sin \beta] R(\theta) \dot{\xi} = 0 \quad (2.6)$$

The velocity of the wheel is $\dot{g} = r \dot{\varphi}$ in Eq. (2.2). Because the rotational velocity $\dot{\varphi}$ is arbitrary, Eq. (2.5) is satisfied at any velocity $\dot{\xi}$. On the other hand, the constraint that the wheel does not skid leads to $\dot{f} = 0$ in Eq. (2.3). Then, Eq. (2.6) becomes a nonholonomic constraint, i.e., the components of $\dot{\xi}$ are subordinate to each other, and the mobile robot is no longer able to move at an arbitrary velocity with three-DOF planar motion. Therefore, robots with fixed or orientable wheels are nonholonomic, and not omnidirectional.

In contrast, if both $\dot{g}$ and $\dot{f}$ can take arbitrary values, neither Eq. (2.2) nor (2.3) yields nonholonomic constraints for $\dot{\xi}$. The equations are established at any $\dot{\xi}$ by substituting appropriate values for $\dot{g}$ and $\dot{f}$. A mobile robot whose wheeled mechanisms satisfy these conditions and provide three or more active DOFs can be an omnidirectional mobile robot and move immediately in an arbitrary direction.

In the first place, regarding the omnidirectional mobile robot, let us think a robot with omni wheels [Fig. 2.1(c)]. Given the radius of the outer rollers r_r and the rotation angle of the outer rollers φ_r , the kinematic equations equivalent to Eqs. (2.2) and (2.3) are as follows:

$$[\sin(\alpha + \beta) \quad -\cos(\alpha + \beta) \quad -l \cos \beta] R(\theta) \dot{\xi} + r \dot{\varphi} = 0 \quad (2.7)$$

$$[\cos(\alpha + \beta) \quad \sin(\alpha + \beta) \quad l \sin \beta] R(\theta) \dot{\xi} - r_r \dot{\varphi}_r = 0 \quad (2.8)$$

In Eqs. (2.7) and (2.8), $\dot{g} = r \dot{\varphi}$ and $\dot{f} = r_r \dot{\varphi}_r$ are substituted for Eqs. (2.2) and (2.3). The value of $\dot{\varphi}_r$ is arbitrary because the outer rollers of the omni wheel can rotate freely. Therefore, the appropriate values of $\dot{\varphi}$ and $\dot{\varphi}_r$ establish Eqs. (2.7) and (2.8) at any $\dot{\xi}$. Thus, a robot with three or more omni wheels can be an omnidirectional mobile robot.

In the Mecanum wheel, the rotation axis of the outer rollers is tilted from the wheel plane. Thus, the rotation of the outer rollers produces the velocity which includes the components of both the directions parallel to the wheel plane and perpendicular to it. As a result, $\dot{g}$ includes $\dot{\varphi}$ and $\dot{\varphi}_r$, and $\dot{f}$ includes $\dot{\varphi}_r$. In any case, the kinematic equations for the Mecanum wheel are satisfied

at any $\dot{\xi}$ with the appropriate $\dot{\varphi}$ and $\dot{\varphi}_r$ as in the omni wheel. The orthogonal wheel has the same kinematic equations as the omni wheel.

In the spherical wheel [Fig. 2.1(d)], the kinematic equations are

$$[\sin(\alpha + \beta) \quad -\cos(\alpha + \beta) \quad -l \cos \beta]R(\theta)\dot{\xi} + r\dot{\varphi} = 0 \quad (2.9)$$

$$[\cos(\alpha + \beta) \quad \sin(\alpha + \beta) \quad l \sin \beta]R(\theta)\dot{\xi} - r\dot{\varphi}_s = 0 \quad (2.10)$$

where φ_s is the rotation angle of the wheel perpendicular to φ . The same discussion is held as the omni wheel, and $\dot{\xi}$ can take an arbitrary value. In the case of the spherical wheel with two active DOFs, we can choose the value of β arbitrarily because the wheel is completely symmetrical.

The caster wheel [Fig. 2.1(e)] has an offset distance d between the ground contact point and the steering axis. Caster wheels rotate around the vertical axis, and so β is variable. Eqs. (2.2) and (2.3) are written specifically as follows [68], [69]:

$$[\sin(\alpha + \beta) \quad -\cos(\alpha + \beta) \quad -l \cos \beta]R(\theta)\dot{\xi} + r\dot{\varphi} = 0 \quad (2.11)$$

$$[\cos(\alpha + \beta) \quad \sin(\alpha + \beta) \quad d + l \sin \beta]R(\theta)\dot{\xi} + d\dot{\beta} = 0 \quad (2.12)$$

In comparison with Eqs. (2.2) and (2.3), $\dot{g} = r\dot{\varphi}$ and $\dot{f} = -d(\dot{\beta} + \dot{\theta})$ are given. Because $\dot{\theta}$ in $\dot{f}$ is a component of $\dot{\xi}$, $\dot{\xi}$ is arbitrary in Eqs. (2.11) and (2.12) if the steering angular velocity $\dot{\beta}$ is arbitrary.

2.2.2 Synthesis of a Slidable Wheel

As shown in the previous section, the generalized kinematic conditions for omnidirectional movement is that $\dot{g}$ and $\dot{f}$ can take arbitrary values so that Eqs. (2.2) and (2.3) are established at any $\dot{\xi}$. In general, $\dot{g}$ includes $\dot{\varphi}$ and then $\dot{g}$ becomes arbitrary whenever the wheel can rotate around its axle at an arbitrary velocity. Therefore, the mechanism to allow $\dot{f}$ to take an arbitrary value should be considered.

The wheeled mechanisms described in the previous section utilize the rotation of outer rollers $\dot{\varphi}_r$ (in the omni wheel), the rotation of the wheel in another direction $\dot{\varphi}_s$ (in the spherical wheel), or the rotation around the steering axis $\dot{\beta}$ (in the caster wheel) to meet the condition for $\dot{f}$. This research synthesizes a novel wheeled mechanism for omnidirectional movement by adopting other parameters as $\dot{f}$ instead of them. The use of a normal wheel is assumed in order to keep the structure simple. Here, one of the two parameters in Fig. 2.2(b), α or l , can be used: if α is chosen as a variable, however, the effect of $\dot{\alpha}$ in $\dot{f}$ will be close to that of $\dot{\beta}$ in the caster wheel. Then, let us focus on a wheeled mechanism in which l is a variable and $\dot{l}$ is used for $\dot{f}$.

A slidable wheel, whose sliding joint enables l to vary, is proposed. Fig. 2.3 shows the schematic structure and parameters for the slidable wheel. The rail of the sliding joint is located at

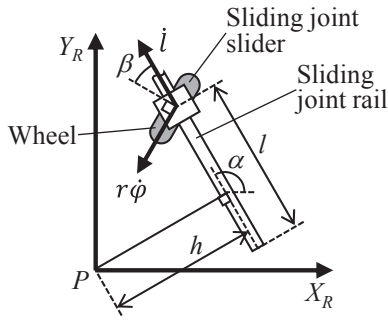


Fig. 2.3

Proposed wheel mechanism using a sliding joint, slidable wheel, and its parameters.

a distance h from P , and tilted at an angle α from X_R . The normal wheel can move passively along the sliding joint relative to the rail. The wheel is at distance l on the sliding joint from the foot of the perpendicular from P . The angle between the wheel axle and the sliding joint rail is β , whose value is either fixed or changed actively. Then, the kinematic equations equivalent to Eqs. (2.2) and (2.3) are as follows:

$$[\sin(\alpha + \beta) \quad -\cos(\alpha + \beta) \quad h \sin \beta - l \cos \beta]R(\theta)\dot{\xi} - r\dot{\phi} + \dot{l} \sin \beta = 0 \quad (2.13)$$

$$[\cos(\alpha + \beta) \quad \sin(\alpha + \beta) \quad h \cos \beta + l \sin \beta]R(\theta)\dot{\xi} + \dot{l} \cos \beta = 0 \quad (2.14)$$

In Eqs. (2.13) and (2.14), $\dot{g} = r\dot{\phi} - \dot{l} \sin \beta - h\dot{\theta} \sin \beta$ and $\dot{f} = \dot{l} \cos \beta + h\dot{\theta} \cos \beta$ are given. By considering $\dot{l}$ as arbitrary, Eqs. (2.13) and (2.14) are applicable to any $\dot{\xi}$ under the condition $\cos \beta \neq 0$. Omnidirectional mobile robots can be realized by using three or more slidable wheels.

Note that the condition $\cos \beta \neq 0$, namely $\beta_i = \pm\pi/2$, corresponds to a singular configuration, where Eq. (2.14) causes a nonholonomic constraint for $\dot{\xi}$. In this situation, the movement direction of the wheel is parallel to the rail of the sliding joint, and the robot no longer can move in the direction perpendicular to the wheel plane. This singular configuration restricts the movable direction of the robot, but does not make the robot states out of control and $\dot{\xi}$ can be decided uniquely. In other words, this singular configuration is unfavorable but acceptable to the mobile robot. The unacceptable singular configuration will occur if the rails of the slidable wheels in a robot are all parallel, but this situation is avoidable in the design.

2.2.3 Comparisons of Wheeled Mechanisms

The proposed slidable wheel is compared with other wheeled mechanisms. Table 2.1 summarizes the characteristics of the wheeled mechanisms as explained above. First of all, the slidable wheel can make omnidirectional movement by using a normal wheel, as well as the caster wheel. This means that the slidable wheel establishes both superb mobility and structural simplicity. The other omnidirectional wheeled mechanisms with two active DOFs, such as the AOW, spherical wheel,

Table 2.1
Characteristics of wheeled mechanisms.

	Fixed wheel		Orientable wheel		Omni wheel		Spherical wheel		Caster wheel		Slidable wheel
	1 DOF	0 DOF	2 DOF	0 DOF	1 DOF	0 DOF	1 DOF	0 DOF	1 DOF	0 DOF	
Active	1 DOF	0 DOF	2 DOF	0 DOF	1 DOF	0 DOF	1 DOF	0 DOF	1 DOF	0 DOF	2 DOF
Passive	0 DOF	1 DOF	0 DOF	1 DOF	1 DOF	0 DOF	1 DOF	0 DOF	1 DOF	0 DOF	1 DOF
Mobility	Impossible to move sideways	Low	Unlimited but non-immediate	Low	Omni-directional	High	Omni-directional	Medium/high	Omni-directional	Low	Omni-directional
Structural complexity	Low	Low	Low	Low	High	High	Medium/high	Medium	Low	Low	Low
Over-constraint	By two or more wheels	By two or more wheels	By two or more wheels	By two or more wheels	By four or more wheels	By two or more wheels	By four or more wheels	By two or more wheels	By four or more wheels	By two or more wheels	By four or more wheels
Passive elements	Not included	Not included	Not included	Not included	Included and uncontrollable	Not included	Included and uncontrollable	Not included	Included and uncontrollable	Not included	Included but controllable
Control complexity	Low	Low	Low	Low	Low	Low	Low	Relatively low	Relatively high	Relatively high	High
Singular configuration	-	-	-	-	Avoidable	Never	Possible but acceptable	Never	Possible and unacceptable	Never	Possible but acceptable
Structural isotropy	Longitudinal and lateral	Longitudinal and lateral	Longitudinal and lateral	Longitudinal and lateral	Longitudinal	Longitudinal	All-directional	All-directional	Lateral	Lateral	Longitudinal and lateral
Occupation volume	Small	Small	Small	Small	Medium	Medium	Large	Large	Small/medium	Small/medium	Small/medium

and caster wheel, causes overconstraint-like situation by using at least two wheels, which cannot keep the robot standing statically by themselves. On the other hand, three slidable wheels do not bring the constraint due to the passive sliding joints. Passive elements may disturb the motion if uncontrollable, like in the omni wheel and spherical wheel with one active DOF, but the sliding joint movement in the slidable wheel is controllable through steering the wheel actively, as described in the next section. To control the sliding joint makes the control system somewhat complicated, which is discussed in Chapter 3. The slidable wheel has a singular configuration, but it is acceptable as explained above, unlike the caster wheel with one active DOF. The structure of the slidable wheel is practically the orientable wheel with a slider, so that the structural isotropy is inferior to that of the spherical wheel but an advantage in volume remains. By steering the wheel, the slidable wheel can face to steps or obstacles and use its maximum radius to surmount them, and therefore the low isotropy may not be a serious disadvantage.

Thus, the slidable wheel assumes characteristics different from those of the other wheeled mechanisms, especially in its simple structure and less constraint.

2.3 Kinematic Analysis of Omnidirectional Mobile Robot Using Slidable Wheels

To show the usage of the proposed slidable wheel concretely, a novel omnidirectional mobile robot with three slidable wheels is proposed.

2.3.1 Slidable Wheeled Omnidirectional Mobile Robot (SWOM)

The robot, named slidable-wheeled omnidirectional mobile robot (SWOM), is shown in Fig. 2.4. The main body of SWOM consists of a base block and three rails of sliding joints. The three rails are fixed to the base block at equal intervals. Each driving unit of SWOM is like an orientable wheel with a slider of the sliding joint. The wheel rotates actively around both the axle and the ground contact point by the driving motor and the steering motor, respectively. The sliding joint is

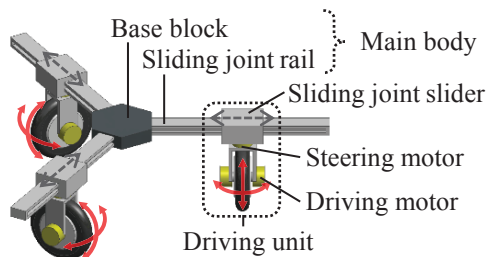


Fig. 2.4
Proposed omnidirectional mobile robot, SWOM.

a passive joint that moves depending on the driving unit movement, which changes the distance between the base block and the driving unit.

Next, the omnidirectional movement of SWOM is considered. Fig. 2.5 schematically shows the motion of SWOM: (a) forward translation, (b) right translation, (c) diagonal translation, and (d) turning. In this figure, although the direction of each wheel is always perpendicular to the rail of the sliding joint, SWOM moves in any direction. Each driving unit using a normal wheel can move in a direction parallel to the wheel plane and steer the wheel, but cannot move in the direction perpendicular to the wheel plane. However, the relative movements of the sliding joints enable the main body to move even in the direction perpendicular to each wheel plane. As a result, the main body is capable of three-DOF planar motion by rotating the wheels at appropriate velocities. Additionally, the moving direction of the main body can change immediately without adjusting the directions of the wheels because the steering angle of each wheel is constant in every movement shown in Fig. 2.5. Thus, SWOM achieves omnidirectional movement.

Another moving situation is shown in Fig. 2.6, in which the steering angles of the wheels are changed according to the movement direction of the main body. The movement direction of each wheel is the same as that of the main body during the translation, such as in (a) forward translation, (b) right translation, and (c) diagonal translation, or the circumference direction of the rotation in (d) turning. In this way, SWOM can move in each direction without relative movements of the sliding joints. Considering the movements in Fig. 2.5, the omnidirectional movement of SWOM basically depends on the relative movements of the sliding joints. This means that the reachable region of SWOM is limited to the region where the relative movements are feasible, i.e., where the sliders of the sliding joints are away from the ends of the rails. On the other hand, in Fig. 2.6, the movable range of the sliding joints does not restrict the reachable region of SWOM, and so SWOM can continue with movement unlimitedly. Note that SWOM cannot switch the motion in Fig. 2.6 immediately because it is necessary to steer the wheels.

The situation between Fig. 2.5 and Fig. 2.6 should be discussed. It means that the DOFs of steering the wheels enables SWOM to manipulate the relative movement of the sliding joints in addition to the motion of the main body. Fig. 2.7 shows the case where SWOM is moving forward. As shown in Fig. 2.7(a), when the traveling directions of the wheels are set inward to the moving direction of the main body (i.e., forward direction), the sliding joints move toward the base block while the main body moves forward. In contrast, when the traveling directions of the wheels are set outward, as shown in Fig. 2.7(b), the sliding joints move away from the base block. When the traveling directions of the wheels agree with the moving direction of the main body, as shown in Fig. 2.7(c), the main body moves but the sliding joints do not. Thus, SWOM can control the relative position of the sliding joints by steering each wheel appropriately while the main body

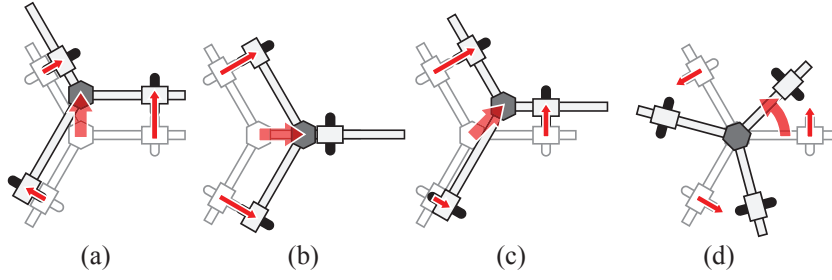


Fig. 2.5

Omnidirectional movement of SWOM using the relative movements of the sliding joints: (a) Forward translation. (b) Right translation. (c) Diagonal translation. (d) Turning. The upward direction in the figure is the forward direction.

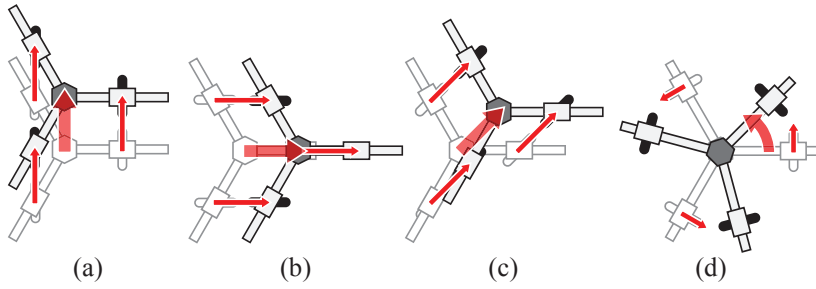


Fig. 2.6

Movement of SWOM when the movement direction of each wheel is changed depending on the movement direction of the main body. (a) Forward translation. (b) Right translation. (c) Diagonal translation. (d) Turning. The upward direction in the figure is the forward direction.

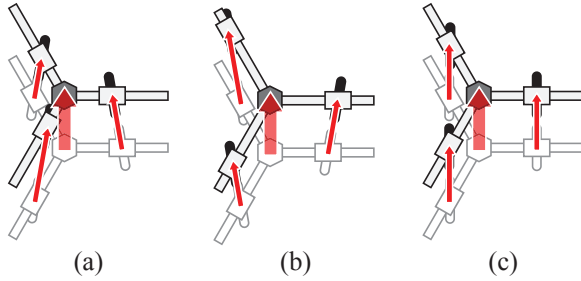


Fig. 2.7

Differences in the relative movements of the sliding joints depending on the steering angle of each wheel when SWOM is moving forward (toward the top of the figure). (a) Inward slider displacement. (b) Outward slider displacement. (c) No slider displacement.

moves along the desired trajectory. When the relative positions of the sliding joints are controlled within a certain range, SWOM can move without restriction, even with finite rail lengths.

2.3.2 Kinematics of SWOM

Fig. 2.8 shows the coordinate systems for the kinematic analysis of SWOM. The reference frames are basically the same as those in Fig. 2.2, except the origin of the local reference frame P is on the center of the base block, and X_R is parallel to one of the sliding joints. The driving unit in the

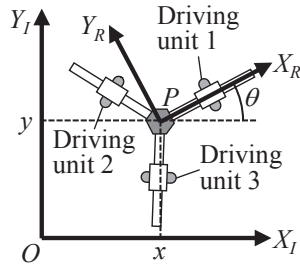


Fig. 2.8
SWOM and reference frames for kinematic analysis.

sliding joint parallel to X_R is assigned as driving unit 1, and the others as driving units 2 and 3 counterclockwise.

First, the kinematics is explained by pairing a sliding joint and a driving unit. By substituting $h = 0$ in Eqs. (2.13) and (2.14), we obtain the following six conditional expressions, where subscript i represents the parameter belonging to driving unit i .

$$[\sin(\alpha_i + \beta_i) \quad -\cos(\alpha_i + \beta_i) \quad -l_i \cos \beta_i] R(\theta) \dot{\xi} + r \dot{\phi}_i + \dot{l}_i \sin \beta_i = 0 \quad (2.15)$$

$$[\cos(\alpha_i + \beta_i) \quad \sin(\alpha_i + \beta_i) \quad l_i \sin \beta_i] R(\theta) \dot{\xi} + \dot{l}_i \cos \beta_i = 0 \quad (2.16)$$

Here, $\alpha_1 = 0$, $\alpha_2 = 2\pi/3$, and $\alpha_3 = 4\pi/3$ are constants. If the rotational velocity of the wheel $\dot{\phi}_i$ and the relative velocity of the driving unit on the sliding joint $\dot{l}_i$ are arbitrary, Eqs. (2.15) and (2.16) give zero nonholonomic constraints for $\dot{\xi}$ under the condition $\beta_i \neq \pm\pi/2$. This proves theoretically that SWOM can move at an arbitrary velocity with regard to three-DOF planar motion and can perform as an omnidirectional mobile robot.

By eliminating $\dot{l}_i$ from Eqs. (2.15) and (2.16), the following equation is obtained.

$$\begin{bmatrix} -\sin \alpha_1 & \cos \alpha_1 & l_1 \\ -\sin \alpha_2 & \cos \alpha_2 & l_2 \\ -\sin \alpha_3 & \cos \alpha_3 & l_3 \end{bmatrix} R(\theta) \dot{\xi} - \begin{bmatrix} r \cos \beta_1 & 0 & 0 \\ 0 & r \cos \beta_2 & 0 \\ 0 & 0 & r \cos \beta_3 \end{bmatrix} \begin{bmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \\ \dot{\phi}_3 \end{bmatrix} = 0 \quad (2.17)$$

The forward and inverse kinematics are represented by Eqs. (2.18) and (2.19), respectively, when the input is the velocities of the driving units and the output is the velocity of the main body. Here, $\beta_i = \pm\pi/2$ is excluded in (2.19).

$$\dot{\xi} = R(\theta)^{-1} \begin{bmatrix} -\sin \alpha_1 & \cos \alpha_1 & l_1 \\ -\sin \alpha_2 & \cos \alpha_2 & l_2 \\ -\sin \alpha_3 & \cos \alpha_3 & l_3 \end{bmatrix}^{-1} \begin{bmatrix} r \cos \beta_1 & 0 & 0 \\ 0 & r \cos \beta_2 & 0 \\ 0 & 0 & r \cos \beta_3 \end{bmatrix} \begin{bmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \\ \dot{\phi}_3 \end{bmatrix} \quad (2.18)$$

$$\begin{bmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \\ \dot{\phi}_3 \end{bmatrix} = \begin{bmatrix} r \cos \beta_1 & 0 & 0 \\ 0 & r \cos \beta_2 & 0 \\ 0 & 0 & r \cos \beta_3 \end{bmatrix}^{-1} \begin{bmatrix} -\sin \alpha_1 & \cos \alpha_1 & l_1 \\ -\sin \alpha_2 & \cos \alpha_2 & l_2 \\ -\sin \alpha_3 & \cos \alpha_3 & l_3 \end{bmatrix} R(\theta) \dot{\xi} \quad (2.19)$$

SWOM falls into the singular configuration when the movement direction of the wheel is parallel to the rail of the sliding joint, as explained in Section 2.2.2. This is because the motion of the driving unit is never transmitted to the main body through the sliding joint. When driving unit i causes the singular configuration, $\beta_i = \pm\pi/2$, Eq. (2.16) gives a nonholonomic constraint for $\dot{\xi}$. Every driving unit with $\beta_i = \pm\pi/2$ generates this constraint, and reduces the DOFs of the main body by one DOF, so that omnidirectional movement is no longer feasible. For example, when driving unit 1 is in the singular configuration, the DOFs of SWOM is two: translation in the X_R direction and rotation around driving unit 1. When two driving units fall into the singular configuration at the same time, SWOM motion is only one-DOF rotation around a certain point. Thus, the movable direction is restricted in the singular configuration, but SWOM can move in the accepted directions. When all the driving units are in the singular configuration, SWOM cannot move at all. However, SWOM can avoid the singular configuration just by steering the driving units slightly from $\beta_i = \pm\pi/2$.

2.3.3 Reachable Region under Fixed Steering Angles

The reachable region of SWOM is limited by the movable range of the sliding joints when SWOM makes an omnidirectional movement with fixed steering angles. In this section, the reachable region is evaluated quantitatively. The analysis target excludes the singular configuration, $\beta_i = \pm\pi/2$.

First, the translation without rotation of SWOM is examined. If $\theta = \dot{\theta} = 0$ is given in Eq. (2.16) and the global and local reference frames are in agreement at the initial condition, the relative position of the driving unit on the sliding joint l_i is given as follows:

$$l_i = -\frac{\cos(\alpha_i + \beta_i)}{\cos \beta_i} x - \frac{\sin(\alpha_i + \beta_i)}{\cos \beta_i} y + l_{i0} \quad (2.20)$$

where l_{i0} denotes the relative position of driving unit i on the sliding joint at the initial condition. When the movable range of the sliding joints is represented by $l_{\min} \leq l_i \leq l_{\max}$, the reachable region of the main body of SWOM is obtained.

$$-(l_{i0} - l_{\min}) \leq -\frac{\cos(\alpha_i + \beta_i)}{\cos \beta_i} x - \frac{\sin(\alpha_i + \beta_i)}{\cos \beta_i} y \leq l_{\max} - l_{i0} \quad (2.21)$$

Under the conditions $\beta_1 = \beta_2 = \beta_3 = 0$ and $l_{10} = l_{20} = l_{30}$, the reachable region of the main body in translation, represented by the center of the base block, is shown in Fig. 2.9(a). The six inequalities of Eq. (2.21) shape the reachable region into a hexagon. The movable range of the sliding joint connected with driving unit i defines two boundaries of the region. The boundaries are parallel to the wheel plane of driving unit i and apart from the center of the base block by

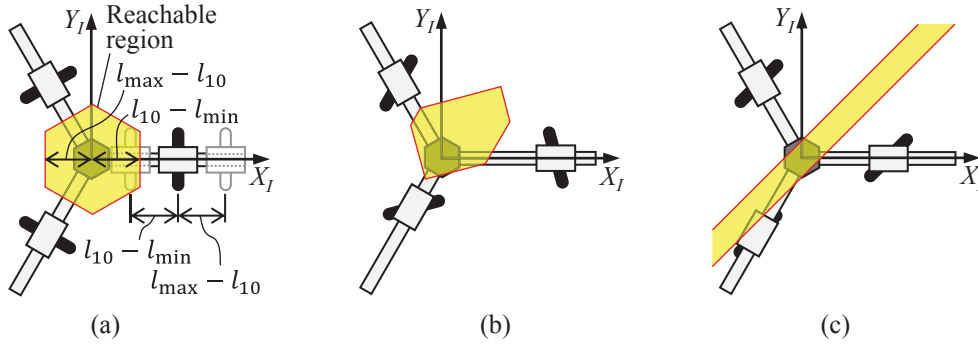


Fig. 2.9

Reachable region in translation of SWOM with fixed steering angles of the wheels. (a) $\beta_1 = \beta_2 = \beta_3 = 0$, $l_{10} = l_{20} = l_{30}$. (b) $\beta_1 = \pi/12$, $\beta_2 = \pi/6$, $\beta_3 = \pi/4$, $l_{10} > l_{20} > l_{30}$. (c) $\alpha_1 + \beta_1 = \alpha_2 + \beta_2 = \alpha_3 + \beta_3 = \pi/4$, $l_{10} = l_{20} = l_{30}$.

$l_{\max} - l_{i0}$ and $l_{i0} - l_{\min}$. When β_i and l_{i0} are varied, as shown in Fig. 2.9(b), a reachable region remains in all directions, while the reachable distance depends on the direction. In the case that the movement directions of the three wheels match each other, as shown in Fig. 2.9(c), the gradients of the six boundaries of Eq. (2.21) are equivalent. As a result, the reachable region becomes open, and SWOM can move unlimitedly in the movement direction of the wheel. In the other directions, the reachable distance is limited. Thus, in any direction of the wheels, except for the singular configuration, a reachable region exists in all directions and SWOM can be translated in any direction immediately.

Next, the reachable region when SWOM rotates around the center of the base block without translation is considered. When $\dot{x} = \dot{y} = 0$ is given in Eq. (2.16) and the initial condition is the same as in the translation discussed above, l_i is obtained:

$$l_i = l_{i0} \exp(-\theta \tan \beta_i) . \quad (2.22)$$

Therefore, the reachable region of the main body of SWOM is represented by the following inequalities.

$$\begin{cases} -\frac{1}{\tan \beta_i} \ln \frac{l_{\min}}{l_{i0}} \leq \theta \leq -\frac{1}{\tan \beta_i} \ln \frac{l_{\max}}{l_{i0}} & \text{if } -\frac{\pi}{2} < \beta_i < 0 \\ -\infty < \theta < \infty & \text{if } \beta_i = 0 \\ -\frac{1}{\tan \beta_i} \ln \frac{l_{\max}}{l_{i0}} \leq \theta \leq -\frac{1}{\tan \beta_i} \ln \frac{l_{\min}}{l_{i0}} & \text{if } 0 < \beta_i < \frac{\pi}{2} \end{cases} \quad (2.23)$$

Eq. (2.23) shows SWOM can rotate in either direction with any steering angle of the wheels except for the singular configuration.

2.4 Example of Unlimited Travel: Unlimited Translation

In the practical use of mobile robots, the reachable range of the robots should not be restricted. Then, in this section, how the SWOM can move in an arbitrary direction immediately and unlimitedly with handling the relative movements of the sliding joints is discussed.

2.4.1 Fundamental Traveling Strategy

Based on the discussion in Section 2.3.1, the basic strategy to calculate the trajectories for the unlimited travel of SWOM is as follows. SWOM has six-DOF mobility: the three-DOF planar motion of the main body and three-DOF relative movements of the sliding joints. Though the former planar motion is the main target in traveling, the latter relative movements should be taken into account at the same time. The inverse kinematics [Eq. (2.19)] gives the required rotational velocities of the wheels $\dot{\phi}_i$ to realize the desired movement of the planar motion $\dot{\xi}$ depending on the relative position of the sliding joints l_i and the steering angles of the wheels β_i . Moreover, l_i should be kept within its movable range by adjusting β_i appropriately while SWOM moving.

Considering the transition from a certain initial condition to a target condition where SWOM moves at a certain desired velocity. In the initial condition, β_i , ϕ_i , and $\dot{\beta}_i$ are arbitrary. In the target condition, from the perspective of unlimited traveling, the steering angles of the wheels and the relative positions of the sliding joints should be fixed at the states where SWOM can move unlimitedly, as in Fig. 2.6. The transition process is as follows: First, we accelerate the main body for the target velocity by adjusting $\dot{\phi}_i$ according to Eq. (2.19). As shown in Section 2.3.3, the reachable region where SWOM can move immediately exists in all directions, regardless of the values of β_i (except in the singular configuration). Thus, SWOM can start acceleration immediately inside the reachable region. At the same time that SWOM moves, we change β_i to control l_i . The reachable region changes depending on β_i , as shown in Fig. 2.9, but SWOM can continue moving by keeping at least the target direction in the reachable region at each instant. Finally, SWOM reaches the target condition and can continue moving arbitrarily. In this way, SWOM can start moving immediately with arbitrary β_i , ϕ_i , and $\dot{\beta}_i$, and continue moving unlimitedly in an arbitrary direction.

The subsequent sections deal with unlimited translation for the sake of analytic solution of the trajectories. The state of unlimited translation is defined as the state where the directions of all wheels are parallel and SWOM can move in a direction unlimitedly, as in Fig. 2.6(a)–(c). It is assumed that the initial condition is the state of unlimited translation in a certain initial direction and the target condition is the state of unlimited translation in another target direction.

2.4.2 Analysis and Simulation

To perform unlimited translation, the trajectories that satisfy the constraints are examined. By solving Eq. (2.16), β_i is obtained.

$$\beta_i = \tan^{-1} \left(\frac{\dot{X} \cos \alpha_i + \dot{Y} \sin \alpha_i + \dot{l}_i}{\dot{X} \sin \alpha_i - \dot{Y} \cos \alpha_i - l_i \dot{\theta}} \right) \quad (2.24)$$

where $R(\theta)\dot{\xi} = [\dot{X} \ \dot{Y} \ \dot{\theta}]^T$ is applied. According to Eq. (2.24), β_i becomes continuous when $\dot{X}$, $\dot{Y}$, $\dot{\theta}$, and $\dot{l}_i$ are continuous, because the argument of the arctangent on the right side becomes continuous, except for $\dot{X} \sin \alpha_i - \dot{Y} \cos \alpha_i - l_i \dot{\theta} = 0$. By using Eq. (2.19), the discontinuous condition can be rewritten as follows:

$$r_i \dot{\phi}_i \cos \beta_i = 0 \quad (2.25)$$

Thus, SWOM can move continuously when β_i is in the trajectory where $\dot{X}$, $\dot{Y}$, $\dot{\theta}$, and $\dot{l}_i$ are continuous and neither $\dot{\phi}_i = 0$ nor $\beta_i = \pm\pi/2$ is passed through. Additionally, the steering angle is arbitrary when the wheel is stopping, so that $\dot{\phi}_i = 0$ can be passed through when β_i is continuous before and after $\dot{\phi}_i = 0$. The singular configuration $\beta_i = \pm\pi/2$ in unlimited translation is dealt with in the next section and $\beta_i \neq \pm\pi/2$ is assumed here.

When Eq. (2.16) is considered as a differential equation with respect to l_i , the integration form is as follows.

$$l_i = \left[\int \left\{ -\frac{\cos(\alpha_i + \beta_i)}{\cos \beta_i} \dot{X} - \frac{\sin(\alpha_i + \beta_i)}{\cos \beta_i} \dot{Y} \right\} \exp\left(\int \dot{\theta} \tan \beta_i dt\right) dt + C \right] \cdot \exp\left(-\int \dot{\theta} \tan \beta_i dt\right) \quad (2.26)$$

where C denotes the integral constant. It is impossible to calculate the integration of Eq. (2.26) analytically, because the slidable wheels are nonholonomic. As shown in Eqs. (2.20) and (2.22), however, we can obtain the analytical solution in the case that the trajectory is either translation or rotation only.

By using the equations derived above, the trajectories of the driving units are obtained through the following process. First, we calculate the target value of β_i in the target condition by using Eq. (2.24). Next, we find the trajectory of β_i that satisfies the boundary conditions such that β_i , $\dot{\beta}_i$, and $\dot{l}_i$ are continuous based on Eq. (2.26). Finally, $\dot{\phi}_i$ is given by substituting the calculated β_i and $\dot{l}_i$ in Eq. (2.19). Thus, the trajectories of rotation and steering of the wheels between the initial and target conditions are provided. The trajectories should be chosen to satisfy the condition that SWOM is inside the reachable region, i.e., $l_{\min} < l_i < l_{\max}$ and $\beta_i \neq \pm\pi/2$.

An example of the trajectory generation of unlimited translation is now shown by using the expressions obtained above. The process of transition from the initial condition to the target

condition is divided into two steps. Additionally to $\theta = \dot{\theta} = 0$, a part of the boundary conditions and the trajectories is given as follows.

$$\dot{x}(t) = \begin{cases} V_0 \cos \chi_0 & \text{at } t = t_0 \\ V_2 \cos \chi_2 & \text{for } t_1 \leq t \leq t_2 \end{cases}$$

$$\dot{y}(t) = \begin{cases} V_0 \sin \chi_0 & \text{at } t = t_0 \\ V_2 \sin \chi_2 & \text{for } t_1 \leq t \leq t_2 \end{cases}$$

$$\beta_i(t) = \begin{cases} \beta_{i0} & \text{for } t_0 \leq t \leq t_1 \\ \beta_{i2} & \text{at } t = t_2 \end{cases}$$

$$l_i(t_0) = l_{i0}, \quad l_i(t_2) = l_{i2}$$

$$\dot{\beta}_i(t_1) = \dot{\beta}_i(t_2) = 0$$

$$\dot{l}_i(t_0) = \dot{l}_i(t_2) = 0$$

SWOM takes the initial condition, i.e., the state of unlimited translation in the initial direction for $t \leq t_0$, and then switches from moving in the initial direction to changing the direction to the target direction at $t = t_0$. First, for $t_0 \leq t \leq t_1$, the main body accelerates to the target velocity inside the reachable region with the steering angles of the wheels fixed. Next, for $t_1 \leq t \leq t_2$, the driving units change β_i and control $\dot{l}_i$ while keeping the velocity of the main body. Finally, SWOM achieves the target condition at $t = t_2$ and then continues unlimited translation in the target direction. Here, V_0 and V_2 are the velocities of the main body at $t = t_0$ and $t = t_2$, respectively; χ_0 and χ_2 denote the angles between X_i and the initial or target direction, respectively.

The steering angles of the wheels in the state of unlimited translation, β_{i0} and β_{i2} , are obtained from Eq. (2.24).

$$\beta_{i0} = \chi_0 - \alpha_i \pm \frac{\pi}{2} \quad \beta_{i2} = \chi_2 - \alpha_i \pm \frac{\pi}{2} \quad (2.27)$$

The plus/minus signs in Eq. (2.27) correspond to the reverse and forward rotation of the wheels, but are not distinguished structurally in SWOM. Then, β_{i0} or β_{i2} that satisfies $-\pi/2 < \beta_i(t) < \pi/2$ is chosen. Next, $l_i(t)$ is calculated from Eq. (2.26) by considering the boundary conditions.

$$l_i = \begin{cases} -\frac{\cos(\alpha_i + \beta_{i0})}{\cos \beta_{i0}}(x - x_0) - \frac{\sin(\alpha_i + \beta_{i0})}{\cos \beta_{i0}}(y - y_0) + l_{i0} & \text{for } t_0 \leq t \leq t_1 \\ -V_2 \cos(\alpha_i - \chi_2)(t - t_2) + V_2 \sin(\alpha_i - \chi_2) \int_{t_1}^t \tan \beta_i(\tau) d\tau + l_{i1} & \text{for } t_1 < t \leq t_2 \end{cases} \quad (2.28)$$

where $x(t_0) = x_0$, $y(t_0) = y_0$, and $l_i(t_1) = l_{i1}$ are constants. As shown in Eq. (2.28), the analytic solutions for this trajectory are given when $\tan \beta_i(t)$ can be integrated, i.e., when $\beta_i(t)$ satisfies $-\pi/2 < \beta_i(t) < \pi/2$ for $t_1 \leq t \leq t_2$. By considering the continuities of $\beta_i(t)$ and $l_i(t)$ at $t = t_1$, t_2 , the boundary conditions that $\beta_i(t)$ should satisfy are the following five expressions.

$$\beta_i(t_1) = \beta_{i0}, \beta_i(t_2) = \beta_{i2}, \dot{\beta}_i(t_1) = \dot{\beta}_i(t_2) = 0, \int_{t_1}^{t_2} \tan \beta_i(t) dt = \frac{(l_{i2} - l_{i1}) + V_2 \cos(\alpha_i - \chi_2)(t_2 - t_1)}{V_2 \sin(\alpha_i - \chi_2)} \quad (2.29)$$

The following simulation supposes $\tan \beta_i(t)$ is a quartic function satisfying Eq. (2.29). Thus, $\beta_i(t)$ and $l_i(t)$ are determined, and then the rotational velocity of the wheels $\dot{\phi}_i(t)$ is obtained from Eq. (2.19).

The results of the numerical simulation are shown. The values of the parameters are set as follows: $t_0 = 0.0$ s, $t_1 = 2.0$ s, $t_2 = 7.0$ s; $V_0 = 0$ mm/s, $V_2 = 20$ mm/s; $l_{i0} = l_{i2} = 300$ mm, $l_{\min} = 100$ mm, $l_{\max} = 500$ mm; $x_0 = y_0 = 0$ mm; and $\chi_0 = -\pi/6$, $\chi_2 = \pi/2$. The movement velocities $\dot{x}(t)$ and $\dot{y}(t)$ for $t_0 \leq t \leq t_1$ are linear functions that are continuous at $t = t_0, t_1$. These conditions give the trajectories shown in Fig. 2.10. Fig. 2.10(a) shows the paths of the main body and three driving units. Fig. 2.10(b) shows the change of l_i and β_i over time.

For $t = 0.0$ – 2.0 s, by accelerating the wheels while maintaining the initial steering angles, the main body of SWOM starts to move in the target direction with the relative movement of the sliding joints inside the reachable region. For $t = 2.0$ – 7.0 s, the directions of the wheels are

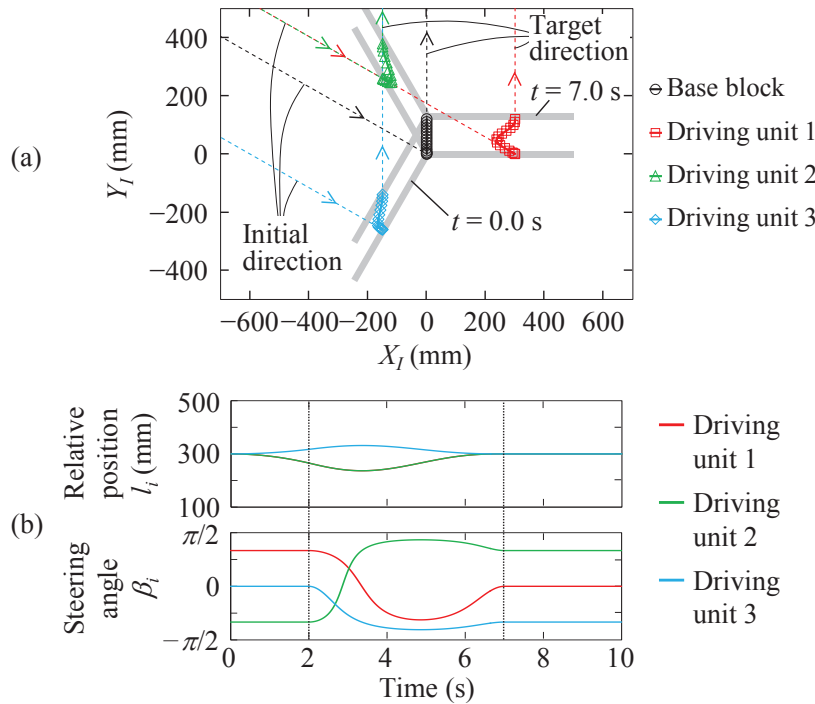


Fig. 2.10

Simulation results of movement direction change in unlimited translation of SWOM. (a) Trajectories of SWOM with the colored solid lines and markers at 0.5-s intervals during the change of the movement direction and with the colored dashed lines during unlimited translation in the initial and target conditions. (b) Graphs of the relative position of the sliding joints and the steering angles of the driving units. Note that $l_1 = l_2$ occurs for the entire time.

changing. The main body moves at a constant velocity in the target direction with the relative positions of the driving units controlled. After $t = 7.0$ s, the directions of the wheels and the relative positions of the driving units are all constant; this is the state of unlimited translation in the target direction. As a result, although the trajectories of the driving units are smooth curves, the main body moves straight from the beginning of the change of the movement direction. That is, the main body moves in the target direction immediately in the initial condition. In this simulation, the boundary conditions are also set so that the relative positions of the driving units at the initial and target conditions are in agreement. By keeping the driving units as far from the ends of the sliding joints as possible, the driving units are likely to stay within the movable range of the sliding joints in the following movement.

Thus, it is verified that the appropriate trajectory continuously connects the state of unlimited translation in a certain direction with that in another direction.

2.4.3 Singular Configuration in Unlimited Translation

The singular configuration is explained additionally. When the initial and target conditions are not singular in unlimited translation, SWOM can definitely choose trajectories between them that do not pass through $\beta_i = \pm\pi/2$, and so can avoid the singular configuration. In the following, the case that the initial or target condition is in the singular configuration is considered.

If the target condition is singular, the same strategy as the nonsingular case is available immediately before the target condition, when the movement direction of the main body becomes parallel to the sliding joint connected with the driving unit that is singular in the target condition. At this point, the relative movement of the sliding joint allows the driving unit to stop temporarily without interruption of movement of the main body. Meanwhile, the driving unit can steer the wheel to $\beta_i = \pm\pi/2$. After that, when the driving unit resumes moving, the target condition is achieved. Thus, the singular target condition does not matter because the main body of SWOM can move without any problem.

In the case that the initial condition is in the singular configuration, SWOM cannot move immediately in some directions. To move in those directions, it is necessary to change the direction of the wheel of the driving unit in the singular configuration from $\beta_i = \pm\pi/2$. After the driving unit avoids the singular configuration, the situation is just nonsingular trajectory generation. The arrangements cost extra time to slightly steer the wheel, but it takes only a short time.

Consequently, the singular configuration does not affect the movement of SWOM except in the case of the singular initial condition. Additionally, the singular configuration, $\beta_i = \pm\pi/2$, is seldom required; it is required only when SWOM moves in the direction parallel to the sliding joints, i.e., $0, \pi/3, 2\pi/3, \pi, 4\pi/3$, and $5\pi/3$ from X_R . Furthermore, even if the initial condition

is singular, SWOM can move by taking some preparatory time to slightly change the direction of the wheel, as mentioned above.

2.5 Experiments

To show that SWOM can really move as described, experiments were conducted to demonstrate SWOM movements by using a prototype.

2.5.1 SWOM Prototype

Fig. 2.11 shows the prototype of SWOM. Each sliding joint consists of two parallel linear guides. The driving unit, which has an upper part and a lower part, changes the direction of the wheel by turning the lower part relative to the upper part around a vertical axis. The upper part of the driving unit is attached to the sliders of the linear guides. Two types of motors are installed in the lower part of the driving unit: one motor turns the lower part relative to the upper part, and the other two motors rotate the wheel. Note that two driving motors are installed to increase the power, and these motors always rotate at the same velocity, just like using one motor. The specifications of the prototype are shown in Table 2.2.

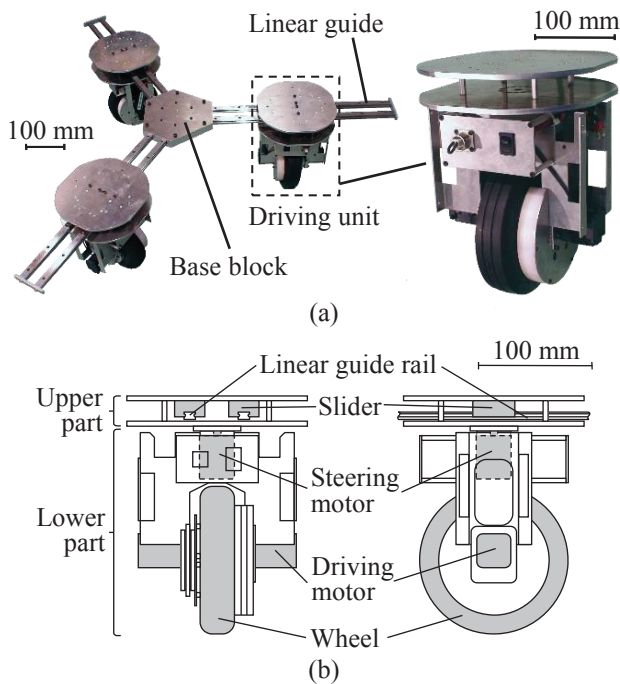


Fig. 2.11
(a) Prototype of SWOM. (b) Structure of the driving unit.

Table 2.2
Specifications of wheeled mechanisms.

Whole system	Radius	542 mm
	Weight	16.4 kg
Driving unit	Radius	100 mm
	Height	265 mm
	Weight	3.5 kg
Wheel	Radius	78 mm
Driving motor/ steering motor	Model	Futaba RS405CB
	Output torque	48.0 kgf·cm
	Rotational speed	0.21 s / 60 deg.
Rotary encoder	Resolution	360 ppr

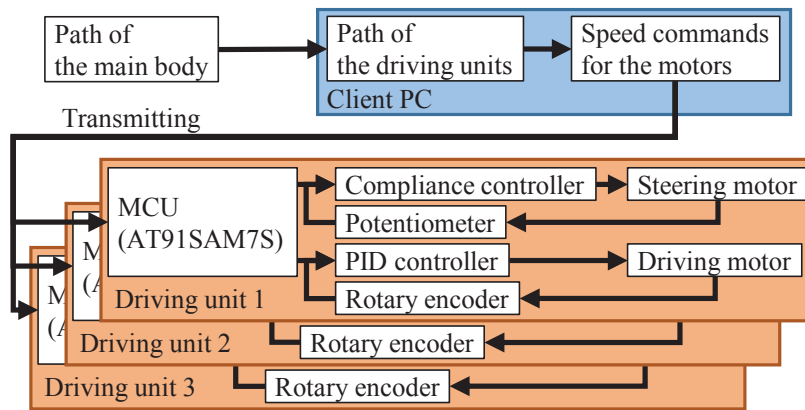


Fig. 2.12
Schematic diagram of the system of the SWOM prototype.

The controllers of the motors are independent of each other as in Fig. 2.12. The speed commands for each motor are calculated and sent in advance, and then the motors start moving at the same time when they receive a start command. This means that the prototype basically has a feedforward-control system except for each individual motor. This control system was designed to make the hardware and software simple because the purpose of the prototype is to verify the mechanism of SWOM.

2.5.2 Experiments of Fundamental Movements

First, whether SWOM can perform translation or rotation in any direction without steering the wheels, as shown in Fig. 2.5, is confirmed. Fig. 2.13 shows the motions of the SWOM prototype. Each case starts from $l_i = 300$ mm. Fig. 2.13(a), (b), and (c) show the movement when the steering angles of every wheels are fixed at $\beta_i = 0$, which corresponds to Fig. 2.5(a), (b), and (c). As a result, SWOM achieves each movement: translation in the X_I direction, translation in the Y_I direction, and turning. The experiment confirmed that SWOM can make three-DOF planar motion independently of the steering angles of the wheels.

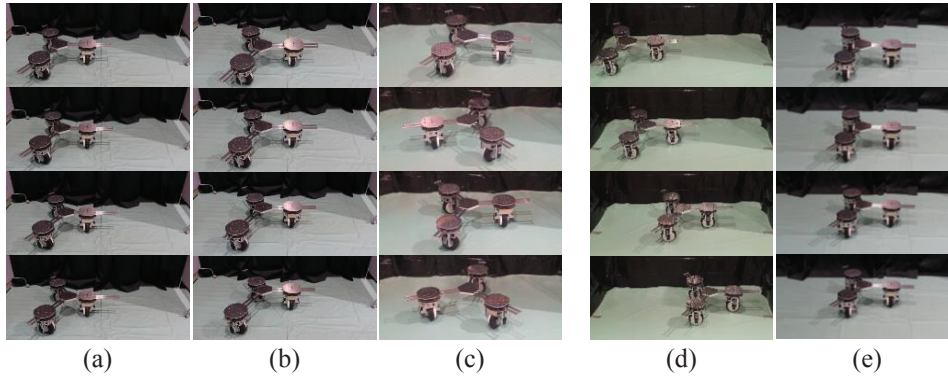


Fig. 2.13

Motions of SWOM prototype with fixed steering angles. (a) Translation in X_I direction with $\beta_i = 0$. (b) Translation in Y_I direction with $\beta_i = 0$. (c) Turning with $\beta_i = 0$. (d) Translation in X_I direction with $\alpha_i + \beta_i = \pi/2$. (e) Translation in Y_I direction with $\alpha_i + \beta_i = 0$. The right and depth directions are the X_I and Y_I directions, respectively.

In Fig. 2.13(d), SWOM shows the state of unlimited translation in the X_I direction, where the steering angles of the driving units satisfy $\alpha_i + \beta_i = \pi/2$. Additionally, Fig. 2.13(e) also shows the state of unlimited translation in the Y_I direction, and Fig. 12(c) corresponds to the unlimited rotatable state. Each situation represents Fig. 2.6(a), (b), and (d), respectively. In these movements, the relative positions of the driving units on the sliding joint rails are kept constant, as expected. Actually, the situation of Fig. 2.13(d) is the singular configuration because driving unit 1, the right driving unit in the figure, has $\beta_1 = \pi/2$. However, SWOM certainly moves in the X_I direction without any problem.

The above results verify that the prototype of SWOM performed three-DOF planar motion according to the theory.

2.5.3 Experiments of Omnidirectional Movements

Next, an experiment on the omnidirectional movement is conducted. The experiment deals with three kinds of motions. One is a translation on a triangular path. The prototype moves in the right direction for 4.0 s at 30 mm/s, in the forward left direction for 4.0 s at about 42.3 mm/s, and in the backward direction for 4.0 s at 30 mm/s, in order. During this motion, all driving units keep the steering angles fixed at $\beta_i = 0$. Second, the prototype moves in the right direction at 10 mm/s while rotating clockwise at about 0.157 rad/s. The steering angles of the driving units are fixed at $\beta_i = 0$, too. These two motions show the omnidirectional mobility of SWOM under the fixed steering angles, as well as the previous section, but several kinds of the movements are combined together. Third, the unlimited translation discussed in Section 2.4 is conducted. The conditions are basically the same as those in the simulation, but the prototype continues moving in the steady state until $t = 10.0$ s after finishing the change in the movement direction at $t = 7.0$ s. The motion

of the prototype is measured by using the motion capture camera VICON MX (Vicon Motion Systems Ltd) with two Bonita 10 cameras and four Bonita 3 cameras to evaluate the movements quantitatively.

Fig. 2.14(a), (b), and (c) shows the experimental results of translation on a triangular path, translation with rotation, and unlimited translation, respectively. The figure shows that the measured motions generally agree with the desired motions. Thus, it is verified that the prototype of SWOM performed omnidirectional movement according to the theory derived in the previous sections and that the proposed mechanism and movement method are effective fundamentally.

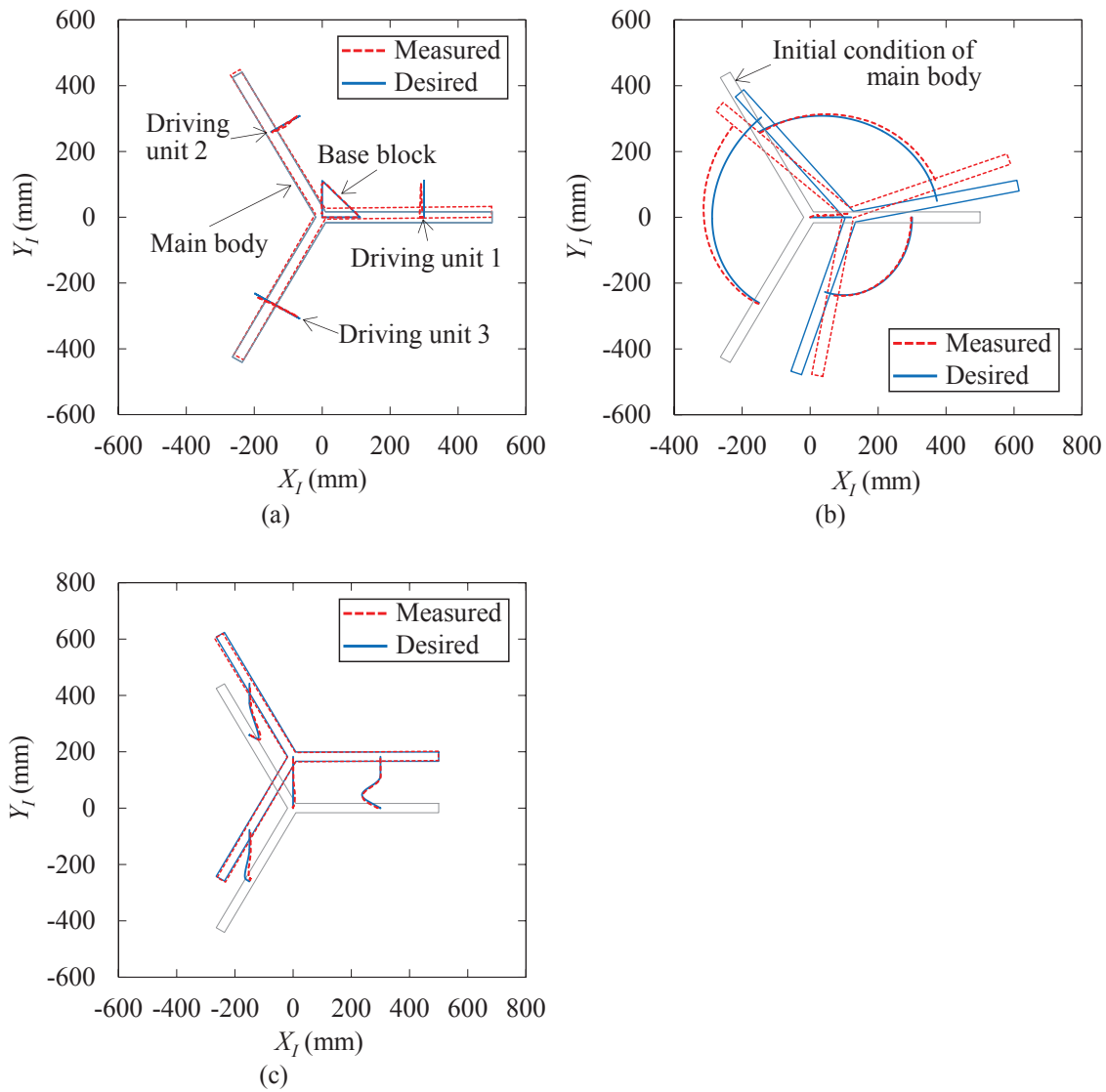


Fig. 2.14

Motions of SWOM prototype. (a) Translation on a triangular path. (b) Translation in X_I direction with clockwise rotation. (c) Unlimited translation from $\chi_0 = -\pi/6$ to $\chi_2 = \pi/2$. The main body is represented by the Y-shaped object. The blue solid lines are the desired paths of the driving units and the center of the base block, or the desired final condition of the main body. The red dashed lines are measured paths or the measured final condition.

However, by taking a closer look at the result of Fig. 2.14(b), the main body certainly moves in the right direction and rotates clockwise, but some motion errors occur. The errors in the results are analyzed numerically. Fig. 2.15 shows the errors in the measured paths. The upper graphs indicate the errors of the main body in the X_I , Y_I , and θ [only in Fig. 2.15(b)] directions over time. The lower graphs indicate the errors of the driving units in the sideways directions, i.e., the directions perpendicular to each wheel plane.

In Fig. 2.15(a) and (c), the errors of the main body are so small that the values keep smaller than 5% for the entire time. The assumed reason is that the errors of the driving units do not always affect the errors of the main body. Actually, in Fig. 2.15(a), the errors of the driving units occasionally change widely and become larger than 5%, especially when the movement direction of the main body is nearly parallel to the rail of the driving unit (driving unit 1 for $t = 0.0\text{--}4.0$ s and driving unit 2 for $t = 4.0\text{--}8.0$ s). The sideways errors of the driving units cause the relative position errors on the rails. According to Eq. (2.19), however, the desired velocities of the driving units are independent of l_i under the condition $\dot{\theta} = 0$. Therefore, the errors of the driving units in the rail directions do not disturb the translation of the main body. By observing Fig. 2.15(a) and (c) in more detail, the errors of the main body become relatively large during acceleration: before and

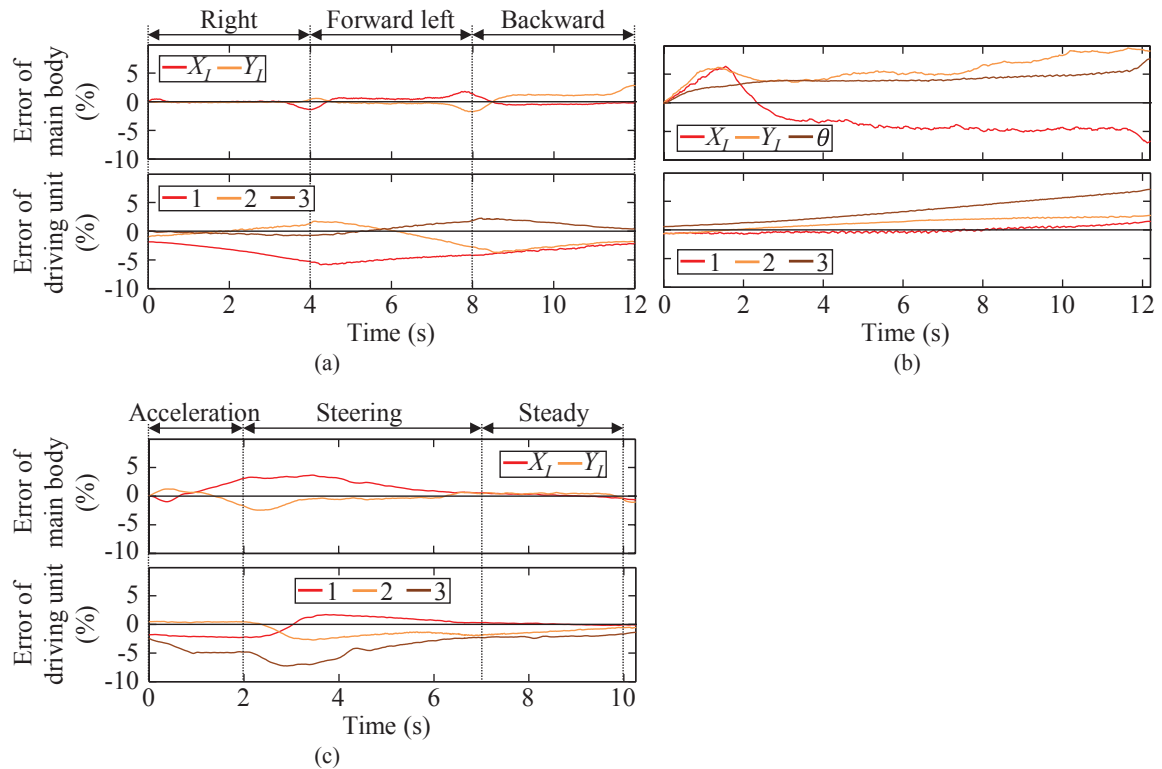


Fig. 2.15

Motion errors in experimental results. Upper graphs show the relative errors of the main body in the X_I , Y_I , and θ directions to the entire desired movement distance over time. Lower graphs show the relative errors of driving units 1, 2, or 3 in the sideways directions to each entire desired movement distance over time. (a) Result of the translation on a triangular path. (b) Result of the translation with rotation. (c) Result of the unlimited translation.

after $t = 0.0, 4.0, 8.0, 12.0$ s in Fig. 2.15(a) and for $t = 0.0-2.0$ s in Fig. 2.15(c). The errors might be caused partly by the problem of the control system compensating for the rotation angle errors of the wheels.

In Fig. 2.15(b), the errors of the main body become large not only in acceleration but also after acceleration. In this case, $\dot{\theta} \neq 0$, the motion of the main body receives the effect of the change of l_i , and unfortunately, the errors of the driving units monotonically increase for the entire time. This is because the wheels generally tend to deviate in the centrifugal directions in the rotation due to the resistance force to the turning of the wheels. The phenomenon also seems to cause the error fluctuation in Fig. 2.15(c) when the wheels are steered. It is difficult to predict how the wheels deviate because the wheels are affected by the friction condition or the load applied to each driving unit. However, it is thought that if sensors are installed to measure the positions of the driving units on the rails and feedback of their changes is supplied, those errors can be compensated. Especially in indoor tasks, where the friction condition does not fluctuate widely, such a feedback strategy would be effective.

Fig. 2.16 shows the speeds of the main body and driving units in the unlimited translation shown in Fig. 2.14(c). The speeds are given from the increments of the position vectors at 0.01-s intervals. The median filter with the size of 0.10 s is applied to the calculated speeds. The measured speeds of the driving units basically match the desired values, but the speeds do not fully follow the desired values in large acceleration. To address this problem, the performance of the motors or the control method needs to be reconsidered. With respect to the speed of the main body, the errors are not so large compared to those of driving unit 3 for $t = 0.0-1.0$ s. This means that

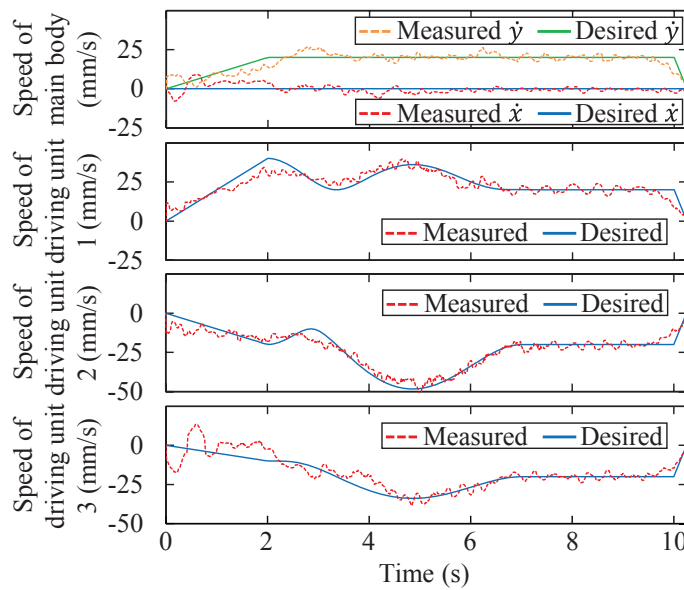


Fig. 2.16

Measured and desired speeds of the main body and driving units over time in the experimental result of unlimited translation.

the sliding joints of SWOM do not add together the errors of the driving units in the velocities, as well as in the displacement mentioned above.

2.6 Summary

In order to realize a wheeled mechanism with high mobility, simple structure, active drive, and fewer constraints, the analysis and synthesis of wheeled mechanisms were carried out. Kinematic analyses and experiments on the robot using the proposed wheeled mechanisms were also conducted. Then, the following results were obtained.

- The characteristics of existing wheeled mechanisms were compared. Through the analysis of these mechanisms, generalized kinematic conditions for omnidirectional movement were derived.
- Based on the derived conditions, a novel omnidirectional wheeled mechanism, a slidable wheel, was proposed which can use a normal wheel and avoid the overconstraint-like situation due to the passive sliding joint.
- A mobile robot with the slidable wheels named SWOM was proposed and the kinematic equations for SWOM were derived. It was proved theoretically that SWOM is able to make an omnidirectional movement with respect to three-DOF planar motion.
- The reachable region of SWOM was shown by discussing the movable range of the sliding joints. The results demonstrated that the reachable region existed in any direction of the wheels except in the singular configuration.
- A trajectory generation method was offered that enabled SWOM to change its movement direction while keeping the reachable region and then to continue omnidirectional movement unlimitedly. The simulation showed that the calculated trajectories enabled SWOM to make the unlimited translation in an arbitrary direction, as determined theoretically.
- A prototype of SWOM was developed and demonstrative experiments were conducted. The results verified that SWOM was able to make omnidirectional movement and to achieve unlimited translation by using the proposed method.

Chapter 3

Feedback Control of an Omnidirectional Mobile Robot with Slidable Wheels

3.1 Introduction

A slidable wheel and an omnidirectional mobile robot named SWOM has been proposed in Chapter 2. As a next step, it is needed to construct a control system for SWOM. The fundamental trajectory generation method, which is useful for feedforward control, has been already proposed, although feedback control is often necessary for practical use to endure noise or other disturbing factors. For this reason, this chapter tackles the design of a feedback controller for SWOM.

The description of SWOM behavior includes nonlinearity resulting from the sliding joints and nonholonomic constraints of the wheels. This type of system does not have a generally available control method, so designing a controller for the system is difficult. To deal with this task, feedback linearization method, which has been used to control a variety of machines including conventional wheeled mobile robots with nonholonomic constraints [70], [71], is used to design a controller for SWOM.

This chapter derives the exactly linearized state equation of SWOM using feedback and coordinate transformation based on a kinematic model, and then a stabilizing controller for SWOM considering singular configurations is designed. A numerical simulation and an experiment are conducted to verify its effectiveness.

3.2 Design of control system for SWOM

In this section, the construction of a controller for SWOM is discussed based on kinematics.

3.2.1 Kinematic Model

The kinematic equations of SWOM are given by Eqs. (2.15) and (2.16). By transforming these equations, the following equations are yielded:

$$\begin{bmatrix} A_1 R & O \\ A_2 R & I \end{bmatrix} \dot{q} = - \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix} \quad (3.1)$$

where

$$q = [x \ y \ \theta \ l_1 \ l_2 \ l_3]^T$$

$$A_1(l_1, l_2, l_3) = \begin{bmatrix} \sin \alpha_1 & -\cos \alpha_1 & -l_1 \\ \sin \alpha_2 & -\cos \alpha_2 & -l_2 \\ \sin \alpha_3 & -\cos \alpha_3 & -l_3 \end{bmatrix}, B_1(\beta_1, \beta_2, \beta_3) = \begin{bmatrix} r \cos \beta_1 & 0 & 0 \\ 0 & r \cos \beta_2 & 0 \\ 0 & 0 & r \cos \beta_3 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \cos \alpha_1 & \sin \alpha_1 & 0 \\ \cos \alpha_2 & \sin \alpha_2 & 0 \\ \cos \alpha_3 & \sin \alpha_3 & 0 \end{bmatrix}, B_2(\beta_1, \beta_2, \beta_3) = \begin{bmatrix} r \sin \beta_1 & 0 & 0 \\ 0 & r \sin \beta_2 & 0 \\ 0 & 0 & r \sin \beta_3 \end{bmatrix}.$$

The rotation matrix $R(\theta)$ is always regular, and the matrix $A_1(l_1, l_2, l_3)$ is also regular under the condition $l_i > 0$. Then, Eq. (3.1) can be rewritten as

$$\dot{q} = - \begin{bmatrix} R^{-1} A_1^{-1} & O \\ -A_2 A_1^{-1} & I \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix} \equiv \begin{bmatrix} J_1 \\ J_2 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix}, \quad (3.2)$$

where

$$J_1(\theta, l_1, l_2, l_3, \beta_1, \beta_2, \beta_3) := -R^{-1}(\theta) A_1^{-1}(l_1, l_2, l_3) B_1(\beta_1, \beta_2, \beta_3)$$

$$J_2(l_1, l_2, l_3, \beta_1, \beta_2, \beta_3) := A_2 A_1^{-1}(l_1, l_2, l_3) B_1(\beta_1, \beta_2, \beta_3) - B_2(\beta_1, \beta_2, \beta_3).$$

3.2.2 Feedback Linearization

A linearized state equation is derived from the kinematics described in the previous section. The time derivative of Eq. (3.2) is given by

$$\ddot{q} = \begin{bmatrix} J_1 \\ J_2 \end{bmatrix} \begin{bmatrix} \ddot{\phi}_1 \\ \ddot{\phi}_2 \\ \ddot{\phi}_3 \end{bmatrix} - \begin{bmatrix} R^{-1} A_1^{-1} & O \\ -A_2 A_1^{-1} & I \end{bmatrix} \begin{bmatrix} \dot{B}_1 \\ \dot{B}_2 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix} - \begin{bmatrix} R^{-1} A_1^{-1} & O \\ -A_2 A_1^{-1} & I \end{bmatrix} \begin{bmatrix} \dot{A}_1 R + A_1 \dot{R} & O \\ A_2 \dot{R} & O \end{bmatrix} \begin{bmatrix} J_1 \\ J_2 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix}. \quad (3.3)$$

The time derivatives of the matrices in this equation are calculated as follows:

$$A_1 = - \begin{bmatrix} \dot{l}_1 \\ \dot{l}_2 \\ \dot{l}_3 \end{bmatrix} [0 \ 0 \ 1] = -J_2 \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix} [0 \ 0 \ 1]$$

$$\dot{B}_1 = -B_2 \text{diag}(\dot{\beta}_1, \dot{\beta}_2, \dot{\beta}_3) \quad (3.4)$$

$$\dot{B}_2 = B_1 \text{diag}(\dot{\beta}_1, \dot{\beta}_2, \dot{\beta}_3) .$$

Then Eq. (3.3) is arranged as

$$\ddot{q} = \begin{bmatrix} J_1 \\ J_2 \end{bmatrix} \begin{bmatrix} \ddot{\phi}_1 \\ \ddot{\phi}_2 \\ \ddot{\phi}_3 \end{bmatrix} - \begin{bmatrix} -R^{-1}A_1^{-1}B_2 \\ A_2A_1^{-1}B_2 + B_1 \end{bmatrix} \text{diag}(\dot{\phi}_1, \dot{\phi}_2, \dot{\phi}_3) \begin{bmatrix} \dot{\beta}_1 \\ \dot{\beta}_2 \\ \dot{\beta}_3 \end{bmatrix} - \left(\begin{bmatrix} R^{-1}\dot{R}J_1 \\ O \end{bmatrix} - \begin{bmatrix} R^{-1}A_1^{-1} \\ -A_2A_1^{-1} \end{bmatrix} J_2 \begin{bmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \\ \dot{\phi}_3 \end{bmatrix} J_{1,3} \right) \begin{bmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \\ \dot{\phi}_3 \end{bmatrix} \quad (3.5)$$

where $J_{1,3}(\theta, l_1, l_2, l_3, \beta_1, \beta_2, \beta_3)$ is the third row vector of $J_1(\theta, l_1, l_2, l_3, \beta_1, \beta_2, \beta_3)$. Here, a part of the state variables of the system is set as $w = [\dot{\phi}_1 \ \dot{\phi}_2 \ \dot{\phi}_3 \ \beta_1 \ \beta_2 \ \beta_3]^T$ and the input vector to the system as $u = [\ddot{\phi}_1 \ \ddot{\phi}_2 \ \ddot{\phi}_3 \ \dot{\beta}_1 \ \dot{\beta}_2 \ \dot{\beta}_3]^T$. According to Eq. (3.1), w assumes a function of q and $\dot{q}$. As a result, the following equation is given:

$$\ddot{q} = N(q, w(q, \dot{q}))u + k(q, \dot{q}, w(q, \dot{q})) \quad (3.6)$$

where

$$N(q, w(q, \dot{q})) = \begin{bmatrix} J_1 & R^{-1}A_1^{-1}B_2 \text{diag}(\dot{\phi}_1, \dot{\phi}_2, \dot{\phi}_3) \\ J_2 & -(A_2A_1^{-1}B_2 + B_1) \text{diag}(\dot{\phi}_1, \dot{\phi}_2, \dot{\phi}_3) \end{bmatrix}$$

$$k(q, \dot{q}, w(q, \dot{q})) = - \left(\begin{bmatrix} R^{-1}\dot{R}J_1 \\ O \end{bmatrix} - \begin{bmatrix} R^{-1}A_1^{-1} \\ -A_2A_1^{-1} \end{bmatrix} J_2 \begin{bmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \\ \dot{\phi}_3 \end{bmatrix} J_{1,3} \right) \begin{bmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \\ \dot{\phi}_3 \end{bmatrix} .$$

Next, a new input vector v is introduced that satisfies the following condition:

$$u = N^{-1}(q, w(q, \dot{q}))\{v - k(q, \dot{q}, w(q, \dot{q}))\} . \quad (3.7)$$

Using this input, Eq. (3.6) is transformed into

$$\ddot{q} = v . \quad (3.8)$$

When $\tilde{\xi} = [q \ \dot{q}]^T$ is defined, the linearized state equation with the state $\tilde{\xi}$ and the input v is given as follows:

$$\dot{\tilde{\xi}} = C\tilde{\xi} + Dv \quad (3.9)$$

where

$$C = \begin{bmatrix} O & I \\ O & O \end{bmatrix}, D = \begin{bmatrix} O \\ I \end{bmatrix} .$$

This derives the exactly linearized state equation for SWOM.

Finally, the controller stabilizing the system described by Eq. (3.9) is designed. When the desired value of the state $\tilde{\xi}$ is expressed as $\tilde{\xi}_{\text{ref}}$, the state feedback is constructed as follows:

$$v = -K(\tilde{\xi} - \tilde{\xi}_{\text{ref}}) \quad (3.10)$$

where $K \in \mathbb{R}^{6 \times 12}$ is the feedback gain. The system is stabilized by the feedback gain given by

$$K = [\text{diag}(k_{11}, k_{12}, \dots, k_{16}) \quad \text{diag}(k_{21}, k_{22}, \dots, k_{26})] \quad (3.11)$$

$$k_{mn} > 0 \quad (m = 1, 2, n = 1, 2, \dots, 6) .$$

When this linear controller is used, the input to the original system u is calculated by Eqs. (3.7) and (3.10).

$$u = -N^{-1}(\xi)\{K(\xi - \xi_{\text{ref}}) + k(\xi)\} . \quad (3.12)$$

The overall structure of the constructed control system is shown in Fig. 3.1. This feedback controller is called the normal controller in the following discussions.

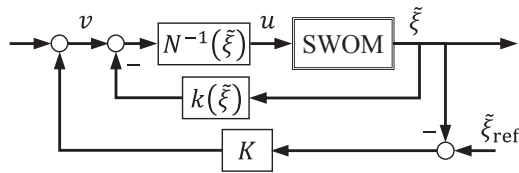


Fig. 3.1
Structure of the feedback controller for SWOM

3.2.3 Singular Configuration Avoidance

In Eq. (3.12), $N^{-1}(\xi)$ is calculated as follows:

$$N^{-1}(\xi) = - \begin{bmatrix} \frac{\sin(\alpha_1 + \beta_1 + \theta)}{r} & -\frac{\cos(\alpha_1 + \beta_1 + \theta)}{r} & -\frac{l_1 \cos \beta_1}{r} & \frac{\sin \beta_1}{r} & 0 & 0 \\ \frac{\sin(\alpha_2 + \beta_2 + \theta)}{r} & -\frac{\cos(\alpha_2 + \beta_2 + \theta)}{r} & -\frac{l_2 \cos \beta_2}{r} & 0 & \frac{\sin \beta_2}{r} & 0 \\ \frac{\sin(\alpha_3 + \beta_3 + \theta)}{r} & -\frac{\cos(\alpha_3 + \beta_3 + \theta)}{r} & -\frac{l_3 \cos \beta_3}{r} & 0 & 0 & \frac{\sin \beta_3}{r} \\ \frac{\cos(\alpha_1 + \beta_1 + \theta)}{r\dot{\phi}_1} & \frac{\sin(\alpha_1 + \beta_1 + \theta)}{r\dot{\phi}_1} & \frac{l_1 \sin \beta_1}{r\dot{\phi}_1} & \frac{\cos \beta_1}{r\dot{\phi}_1} & 0 & 0 \\ \frac{\cos(\alpha_2 + \beta_2 + \theta)}{r\dot{\phi}_2} & \frac{\sin(\alpha_2 + \beta_2 + \theta)}{r\dot{\phi}_2} & \frac{l_2 \sin \beta_2}{r\dot{\phi}_2} & 0 & \frac{\cos \beta_2}{r\dot{\phi}_2} & 0 \\ \frac{\cos(\alpha_3 + \beta_3 + \theta)}{r\dot{\phi}_3} & \frac{\sin(\alpha_3 + \beta_3 + \theta)}{r\dot{\phi}_3} & \frac{l_3 \sin \beta_3}{r\dot{\phi}_3} & 0 & 0 & \frac{\cos \beta_3}{r\dot{\phi}_3} \end{bmatrix} . \quad (3.13)$$

Focusing on the lower three rows of $N^{-1}(\xi)$, $N(\xi)$ becomes singular for $\dot{\phi}_i = 0$ and the feedback input $\dot{\beta}_i$ diverges according to Eqs. (3.12) and (3.13). However, the feedback input $\dot{\phi}_i$, which is determined by the upper three rows of $N^{-1}(\xi)$, never diverges, so that at least three-DOF motion can be controlled even when the configuration is singular as $\dot{\phi}_i = 0$. Thus, let us consider a strategy to control the position and orientation of the main body with priority by switching the feedback controller to another one neglecting the state values l_i temporarily near $\dot{\phi}_i = 0$. The concrete form of the switching controller is described below.

When $q_1 = [x \ y \ \theta]^T$, $u_1 = [\ddot{\varphi}_1 \ \ddot{\varphi}_2 \ \ddot{\varphi}_3]^T$, and $u_2 = [\dot{\beta}_1 \ \dot{\beta}_2 \ \dot{\beta}_3]^T$ are set, the first to third rows of Eq. (3.6) is rewritten.

$$\ddot{q}_1 = J_1(\tilde{\xi})u_1 + R^{-1}(\tilde{\xi})A_1^{-1}(\tilde{\xi})B_2(\tilde{\xi})\text{diag}(\varphi_1, \varphi_2, \varphi_3)u_2 + k(\tilde{\xi}) . \quad (3.14)$$

Here, it is considered that u_2 is controlled independently from u_1 and give a new input vector v'_1 for u_1 to satisfy the following condition:

$$u_1 = J_1^{-1}(\tilde{\xi})\{v'_1 - k(\tilde{\xi}) - R^{-1}(\tilde{\xi})A_1^{-1}(\tilde{\xi})B_2(\tilde{\xi})\text{diag}(\varphi_1, \varphi_2, \varphi_3)u_2\} . \quad (3.15)$$

Then, the input-output relation is given as follows:

$$\ddot{q}_1 = v'_1 . \quad (3.16)$$

This is the same form as Eq. (3.7), so that the linearized state equation and the stabilizing feedback input are derived as well. Specifically, the feedback input is

$$v'_1 = -K'_1 \left(\begin{bmatrix} q_1 \\ \dot{q}_1 \end{bmatrix} - \begin{bmatrix} q_{1\text{ref}} \\ \dot{q}_{1\text{ref}} \end{bmatrix} \right) \quad (3.17)$$

where $K'_1 \in \mathbb{R}^{3 \times 6}$ is the feedback gain and $q_{1\text{ref}}$ and $\dot{q}_{1\text{ref}}$ are the desired value of q_1 and $\dot{q}_1$, respectively. Here, the condition that $J_1(\tilde{\xi})$ does not have an inverse matrix in Eq. (3.15) is given as $\beta_i = \pm \pi/2$. This corresponds to the singular configuration where the traveling direction of the wheel is parallel to the sliding joint rail, as explained in Sections 2.3.2. However, β_i and $\dot{\beta}_i$ are determined arbitrarily here. Therefore, SWOM can avoid the singular configuration $\beta_i = \pm \pi/2$ by using the feedback to make $\beta_i \rightarrow n_i\pi$ ($n_i \in \mathbb{Z}$), for example,

$$u_2 = -K'_2 \begin{bmatrix} \beta_1 - n_1\pi \\ \beta_2 - n_2\pi \\ \beta_3 - n_3\pi \end{bmatrix} \quad (3.18)$$

where $K'_2 \in \mathbb{R}^{3 \times 3}$ is the feedback gain. This feedback controller is called the secondary controller.

Here the control strategy explained in this section is summarized. Away from the singular configuration $\varphi_i = 0$, all of the state variables of SWOM, including l_i , are controlled using the normal controller constructed in Section 3.2.2. Near $\varphi_i = 0$, the controller is switched to the secondary controller proposed in this section, where the position and orientation of the main body are controlled, but l_i is temporarily neglected. In this situation, there are concerns that the driving units move out of the range of the sliding joint rails. However, this does not occur if the driving units are placed sufficiently far away from the ends of the sliding joints and the required movement after switching the controller is adjusted within a specified region before switching the controller. In the secondary controller, there exists the singular configuration $\beta_i = \pm \pi/2$, not $\varphi_i = 0$, but it is avoidable because β_i can be controlled to an arbitrary desired value instead of l_i . Note

that in the normal controller, it is unnecessary to avoid $\beta_i = \pm \pi/2$ because the system does not assume a singular configuration except when $\phi_i = 0$.

3.3 Simulation and Experiment

To verify the effectiveness of the control system constructed as described in the previous section, a simulation and an experiment were conducted.

3.3.1 Simulation

In this simulation, the design parameters were set as follows: $\alpha_1 = 0$, $\alpha_2 = 2\pi/3$, $\alpha_3 = 4\pi/3$, $r = 0.064$, $k_{11} = k_{12} = 0.25$, $k_{13} = 0.16$, $k_{14} = k_{15} = k_{16} = 1.0$, $k_{21} = k_{22} = 1.0$, $k_{23} = 0.80$, $k_{24} = k_{25} = k_{26} = 2.0$, $K'_1 = 0.75 \times [\text{diag}(k_{11}, k_{12}, k_{13}) \quad \text{diag}(k_{21}, k_{22}, k_{23})]$, and $K'_2 = \text{diag}(0.20, 0.20, 0.20)$. The initial state was

$$\tilde{\xi} = [0.00 \ 0.00 \ 0.00 \ 0.20 \ 0.20 \ 0.20 \ 0.00 \ 0.00 \ 0.00 \ 0.00 \ 0.00 \ 0.00 \ 0.00]^T$$

and the desired states at times $t = 20$, 40 , and 60 were

$$\tilde{\xi}_{\text{ref}} = \begin{cases} [0.00 \ 0.50 \ \pi/12 \ 0.22 \ 0.22 \ 0.22 \ 0.00 \ 0.00 \ 0.00 \ 0.00 \ 0.00 \ 0.00 \ 0.00]^T & \text{at } t = 20 \\ [0.43 \ 0.25 \ \pi/6 \ 0.22 \ 0.22 \ 0.22 \ 0.00 \ 0.00 \ 0.00 \ 0.00 \ 0.00 \ 0.00 \ 0.00]^T & \text{at } t = 40 \\ [0.00 \ 0.00 \ \pi/4 \ 0.22 \ 0.22 \ 0.22 \ 0.00 \ 0.00 \ 0.00 \ 0.00 \ 0.00 \ 0.00 \ 0.00]^T & \text{at } t = 60 \end{cases}.$$

Additionally, the following conditions were given in the simulation. First, the condition of switching the controllers was $|\dot{\phi}_i(t)| < \pi/60 \vee \|q_1(t) - q_{1\text{ref}}\| < 0.035$. The former condition defines the vicinity of the singular configuration $\phi_i = 0$, while the latter condition restricts the movement after switching the controller within a sufficiently small region. Next, the limits of the rotational speed and acceleration of the driving and steering motors were set as

$$\dot{\phi}_i(t) = \begin{cases} 9.32 & \text{if } \phi_i(t) > 9.32 \\ \dot{\phi}_i(t) & \text{if } \phi_i(t) \in [-9.32, 9.32], \\ -9.32 & \text{if } \phi_i(t) < -9.32 \end{cases}, \quad \ddot{\phi}_i(t) = \begin{cases} 200 & \text{if } \ddot{\phi}_i(t) > 200 \\ \ddot{\phi}_i(t) & \text{if } \ddot{\phi}_i(t) \in [-200, 200] \\ -200 & \text{if } \ddot{\phi}_i(t) < -200 \end{cases}$$

$$\dot{\beta}_i(t) = \begin{cases} 10.7 & \text{if } \beta_i(t) > 10.7 \\ \dot{\beta}_i(t) & \text{if } \beta_i(t) \in [-10.7, 10.7], \\ -10.7 & \text{if } \beta_i(t) < -10.7 \end{cases}, \quad \ddot{\beta}_i(t) = \begin{cases} 230 & \text{if } \ddot{\beta}_i(t) > 230 \\ \ddot{\beta}_i(t) & \text{if } \ddot{\beta}_i(t) \in [-230, 230] \\ -230 & \text{if } \ddot{\beta}_i(t) < -230 \end{cases}.$$

The simulation is carried out to verify that SWOM can converge to the desired state without difficulty under these conditions.

The simulation results are shown in Fig. 3.2. Fig. 3.2(a) is the trajectory of the main body of SWOM, where each parameter reaches the desired value at $t = 20$, 40 , and 60 . Fig. 3.2(b) shows the time variation of the other parameters. In the upper graph, l_i repeats convergence to the desired value $l_i = 0.22$ and deviation. This behavior represents the switching of the controller, where the state variables of SWOM, including l_i , are controlled to the halfway point but l_i is not controlled around $t = 20$, 40 , and 60 . However, the values of l_i are within $0.20 < l_i < 0.25$ for the entire simulation. Therefore, if the movable range of the sliding joints covers $0.20 < l_i < 0.25$, the driving units do not reach the ends of their sliding joints. With respect to ϕ_i and β_i in the middle and lower graphs, these values varied continuously within a certain range. For β_i especially, the problem of divergence around the singular configuration $\phi_i = 0$ did not occur. These results show that the control system worked as expected.

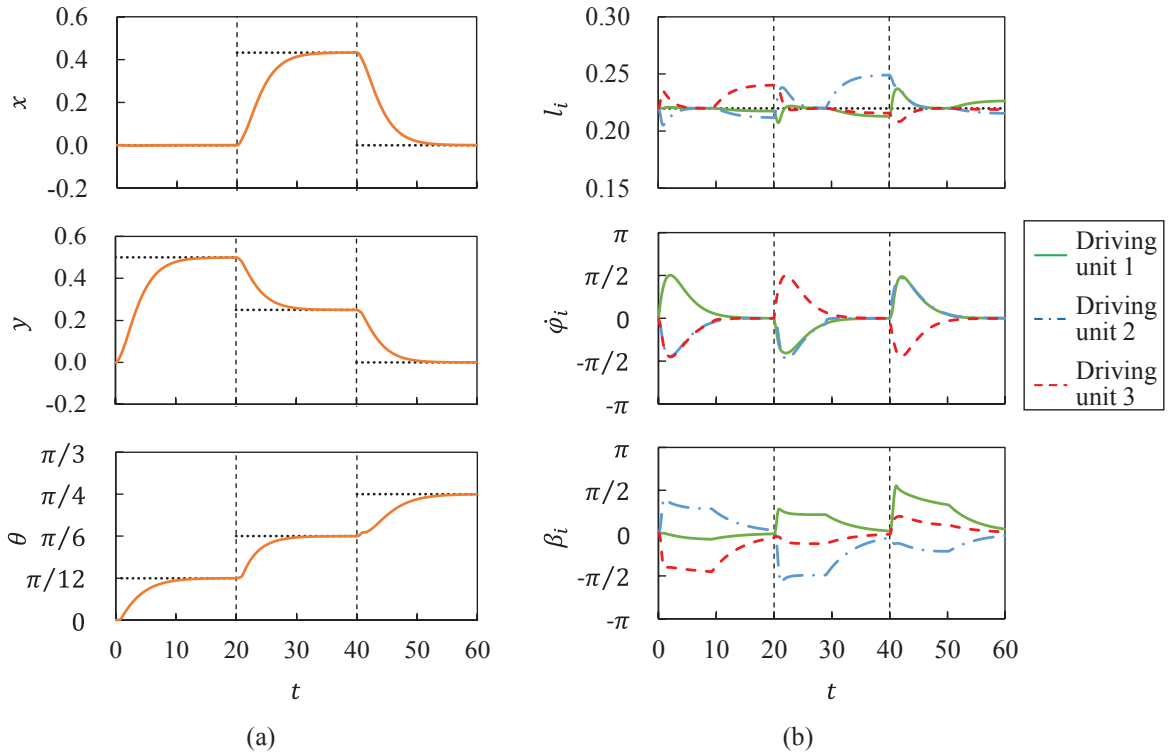


Fig. 3.2

Simulation results: (a) Trajectory of the main body of SWOM. The dotted line indicates the desired value. (b) Time variation of the parameters of each driving unit—relative position l_i , wheel rotational speed ϕ_i , and steering angle β_i .

3.3.2 Experiment

To demonstrate the effectiveness of the developed control system, experiments were conducted using the second SWOM prototype pictured in Fig. 3.3. The base block of the prototype is larger

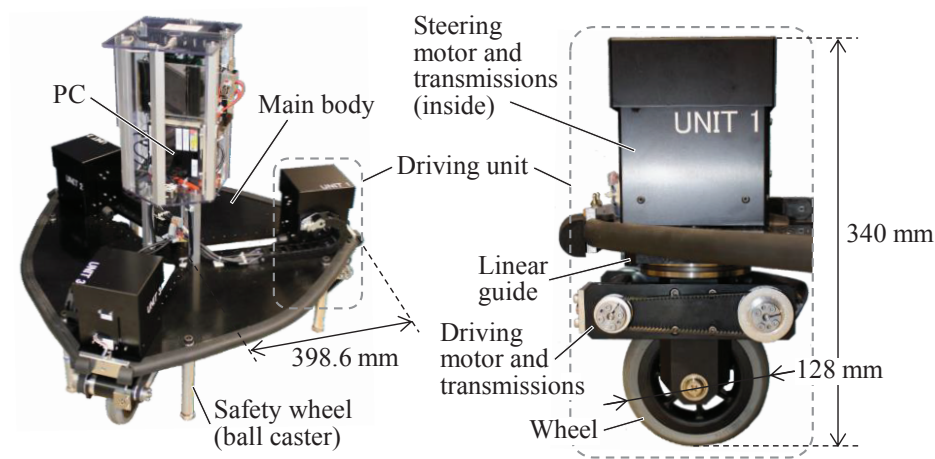


Fig. 3.3
Second SWOM prototype

Table 3.1
Specifications of SWOM prototype

Whole system	Weight	33 kg
Driving unit (driving / steering)	Maximum speed	9.32 / 10.7 rad/s
	Maximum torque	38.0 / 41.7 N·m
	Resolution of encoder	1.35×10^{-5} / 1.54×10^{-5} rad
Linear guide	Movable range	$0.122 \text{ m} \leq l_i \leq 0.314 \text{ m}$
Motion capture	Operation period	1,000 Hz
Linear encoder	Resolution	0.1 μm

than that of the previous version shown in Fig. 2.11(a), but the fundamental structure is the same. Two parallel linear guides placed on the bottom of the main body constitute the sliding joint. The driving unit is composed of the upper and lower parts separated by the linear guides. The traveling direction of the wheel changes by rotating the lower part around the vertical axis. The upper part has a steering motor, and the lower part has a driving motor and a wheel. The output rotation of each motor is transmitted through a gear head and timing belts. This prototype has six ball casters as safety wheels to prevent tipover, but they normally do not affect the motion of the prototype because usually, only the three wheels of the driving units contact the ground. Specifications of the SWOM prototype are shown in Table 3.1.

Fig. 3.4 shows the structure of the control system hardware in the new SWOM prototype. To measure the state variables for state feedback, motion capture cameras for the position of the main body, linear encoders for the displacement of the linear guides, and rotary encoders for the rotating velocity and steering angle/velocity of the wheels are installed. The velocities of the main body and the linear guides were estimated using the complementary filters combining the time difference of each position and the theoretical velocity calculated by Eq. (3.1). The values measured by the sensors were sent to the PC in the prototype, and after calculation, the PC sent

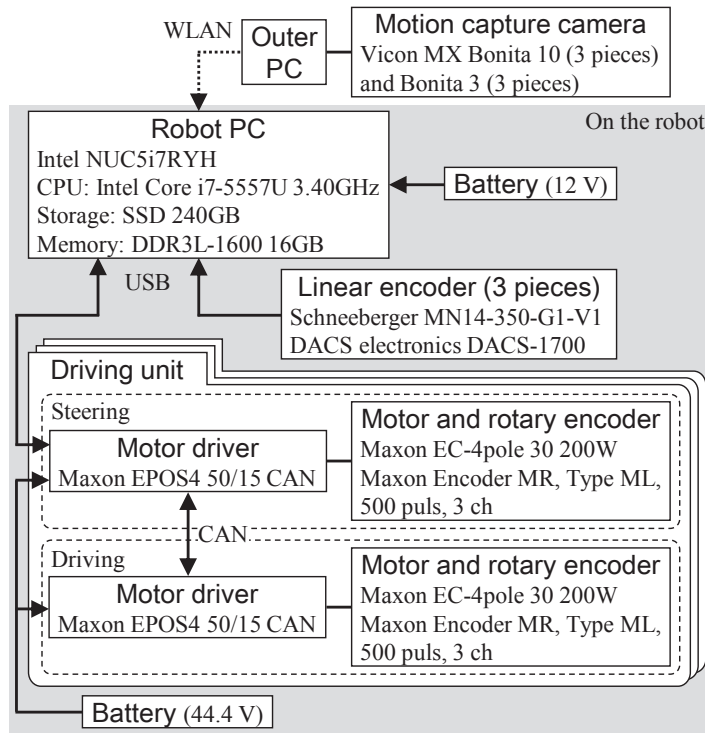


Fig. 3.4
Control system hardware for SWOM prototype

commands to the motors through the motor drivers. The processing was done in a cycle taking about 80 ms.

An experiment was conducted by making this prototype follow the simulated motion described in Section 3.3.1. The results are shown in Fig. 3.5 and Fig. 3.6. Fig. 3.5(a) illustrates the measured path of the SWOM prototype and the simulated path. To compare the measured state and the desired state at $t = 20, 40$, and 60 s, the position and orientation of the Y-shaped object in the figure indicates the position and orientation of the main body and the length of each limb indicates l_i (slider position). The result proves that the prototype reached the desired position and orientation at $t = 20, 40$, and 60 s, though the measured path tended to deviate outward compared to the simulated path. The sequence of the prototype movement for $t = 20 - 40$ s is schematically shown in Fig. 3.5(b). Each wheel moves along a smooth curve while leading the main body to the desired state and keeping the relative position of the sliding joint around the middle of its movable range. Fig. 3.6(a) shows the measured trajectories of the main body, and Fig. 3.6(b) shows the other state variables. Fig. 3.6(a) demonstrates that the variables x, y , and θ all reach their desired values at $t = 20, 40$, and 60 s. As in the upper plot of Fig. 3.6(b), l_i was maintained within the range where the driving units did not reach the ends of the sliding joint rails. Comparison of $\dot{\phi}_i$ and β_i in the middle and lower graphs in Fig. 3.6(b) shows that β_i did not diverge, even near the singular configuration $\dot{\phi}_i = 0$. These results demonstrate that the developed control system works effectively.

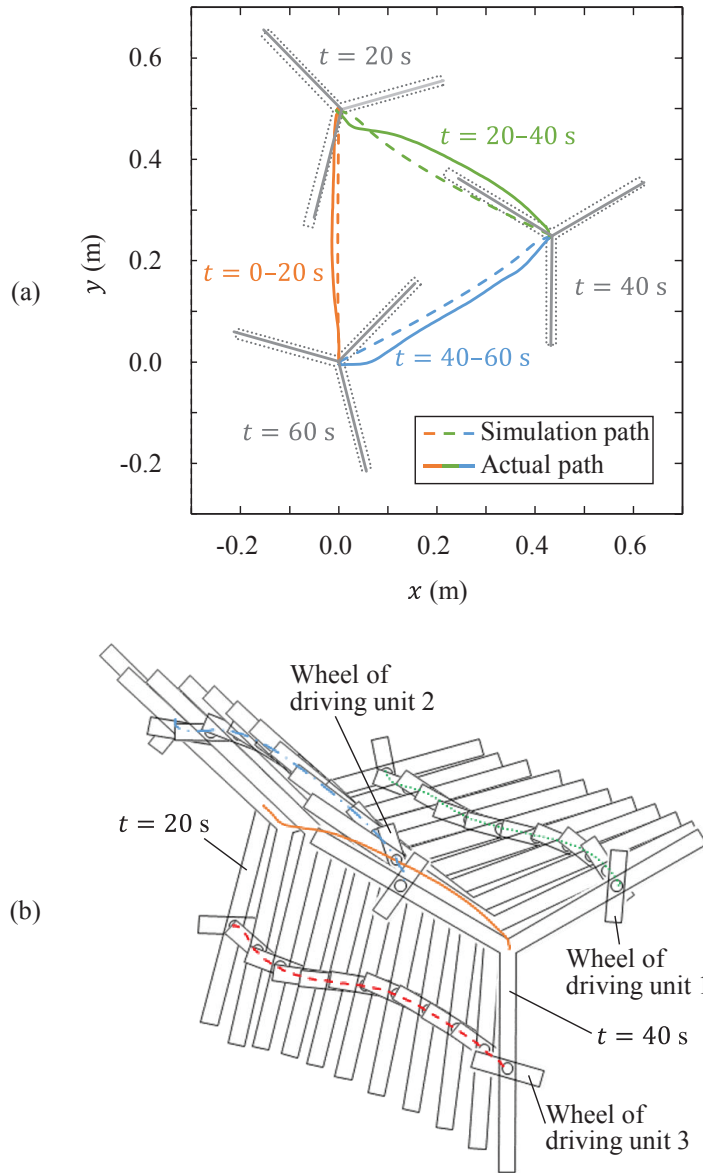


Fig. 3.5

(a) Path of the main body of the SWOM prototype. The solid line indicates the actual path of the experimental result measured by the motion capture cameras. The dashed lines indicate the simulation path. The Y-shaped objects show the states of the SWOM at $t = 20$, 40 , and 60 s, where the solid lines indicates the experimental results and the dotted lines describe the desired states. (b) Motion and paths of the main body and three wheels for $t = 20 - 40$ s.

3.3.3 Discussions

Comparison of Fig. 3.6 and Fig. 3.2 shows that the time variation of β_i in Fig. 3.6 is larger than that in Fig. 3.2. For example, β_3 existed in the range of $-\pi/2 \leq \beta_3 \leq \pi/2$ in Fig. 3.2, whereas β_3 in Fig. 3.6 varied over mainly the range of $-3\pi/2 \leq \beta_3 \leq -\pi/2$. This is not because of the

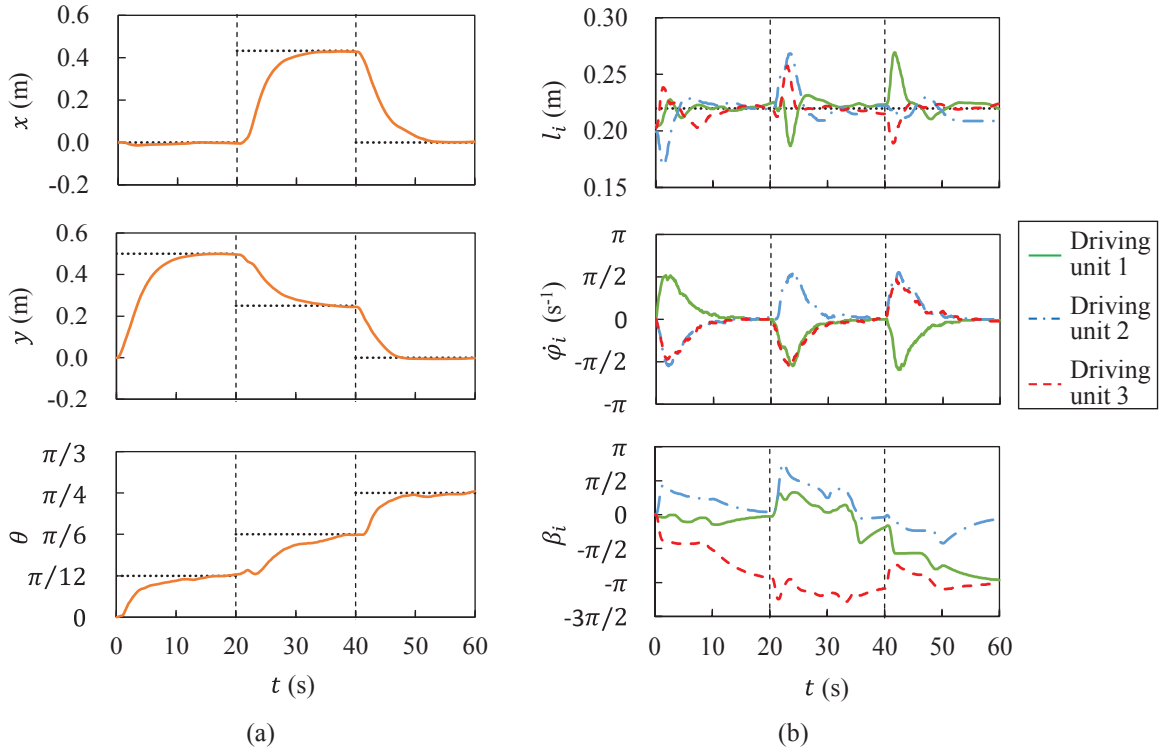


Fig. 3.6

Experimental results: (a) Trajectory of the main body of SWOM given by the motion capture cameras. Dotted lines indicate the desired value. (b) Time variation of the parameters of each driving unit—relative position l_i , wheel rotational speed $\dot{\phi}_i$, and steering angle β_i given by the encoders installed in the prototype.

divergence of β_i in the singular configuration $\dot{\phi}_i = 0$ but rather because of the symmetrical structure of the driving units. In the driving units of SWOM, changing the steering angle of a wheel by $\pm\pi$ is substantially the same as driving the wheel in reverse. Then, the system was designed for β_i to approach the nearest $n_i\pi$ in Eq. (3.18) regardless of what direction the wheel was traveling. Actually, the values of $\dot{\phi}_3$ in Fig. 3.6 and Fig. 3.2 reversed in positivity and negativity for $20\text{ s} \leq t \leq 60\text{ s}$, but this phenomenon does not disrupt the motion and control of the SWOM.

When other parameters are compared, the maximum variation of l_i from the desired value is larger in Fig. 3.6 than in Fig. 3.2. Such l_i peaks do not appear during the secondary controller is applied where l_i is not controlled but during the normal controller is applied where the state variables, including l_i , are controlled. This suggests that some difference between the SWOM prototype and the model used in the control system affects its stability. Considering that the developed control system is based completely on kinematics, the error factors are thought to come mainly from dynamical disturbances, such as wheel slippage, sliding joint friction, and drive mechanism viscosity. Using a disturbance observer or a dynamics-based controller can address these factors and achieve a more stable system. Additionally, the proposed control system handles

the condition that the driving units do not get to the ends of the sliding joints, that is, l_i is limited within a certain range, by giving large feedback gains and making l_i converge as quickly as possible when the normal controller is applied. However, this may decrease the efficiency of the movement because l_i can have priority over the position and orientation of the main body in the control system. Moreover, this control method does not assure that l_i values stay away from the upper and lower limits. Regarding this issue, a feedback controller based on the inequality constraints of l_i , such as a potential method, can be used.

3.4 Summary

The proposed SWOM needs feedback control for practicality. To address this issue, the controller is built by using feedback linearization method and evaluated its effectiveness through a simulation and an experiment. The following results were obtained.

- Based on the kinematic equations of SWOM, which is described as a nonlinear system with nonholonomic constraints, the exactly linearized state equation of SWOM is derived using feedback and coordinate transformation.
- A controller that stabilizes the linearized SWOM system was designed. The unwanted singular configurations in the controller were evaluated and a control strategy to avoid them was proposed.
- The numerical simulations and experiment demonstrated that SWOM could control its state including the position and orientation and the relative positions of the sliding joints as expected, which verified the effectiveness of the developed control system.

Chapter 4

Active Omni Wheel Achieving Seamless Periphery and Simple Structure

4.1 Introduction

In Chapter 2 and 3, a novel omnidirectional wheeled mechanism, a slidable wheel, was proposed and its characteristics in an omnidirectional mobile robot were clarified. The slidable wheel has advantages mainly in a simple structure and less constraint compared to the other omnidirectional wheeled mechanisms with two active DOFs. On the other hand, as shown in Chapter 3, its control complexity seems higher than the other mechanisms. This may not be a serious problem for automated or semi-automated mobile robots that have enough capability to perform, but unfavorable in a certain type of vehicles, such as electric wheelchairs.

Now let us refer the comparison of the wheeled mechanisms in Section 2.2.3 again and focus on the AOW (the omni wheel wit two active DOF). The previously developed AOW and omnidirectional vehicle are shown in Fig. 4.1. The AOW has complexity in structure, but simplicity in control, just oppositely to the slidable wheel. The main problems of the AOW are vertical vibration and a large number of required parts. Vertical vibration of locomotion devices is unacceptable in most cases, especially in transporting people, precision instruments, and chemicals. A large number of parts can lead to an increase in manufacturing cost, size, and failure rate. On the other hand, if these problems are solved, the AOW will become more useful.

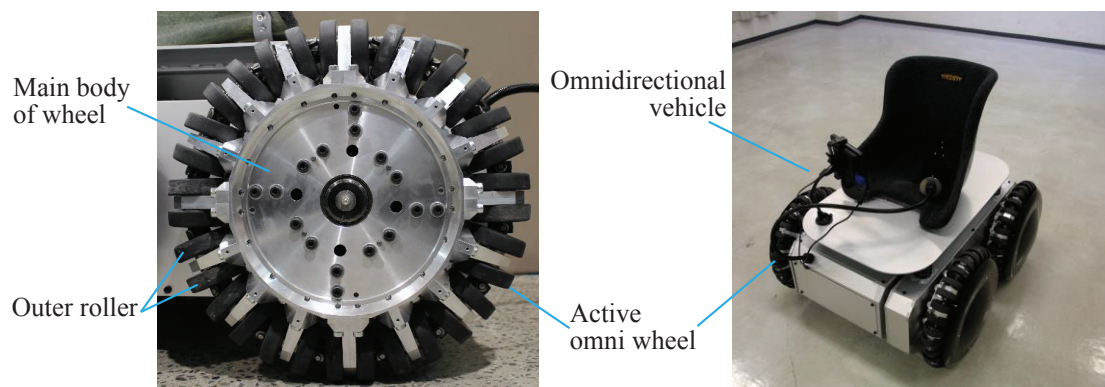


Fig. 4.1
Active omni wheel (AOW) and omnidirectional vehicle developed in [62].

This chapter proposes a new AOW design improved in the suppression of the vertical vibration and reduction of the number of components. The geometry, kinematics, and wheel arrangement of the AOW are discussed. Its benefit is evaluated through experiments by using the prototype of the AOW and the vehicle with it.

4.2 Proposition of New AOW

4.2.1 Conventional AOW

First, the structure and motion of the AOW previously developed are explained. Its structure is shown schematically in Fig. 4.2, where Fig. 4.2(a) is a cross-section of the AOW through the rotation axis of the wheel main body and Fig. 4.2(b) is a cross-section parallel to the wheel main body. The outer circumference of the AOW consists mainly of outer rollers, like a normal omni wheel. Inside the AOW, a differential gear mechanism composed of input and output bevel gears is installed. Each input bevel gear is connected to a motor by an input shaft, and the output bevel gear is connected to the outer roller drive system by an output shaft. The outer roller drive system links the output shaft with the outer rollers to transmit the rotation. All of the outer rollers are joined by rotational couplings, and they rotate simultaneously. Bearings are placed between the main body of the wheel and the input and output shafts.

Next, the motion of the AOW is explained. When the two input shafts are rotated by the motors at the same speed in the same direction, as shown in Fig. 4.3(a), the output shaft revolves around the same rotation axis as the input shafts (axis A), and consequently, the wheel main body also

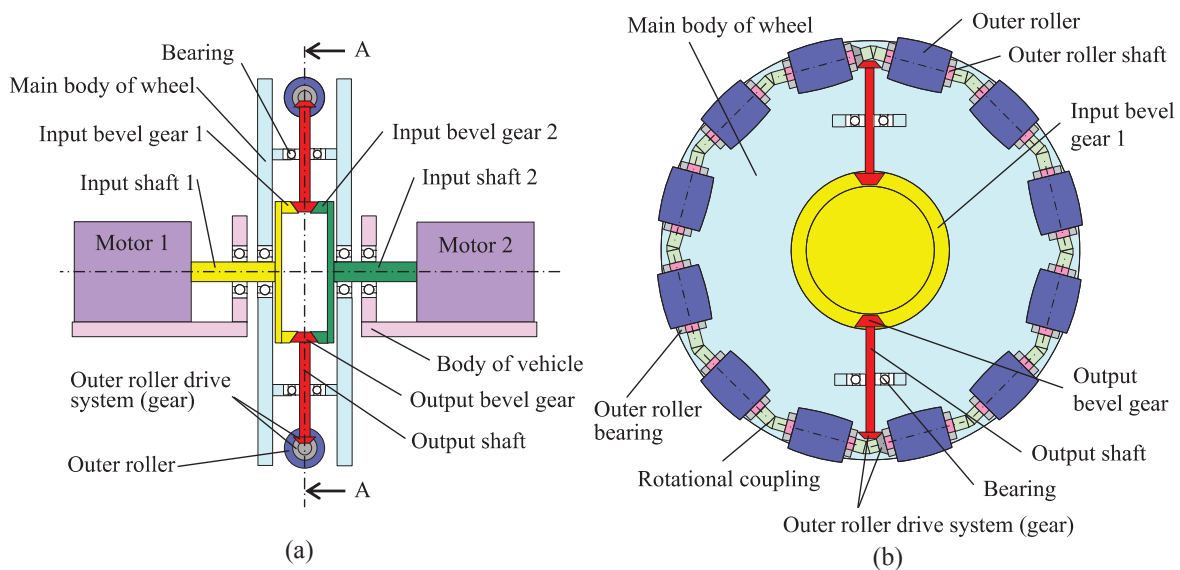


Fig. 4.2 Structure of previously developed AOW [62]. (a) Cross-section through the rotation axis of the main body of the wheel. (b) Cross-section A-A.

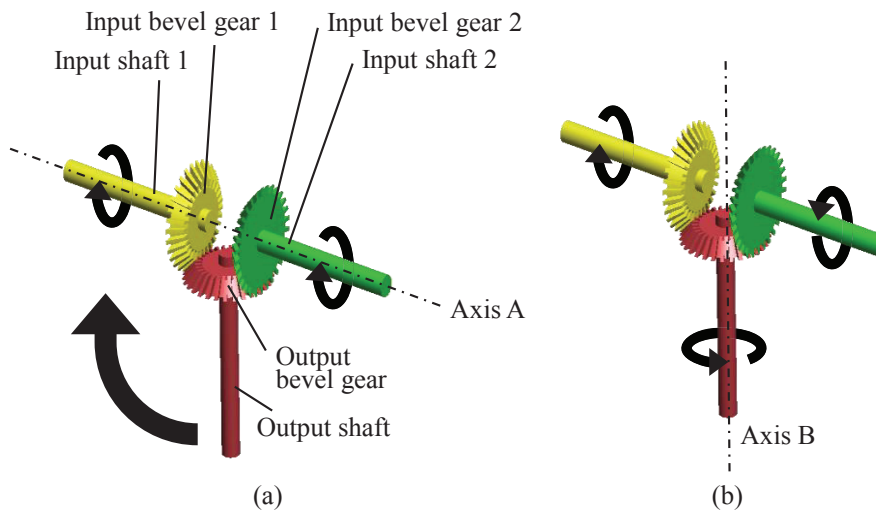


Fig. 4.3

Motion transmission in differential gear mechanism of AOW [62]. (a) Rotation of input shafts in the same direction. (b) Rotation of input shafts in opposite directions.

rotates around axis A. Then, the AOW travels in a forward or backward direction. Oppositely, when the two input shafts are rotated at the same speed in opposite directions, as shown in Fig. 4.3(b), the output shaft rotates around the axis in a radial direction (axis B). It rotates the outer rollers around their axes via the outer roller drive system. Then, the AOW travels in a right or left direction. Accordingly, when the two input shafts are rotated at different speeds, the output shaft rotates around both axes A and B, and both the main body of the wheel and the outer rollers rotate at the same time. Then, the AOW travels in a diagonal direction. Thus, the AOW can travel actively in an arbitrary direction by appropriate rotations of the input shafts.

The structure of the conventional AOW, however, creates some problems. The outer circumference of the AOW consists of outer rollers arranged annularly. To avoid interference between neighboring outer rollers, they are separated by spaces. These spaces are also necessary to provide structural supports and drive systems for the outer rollers, as shown in Fig. 4.2(b). For these reasons, the outer circumference of the AOW does not form a smooth circle. This means that the distance between the axle and the ground varies as the main body of the wheel rotates, which causes a vibration from this vertical motion as the AOW moves forward, backward, or diagonally. To address this issue, the conventional AOW design incorporated as many outer rollers as possible to make the gaps smaller, but this led to an increased number of parts although the omni-wheel-like structure itself often required many components compared to other wheeled mechanisms.

4.2.2 Structure and Motion of New AOW

To solve the problems in the conventional AOW, a new AOW design is developed, whose structure is shown schematically in Fig. 4.4. The new AOW has barrel-shaped outer rollers on both sides of the wheel. The figure shows the design where six outer rollers are installed. All outer rollers have the same shape with a constant surface curvature. These outer rollers are arranged alternately on the left and right sides of the wheel at equal intervals so as to make the circumference of the AOW, as seen from the side, form a full smooth circle. This circle is called the main wheel in the following discussion. Each outer roller has structural supports at both ends to rotate freely and contact a transmission roller in the middle. The drive mechanism of this AOW is basically the same as that of the conventional AOW: Rotation of the two input shafts drives a differential gear mechanism inside the AOW, which rotates the main body of the wheel and the transmission rollers.

Next, the motion of the new AOW is explained. When the two input shafts are rotated at the same speed in the same direction, the main body of the wheel rotates as shown in Fig. 4.3(a), and the AOW travels in a forward or backward direction. When the two input shafts rotate at the same speed in opposite directions, the transmission rollers rotate as shown in Fig. 4.3(b). The rollers transmit their rotation to the outer rollers by friction force because they are in contact, as indicated in Fig. 4.4. Then, the outer rollers rotate and consequently, the AOW can move to the right or left. When the two input shafts rotate at different speeds, both the main body of the wheel and the outer rollers rotate, and the AOW can travel diagonally. Thus, the new AOW can actively travel in an arbitrary direction just as well as the previous AOW design.

Here, the design concept of the new AOW is discussed in detail. As mentioned above, the objectives are summarized into the following two points: to realize seamless periphery for vibration suppression and to reduce the number of parts. Regarding the seamless periphery, the barrel-shaped rollers are adopted. By staggering the outer rollers on the left and right sides, the

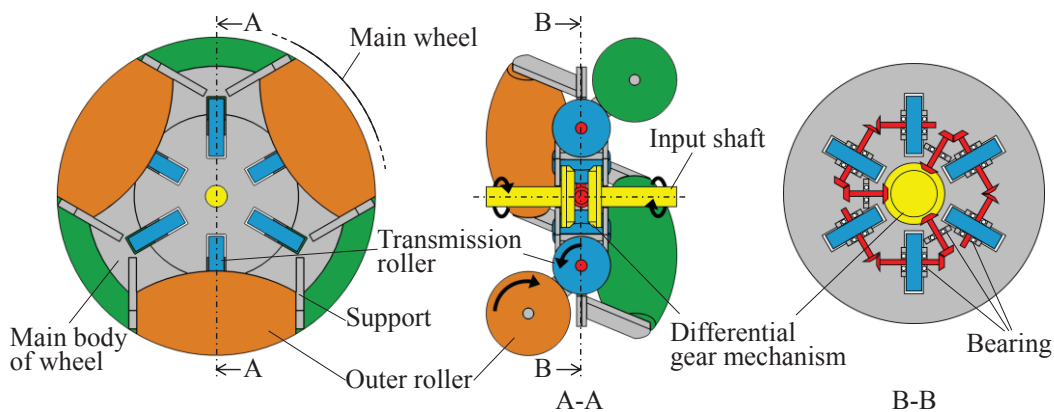


Fig. 4.4
New AOW design with seamless periphery.

rollers eliminate the gaps created by the structural supports at both ends of each roller. Also, using friction instead of gears to drive the rollers eliminates the gaps previously required for the outer roller drive systems. Thus, the periphery of the new AOW forms a seamless circle without gaps between rollers, and the AOW can travel smoothly without vibration.

With respect to parts reduction in the AOW, the barrel shape of the outer rollers makes it easier to reduce the number of outer rollers because each outer roller can cover the larger range of the periphery than in the conventional AOW; actually, as few as four outer rollers can be employed at least in the new AOW. The friction drive with the transmission rollers makes the outer roller drive system simpler than the gear-driven system. Additionally, the drive mechanism is shared by the outer rollers on both sides.

Thus, the proposed new AOW can conquer the problems in the conventional design.

4.3 Analysis of New AOW

In this section, analyses to clarify the structural conditions and kinematical characteristics of the new AOW are described.

4.3.1 Geometry

The barrel-shaped outer rollers of the AOW do not have a constant radius. Here, how this variation in radius should be addressed is described. First, the radius at an arbitrary section of the outer roller is calculated. According to Fig. 4.5, the radius r_r of the section at central angle φ from the section with maximum radius is represented by

$$r_r = r_{r\max} - r(1 - \cos \varphi), \quad -\frac{\pi}{N_r} \leq \varphi \leq \frac{\pi}{N_r}, \quad (4.1)$$

where $r_{r\max}$ is the maximum radius of the outer roller, r is the radius of the main wheel, and N_r is the number of the outer rollers in the AOW.

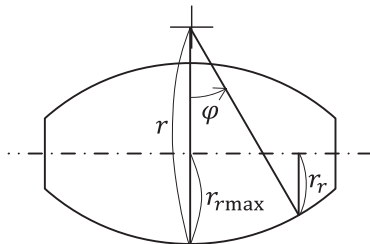


Fig. 4.5
Geometry of barrel-shaped outer roller.

To allow the AOW to travel in a right or left direction, no matter what point of the outer roller contacts the ground, the radius of the outer roller needs to be larger than zero at all sections. Considering that the minimum radius exists at the ends of the outer rollers, that is, $\varphi = \pm \pi/N_r$, this condition is given by

$$r_{r\max} - r \left(1 - \cos \frac{\pi}{N_r}\right) > 0 \Leftrightarrow r_{r\max} > r \left(1 - \cos \frac{\pi}{N_r}\right). \quad (4.2)$$

In contrast, if the radius is too large, the outer rollers on the same side of the AOW interfere with each other. Such interferences occur either when the ends of the outer rollers interfere as in Fig. 4.6(a) or when the sections with maximum radius interfere as in Fig. 4.6(b). The condition for avoiding these situations is expressed as follows:

$$r_{r\max} - r \left(1 - \cos \frac{\pi}{N_r}\right) < \frac{r}{4 \cos \frac{\pi}{N_r}} \wedge r_{r\max} < \frac{r}{2} \\ \Leftrightarrow r_{r\max} < \min \left\{ r \left(1 - \cos \frac{\pi}{N_r} + \frac{1}{4 \cos \frac{\pi}{N_r}}\right), \frac{r}{2} \right\}. \quad (4.3)$$

Eqs. (4.2) and (4.3) give the fundamental geometrical condition of the outer rollers.

Based on the conditions derived above, the radius variations of the outer roller is evaluated. In the AOW, the radius of the outer roller varies according to the ground contact point, which causes velocity variation in the right or left directions when the outer roller rotates at a constant speed. Therefore, to travel in the right or left direction at a constant speed, the rotational speed of the outer rollers needs to change according to the ground contact point. It is preferable to keep such variations small during operation of the AOW. Then, the minimum to maximum radius ratio of the outer roller is defined as

$$\rho = \frac{r_{r\max} - r \left(1 - \cos \frac{\pi}{N_r}\right)}{r_{r\max}}, \quad (4.4)$$

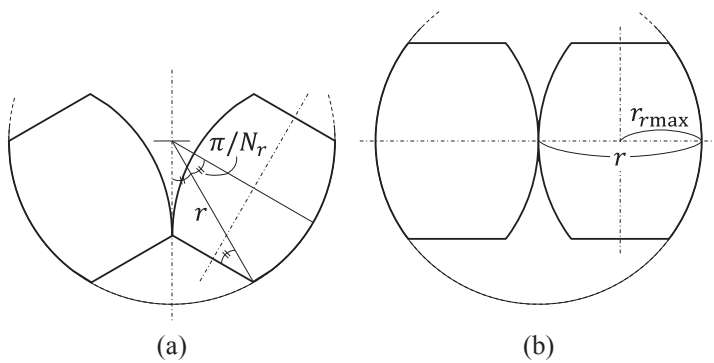


Fig. 4.6
Maximum available space for outer roller radius: (a) Roller end interference. (b) Roller middle interference.

and evaluate the available range of ρ under the conditions of Eqs. (4.2) and (4.3). As ρ approaches 1, these variations become smaller.

The calculation result is shown in Fig. 4.7, where the vertical and horizontal axes represent ρ and N_r , respectively. The lower limit of ρ determined by Eq. (4.2) is $\rho = 0$ and the upper limit of ρ determined by Eq. (4.3) is indicated by the bold black solid line in the graph. The region exceeding this line is no longer a candidate for the AOW design because the outer roller interferences occur. At each N_r , ρ can assume a value between these limits depending on $r_{r\max}/r$. In the graph, the contour lines describe $r_{r\max}/r$ for each ρ and N_r . Note that N_r is restricted to even numbers, but the points are joined to describe contour lines in the graph for clarity. The graph shows that the upper limit of ρ becomes large, that is, the radius variations of the outer rollers becomes small, when N_r is large: In the region where N_r is small, the upper limit of ρ increases remarkably according to the increase of N_r , while in the region where N_r is large, the upper limit of ρ gradually approaches 1. Under the same value of ρ , $r_{r\max}/r$ becomes small as N_r becomes large. For example, the schematics on the right illustrate the geometrical relation of the main wheel and the outer rollers corresponding to points A and B in the graph, which suggests that the radius of the outer rollers becomes large relative to the radius of the main wheel r when N_r becomes small. With respect to the step climbing in the sideways direction, it is preferable to make $r_{r\max}$ as large as possible because the capability depends on the roller radius. At the same time, decreasing N_r , i.e., decreasing the number of the outer rollers leads to decreasing the number of the parts constituting the AOW. In contrast, the small value of $r_{r\max}/r$ contributes to decreasing the horizontal width of the AOW. Therefore, the number and size of outer rollers in the AOW should be designed with this tradeoff in mind.

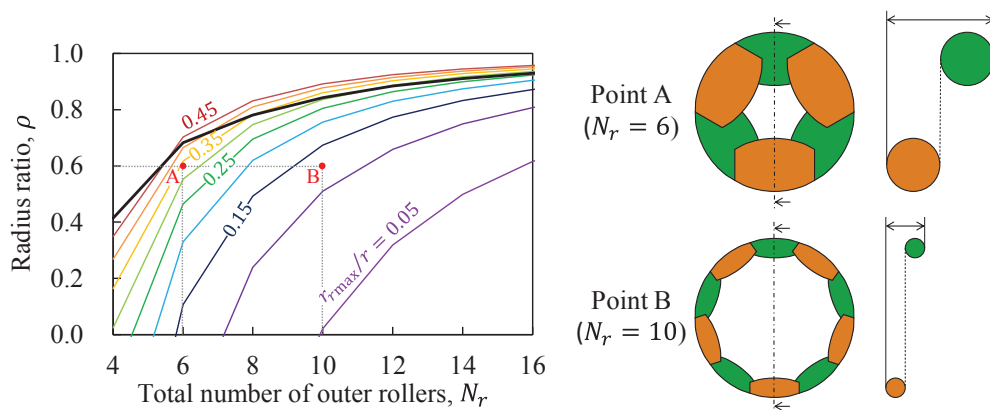


Fig. 4.7

Minimum to maximum radius ratio of the outer roller for each number of outer rollers in the AOW. The bold black line indicates the upper limit of the radius ratio. The colored lines show the contour plots of the ratios between the maximum radius of the outer roller and the radius of the main wheel.

4.3.2 Kinematics

This section explains the kinematics of a single AOW. When the velocity of the AOW in the forward/backward and right/left directions are set as v_Y and v_X , respectively, and the angular velocities input to the two input shafts are ω_1 and ω_2 , respectively, the relation of these velocities is given by

$$\begin{bmatrix} v_X \\ v_Y \end{bmatrix} = \begin{bmatrix} \frac{r_r}{2n_r} & -\frac{r_r}{2n_r} \\ \frac{r}{2} & \frac{r}{2} \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix}, \quad (4.5)$$

where $n_r \in \mathbb{R}$ represents the reduction ratio between the differential gear mechanism and the outer rollers [62].

Eq. (4.5) shows that v_X is proportionate to r_r . As mentioned in Section 4.3.1, the value of r_r varies according to the ground contact point. Therefore, the movement of the AOW is affected by the radius variation when it travels in the right, left, or diagonal direction. Then, the inputs for the AOW to travel at a constant velocity are derived by compensating the variation of r_r . From Eqs. (4.1) and (4.5), the input angular velocities ω_1 and ω_2 to realize certain velocities v_X and v_Y are calculated as follows:

$$\begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} = \begin{bmatrix} \frac{n_r}{r_r} & \frac{1}{r} \\ -\frac{n_r}{r_r} & \frac{1}{r} \end{bmatrix} \begin{bmatrix} v_X \\ v_Y \end{bmatrix} = \begin{bmatrix} \frac{v_Y}{r} + \frac{n_r v_X}{r_{r\max} - r(1 - \cos \varphi)} \\ \frac{v_Y}{r} - \frac{n_r v_X}{r_{r\max} - r(1 - \cos \varphi)} \end{bmatrix}. \quad (4.6)$$

Time differentiation of Eq. (4.6) gives the input angular accelerations $\dot{\omega}_1$ and $\dot{\omega}_2$ as

$$\begin{bmatrix} \dot{\omega}_1 \\ \dot{\omega}_2 \end{bmatrix} = \begin{bmatrix} \frac{\dot{v}_Y}{r} + \frac{n_r \dot{v}_X}{r_{r\max} - r(1 - \cos \varphi)} + \frac{n_r v_X v_Y \sin \varphi}{\{r_{r\max} - r(1 - \cos \varphi)\}^2} \\ \frac{\dot{v}_Y}{r} - \frac{n_r \dot{v}_X}{r_{r\max} - r(1 - \cos \varphi)} - \frac{n_r v_X v_Y \sin \varphi}{\{r_{r\max} - r(1 - \cos \varphi)\}^2} \end{bmatrix}, \quad (4.7)$$

where the relation $r\dot{\varphi} = v_Y$ is used. Considering that the outer roller contacting the ground changes as the main wheel rotates, φ needs to be replaced to meet $-\pi/N_r \leq \varphi \leq \pi/N_r$ according to circumstances, namely,

$$\varphi \rightarrow \varphi - \frac{2j\pi}{N_r}, \quad j \in \mathbb{Z}.$$

By using Eqs. (4.6) and (4.7), the inputs needed to drive the AOW at a constant velocity are calculated, and the result is shown in Fig. 4.8. Here, the situation is chosen where the AOW travels diagonally at a constant velocity with $r = 0.15$, $r_{r\max} = 0.04$, $n_r = 2/3$, $N_r = 6$, $v_X = v_Y = \sqrt{2}/4$, and $\dot{v}_X = \dot{v}_Y = 0$. The graphs show that the input angular velocity and acceleration need to be changed according to the rotational angle of the main wheel. During diagonal travel, especially, the

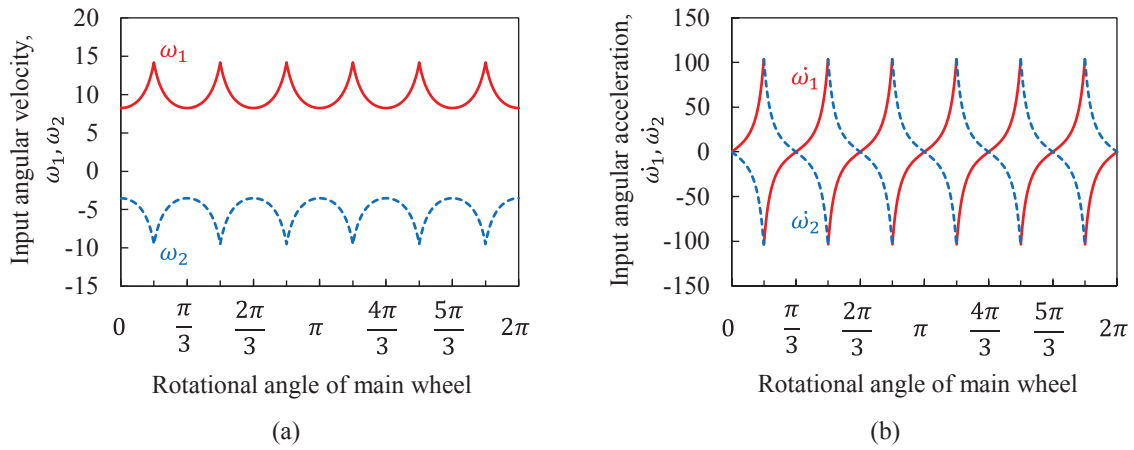


Fig. 4.8

(a) Input angular velocity and (b) acceleration versus rotational angle of the main wheel when the AOW moves diagonally at $v_x = v_y = \sqrt{2}/4$ and $\dot{v}_x = \dot{v}_y = 0$.

rotational angle of the main wheel changes over time, and therefore the inputs also must change. The graphs indicate that the absolute values of the input angular velocity and acceleration take maximums at $\varphi = \pm \pi/N_r$, when the end of an outer roller contacts the ground. Before and after $\varphi = \pm \pi/N_r$, the angular acceleration input needs to change discontinuously because the outer roller contacting the ground changes.

When the AOW travels at a constant speed in a forward, backward, right, or left direction (but not diagonally), Eq. (4.7) gives $\omega_1 = \omega_2 = 0$, and then the input angular velocity becomes constant over time. During right or left travel, however, the input rotational velocity needs to be changed according to the rotational angle of the main wheel as well as in Fig. 4.8(a) because v_x is proportionate to r_r . In contrast, forward or backward travel is not affected by the ground contact, because v_y is independent of r_r .

4.3.3 Acceleration Limit

In the new AOW, the input angular velocity needs to be temporally changed depending on the direction of travel, as discussed above. However, motors can generally accelerate their rotation only within certain limits, which imposes a restriction on the variation of angular velocity that the motors can impart to the input shafts of the AOW. Accordingly, the effect of the input angular acceleration limit on the velocity and acceleration of the AOW is evaluated.

Differentiating Eq. (4.5) by time yields

$$\begin{bmatrix} \dot{v}_x \\ \dot{v}_y \end{bmatrix} = \begin{bmatrix} \frac{r_r}{2n_r}(\omega_1 - \omega_2) - \frac{v_x v_y \sin \varphi}{r_r} \\ \frac{r}{2}(\omega_1 + \omega_2) \end{bmatrix}. \quad (4.8)$$

Here, the situation is considered where the AOW travels at the speed $\bar{v}$ in direction ψ , which is defined as $v_X = \bar{v} \cos \psi$ and $v_Y = \bar{v} \sin \psi$. When the AOW accelerates with an acceleration $\bar{a}$ in the same direction as the traveling direction, that is, $\dot{v}_X = \bar{a} \cos \psi$ and $\dot{v}_Y = \bar{a} \sin \psi$, the following relations are given:

$$\begin{cases} \bar{a} = \dot{v}_X \cos \psi + \dot{v}_Y \sin \psi \\ \quad = \left(\frac{r \sin \psi}{2} + \frac{r_r \cos \psi}{2n_r} \right) \dot{\omega}_1 + \left(\frac{r \sin \psi}{2} - \frac{r_r \cos \psi}{2n_r} \right) \dot{\omega}_2 - \frac{\sin \varphi \cos^2 \psi \sin \psi}{r_r} \bar{v}^2 \\ 0 = \dot{v}_X \sin \psi - \dot{v}_Y \cos \psi \\ \quad = - \left(\frac{r \cos \psi}{2} - \frac{r_r \sin \psi}{2n_r} \right) \dot{\omega}_1 - \left(\frac{r \cos \psi}{2} + \frac{r_r \sin \psi}{2n_r} \right) \dot{\omega}_2 - \frac{\sin \varphi \cos \psi \sin^2 \psi}{r_r} \bar{v}^2 \end{cases} \quad (4.9)$$

Eliminating $\dot{\omega}_2$ from these two equations results in

$$\bar{a} = \frac{r r_r}{n_r r \cos \psi + r_r \sin \psi} \dot{\omega}_1 - \frac{n_r r \sin \varphi \cos \psi \sin \psi}{r_r (n_r r \cos \psi + r_r \sin \psi)} \bar{v}^2 \quad (4.10)$$

Considering the symmetry, the region of $0 \leq \psi \leq \pi/2$ and $\bar{v} \geq 0$ is focused on. At this time, $\bar{a}$ monotonically increases with respect to $\dot{\omega}_1$. When the upper limits of the input angular accelerations are set as $\dot{\omega}_{1\max}$ and $\dot{\omega}_{2\max}$, that is to say,

$$|\dot{\omega}_1| \leq \dot{\omega}_{1\max}, |\dot{\omega}_2| = \left| -\frac{n_r r \cos \psi - r_r \sin \psi}{n_r r \cos \psi + r_r \sin \psi} \dot{\omega}_1 - \frac{2n_r \sin \varphi \cos \psi \sin^2 \psi}{r_r (n_r r \cos \psi + r_r \sin \psi)} \bar{v}^2 \right| \leq \dot{\omega}_{2\max} \quad (4.11)$$

the maximum value of $\dot{\omega}_1$ is calculated as

$$\begin{cases} \min \left(\dot{\omega}_{1\max}, \frac{n_r r \cos \psi + r_r \sin \psi}{n_r r \cos \psi - r_r \sin \psi} \dot{\omega}_{2\max} - \frac{2n_r \sin \varphi \cos \psi \sin^2 \psi}{r_r (n_r r \cos \psi - r_r \sin \psi)} \bar{v}^2 \right) & \text{if } \frac{n_r r}{r_r} > \tan \psi \\ \dot{\omega}_{1\max} & \text{if } \frac{n_r r}{r_r} = \tan \psi \\ \min \left(\dot{\omega}_{1\max}, -\frac{n_r r \cos \psi + r_r \sin \psi}{n_r r \cos \psi - r_r \sin \psi} \dot{\omega}_{2\max} - \frac{2n_r \sin \varphi \cos \psi \sin^2 \psi}{r_r (n_r r \cos \psi - r_r \sin \psi)} \bar{v}^2 \right) & \text{if } \frac{n_r r}{r_r} < \tan \psi \end{cases} \quad (4.12)$$

By substituting Eq. (4.12) into Eq. (4.10), we obtain the maximum value of acceleration $\bar{a}$.

The calculation results of the maximum acceleration at each pair of values for $\bar{v}$ and ψ are shown in Fig. 4.9. The parameters used were $r = 0.15$, $r_{r\max} = 0.04$, $n_r = 2/3$, and $N_r = 6$. The maximum input angular velocities were set as $\dot{\omega}_{1\max} = \dot{\omega}_{2\max} = 50\pi$. Note that the maximum acceleration in the graph is normalized by using the ratio to the value at $\bar{v} = 0$ for each ψ to compare the effect of the radius variation of the outer rollers on the traveling acceleration. The graph shows that the maximum acceleration monotonically decreases as $\bar{v}$ increases, except when $\psi = 0$ and $\pi/2$. This is because the input angular velocity needs to be changed periodically during diagonal travel, as shown in Fig. 4.8, and the required angular velocity becomes larger as $\bar{v}$ becomes larger. The value of $\bar{v}$ when the maximum acceleration is zero represents the maximum velocity of the AOW in each direction. The decrease of the maximum acceleration increases as ψ approaches $\pi/4$. Meanwhile, the maximum acceleration is constant independent of $\bar{v}$ at $\psi = 0$ and $\pi/2$ because the input angular velocity does not need to change temporally for non-diagonal

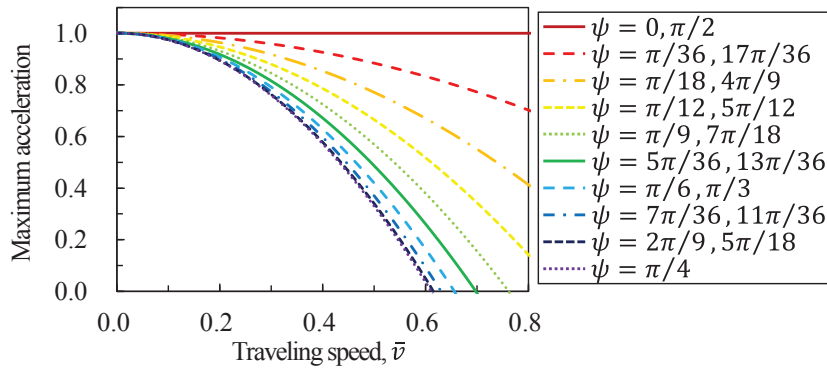


Fig. 4.9

Maximum acceleration of AOW in various different directions when it travels at various speeds in the initial direction. The acceleration indicates the ratio to the value at $\bar{v} = 0$.

travel (forward/backward and right/left). Thus, the maximum velocity and acceleration of the AOW are different depending on the direction of travel. Therefore, it is needed to design the AOW so that it can travel at the required speed in each direction. Furthermore, it is needed to select motors and generate commands that allow the maximum rotational acceleration to vary depending on the rotational speed especially under high-speed and high-acceleration conditions.

4.4 Two-wheel-drive Vehicle Using AOW

The new AOW can be used to realize an omnidirectional vehicle that achieves three-DOF planar motion, including arbitrary-directional translation and turns. This section discusses the structure and motion of the omnidirectional vehicle.

The AOW itself has two active DOFs, so that a driving wheel with at least one active DOF is necessary in addition to the AOW to make a three-DOF vehicle. The most basic idea is to use two AOWs, but it causes the overconstraint-like situation as explained in Section 2.1 and high-accurate harmonization of the motors are required. To avoid this situation, the other wheel must have only one active DOF. Moreover, for omnidirectional movement, the driving wheel must not impose nonholonomic constraints on the vehicle motion. By considering these conditions, a two-wheel-drive (2WD) vehicle with one AOW and one omni wheel (an omni wheel with one active DOF and one passive DOF) is proposed.

Thus the combination of the wheeled mechanisms for an omnidirectional vehicle is determined. However, the flexibility of the wheel arrangement is infinite. Then, it is next discussed based on kinematics. As shown in Fig. 4.10, each wheel is located at an angle α_i and distance l_i from the center of the vehicle directed at β_i , where $\alpha_i = 0$ corresponds to the right direction of the vehicle and $\beta_i = 0$ corresponds to the situation when the line passing through the centers of the vehicle

and the wheel cross the wheel plane at right angles. These nomenclature is the same as in the previous chapters. The index $i = 1$ represents the parameters belonging to the AOW, and $i = 2$ represents the parameters belonging to the omni wheel. When the forward or backward velocity of the vehicle is set as $\dot{Y}$, the right or left velocity as $\dot{X}$, the angular velocity as $\dot{\theta}$, and the forward or backward velocity of the omni wheel as v_w , the relation between the velocities of the wheels and the vehicle is given as follows based on Eqs. (2.7) and (2.8):

$$\hat{A} \begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} v_Y \\ v_X \\ v_w \end{bmatrix} \quad (4.13)$$

$$\hat{A} = \begin{bmatrix} -\sin(\alpha_1 + \beta_1) & \cos(\alpha_1 + \beta_1) & l_1 \cos \beta_1 \\ \cos(\alpha_1 + \beta_1) & \sin(\alpha_1 + \beta_1) & l_1 \sin \beta_1 \\ -\sin(\alpha_2 + \beta_2) & \cos(\alpha_2 + \beta_2) & l_2 \cos \beta_2 \end{bmatrix}.$$

Eq. (4.13) expresses the kinematics of the proposed omnidirectional vehicle.

Based on the derived kinematics, the wheel arrangement in the proposed omnidirectional vehicle is evaluated. The determinant of the matrix $\hat{A}$ in Eq. (4.13) is calculated as

$$\begin{aligned} \det \hat{A} &= l_1 \cos(\alpha_1 - \alpha_2 - \beta_2) - l_2 \cos \beta_2 \\ &= \{l_1 \cos(\alpha_2 - \alpha_1) - l_2\} \cos \beta_2 - l_1 \sin(\alpha_2 - \alpha_1) \sin \beta_2. \end{aligned} \quad (4.14)$$

When the determinant is zero, namely,

$$\det \hat{A} = 0 \Leftrightarrow \tan \beta_2 = \frac{l_1 \cos(\alpha_2 - \alpha_1) - l_2}{l_1 \sin(\alpha_2 - \alpha_1)} \quad (4.15)$$

is given, Eq. (4.13) assumes a singular configuration where the forward kinematics is not established. At this time, the motion of the vehicle is not determined by any wheel velocity. The condition for Eq. (4.15) indicates the situation when the AOW is located in the wheel plane of the omni wheel, as shown in Fig. 4.11. In this case, the vehicle cannot control the rotation around the AOW at all. Therefore, in the design of the vehicle, the states of or near those shown in Fig. 4.11

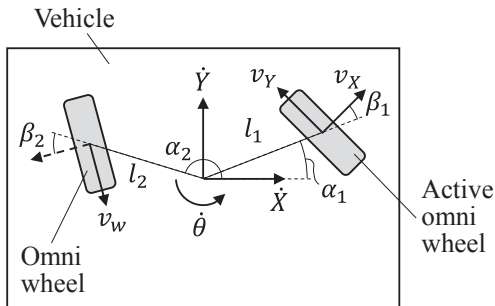


Fig. 4.10

Kinematic model for omnidirectional vehicle with one AOW and one omni wheel.

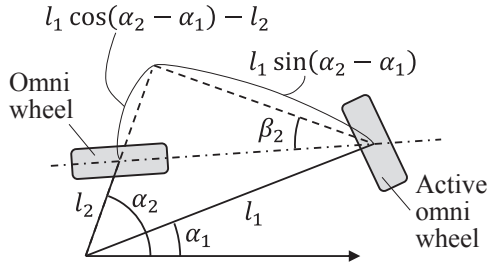


Fig. 4.11

Singular configuration in the omnidirectional vehicle, where the center of the AOW is on the wheel plane of the omni wheel.

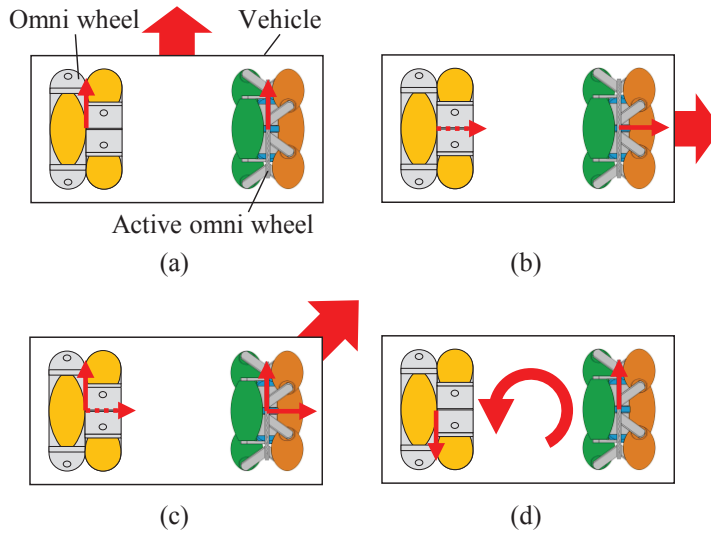


Fig. 4.12

Omnidirectional movement of the two-wheel-drive (2WD) vehicle using the new AOW and an omni wheel. (a) Forward translation. (b) Right translation. (c) Right-forward (diagonal) translation. (d) Turning.

must be avoided because the motion becomes unstable. In contrast, arranging the AOW and omni wheel in parallel achieves the state farthest from the singular configuration under the designated α_1 , α_2 , l_1 , and l_2 , and the vehicle is more likely to perform well. Additionally, $\det \hat{A}$ expressed by Eq. (4.14) is equivalent to the distance between the wheel plane of the omni wheel and the center point of the AOW. Then, the parameters α_1 , α_2 , l_1 , and l_2 should be designed to make the distance as long as possible.

Based on this result, the wheel arrangement where the AOW and the omni wheel are arranged coaxially in parallel is adopted. The vehicle's omnidirectional movement is shown in Fig. 4.12. When both wheels travel forward at the same speed, as shown in Fig. 4.12(a), the vehicle moves forward. When the AOW travels to the right, as in Fig. 4.12(b), the omni wheel also travels to the right at the same speed because it travels on the passively rotating rollers. As a result, the vehicle moves to the right at a speed determined by the AOW. This means that the passive rotation of the rollers in the omni wheel can be controlled by the active rotation of the AOW, which makes it easy

to compensate the undesired effect of resistance in the passive rollers. As shown in Fig. 4.12(c), when the AOW travels diagonally and the omni wheel travels forward in the forward direction component velocity of the AOW, the vehicle performs the diagonal translation. Fig. 4.12(d) shows that rotating the AOW and the omni wheel in opposite directions makes the vehicle turn. Thus, the vehicle can realize planar motion in three DOF, omnidirectional movement, by combining these movements.

4.5 Development and Experiments

To verify that the new AOW installed in an omnidirectional vehicle performs as expected, a prototype is developed and experiments are conducted.

4.5.1 Design and Development of Prototype

First, the design specifications of the AOW prototype are explained. Concerning the design objectives of the new AOW described in Section 4.2.2, it is aimed to reduce the magnitude of vibration caused by the gaps between the outer rollers to smaller than a tenth of that in the conventional AOW (Fig. 4.1), and to decrease the number of main parts necessary for the fundamental structure and function to the half or less. Besides them, it is intended that the load capacity is greater than 900 N, the diameter of the AOW is smaller than 0.35 m (approximately 75% of the conventional AOW diameter), and the driving force capacity is larger than 200 N. The prototype is assumed to be used on a flat ground. Based on these specifications, the AOW prototype was designed.

Fig. 4.13 shows the developed AOW. As shown in Fig. 4.13(a), its structure is basically the same as the structure shown in Fig. 4.4. Six outer rollers ($N_r = 6$) are arranged alternately on the left and right sides at 60° intervals. Seen from the side, the arcs of the outer rollers are connected smoothly, which creates a circular wheel circumference. The transmission system shown inside the wheel in Fig. 4.13(b) is divided into three paths after the differential gear mechanism, each of which rotates the adjacent two transmission rollers. This means that the outer rollers on both sides share the same power transmission path because the transmission rollers are located in the same plane. Thus, the new AOW has fewer outer rollers, a simpler transmission system, and fewer parts than the conventional AOW shown in Fig. 4.1. With respect to the vibration and the number of the parts in the AOW, a quantitative evaluation is made in Section 4.5.3. The diameter of the developed AOW is 0.3 m, which meets the design specification. The material, size and shape of

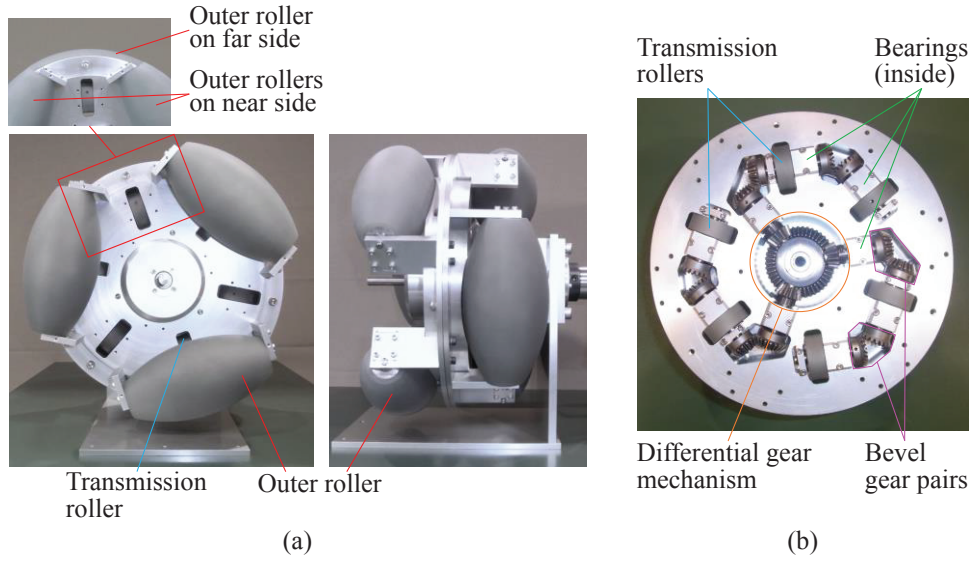


Fig. 4.13

(a) Developed new AOW. (b) Transmission system inside the AOW.

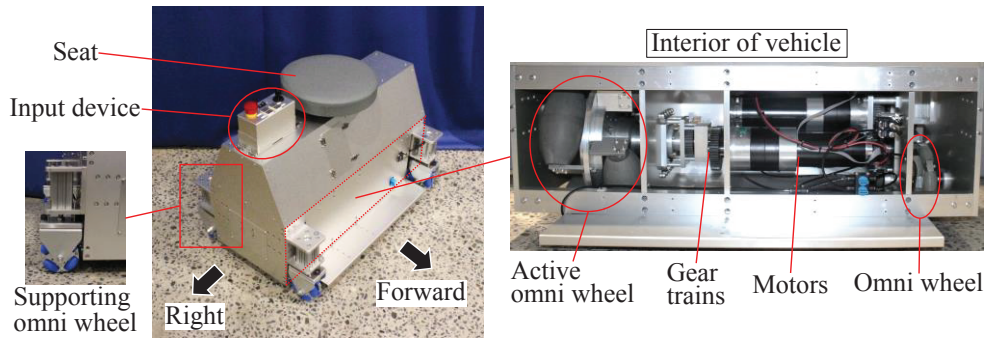


Fig. 4.14

Developed 2WD omnidirectional vehicle with one new AOW and one omni wheel.

each part are designed to satisfy the specifications of the load capacity and the driving force capacity. The parameters of the developed AOW are $r = 0.15$ m, $r_{\max} = 0.04$ m, and $n_r = 2/3$.

Fig. 4.14 shows the 2WD omnidirectional vehicle developed using the new AOW. The right driving wheel is the AOW and the left driving wheel is the omni wheel. These wheels are arranged in parallel and their axles are in the same vertical plane. The two input shafts of the AOW and the one input shaft of the omni wheel are driven independently by three motors. In addition to the driving wheels, supporting wheels are attached at the four corners of the vehicle to stabilize it horizontally. The supporting wheels are omni wheels whose main body and outer rollers all rotate passively. The parameters of the developed vehicle are $\alpha_1 = 0$, $l_1 = 0.275$ m, $\beta_1 = 0$, $\alpha_2 = \pi$, $l_2 = 0.330$ m, and $\beta_2 = -\pi$. The sizes of other parts are as follows: The height from the ground to the seat is 0.55 m, the total width of the vehicle is 0.80 m, the radius of the driving omni wheel is 0.10 m, and the radii of the supporting omni wheels are 0.05 m.

Fig. 4.15 shows the structure of the control system hardware of the developed omnidirectional vehicle. The vehicle is operated using a three-axis joystick. The tilt and twist angles input with the joystick are sent to the controller and converted into the target velocity of the vehicle. The input angular velocity for each motor is generated from the target velocity using the inverse kinematics, such as Eqs. (4.6) and (4.13). The angular velocity commands are sent to each motor driver. The angular velocity commands are restricted not to exceed the limits of the rotational acceleration, as discussed in Section 4.3.3. The rotational angle of the main wheel of the AOW is measured by a rotary encoder and used in the calculation of the inverse kinematics.

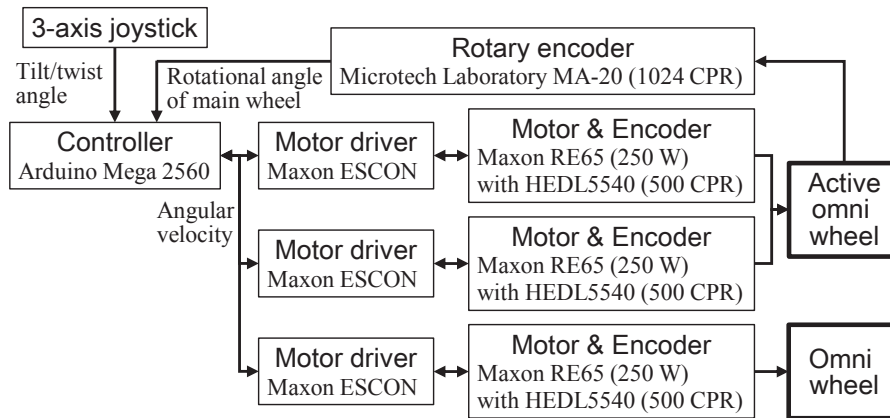


Fig. 4.15
Control system hardware for developed omnidirectional vehicle.

4.5.2 Experiments

Mobility testing of the developed vehicle is pictured in Fig. 4.16. In this experiment, a person rode on and operated the vehicle. The operator assented after being informed about the experiments prior to participation. The figure shows (a) forward translation, (b) right translation, (c) diagonal translation, and (d) turning. All of these movements were achieved as expected.

The input angular velocities of the AOW ω_1 and ω_2 need to be changed according to the changing ground contact point of the outer rollers during diagonal translation, as discussed in Section 4.3. To demonstrate that this behavior was actually realized in the developed vehicle, the rotational speeds of the motors were measured with the rotary encoder attached to each motor, and compared them with the commanded values. Fig. 4.17 shows the measurement result when the target velocity was $\dot{X} = \dot{Y} = 0.28$ m/s, which is the same movement as in Fig. 4.16(c). The graph extracts the region where the velocity became almost constant after a certain period of time from the start at 0 s. It shows that the measured and commanded values of ω_1 and ω_2 agreed with each other and verifies that the developed mechanisms work according to the theory.

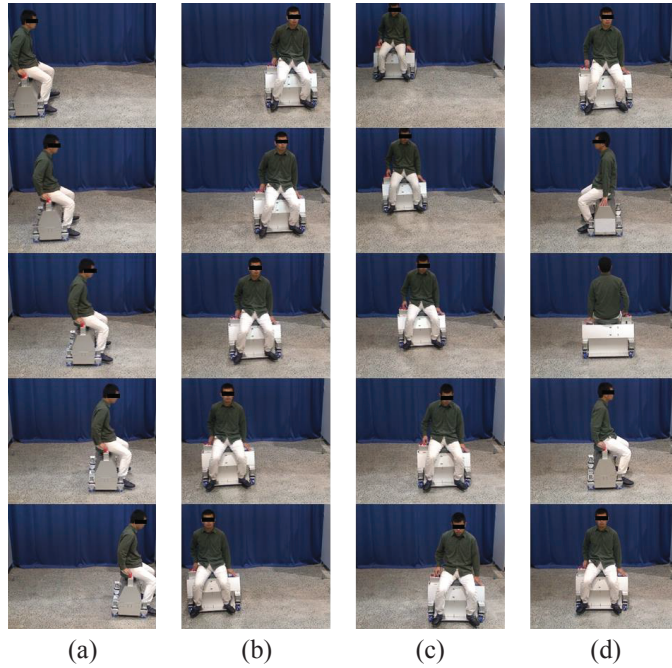


Fig. 4.16
Mobility testing of developed omnidirectional vehicle. (a) Forward translation. (b) Right translation. (c) Diagonal translation. (d) Turning.

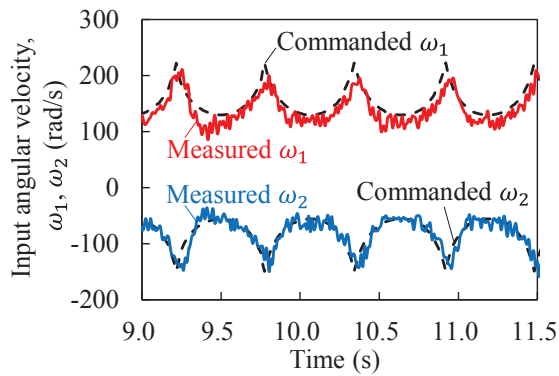


Fig. 4.17
Measured values of angular velocities ω_1 and ω_2 compared to values commanded by the AOW controller.

When the result of Fig. 4.17 is evaluated in detail by focusing on the peaks of ω_1 and ω_2 , the measured values seem delayed compared to the commanded values by about 50 ms. Such delays cause a difference between the ground contact points of the outer rollers in the calculation and the actual state, which leads to errors in the right/left velocity of the vehicle $\dot{X}$. Fig. 4.18 shows the simulation result of $\dot{X}$ when ω_1 and ω_2 are input with a 50-ms delay. The graph clarifies that $\dot{X}$ varies around the target value $\dot{X} = 0.28$ m/s. In fact, the vehicle operator in the experiment perceived lateral vibration when moving diagonally. Regarding this issue, dynamics-based feedforward control is thought to be useful for solving the delay problem.

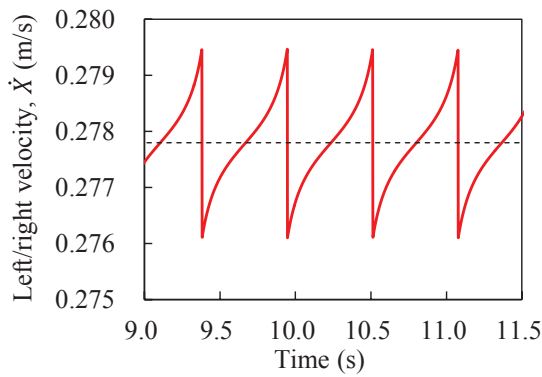


Fig. 4.18

Variation in the right/left velocity of the vehicle $\dot{X}$ when a 50-ms delay exists between the actual and commanded input angular velocities.

4.5.3 Comparison of Conventional and New AOWs

To show the effectiveness of the developed new AOW, it is compared with the conventional AOW shown in Fig. 4.1. The problem in the conventional AOW is that it has gaps between the outer rollers, which cause vibration during traveling, and it requires a large number of parts to make the gaps smaller. The new AOW is designed to solve these problems.

First, in order to evaluate the traveling vibration, the vertical vibration of the vehicles equipped with the conventional and new AOWs (Fig. 4.1 and Fig. 4.14, respectively) was measured. An acceleration meter (Cosmic ME Co., Inc., Sorocaba-1) was fixed on the center of the upper plane of the vehicles after removing the seats. The sampling range was 20 m/s^2 , the sampling rate was 1 ms , and the sampling number was $2^{14} = 16384$. The vehicles traveled forward at $\dot{Y} = 0.3 \text{ m/s}$. Fig. 4.19 shows the measurement result of the vibration spectra. In the figure, the dotted lines indicate the multiples of the specific frequency f_s of the vibration resulting from the gaps between the outer rollers. Namely, f_s is given by the following equation.

$$f_s = \frac{\dot{Y}}{2\pi r} N_r \quad (4.16)$$

In the conventional AOW, $r = 0.23 \text{ m}$ and $N_r = 32$ is applied, and then $f_s = 6.57 \text{ Hz}$ is supposed. In the new AOW, $r = 0.15 \text{ m}$ and $N_r = 6$ result in $f_s = 1.91 \text{ Hz}$.

Focusing on the vibration at the frequency of f_s , the conventional AOW caused a relatively large peak around 0.40 m/s^2 , while the amplitude of the new AOW is even smaller than 0.01 m/s^2 . This result suggests that the new AOW can suppress the vibration caused by the gaps between outer rollers as expected, and the design specification is satisfied. In the figure, the recommended amplitude limit proposed by Janeway [72] is drawn by a black line as an example of an evaluation index. This limit represents the threshold between comfort and discomfort in human reactions to

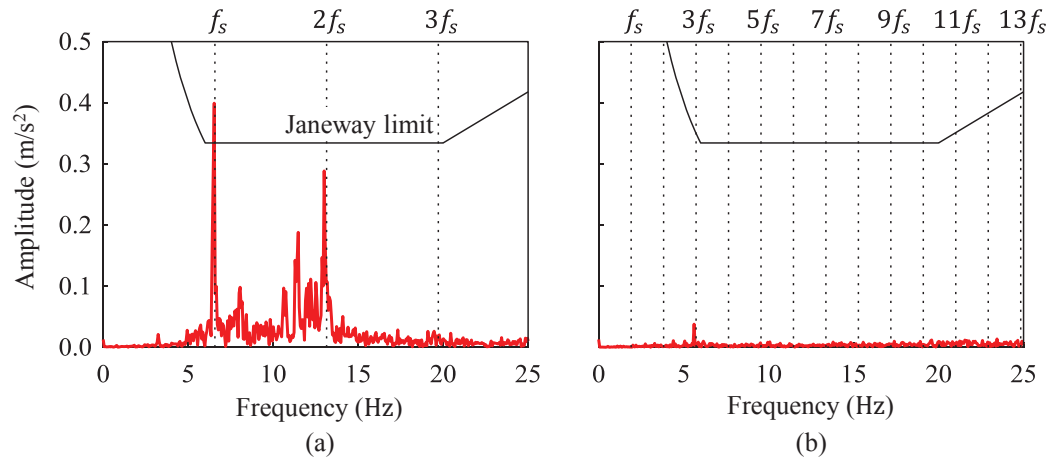


Fig. 4.19

Measurement results of vertical vibration in vehicle motion. (a) Vibration of the vehicle with the conventional AOW ($f_s = 6.57$ Hz). (b) Vibration of the vehicle with the new AOW ($f_s = 1.91$ Hz). The black line indicates Janeway's recommended amplitude limit [71].

Table 4.1

Numbers of parts in the conventional and new AOWs

	Conventional AOW	New AOW
Rollers	32	12
Gears	26	17
Couplings	32	0
Shafts	30	17
Outer roller's supports	20	12
Main body	2	2
Total	142	60

vibration. Here, the vibration of the conventional AOW is beyond the limit, which means that the vibration may make sensitive persons feel discomfort. The conventional AOW also assumes a peak close to the limit at the frequency of $2f_s$. On the other hand, though the vibration of the new AOW has a peak at the frequency of $3f_s$, its amplitude reaches no more than 11% of the limit. It must be considered that the fair comparison of these results is difficult because the structures of the two vehicles are different and the factors other than the AOWs may affect the vibrations. Nevertheless, the fact that the vehicle with the new AOW can travel with small vibration verifies that the new AOW design can avoid the vibration problem of the conventional AOW as expected, and that it can be successfully incorporated into an omnidirectional vehicle.

Next, the number of parts consisting of the AOWs is compared. Table 4.1 shows the numbers of main parts, i.e., the rollers, gears, couplings, shafts, outer roller's supports, and the main body in each AOW. As the table shows, the number of each part in the new AOW is equal to or less than that of the conventional AOW. In total, the new AOW has less than a half of the parts the conventional AOW has. Thus, the design specification on the number of parts is satisfied. This is

mainly because the new AOW does not need so many outer rollers to compensate the gaps as the conventional AOW does. When the number of the outer rollers is small, the number of the transmission paths in the AOW becomes small, and accordingly, the number of the parts such as the gears, shafts, and supports is kept small.

The above results conclude that the new AOW design proposed in the present research achieves the objectives, that is, to realize seamless periphery for vibration suppression and to reduce the number of parts, and that the design is effective to solve the problems of the conventional AOW.

4.6 Summary

In order to solve the problems of vertical vibration and a large number of parts in the conventional AOW, a new AOW design possessing a seamless periphery and simplified structure was developed. Through the analysis and experiments of the new AOW and an omnidirectional vehicle using it, the following results were obtained:

- A new AOW design without gaps between the outer rollers, which are barrel-shaped, friction-driven, and arranged alternately on the left and right sides of the wheel, was proposed. This structure also contributed to the simplification of the transmission system and parts reduction.
- The analysis of the proposed AOW clarified the geometric condition of the AOW design and the characteristics of kinematic behavior and acceleration limit.
- The construction of an omnidirectional vehicle using the proposed AOW was discussed based on the characteristics of wheeled mechanisms and a 2WD vehicle with the AOW and an omni wheel was proposed. Through the kinematic analysis, a wheel arrangement for the stable motion was clarified.
- The prototype of the omnidirectional vehicle with the proposed AOW was developed and tested. Mobility experiments verified that the developed vehicle could perform omnidirectional movement as expected. The comparison with the conventional AOW proved the effectiveness of the new AOW in vibration suppression and parts reduction.

Chapter 5

Transmission Mechanism to Drive Active Omni Wheels with Fewer Motors

5.1 Introduction

The discussion on the improvement of the AOW is continued from a different viewpoint. In general, omnidirectional mobile vehicles need a large number of motors to drive the wheels compared to other conventional vehicles. Namely, at least three motors are required. This is because one motor holds one DOF in the movement of the vehicles and three DOFs are needed in planar motion. However, the large number of motors leads to an increase in the size, weight, and manufacturing cost. Therefore, there arises the problem of how the motors can be reduced with maintaining the mobility.

This chapter tackles this problem. Based on the relation between the input from the motor and the output of the wheel, a transmission mechanism to drive the AOW with fewer motors is proposed. By using the proposed mechanism, several structures of a 2WD vehicle capable of both longitudinal (forward and backward) and lateral (right and left) translation are introduced. Experiments verify the effectiveness of the proposed mechanism.

5.2 Transmission Mechanism for Fewer Motors

5.2.1 Basic Concept

To address the abovementioned problem, let us focus on the simplicity in the control of the AOW. In the AOW, its velocities in the longitudinal and lateral directions are decoupled in its mechanism. Namely, in Eq. (4.5), the velocity in the longitudinal direction v_Y is always in proportion to the sum of the two input angular velocities $\omega_1 + \omega_2$, while the velocity in the lateral direction v_X is always in proportion to the difference of the two input angular velocities $\omega_1 - \omega_2$. Here, if the two input shafts of the AOW share one motor and the same rotation is input to the two input shafts, the AOW always moves in the longitudinal direction. Oppositely, if the two input shafts of the AOW share one motor but the rotational direction is contrary to each other, the AOW always moves in

the lateral direction. Therefore, if the direction of rotation between the motor and one input shaft can be switched in the transmission mechanism, the AOW can achieve both longitudinal and lateral movements by using only one motor.

Fig. 5.1 illustrates this idea. Here, in the AOW, input shaft 1 is a hollow shaft, and input shaft 2 is a solid shaft that runs through input shaft 1, so that both input shafts are placed on one side of the AOW, which is advantageous to the arrangement of the wheel in the vehicle. The drive mechanism has three transmission paths between the input shafts and driving shaft 1, which is driven by motor 1. Driving shaft 1 and input shaft 1 are connected by one transmission path, where a gear pair always transmits the rotation of motor 1 to input shaft 1. The other two transmission paths connect driving shaft 1 and input shaft 2 via clutches 1 and 2. When clutch 1 is on (in the transmitting state) and clutch 2 is off (in the cutting-off state), the rotation of motor 1 is transmitted to input shaft 2 by one gear train, which consists of two gears. When clutch 1 is off and clutch 2 is on, the rotation of motor 1 is transmitted to input shaft 2 by the other gear train, which consists of three gears, including one idler gear. Therefore, when only clutch 1 is on, input shafts 1 and 2 rotate at the same speed and in the same direction, and then, the AOW moves in the longitudinal direction. When only clutch 2 is on, the two input shafts rotate at the same speed but in opposite directions, and the AOW moves in the lateral direction.

Next, the characteristics of the proposed drive mechanism are explained. As shown in the previous chapters, the AOW is driven by two motors in the conventional setup. On the other hand, the proposed drive mechanism in Fig. 5.1(a) uses just one motor to drive the AOW in the longitudinal and lateral directions. Thus, the number of motors needed to drive the AOW is decreased. Note that the AOW driven by the mechanism in Fig. 5.1(a) can move in the longitudinal and lateral directions, but, unlike the conventional AOW, cannot move in the diagonal direction because the clutches are switched only when the vehicle stops. However, if the transmission paths are expanded, the diagonal movement becomes possible as discussed in Section 5.3.

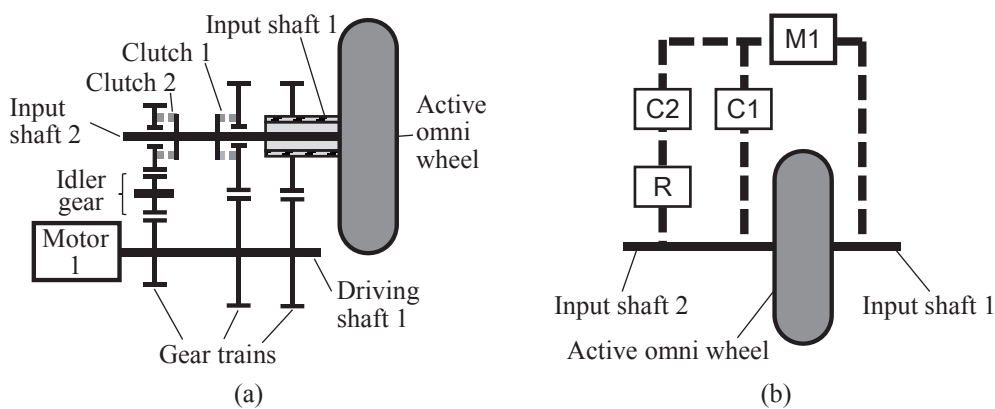


Fig. 5.1

Basic concept of the proposed transmission mechanism driving one AOW by one motor. (a) Concrete structure. (b) Conceptual equivalent.

The structure of the proposed drive mechanism is described conceptually later herein. For example, Fig. 5.1(b) corresponds to the mechanism of Fig. 5.1(a). The dashed line in Fig. 5.1(b) represents the transmission path. The rotation of motor 1 (M1) is directly transmitted to input shaft 1. Two transmission paths from motor 1 to input shaft 2 are switchable by clutches. The rotation is transmitted without changing the direction on the path with clutch 1 (C1), while reversing the direction by the reverse-rotation mechanism (R) on the path with clutch 2 (C2).

5.2.2 Kinematics and Constraints

The input-output relation and the establishment condition in the vehicle equipped with the proposed transmission mechanism are derived based on the kinematics. The kinematic equation of the AOW is given by Eq. (4.5). Next, the input-output relation between the motor and the AOW in the proposed drive mechanism in Fig. 5.1 is discussed. Here, the on/off state of the clutches is represented by

$$c_{21} = \begin{cases} 1 & \text{when clutch 1 is transmitting} \\ -1 & \text{when clutch 2 is transmitting} \end{cases} . \quad (5.1)$$

Specifically, the replacement of $\omega_2 \rightarrow c_{21}\omega_1$ in Eq. (5.1) yields the following equation:

$$\begin{bmatrix} v_X \\ v_Y \end{bmatrix} = \begin{bmatrix} r_r/2n_r & -r_r/2n_r \\ r/2 & r/2 \end{bmatrix} \begin{bmatrix} 1 \\ c_{21} \end{bmatrix} \omega_1 = \begin{bmatrix} \frac{r_r}{2n_r}(1 - c_{12}) \\ \frac{r}{2}(1 + c_{12}) \end{bmatrix} \omega_1 . \quad (5.2)$$

Two values that can be assigned to c_{21} realize two modes of the motion given by Eq. (5.2): a mode in which the longitudinal movement with $v_X = 0$ and $v_Y = r\omega_1$ is feasible for $c_{21} = 1$, and a mode in which the lateral movement with $v_Y = r_r\omega_1/n_r$ and $v_X = 0$ is feasible for $c_{21} = -1$. By generalizing Eqs. (5.1) and (5.2), the kinematics of the drive mechanisms with other transmission paths can be described.

The input-output relation of a vehicle with two parallel driving wheels, as shown in Fig. 5.2, is discussed. This wheel arrangement is chosen for the same reason as in Section 4.4, but the type of the wheels is not limited, i.e., either AOW or omni wheel is feasible in this chapter. The moving velocity of the vehicle body is set as $V_v = [\dot{X} \ \dot{Y} \ \dot{\theta}]^T$, and the traveling velocities of the two wheels are represented by $v_w = [v_{1X} \ v_{1Y} \ v_{2X} \ v_{2Y}]^T$, where v_{1X} and v_{1Y} are the traveling velocity of the right wheel in the right and forward directions, respectively, and v_{2X} and v_{2Y} are the traveling velocity of the left wheel in the right and forward directions, respectively. When the distance

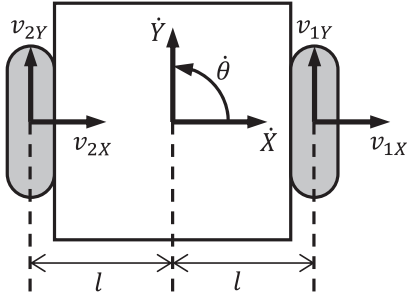


Fig. 5.2
Kinematic model of a 2WD vehicle

between the center of the vehicle body and the wheel is set as both l , the input-output velocity relation of the vehicle is given as follows:

$$\tilde{A}V_v = v_w \quad (5.3)$$

$$\tilde{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & l \\ 1 & 0 & 0 \\ 0 & 1 & -l \end{bmatrix}.$$

With respect to Eq. (5.3), the principle of virtual work

$$V_v^T F_v = v_w^T f_w \quad (5.4)$$

yields the input-output relation of the static force as follows:

$$F_v = \tilde{A}^T f_w \quad (5.5)$$

where $F_v \in \mathbb{R}^3$ and $f_w \in \mathbb{R}^4$ are the force vectors corresponding to V_v and v_w , respectively. When the kernel of the matrix $\tilde{A}^T$ is calculated, an arbitrary vector $f_{w0} \in \ker \tilde{A}^T$ can be represented by the following equation with a constant $C \in \mathbb{R}$:

$$f_{w0} = [C \ 0 \ -C \ 0]^T. \quad (5.6)$$

This relation proves that the opposing forces of the two wheels in the lateral direction, i.e., the forces pushing/pulling each other, are arbitrary in magnitude. Considering that these forces resulting from redundancy just compress/expand the vehicle and never contribute to the vehicle motion, the vehicle should move according to the constraint as

$$v_w^T f_{w0} = 0 \Leftrightarrow v_{1X} = v_{2X}. \quad (5.7)$$

Under the condition for Eq. (5.7), Eq. (5.3) is rewritten as follows:

$$V_v = \tilde{A}^\dagger v_w \quad (5.8)$$

$$\tilde{A}^\dagger = \begin{bmatrix} 1/2 & 0 & 1/2 & 0 \\ 0 & 1/2 & 0 & 1/2 \\ 0 & 1/2l & 0 & -1/2l \end{bmatrix}.$$

5.3 Synthesis of Two-wheel-drive Vehicles with Fewer Motors

This section introduces several structures of 2WD vehicles using the proposed transmission mechanism that satisfies the condition derived in Section 5.2.2.

5.3.1 Vehicle with One Motor and Two AOWs

The vehicle with the mechanism driving two AOWs by one motor is discussed. From Eqs. (4.5) and (5.8), the kinematic equation of this vehicle is given by

$$V_v = \tilde{A}^\dagger \begin{bmatrix} r_r/2n_r & -r_r/2n_r & 0 & 0 \\ r/2 & r/2 & 0 & 0 \\ 0 & 0 & r_r/2n_r & -r_r/2n_r \\ 0 & 0 & r/2 & r/2 \end{bmatrix} \begin{bmatrix} c_{11} \\ c_{21} \\ c_{31} \\ c_{41} \end{bmatrix} \omega_1 \quad (5.9)$$

where

$$c_{i1} = \begin{cases} 1 \\ 0 \\ -1 \end{cases} \quad (i = 1, 2, 3, 4) \quad (5.10)$$

applies $c_{i1} = 1$ when input shaft i is directly connected to motor 1, $c_{i1} = 0$ when input shaft i is not connected to motor 1, and $c_{i1} = -1$ when input shaft i is connected to motor 1 through a reverse-rotation mechanism. Eq. (5.9) gives various movement modes according to the combination of $\{c_{i1}\}$. At this time, the condition for Eq. (5.7) becomes as follows:

$$\frac{r_r}{2n_r}(c_{11} - c_{21})\omega_1 = \frac{r_r}{2n_r}(c_{31} - c_{41})\omega_1 \Leftrightarrow c_{11} - c_{21} = c_{31} - c_{41} . \quad (5.11)$$

Fig. 5.3 shows the structure of the vehicle with the drive mechanism represented by Eq. (5.11). In this mechanism, all of the combinations of $\{c_{i1}\}$ can be realized by switching the on/off states of clutches 1 through 11. Note that the transmission path including a reverse-rotation mechanism is omitted between motor 1 and one of the input shafts (input shaft 1 in the figure) because the

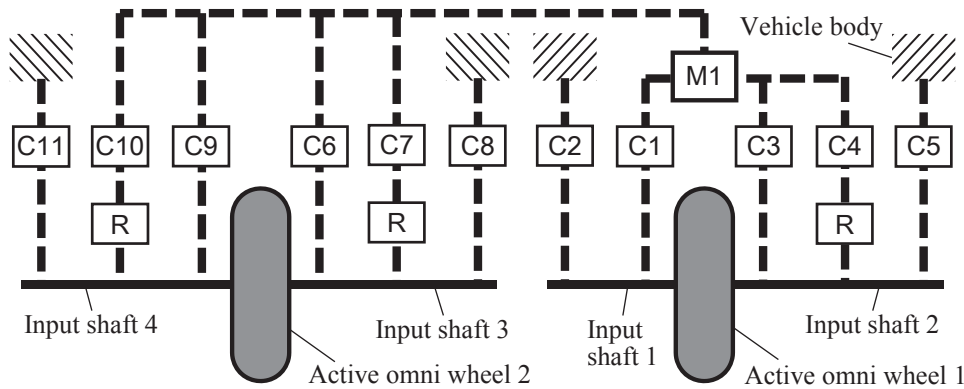


Fig. 5.3

Structure of the vehicle with a drive mechanism for two AOWs and one motor.

motors can generally rotate bidirectionally. Thus, c_{11} takes a value of either 1 or 0 here. In order to constrain the input shaft from rotating for the case in which the input shaft is not connected to motor 1, i.e., $c_{11} = 0$, each input shaft can be braked by connecting the shaft to the vehicle body through a clutch.

Table 5.1 shows the possible movement modes of the vehicle in Fig. 5.3 under the condition for Eqs. (5.9) through (5.11). This vehicle can realize 13 movement modes by switching the transmitting clutches, including three overlapping modes when considering the bidirectional rotation of the motor. The other 10 modes cover longitudinal translation, lateral translation, turns, and their combinations. In addition, when all input shafts are constrained from rotating, the vehicle brakes.

Table 5.1
Movement modes of the vehicle in Fig. 5.3 according to the switching pattern of the clutches.

$\{c_{11}, c_{21}, c_{31}, c_{41}\}$	Transmitting clutches	V_v^T	Vehicle motion
$\{1, 1, 1, 1\}$	C1, C3, C6, C9	$[0 \quad r\omega_1 \quad 0]$	Longitudinal translation
$\{1, 1, -1, -1\}$	C1, C3, C7, C10	$[0 \quad 0 \quad \frac{r\omega_1}{l}]$	Spin turn
$\{1, 1, 0, 0\}$	C1, C3, C8, C11	$[0 \quad \frac{r\omega_1}{2} \quad \frac{r\omega_1}{2l}]$	Pivot turn around the left wheel
$\{1, -1, 1, -1\}$	C1, C4, C6, C10	$[\frac{r_r\omega_1}{n_r} \quad 0 \quad 0]$	Lateral translation
$\{1, 0, 1, 0\}$	C1, C5, C6, C11	$[\frac{r_r\omega_1}{2n_r} \quad \frac{r\omega_1}{2} \quad 0]$	Diagonal translation (right front/left rear)
$\{1, 0, 0, -1\}$	C1, C5, C8, C10	$[\frac{r_r\omega_1}{2n_r} \quad 0 \quad \frac{r\omega_1}{2l}]$	Right/left translation with left/right spin
$\{0, 1, 0, 1\}$	C2, C3, C8, C9	$[-\frac{r_r\omega_1}{2n_r} \quad \frac{r\omega_1}{2} \quad 0]$	Diagonal translation (left front/right rear)
$\{0, 1, -1, 0\}$	C2, C3, C7, C11	$[-\frac{r_r\omega_1}{2n_r} \quad 0 \quad \frac{r\omega_1}{2l}]$	Right/left translation with right/left spin
$\{0, -1, 1, 0\}$	C2, C4, C6, C11	$[\frac{r_r\omega_1}{2n_r} \quad 0 \quad -\frac{r\omega_1}{2l}]$	Right/left translation with right/left spin
$\{0, -1, 0, -1\}$	C2, C4, C8, C10	$[\frac{r_r\omega_1}{2n_r} \quad -\frac{r\omega_1}{2} \quad 0]$	Diagonal translation (left front/right rear)
$\{0, 0, 1, 1\}$	C2, C5, C6, C9	$[0 \quad \frac{r\omega_1}{2} \quad -\frac{r\omega_1}{2l}]$	Pivot turn around the right wheel
$\{0, 0, 0, 0\}$	C2, C5, C8, C11	$[0 \quad 0 \quad 0]$	Stop (braking)
$\{0, 0, -1, -1\}$	C2, C5, C7, C10	$[0 \quad -\frac{r\omega_1}{2} \quad \frac{r\omega_1}{2l}]$	Pivot turn around the right wheel

5.3.2 Vehicle with Two Motors and Two AOWs

Next, the vehicle with the mechanism driving two AOWs by two motors is discussed. From Eqs. (4.5) and (5.8), the kinematic equation of this vehicle is given by

$$V_v = \tilde{A}^\dagger \begin{bmatrix} r_r/2n_r & -r_r/2n_r & 0 & 0 \\ r/2 & r/2 & 0 & 0 \\ 0 & 0 & r_r/2n_r & -r_r/2n_r \\ 0 & 0 & r/2 & r/2 \end{bmatrix} \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \\ c_{31} & c_{32} \\ c_{41} & c_{42} \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} \quad (5.12)$$

where c_{i2} represents the transmitting state between input shaft i and motor 2, defined in the same manner as Eq. (5.10). Here, two different motors must not drive one input shaft at the same time, so that the following condition is necessary:

$$c_{i1}c_{i2} = 0. \quad (5.13)$$

The condition for Eq. (5.7) is calculated as

$$\begin{aligned} \frac{r_r}{2n_r} \{(c_{11}\omega_1 + c_{12}\omega_2) - (c_{21}\omega_1 + c_{22}\omega_2)\} &= \frac{r_r}{2n_r} \{(c_{31}\omega_1 + c_{32}\omega_2) - (c_{41}\omega_1 + c_{42}\omega_2)\} \\ \Leftrightarrow (c_{11} - c_{21} - c_{31} + c_{41})\omega_1 + (c_{12} - c_{22} - c_{32} + c_{42})\omega_2 &= 0. \end{aligned} \quad (5.14)$$

The available solutions for this condition are as follows:

$$\begin{cases} c_{11} - c_{21} - c_{31} + c_{41} = c_{12} - c_{22} - c_{32} + c_{42} = 0, \quad \forall \omega_1, \omega_2 \\ c_{11} - c_{21} - c_{31} + c_{41} \neq 0, \quad \omega_1 = -\frac{c_{12} - c_{22} - c_{32} + c_{42}}{c_{11} - c_{21} - c_{31} + c_{41}} \omega_2 \\ c_{12} - c_{22} - c_{32} + c_{42} \neq 0, \quad \omega_2 = -\frac{c_{11} - c_{21} - c_{31} + c_{41}}{c_{12} - c_{22} - c_{32} + c_{42}} \omega_1 \end{cases}. \quad (5.15)$$

The first row of Eq. (5.15) corresponds to the situation in which the condition is achieved by arranging the transmission paths appropriately, whereas the second and third rows correspond to the situation in which the condition is achieved by adjusting the input angular velocities, i.e., overconstraint-like situation because of motor redundancy.

Fig. 5.4 shows the structure of this vehicle. The number of the possible movement modes of this mechanism is calculated as follows. When $\{c_{11}, c_{12}\} = \{1, 0\}$ or $\{0, 1\}$ is applied, there are five transmission paths to each of input shafts 2 through 4, and then the number of the modes is $2 \times 5^3 = 250$. When $\{c_{11}, c_{12}\} = \{0, 0\}$ is applied, except in the case of $\{c_{21}, c_{22}, c_{31}, c_{32}, c_{41}, c_{42}\} = \{0, 0, 0, 0, 0, 0\}$, half of the modes come to be the same as the other modes by reversing the rotational directions of the motors. Then, the number of modes excluding such overlap is $(5^3 - 1)/2 + 1 = 63$. Therefore, the total number of movement modes of this vehicle is $250 + 63 = 313$. However, the constraint on the velocity is given by the second and third rows of Eq. (5.15) except for the modes satisfying the condition for the first row.

If not all movement modes are necessary, then the transmission paths other than those used in the required motion can be omitted in order to simplify the drive mechanism. Then, let us try to simplify the vehicle in Fig. 5.4 as an example. Assuming that Eq. (5.14) is established at any ω_1 and ω_2 to avoid the overconstraint, the condition for the first row of Eq. (5.15) should be

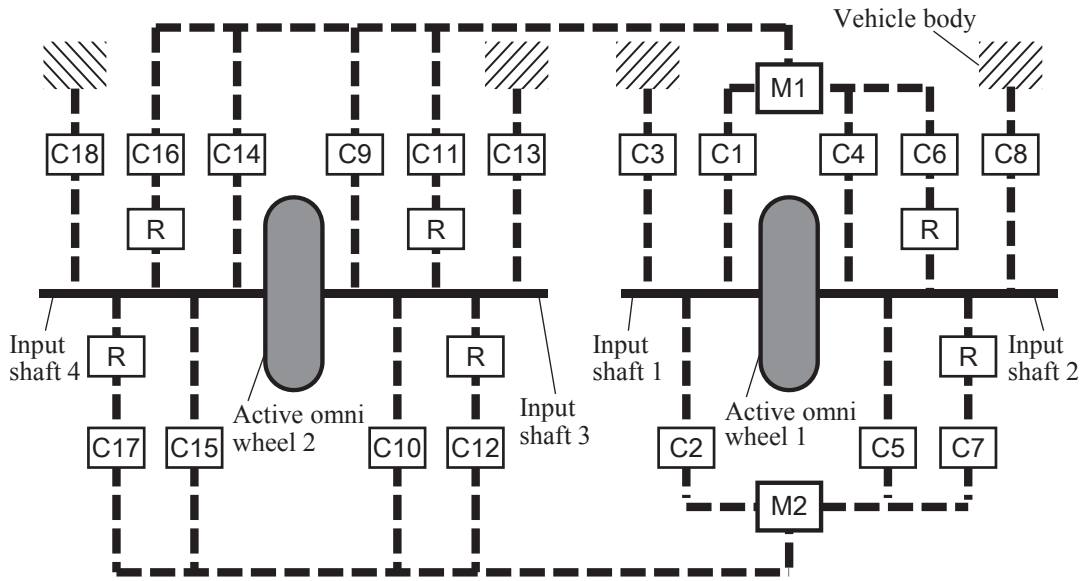


Fig. 5.4
Structure of the vehicle with the drive mechanism for two AOWs and two motors.

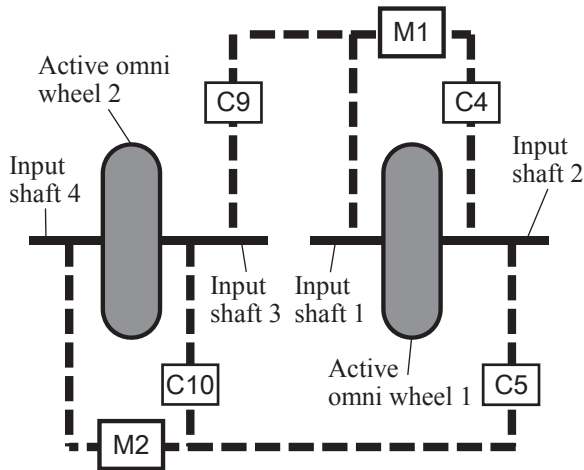


Fig. 5.5
Simplified structure of the vehicle with two AOWs driven by two motors.

satisfied. When $c_{11} = c_{42} = 1$ and $c_{12} = c_{41} = 0$ are set permanently for simplicity, this condition becomes

$$1 - c_{21} - c_{31} = 1 - c_{22} - c_{32} = 0 \Leftrightarrow c_{21} + c_{31} = c_{22} + c_{32} = 1. \quad (5.16)$$

Under the condition for Eqs. (5.13) and (5.16), the possible combination is $\{c_{21}, c_{22}, c_{31}, c_{32}\} = \{1, 0, 0, 1\}$ or $\{0, 1, 1, 0\}$. The simplified structure of the vehicle is shown in Fig. 5.5. Compared to Fig. 5.4, 12 transmission paths and 14 clutches are omitted. Table 5.2 shows the movement modes of this vehicle. In modes $\{c_{21}, c_{22}, c_{31}, c_{32}\} = \{1, 0, 0, 1\}$, the vehicle can move in the longitudinal direction and turn on the spot. In modes $\{c_{21}, c_{22}, c_{31}, c_{32}\} = \{0, 1, 1, 0\}$, the vehicle can move in both the longitudinal and lateral directions, i.e., the vehicle can perform the arbitrary directional translation.

Table 5.2

Movement modes of the vehicle in Fig. 5.5 according to the switching pattern of the clutches.

$\{c_{21}, c_{22}, c_{31}, c_{32}\}$	Transmitting clutches	V_v^T	Vehicle motion
$\{1, 0, 0, 1\}$	C4, C10	$\begin{bmatrix} 0 & \frac{r(\omega_1 + \omega_2)}{2} & \frac{r(\omega_1 - \omega_2)}{2l} \end{bmatrix}$	Longitudinal translation and spin turn
$\{0, 1, 1, 0\}$	C5, C9	$\begin{bmatrix} \frac{r_r(\omega_1 - \omega_2)}{2n_r} & \frac{r(\omega_1 + \omega_2)}{2} & 0 \end{bmatrix}$	Longitudinal and lateral translation

5.3.3 Vehicle with Two Motors, One AOW, and One Omni Wheel

A vehicle having a right wheel that is an AOW and a left wheel that is an omni wheel driven by two motors is discussed. In the velocity vector of the wheels v_w , the elements v_{1X} and v_{1Y} represent the velocity of the AOW, and elements v_{2X} and v_{2Y} represent the velocity of the omni wheel. At this time, v_{2X} can take an arbitrary value, so that

$$v_{1X} = v_{2X} = \dot{X} \quad (5.17)$$

is always established, and the condition for Eq. (5.7) is satisfied. From Eqs. (4.5), (5.8), and (5.17), the kinematic equation of this vehicle is given by

$$V_v = \tilde{A}^\dagger \begin{bmatrix} r_r/2n_r & -r_r/2n_r & 0 & 0 \\ r/2 & r/2 & 0 & 0 \\ r_r/2n_r & -r_r/2n_r & 0 & 0 \\ 0 & 0 & 0 & r \end{bmatrix} \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \\ 0 & 0 \\ c_{31} & c_{32} \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix}. \quad (5.18)$$

Fig. 5.6 shows the structure of this vehicle. The total number of possible movement modes is calculated as $2 \times 5^2 + (5^2 - 1)/2 + 1 = 63$, excluding the overlap resulting from bidirectional rotation of the motors.

The simplified structure of the vehicle in Fig. 5.6 is discussed by considering the example in Fig. 5.7. In this vehicle, nine transmission paths and 11 clutches are omitted from the vehicle in Fig. 5.6. The condition for Eq. (5.13) is always satisfied because $c_{11} = c_{32} = 1$ and $c_{12} = c_{22} = c_{31} = 0$ are applied permanently. When switching the clutches, the movement modes and the movement velocity V_v are changed as shown in Table 5.3. The vehicle can perform longitudinal translation and spin turning in the movement mode of $c_{21} = 1$, and lateral translation and pivot turning around the right wheel in the movement mode of $c_{21} = -1$. Note that pivot turning around each of the wheels can be realized in $c_{21} = 1$ by combining longitudinal translation and spin turning.

As shown above, the proposed transmission mechanism can be designed for various wheel combinations and various numbers of motors to construct 2WD vehicles capable of diverse movement modes, including longitudinal and lateral translation and turns. In addition,

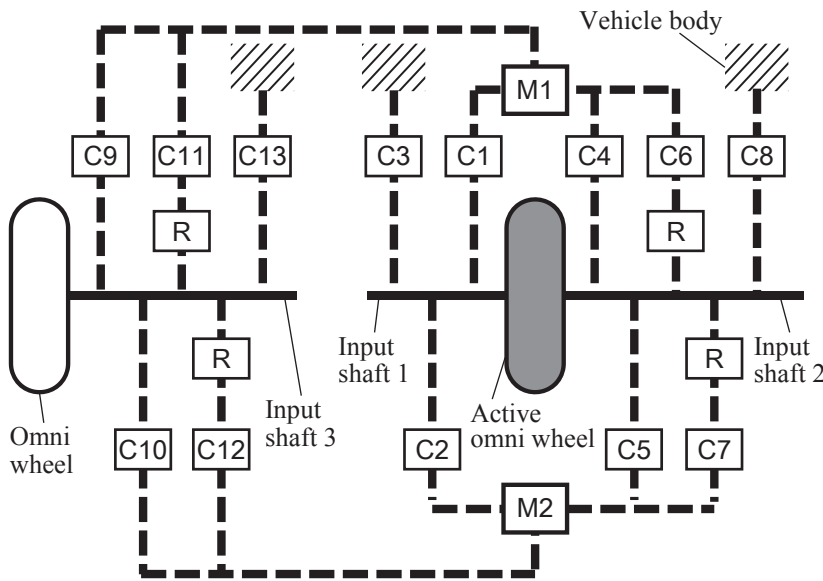


Fig. 5.6
Structure of the vehicle with the drive mechanism for one AOW, one omni wheel, and two motors.

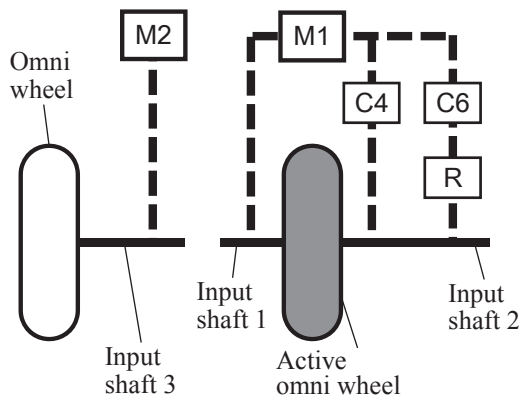


Fig. 5.7
Simplified structure of the vehicle with one AOW and one omni wheel driven by two motors.

Table 5.3
Movement modes of the vehicle in Fig. 5.7 according to the switching pattern of the clutches.

c_{21}	Transmitting clutches	V_v^T	Vehicle motion
1	C4	$\begin{bmatrix} 0 & \frac{r(\omega_1 + \omega_2)}{2} & \frac{r(\omega_1 - \omega_2)}{2l} \end{bmatrix}$	Longitudinal translation and spin turn
-1	C6	$\begin{bmatrix} \frac{r_r \omega_1}{2n_r} & \frac{r\omega_2}{2} & -\frac{r\omega_2}{2l} \end{bmatrix}$	Lateral translation and pivot turn around the right wheel

appropriately omitting the transmission paths in order to simplify the structure of the vehicle can be effective as long as the required movement modes are feasible.

Each of the structures shown in Fig. 5.3 through Fig. 5.7 can be basically realized by expanding the fundamental structure shown in Fig. 5.1(a), but its difficulty is varied. For example, the structures in Fig. 5.3, Fig. 5.4, and Fig. 5.6 have more than ten clutches and transmission paths, so that the difficulty in realization would be relatively high. On the other hand, the structures in

Fig. 5.5 and Fig. 5.7 are the simplified structures of those in Fig. 5.4 and Fig. 5.6, respectively. Therefore, the difficulty is expected to be low. Accordingly, the structure in Fig. 5.7, which has fewer clutches than that in Fig. 5.5, is chosen as an experimental device in the next section.

5.4 Experiments

In order to demonstrate that the proposed transmission mechanism works as expected, a 2WD vehicle is developed, and experiments are conducted.

5.4.1 Developed Two-wheel-drive Vehicle

Fig. 5.8 shows the developed 2WD vehicle. The right driving wheel is an omni wheel and the left driving wheel is an AOW, the outer shapes of which are the same. In addition to two driving wheels, two supporter omni wheels that move passively are installed thorough air suspensions at the front and back of the vehicle.

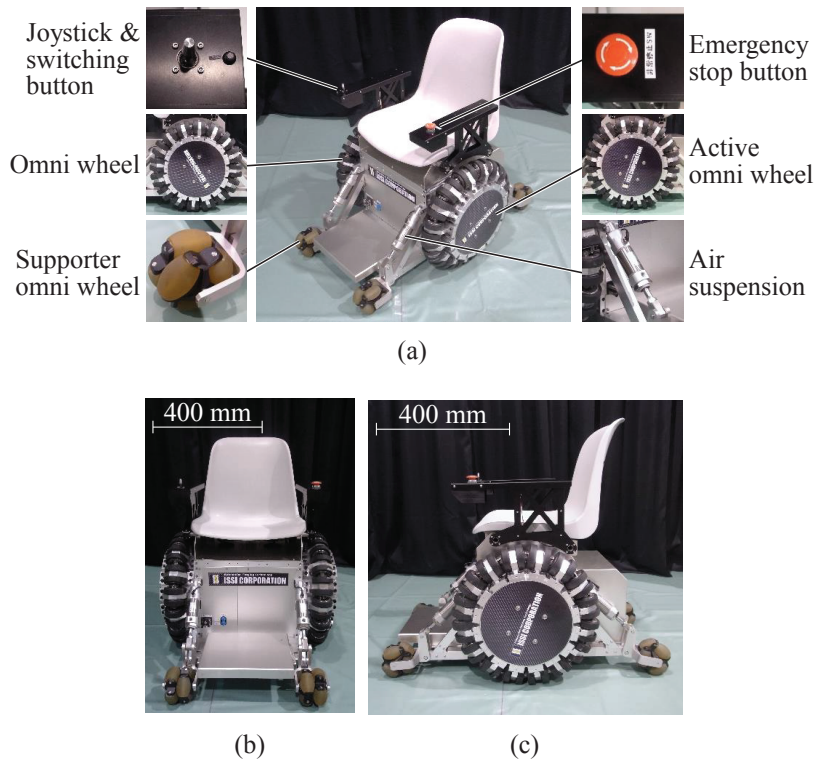


Fig. 5.8
Developed vehicle with the proposed transmission mechanism. (a) Overall view. (b) Front view. (c) Side view.

The transmission mechanism installed in the developed vehicle is shown in Fig. 5.9. This mechanism drives one AOW and one omni wheel by two motors. Its structure is basically the same as that of the vehicle shown in Fig. 5.7 as mentioned above, but the positions of the AOW and the omni wheel are swapped with each other. Motors 1 and 2 (Maxon RE65 250 W) rotate driving shafts 1 and 2, respectively, via gear pairs. The structure between driving shaft 1 and input shafts 1 and 2 of the AOW is identical to that shown in Fig. 5.1(a). Driving shaft 2 and input shaft 3 of the omni wheel are always connected through a gear pair. When clutch 1 is on, the movement mode allows the vehicle to move in the longitudinal direction and to turn on the spot. When clutch 2 is on, the movement mode allows the vehicle to move in the lateral direction and turn around the AOW. For the following discussion, the former is called the longitudinal movement mode and the latter is called the lateral movement mode.

A passenger operates the vehicle with a joystick and a switching button. The moving velocity is commanded by tilting the joystick in two directions, and the movement modes are switched by pushing the switching button. In the longitudinal movement mode, forward/backward tilt of the joystick is converted into the longitudinal translational velocity and right/left tilt into the spin turning velocity, whereas in the lateral movement mode, the former into the pivot turning velocity

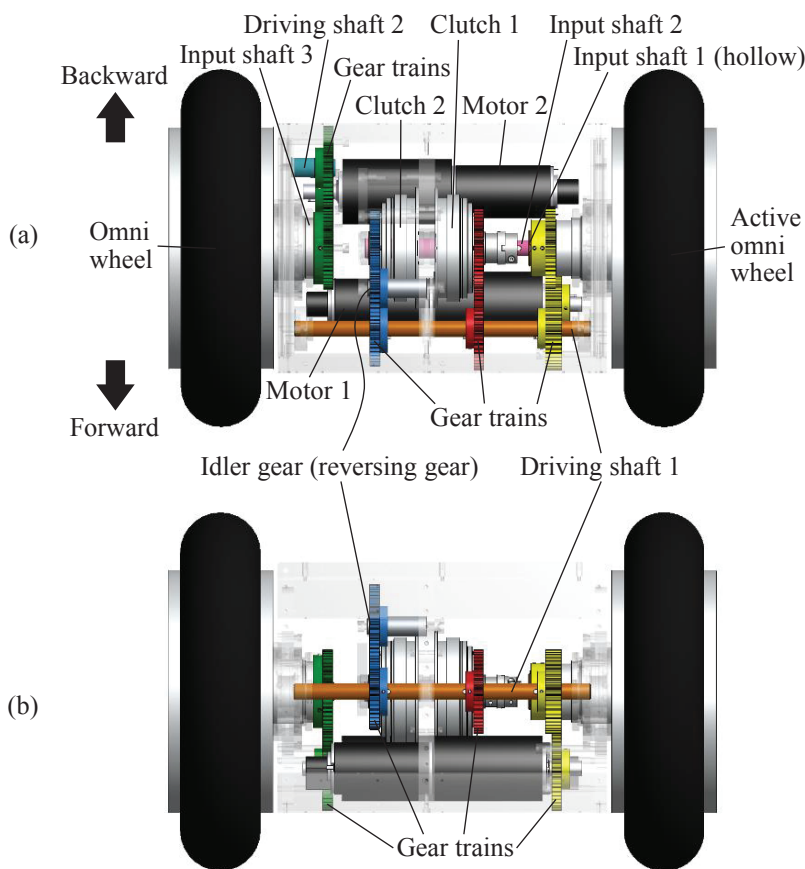


Fig. 5.9
Developed transmission mechanism of the vehicle. (a) Top view. (b) Front view.

around the AOW and the latter into the lateral translational velocity. The vehicle has an emergency stop button for safety. The control system of the developed vehicle is mainly composed of a microcomputer (Arduino Micro), motor drivers (Maxon ESCON-70/10), and a relay for electromagnetic clutches. The microcomputer gets inputs from the joystick and the switching button, calculates the commands, and sends them to the motor drivers and the clutch relay to drive the motors and the clutches. It has three batteries of 13.2 V as a power source.

The parameters of the developed vehicle are $r = 222$ mm, $r_r = 39$ mm, $l = 275$ mm, and $n_r = 0.5$. The dimensions of the entire vehicle are as follows. The total length and total width of the vehicle are 1020 mm and 680 mm, respectively, the height from the floor to the seat surface and the height from the floor to top of the seat are 500 mm and 830 mm, respectively. The diameter of the four supporter omni wheels is 120 mm. The maximum speed is 2.5 km/h in the longitudinal direction and 0.9 km/h in the lateral direction. The vehicle weighs about 138 kg. Note that the developed vehicle was designed to confirm the fundamental functions, rather than to reduce its size and weight. If smaller clutches are used instead, for example, it can achieve downsizing and weight reduction more effectively.

5.4.2 Experiment on the Transmission Mechanism

First, an experiment to confirm the movement of the transmission mechanism was conducted. Fig. 5.10 shows the observation results of the transmission mechanism and the wheels with the vehicle lifted from the ground. Fig. 5.10(a) shows the motion when the vehicle was commanded to perform the forward movement in the longitudinal movement mode, and Fig. 5.10(b) shows the motion when the left movement was commanded in the lateral movement mode. The left side of each figure shows a photograph of the transmission mechanism and the right side shows a photograph of the AOW at the same time. In order to improve the visibility of the rotation, marking seals are attached to the driving shaft, input shafts 1 and 2, and the outer rollers. Four differently colored seals are attached to the outer rollers at 90-degree intervals. The colors are yellow, blue, red, and green, and are attached in this order. In Fig. 5.10(a), the displacement of the seal on the outer rollers indicates that the main body of the AOW rotated but the outer rollers did not. In addition, from the photograph of the drive mechanism, it is confirmed that input shafts 1 and 2 rotated in the same direction. On the other hand, Fig. 5.10(b) shows that the main body of the AOW did not move, and only the outer rollers rotated. In addition, it is confirmed that input shafts 1 and 2 rotated in opposite directions. Fig. 5.10(c) shows the motion of the omni wheel when the vehicle was commanded to move in the forward direction. The figure shows that only the main body of the omni wheel rotated. These results proved that the transmission mechanism performed as expected.

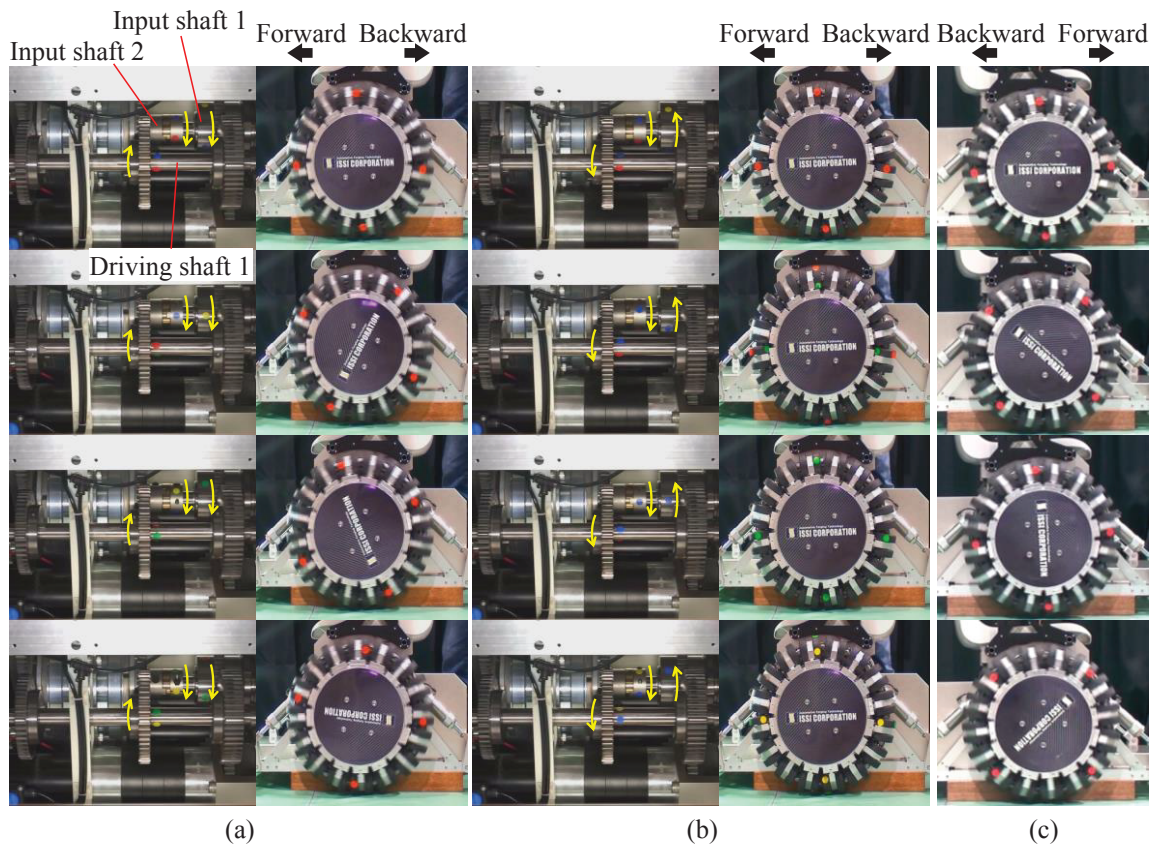


Fig. 5.10

Motions of the transmission mechanism and wheels. (a) Forward movement of the AOW. (b) Left movement of the AOW. (c) Forward movement of the omni wheel.

5.4.3 Experiment on the Vehicle Motion

Next, experiments to verify that the developed vehicle could perform the movement in the longitudinal and lateral directions and turns with a passenger riding on the vehicle were conducted. The passenger operated the vehicle by manipulating the joystick to perform the designated motion. The maximum speed is set as 1.3 km/h in the longitudinal direction and 0.5 km/h in the lateral direction. The passenger assented after being informed about the experiments prior to participation.

Fig. 5.11 shows the motion of the vehicle in the longitudinal movement mode. The forward translation shown in Fig. 5.11(a) is the motion when the left and right driving wheels are driven at the same angular velocity. The spin turn shown in Fig. 5.11(b) is the motion when the left and right driving wheels are driven at the same speed but in opposite directions, in which case the vehicle turns on the spot. When the left and right driving wheels are driven at different speeds, the longitudinal translation with turns shown in Fig. 5.11(c) is possible. Thus, it is verified that the

vehicle can realize two-DOF motion comprising longitudinal translation and spin turning in the longitudinal movement mode, just like conventional wheelchairs. Considering the asymmetry of the drive mechanism of the left and right wheels in the developed vehicle, the differences in inertia and resistance of the wheels could have affected the motion, but did not actually according to the observation. This might be because the motors were controlled based not on torque but on velocity. In turning, the limited speed of the wheels might suppress the effect of the difference in side force.

Fig. 5.12 shows the motion of the vehicle in the lateral movement mode. In the right translation shown in Fig. 5.12(a), the outer rollers of the AOW are rotated actively and the free rollers of the omni wheel are rotated passively. In the right pivot turn shown in Fig. 5.12(b), the rotation pivoting on the AOW is achieved by driving only the omni wheel while the outer rollers of the AOW are not rotated. When the AOW is driven in the right direction and the omni wheel is driven in the forward direction, the right translation with turns shown in Fig. 5.12(c) is possible. Thus, it is verified that the vehicle can realize two-DOF motion comprising lateral translation and pivot turning in the lateral movement mode.

Based on the above results, it was demonstrated that the developed 2WD vehicle with the proposed transmission mechanism performed as expected.



Fig. 5.11
Motion in the longitudinal movement mode. (a) Forward translation. (b) Spin turn. (c) Forward translation with right spin.

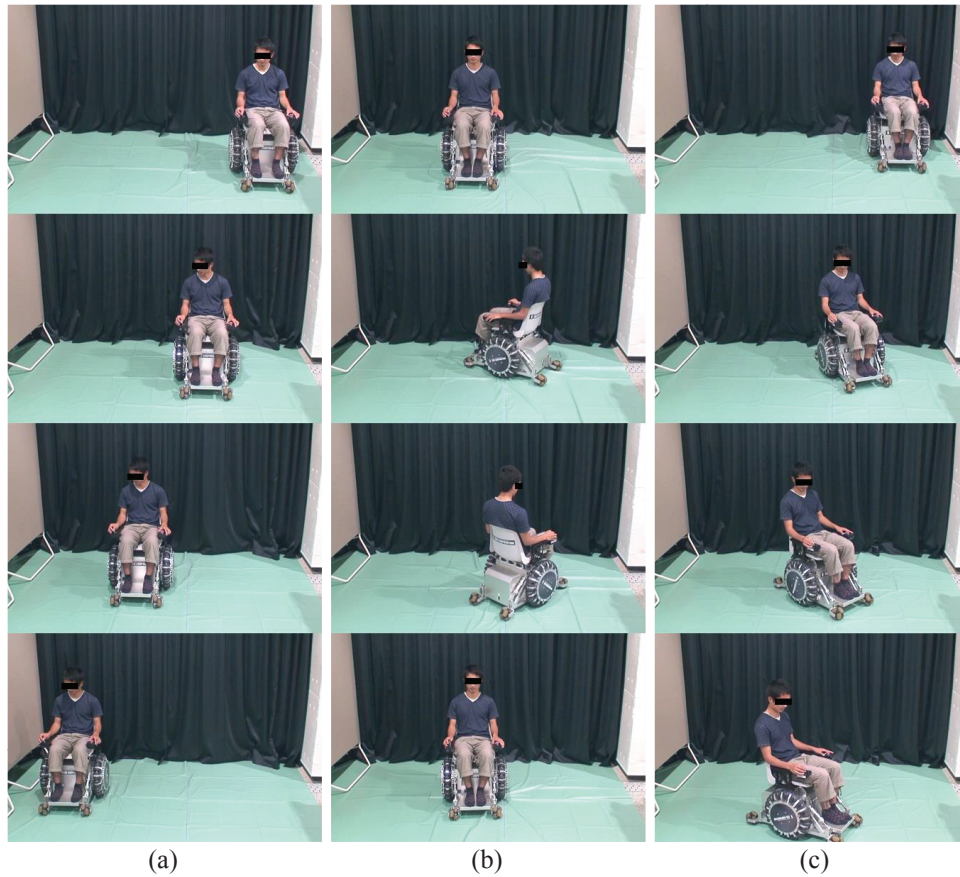


Fig. 5.12

Motion in the lateral movement mode: (a) Right translation, (b) Right pivot turn around the left wheel, and (c) Right translation with pivoting

5.5 Summary

To achieve omnidirectional mobility, a large number of motors are required, whereas it is desirable to reduce the number of motors with respect to the size, weight, and manufacturing cost. Then, a transmission mechanism was proposed to meet both superb mobility and fewer motors by driving AOWs with a small number of motors. As a result of analyses and experiments, the following results were obtained.

- A transmission mechanism that could switch the transmission paths by clutches and drive the AOW and omni wheel by just one or two motors was proposed.
- The kinematic analysis of a 2WD vehicle with the proposed transmission mechanism was carried out, and the input-output velocity relation and the establishment condition were derived.

- The structures of vehicles satisfying the derived condition and the possible movement modes of each vehicle were shown. The simplified structures of the vehicles were also suggested by omitting transmission paths.
- A 2WD vehicle with the proposed mechanism was developed. Experiments proved that the vehicle could perform the longitudinal and lateral movements and turns as expected.

Chapter 6

Reducer-Integrated Motor Using Strain Wave Gearing System

6.1 Introduction

Motors, usually electromagnetic motors, are conventionally used combined with reducers to gain high-precision positioning and large torque output because the motors themselves have high-speed and low-torque output. However, the size of the reducer makes the entire motor system large, especially in the axial direction. Such large motor systems prevent the wheeled locomotion devices from achieving compact design. Small-sized motors will also be useful for industrial robots and other machines.

To miniaturize the entire motor system, reducer-integrated motors, such as a cycloid motor [73] and a wobble motor [74], have been proposed. This type of the reducer-integrated motor is mainly composed of an external gear rotating around the axle, an internal gear meshing with the external gear and revolving around it without rotation, and linear actuators to revolve the internal gear by pushing it from the outside to the center. There is a small tooth number difference between the external and internal gears. Accordingly, when the internal gear revolves around the external gear, the external gear rotates by the difference in teeth number from the internal gear. Thus, the proposed reducer-integrated motors can output reduced rotation directly from the input of the linear actuators and external reducers are no longer unnecessary. It leads the advantage in the downsizing of motors over conventional motor systems that have separate electromagnetic motors and reducers.

In the structure of the reducer-integrated motor, however, there exists a concern about load-supporting capacity and stiffness. The load torque applied to the output part of the motor, i.e., the external gear, brings circumferential force to the linear actuators through the internal gear. Although the linear actuators are suitable to output force in the radial direction, it is difficult for them to support a large force in the circumferential direction. It is also difficult to increase the stiffness in the circumferential direction, and so high-precision positioning becomes a challenge when a large torque is required.

This chapter proposes a new reducer-integrated motor with the aim to establish the small size, high load-supporting capacity, and high stiffness. The principle of a strain wave gearing system

and simultaneous engagement of multiple gear pairs are focused on to meet these requirements. The design condition of the motor is derived and a motor prototype is actually developed to demonstrate its operation.

6.2 Reducer-Integrated Motor

6.2.1 Principle of a Strain Wave Gearing System

The proposed reducer-integrated motor uses the principle of a strain wave gearing system. Fig. 6.1 illustrates its structure and motion. The strain wave gearing system [47] has a flexspline (F/S), a circular spline (C/S), and a wave generator (W/G). The F/S is an elastic and flexible external gear shaped like a thin cylinder, the C/S is a rigid internal gear, and the W/G is an oval cam with thin-raced ball bearing. The C/S has two more teeth than the F/S, in general. When the W/G is put inside the F/S, the F/S is deformed into an oval. Then, the teeth of the F/S and C/S mesh each other near the long axis of the oval and do not near the short axis.

When the rotation of the F/S is fixed and a motor rotates the cum of the W/G, the long axis of the F/S moves as the W/G rotates, and the contact point between the F/S and C/S also moves. While the W/G turns around clockwise once, the C/S rotates clockwise by the difference in teeth number from the F/S, because the rotation of the F/S is fixed. When the rotation of the W/G is set as the input and the rotation of the C/S as the output, the reduction ratio R_s is given as follows [75]:

$$R_s = \frac{z_c}{z_c - z_f} = \frac{z_f}{2} + 1 \quad (6.1)$$

where z_f is the teeth number of the F/S and $z_c = z_f + 2$ is the teeth number of the C/S. Eq. (6.1) suggests that R_s gets large when z_f becomes large. Therefore, the strain wave gearing system can realize a high reduction ratio even in a single stage.

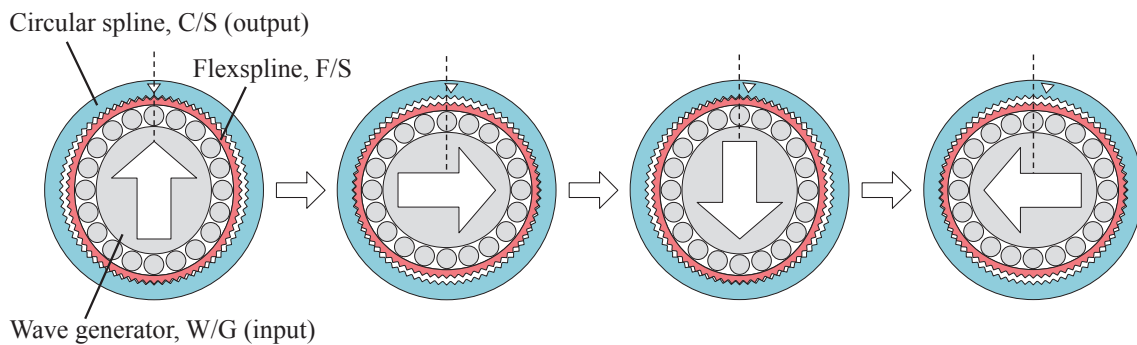


Fig. 6.1
Movement of a strain wave gearing system.

6.2.2 Proposed Reducer-Integrated Motor

A novel reducer-integrated motor is proposed using the principle of a strain wave gearing system. Fig. 6.2 shows the schematic structure of the proposed motor. As shown in Fig. 6.2(a), which is the disassembling view of the main components, one F/S engages simultaneously with two kinds of C/Ss: one C/S-S and two C/S-Ds. The C/S-S has two more teeth than the F/S, and the two C/S-Ds have the same teeth number as the F/S. As shown in Fig. 6.2(b), which illustrates the sectional views, one C/S-D is fixed on the base directly and the other is done through a supporting part, not to rotate. The C/S-Ds are placed coaxially on both sides of the C/S-S. The C/S-S connected with the rotor can rotate around the central axis. Inside the F/S, linear actuators are arranged radially and symmetrically with the central axis, instead of the W/G in the normal strain wave gearing system. Each linear actuator, e.g., a linear solenoid, can actively elongate and shrink. The figure shows the case where twelve linear actuators are arranged at equal intervals.

Next, the movement of the proposed reducer-integrated motor is explained. When one pair of the opposite linear actuators is operated, it pushes the F/S. The F/S is deformed into an oval shape whose long axis agrees with the central line of the operating linear actuators. Then, the deformed

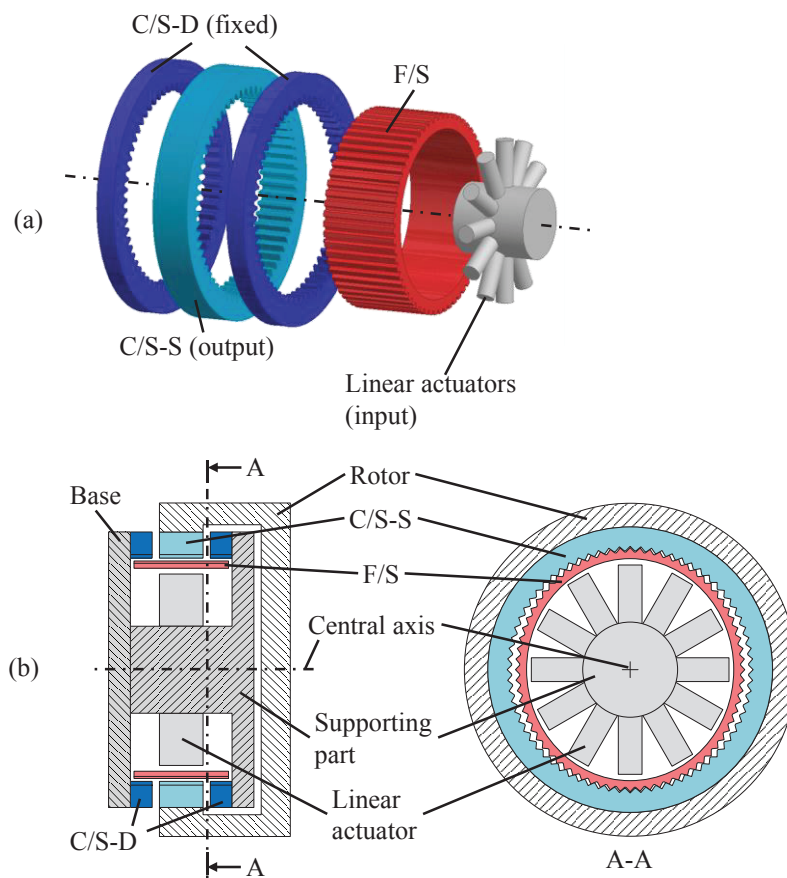


Fig. 6.2
Structure of the proposed reducer-integrated motor. (a) Disassembling view. (b) Sectional views.

F/S contacts the C/S-Ds and C/S-S simultaneously near the long axis, but does not near the short axis. When the operating linear actuators are switched clockwise, as shown in Fig. 6.3, the long axis of the F/S moves clockwise. Accordingly, the contact points between the F/S and the C/Ss also move clockwise, which is the same process as the rotation of the W/G in the strain wave gearing system. The teeth number of the F/S is the same as those of the C/S-Ds fixed on the base, so that the F/S does not rotate. Then, only the C/S-S rotates clockwise by the difference in the teeth number from the F/S in one cycle of switching the operating linear actuators following the principle of the strain wave gearing system. When the switch of operating linear actuators is set as the input and the rotation of the C/S-S as the output, the reduction ratio R_s is represented by the same equation as Eq. (6.1). When the number of linear actuators is set as n_a and the switching interval of the operating linear actuators as T_a , the average rotational speed of the C/S-S $\bar{N}$ is calculated as follows:

$$\bar{N} = \frac{1}{n_a R_s T_a} . \quad (6.2)$$

The characteristics of the proposed motor are discussed. First, the motor can be smaller than the conventional motor systems because the linear actuators are installed inside the reducer. Second, in the proposed motor, the load torque applied to the C/S-S is supported by the C/S-Ds fixed on the base through the F/S, so that the linear actuators do not receive the circumferential force directly. In other words, C/S-Ds can not only prevent the F/S from rotating but also receive the load torque instead of the linear actuators. Therefore, the proposed motor is expected to support higher load torque and gain higher stiffness than the existing reducer-integrated motors. Additionally, because the C/S-Ds are put on both sides of the C/S-S unlike conventional strain wave gearing systems, the torque on the F/S from the C/S-S and C/S-Ds become symmetrical in the

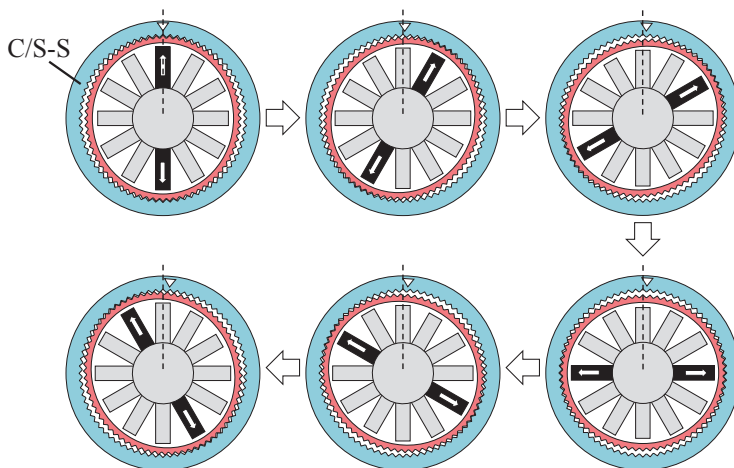


Fig. 6.3
Movement of the proposed reducer-integrated motor. The circular spline S (C/S-S) rotates as the contact point moves.

axial direction as shown in Fig. 6.4. These symmetrical torques are thought to suppress the tilt and torsion of the F/S and increase the stiffness of the motor. From the above, the proposed reducer-integrated motor is expected to establish downsizing, high load-supporting capacity, and high stiffness at the same time. Additionally to this, the motor may hold the advantages of the strain wave gearing system, e.g., backlashes are unnecessary and the pitch error is flattened due to a large number of the teeth in simultaneous engagement.

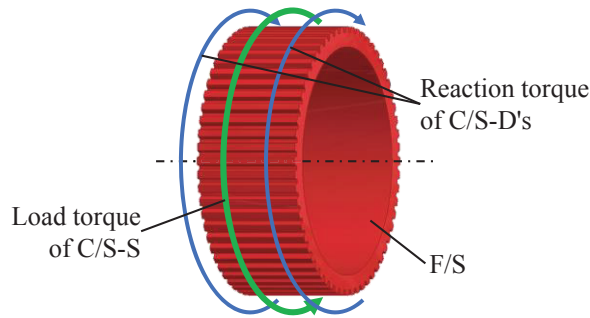


Fig. 6.4
Torque applied to the flexspline (F/S) by the C/S-S and circular spline D (C/S-D).

6.3 Tooth Tip Interference Analysis

In the proposed reducer-integrated motor, the F/S is deformed by operating the linear actuators arranged at intervals, not by the rotation of the W/G in the conventional strain wave gearing system. Here, some design conditions should be satisfied in the number and arrangement of linear actuators. This section discusses the conditions.

6.3.1 Problem Definition

In the proposed motor, at least one pair of linear actuators must be operated to support the load torque, otherwise, the F/S will be restored to a cylindrical shape and the tooth engagement with the C/S-Ds and C/S-S will be lost. When the operating pair of the linear actuators is switched as in Fig. 6.3, for example, the next pair must be operated to keep the tooth engagement before the precedent pair loses the power and the F/S escapes from the C/Ss. At this time, if the interval between the pairs of the linear actuators is too large, the positional relation between the teeth of the F/S and C/Ss may be inappropriate at the point where the next pair of the linear actuators deforms the F/S. This leads to tooth tip interference and failure in the tooth engagement. Based on this concern, the positions of the teeth in engagement is simulated and the positional relations

between the teeth of the F/S deformed into an oval and the teeth of the C/S-Ds and C/S-S are analyzed. Then, the condition for avoiding the tooth tip interference in the deformation of the F/S is clarified.

The following conditions are assumed for the simulation.

- Though the tooth profiles of the F/S and C/Ss in the commercial strain wave gearing systems have a specialized form combining multiple curves [76]–[78], they approximate trapezoids based on the measurement result of an actual strain wave gearing system (Harmonic Drive Systems Inc., FR-50-80-2-G). Fig. 6.5(a) and (b) shows the length of each part of the approximated trapezoidal teeth of the C/S-Ds and F/S when a pitch on the base circle of the C/S-Ds p_d is assumed to be 1, respectively.
- The pitch on the base circle of the C/S-S is the product of p_d times z_f/z_c . The base circle diameter and the length of each part of the teeth of the C/S-S is the same as those of the C/S-Ds.
- The base circle of the F/S deformed into an oval by one pair of opposite linear actuators shapes an ellipse expressed as a conic curve. In the analysis model of the F/S, each of the trapezoidal teeth shown in Fig. 6.5(b) contacts the ellipse at the center of the base line and arranged at regular intervals on the ellipse circumference.
- The teeth of the F/S meshes with those of the C/S-Ds and C/S-S to the deepest part on the long axis. The long diameter of the F/S is longer than the short diameter by the 2.2 times as long as the teeth height of the C/S-Ds and C/S-S. The magnitude of 2.2 times is determined by measuring the counterparts when the F/S is deformed by the W/G in the aforementioned actual strain wave gearing system.

When the reduction ratio R_s of the analyzed strain wave gearing system is determined, the teeth number of the F/S and C/S-Ds z_f and that of the C/S-S z_c are calculated by Eq. (6.1).

$$z_f = 2R_s - 2, \quad z_c = 2R_s \quad (6.3)$$

Given the base circle diameter d_{bd} , the pitch on the base circle of the C/S-Ds p_d is equivalent to the circumference divided by the teeth number.

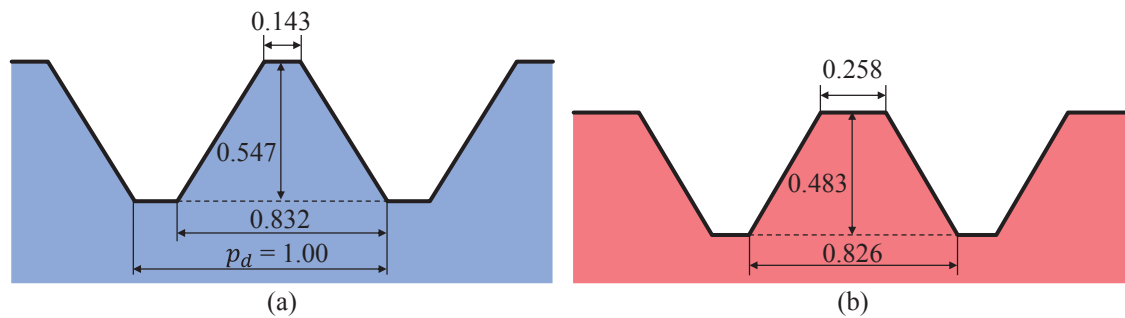


Fig. 6.5

Length of each part of the approximated trapezoidal teeth of (a) C/S-Ds and (b) F/S, when a pitch on the base circle of the C/S-Ds p_d is 1.

$$p_d = \frac{\pi d_{bd}}{z_f} \quad (6.4)$$

From the proportion shown in Fig. 6.5 and the abovementioned assumptions, the long and short diameters of the F/S, l_{af} and l_{bf} , respectively, are determined.

$$l_{af} = d_{bd} - 2 \times 0.547p_d, \quad l_{bf} = l_{af} - 2.2 \times 0.547p_d = d_{bd} - 4.2 \times 0.547p_d \quad (6.5)$$

By using these calculations, the positions of the teeth of the F/S, C/S-D, and C/S-S are determined. Fig. 6.6 shows the simulation result under the condition for $R_s = 81$. In the figure, the teeth and base circle of the F/S (red), C/S-D (light blue), and C/S-S (blue) are drawn. Fig. 6.6(a), (c), and (b) is the enlarged view in the vicinity of the short axis, long axis, and intermediate between them, respectively. Note that the C/S-D almost disappears in Fig. 6.6(c) because its teeth are overlapped with those of the C/S-S.

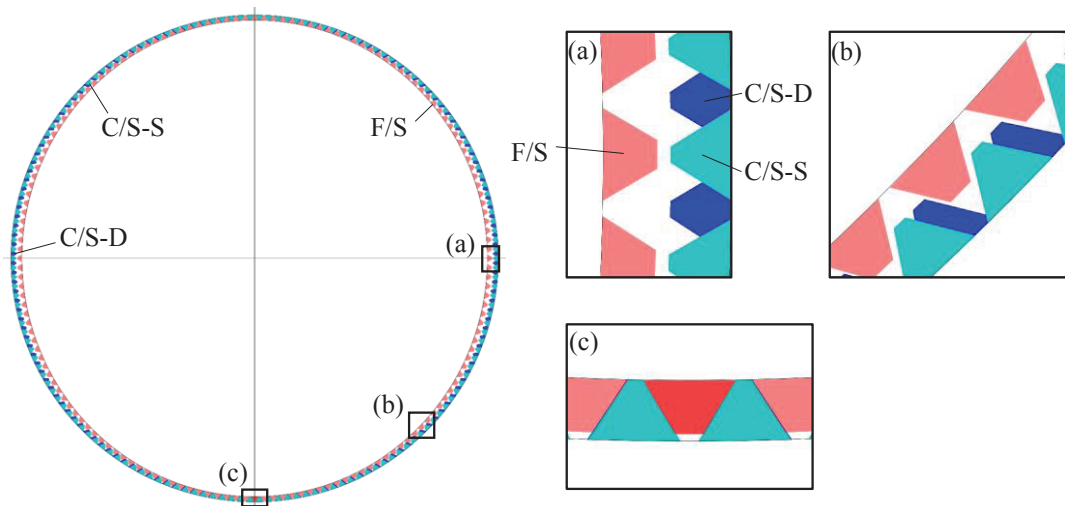


Fig. 6.6
Simulation result of the teeth positions of the F/S, C/S-S, and C/S-D.

6.3.2 Numerical Analysis

In order to evaluate the simulation result, some parameters are prepared. First, the number of each tooth is set as shown in Fig. 6.7. The 0th tooth of the F/S is the tooth on the long axis of the deformed F/S, which meshes with the teeth of the C/S-D or C/S-S to the deepest part. The tooth of the C/S-D or C/S-S neighboring the 0th tooth of the F/S in the clockwise direction is defined as the 0th tooth of the C/S-D or C/S-S. The numbers of the teeth get 0, 1, 2, ... in the counterclockwise direction. Next, the variables $\lambda(i)$, $\psi_s(i)$, and $\psi_d(i)$ are introduced to represent the positional relations between the teeth of the F/S, C/S-Ds, and C/S-S: $\lambda(i)$ is the distance in the radial direction between the i th tooth tip of the F/S and the tip circle of the C/S-D or C/S-S, which is

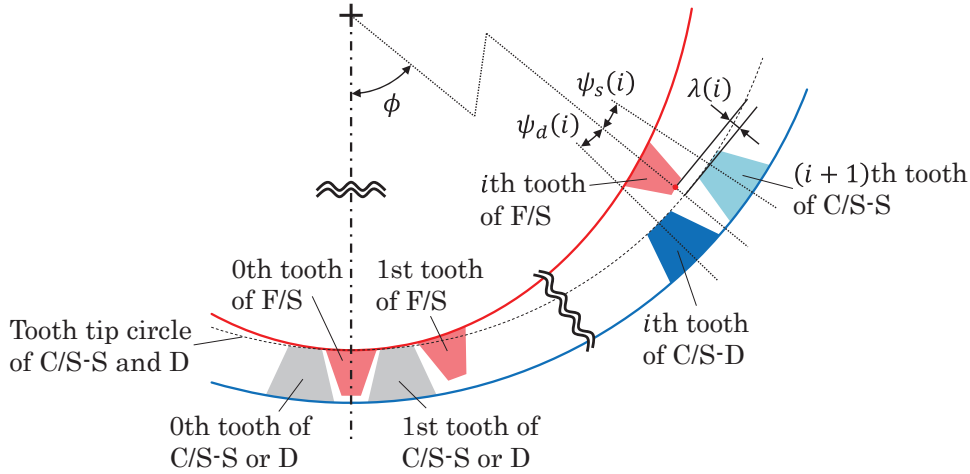


Fig. 6.7

Parameters for an analysis of the positional relations between F/S and C/Ss.

positive when the tooth tip of the F/S is inside the tip circle of the C/S-D or C/S-S and negative when outside; $\psi_s(i)$ is the angle between the line passing through the central axis and the center of the i th tooth tip of the F/S and the line passing through the central axis and the center of the $(i+1)$ th tooth tip of the C/S-S; and $\psi_d(i)$ is the angle between the line passing through the central axis and the center of the i th tooth tip of the F/S and the line passing through the central axis and the center of the i th tooth tip of the C/S-D. The parameters $\psi_s(i)$ [or $\psi_d(i)$] corresponds to the distance between the teeth of the F/S and the C/S-S [or C/S-D] in the circumferential direction. Additionally, the angle between the line passing through the central axis and the center of the i th tooth tip of the F/S and the long axis of the F/S is set as $\phi (= 360^\circ \times i/Z_f)$ to represent the position of the teeth on the F/S.

The analysis was carried out under the condition where $R_s = 81$ and $d_{ba} = 130.7$ mm. After calculating the values of $\lambda(i)$, $\psi_s(i)$, and $\psi_d(i)$ at each i , the ratio of $\lambda(i)$ to the tooth height of the F/S h_f is plotted in Fig. 6.8(a). Similarly, the ratio of $\psi_s(i)$ to $\psi_s(0)$ and the ratio of $\psi_d(i)$ to $\psi_d(0)$ are plotted in Fig. 6.8(b). These graphs cover the range of $0^\circ \leq \phi \leq 90^\circ$, i.e., $i = 0, 1, \dots, 39$, considering the symmetry. In Fig. 6.8(a), the radial distance changes from negative to positive at the point where ϕ is around 64° . As explained above, when the distance is positive, the tooth tip of the F/S exists inside the tip circle of the C/S-Ds or C/S-S. This means that the teeth of the F/S completely escape from the C/S-Ds or C/S-S in the region near the short axis where ϕ becomes larger than 64° . Therefore, an interfere between the teeth of the F/S and the C/S-Ds or C/S-S may occur when the linear actuators push the F/S, depending on the distances between the teeth of the F/S and C/Ss in the circumferential direction.

Fig. 6.8(b) shows that the teeth of the F/S are closest to the teeth of the C/S-D near $\phi = 45^\circ$ in the circumferential direction, while closest to the teeth of the C/S-S near $\phi = 90^\circ$. Now the relation between the tooth tip interference and the circumferential distance of the teeth is discussed

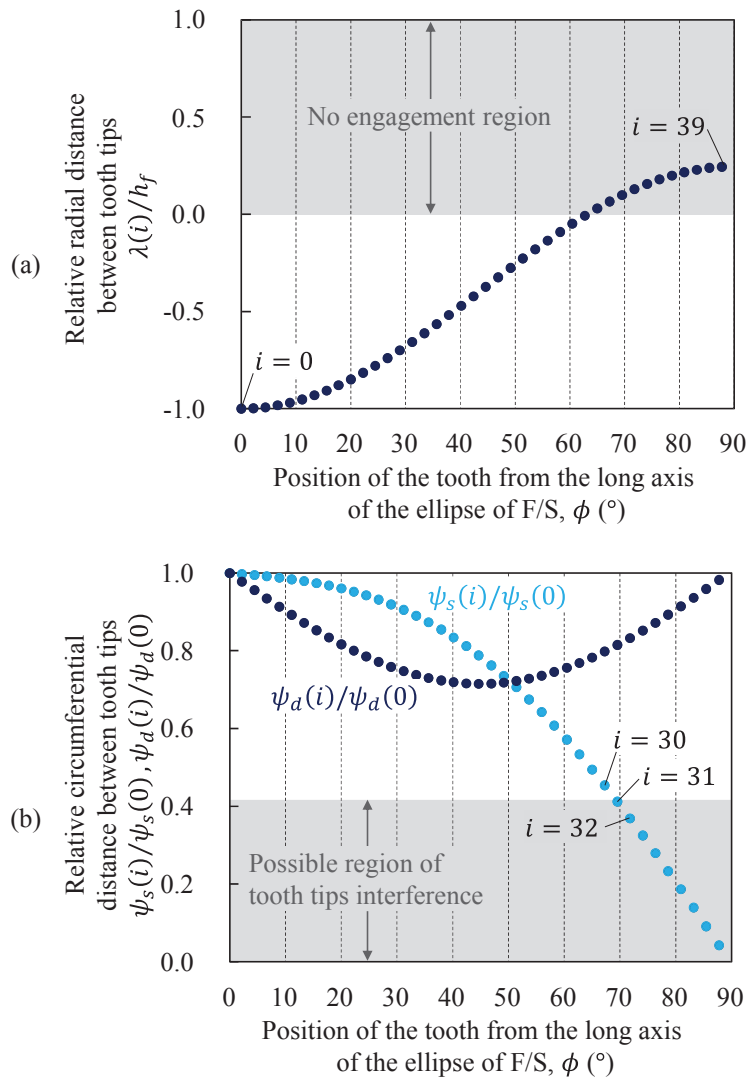


Fig. 6.8

(a) Radial distance and (b) circumferential distance between the tooth tips of F/S and C/S's over the position of the tooth of F/S when the reduction ratio R_s is 81.

by using the model shown in Fig. 6.9. As in Fig. 6.9(a), the situation where a tooth tip of the C/S-D or C/S-S is on the line passing through the central axis and the corner of the tooth tip of the F/S is set as the criterion. Then, the following cases are considered: when the circumferential distance between the teeth of the F/S and C/S-D or C/S-S is larger than that in the criterion [Fig. 6.9(b)], and when it is smaller [Fig. 6.9(c)]. Supposing that a point on the F/S moves in the radial direction when the F/S is pushed and deformed by the linear actuators, for simplicity, in the case of Fig. 6.9(b), the tooth surface of the F/S will contacts that of the C/S-Ds and C/S-S, so that the tooth engagement will be achieved. In the case of Fig. 6.9(c), on the other hand, the tooth tip of the F/S will interfere with that of the C/S-Ds or C/S-S, and then, the tooth engagement will not be achieved. Based on this model, the region where the interference could occur similarly to the situation in Fig. 6.9(c) is calculated, which was expressed as a shaded region in Fig. 6.8(b). The

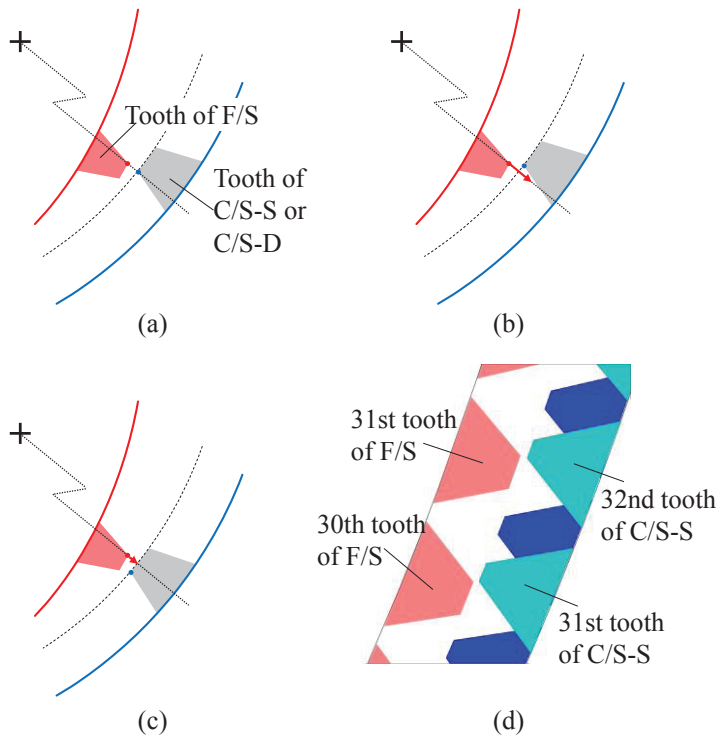


Fig. 6.9 Possibility of tooth tips interference between F/S and C/S's. (a) Criterion of interference possibility. (b) Proper distance. (c) Too close distance. (d) Interference possibility points in simulation result.

figure suggests that the 31st and later teeth of the F/S are likely to interfere with the C/S-S. Actually, in the simulation result shown in Fig. 6.9(d), the corner of the 30th tooth tip of the F/S is slightly apart from that of the 31st tooth of the C/S-S in the circumferential direction, while the corner of the 31st tooth tip of the F/S faces to the 32nd tooth tip of the C/S-S.

Additionally to the analysis for $R_s = 81$ shown in the above, the same analysis for $R_s = 51, 101, 151, 201, 251$, and 301 was conducted. The results were almost the same as the result for $R_s = 81$, namely, the values of ϕ where the radial distance changes from negative to positive shown in Fig. 6.8(a) and where the tooth tip interference may occur in Fig. 6.8(b) were almost the same. Therefore, the result shown in Fig. 6.8 can be used in any case.

6.3.3 Structural Condition for Actuators

Based on the analytical results, the arrangement of the linear actuators to deform the F/S appropriately without tooth tip interference in the proposed reducer-integrated motor is discussed. As explained in the previous section, the tooth tip of the F/S can interfere with the tooth tip of the C/S-Ds or C/S-S when the pair of the linear actuators to operate next pushes and deforms the F/S at the point where either of $\lambda(i)/h_f$, $\psi_s(i)/\psi_s(0)$, or $\psi_a(i)/\psi_a(0)$ is in the shaded region of Fig.

6.8(a) or (b). Hence, in order to avoid the interference, the next pair of the linear actuators should push the F/S in the region of $\phi \leq 60^\circ$, where all of the indices are out of the shaded region. In other words, the condition is that the next pair of the linear actuators must locate at the interval of smaller than 60° from the operating pair of the linear actuators.

When the linear actuators are arranged at regular intervals, the intervals become smaller than 60° if the number of linear actuators is six or larger. Additionally, the number of the linear actuators must be an even number because two linear actuators opposite in between the central axis are operated at the same time to deform the F/S. From the above, the number of the linear actuators should be six or larger even number. According to Fig. 6.8, the interference possibility becomes smaller as the number of the linear actuators is larger because the pair of the linear actuators to operate next push the F/S at the point of smaller ϕ . At the same time, the resolution of the output rotation will be improved since the position of the long axis of the F/S can be adjusted more finely. However, the size and weight of the motor will be increased when many linear actuators are installed. The number of the linear actuators should be chosen with considering these conditions.

6.4 Input-Output Relation Analysis

It was assumed that the deformed F/S shaped an ellipse expressed as a conic curve in Section 6.3. However, the deformation of the F/S actually depends on the pushing force of the linear actuators, and its shape is not always an ellipse. Then, the deformation of the F/S is described based on the theory of material mechanics and derive the fundamental input-output relation of the proposed reducer-integrated motor.

The model for deformation analysis is shown in Fig. 6.10. The F/S is assumed to be a circular curved beam put on the xy -plane whose curvature radius is ρ_0 . The pushing force of the linear actuators is thought to be f_a in y -direction. Here, the axial force N_b and the bending moment M_b at an angle ϕ_t from the x -axis are calculated as follows ($0^\circ \leq \phi_t \leq 90^\circ$) [79]:

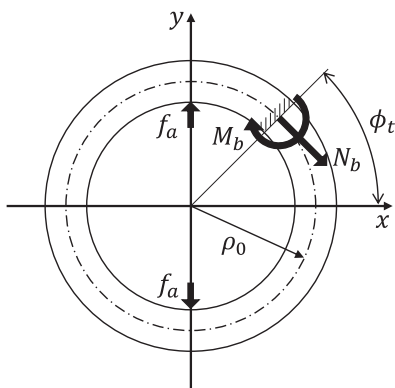


Fig. 6.10
Deformation model of the F/S.

$$N_b = \frac{f_a}{2} \cos \phi_t \quad (6.6)$$

$$M_b = -\frac{f_a \rho_0}{2} \left\{ \cos \phi_t - \frac{2}{\pi(1+\kappa)} \right\} . \quad (6.7)$$

In addition, when the deflection in x - and y -directions of the point (x_1, y_1) at $\phi_t = \phi_{t1}$ is set as δ_{x1} and δ_{y1} , they are given as follows [79]:

$$\delta_{x1} = -\int_0^{\phi_{t1}} \frac{1}{E_b A_b} \left\{ N_b + \frac{M_b}{\rho_0} \left(1 + \frac{1}{\kappa} \right) \right\} (y_1 - y) d\phi_t + \int_0^{x_1} \frac{1}{E_b A_b} \left(N_b + \frac{M_b}{\rho_0} \right) dx + \delta_{x0} \quad (6.8)$$

$$\delta_{y1} = -\int_0^{\phi_{t1}} \frac{1}{E_b A_b} \left\{ N_b + \frac{M_b}{\rho_0} \left(1 + \frac{1}{\kappa} \right) \right\} (x_1 - x) d\phi_t + \int_0^{y_1} \frac{1}{E_b A_b} \left(N_b + \frac{M_b}{\rho_0} \right) dy + \delta_{y0} \quad (6.9)$$

where δ_{x0} and δ_{y0} are the deflection at $\phi_t = 0^\circ$, E_b is the longitudinal elasticity modulus, κ is the section module, and A_b is the cross section of the beam, respectively. When Eqs. (6.6) and (6.7) are substituted into Eqs. (6.8) and (6.9) and the boundary conditions are taken into account, the deflection are finally calculated as follows:

$$\delta_x(f_a, \phi_t) = \frac{f_a \rho_0}{2 E_b A_b \kappa} \left\{ \frac{1}{2} (1 + \sin^2 \phi_t) - \frac{2}{\pi} \phi_t \sin \phi_t - \frac{2}{\pi(1+\kappa)} \cos \phi_t \right\} \quad (6.10)$$

$$\delta_y(f_a, \phi_t) = \frac{f_a \rho_0}{2 E_b A_b \kappa} \left\{ -\frac{1}{4} \sin 2\phi_t + \frac{2}{\pi} \phi_t \cos \phi_t + \frac{1}{2} \phi_t - \frac{2}{\pi(1+\kappa)} \sin \phi_t \right\} . \quad (6.11)$$

From these equations, the deflection in the radial direction $\delta_r(f_a, \phi_t)$ is obtained.

$$\delta_r(f_a, \phi_t) = \delta_x(f_a, \phi_t) \cos \phi_t + \delta_y(f_a, \phi_t) \sin \phi_t = \frac{f_a \rho_0}{E_b A_b \kappa} \left\{ \frac{1}{4} \cos \phi_t + \frac{\phi_t}{4} \sin \phi_t - \frac{1}{\pi(1+\kappa)} \right\} \quad (6.12)$$

If the beam is assumed to be thin, i.e., $\kappa A_b \rho_0^2 \rightarrow I_b$, $\kappa \rightarrow 0$, Eq. (6.12) is rewritten as follows:

$$\delta_r(f_a, \phi_t) = \frac{f_a \rho_0^3}{E_b I_b} \left\{ \frac{1}{4} \cos \phi_t + \frac{\phi_t}{4} \sin \phi_t - \frac{1}{\pi} \right\} \quad (6.13)$$

where I_b is the sectional secondary moment.

By using this equation, the deformation of the F/S can be simulated. For example, if the two pairs of the linear actuators are operating at the same time and the F/S and C/Ss contact each other at an angle θ_s from the y -axis, the following two conditions should be satisfied.

$$\delta_r(f_{a1}, \pi/2 - |\theta_s|) + \delta_r(f_{a2}, \pi/2 - |\theta_s - 2\pi/n_a|) = \delta_{\max} \quad (6.14)$$

$$\frac{\partial \delta_r}{\partial \phi_t}(f_{a1}, \pi/2 - |\theta_s|) + \frac{\partial \delta_r}{\partial \phi_t}(f_{a2}, \pi/2 - |\theta_s - 2\pi/n_a|) = 0 \quad (6.15)$$

Here, f_{a1} and f_{a2} are the pushing force of each pair of the linear actuators, n_a is the total number of the linear actuators, and $\delta_{\max}$ is the difference in the radii of the F/S and C/Ss. When $n_a = 12$, $\delta_{\max} = 1.0$, and $\rho_0^3/E_b I_b = 0.0672$ are set, whose values are based on the measurement result of the actual strain wave gearing system used in Section 6.3.1, the relationship between f_{a1} , f_{a2} , and θ_s is given as Fig. 6.11(a). For another example, the case where three pairs of the linear actuators are operated can be solved in a similar way. The required conditions are

$$\delta_r(f_{a1}, \pi/2 - |\theta_s|) + \delta_r(f_{a2}, \pi/2 - |\theta_s - 2\pi/n_a|) + \delta_r(f_{a3}, \pi/2 - |\theta_s + 2\pi/n_a|) = \delta_{\max} \quad (6.16)$$

$$\frac{\partial \delta_r}{\partial \phi_t}(f_{a1}, \pi/2 - |\theta|) + \frac{\partial \delta_r}{\partial \phi_t}(f_{a2}, \pi/2 - |\theta_s - 2\pi/n_a|) + \frac{\partial \delta_r}{\partial \phi_t}(f_{a3}, \pi/2 - |\theta_s + 2\pi/n_a|) = 0 \quad (6.17)$$

When the condition $f_{a1} = f_{a2} + f_{a3}$ is added, as one example, the result of the analysis is as in Fig. 6.11(b). Thus, the required input force and deformation of the F/S at any output rotation angle can be calculated in various input patterns, i.e., how many linear actuators are used under what conditions.

The same analysis of the tooth tip interference as in Section 6.3 was also carried out by using the deformation of the F/S calculated above instead of the approximation as an ellipse. As a result,

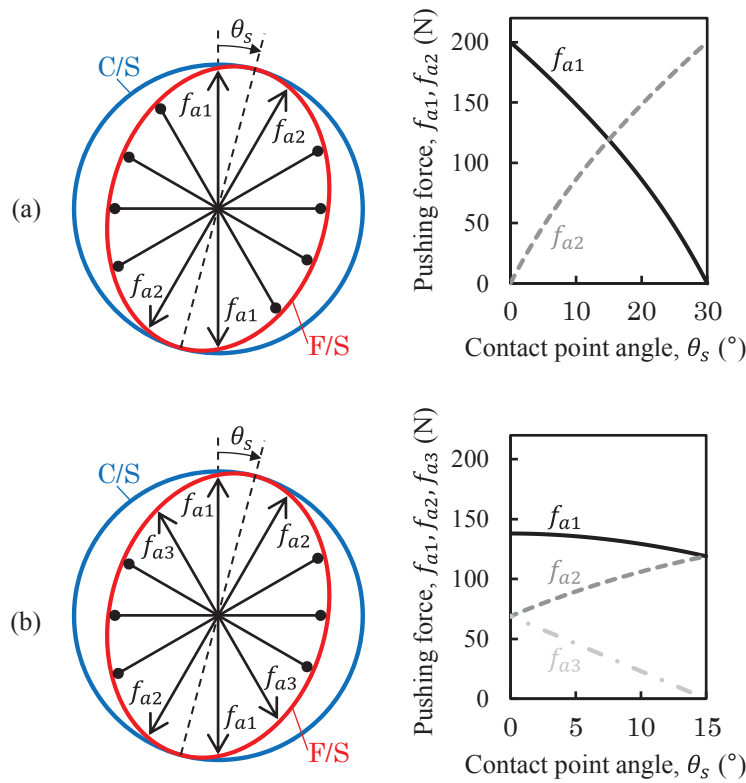


Fig. 6.11 Relationship between the contact point of the F/S and C/Ss and the pushing force of the linear actuators. (a) Operation by two pairs of the linear actuators. (b) Operation by three pairs of the linear actuators.

the condition for the arrangement of the linear actuators was moderate compared to Fig. 6.8. Then, the condition derived in the previous section should be followed in the design of the proposed reducer-integrated motor.

6.5 Experiments

To verify that the proposed reducer-integrated motor works as expected, the prototype is developed and experiments are conducted.

6.5.1 Proposed Motor Prototype

The overview and properties of the developed prototype are shown in Fig. 6.12 and Table 6.1. In Fig. 6.12(a), the C/S-S is exposed by removing one of the two C/S-Ds to show the internal structure. The basic structure of the prototype is the same as that in Fig. 6.2, and 12 linear actuators are installed. Though linear solenoids were used as the linear actuators to gain large force output in order to confirm the movement in this prototype, the linear solenoids are put outside the gears because of their size. Hence, as shown in Fig. 6.13, the output of the linear solenoids is transmitted to the F/S through the connecting parts, which are supported by the linear guides to

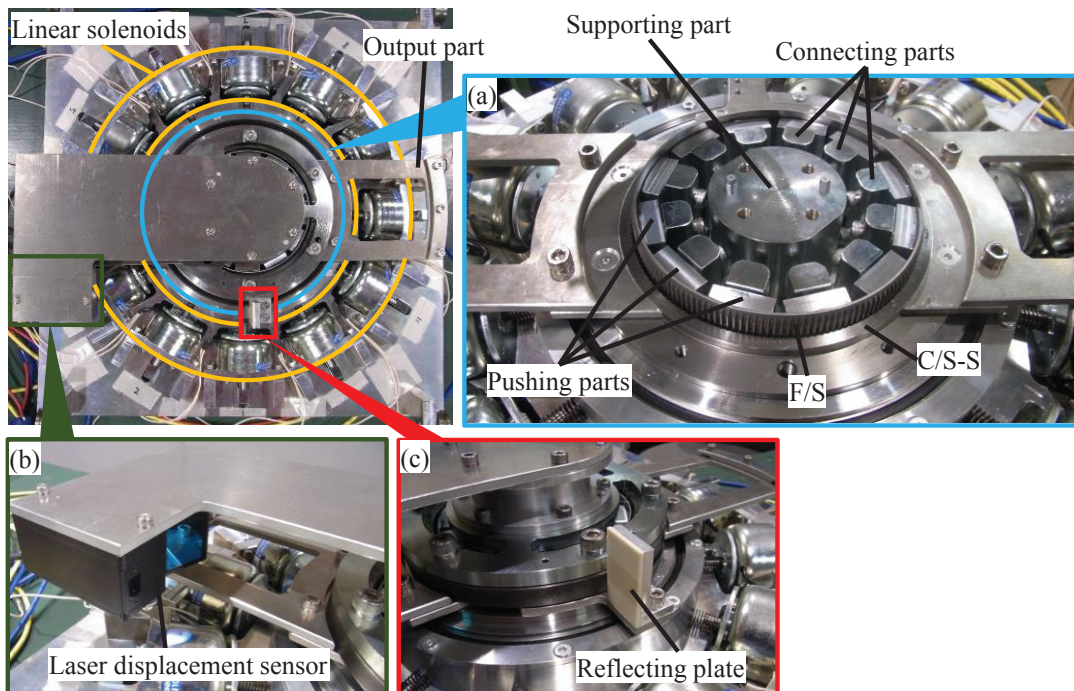


Fig. 6.12 Overview of the proposed reducer-integrated motor prototype. (a) Internal structure. (b) Laser displacement sensor. (c) Reflecting plate for the laser displacement sensor.

Table 6.1

Properties of the proposed reducer-integrated motor prototype.

Entire motor	Width	450 mm
	Depth	450 mm
	Height	150 mm
Strain wave gearing system	Harmonic Drive Systems Inc., FR-50-80-2-G	
	Outer diameter of C/S-S and C/S-D	170 mm
	Inner diameter of F/S	122.4 mm
	Reduction ratio	81
Linear solenoid	Shindengen Mechatronics Co., Ltd., 6EF AWG. No. 20	
	Input power	128 W
Laser displacement sensor	KEYENCE Corporation, LK-G150	
	Resolution	1 μm
	Sampling period	1 ms

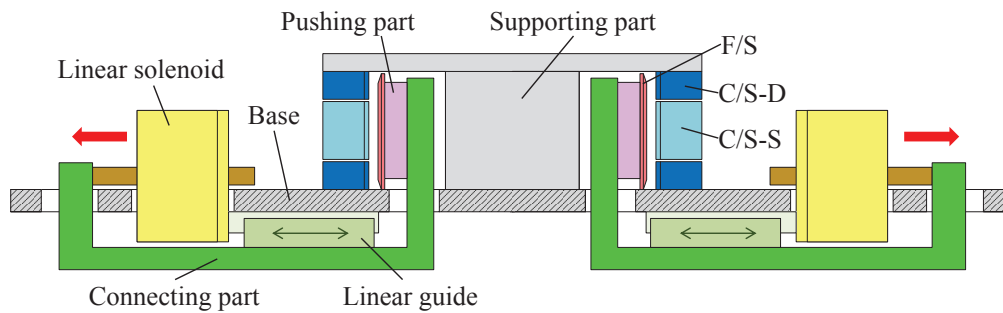


Fig. 6.13

Force transmission from the linear solenoids to the F/S through the linear guides, connecting parts, and pushing parts.

move in the radial direction, and the pushing parts, which have a semi-cylindrical shape and contact the F/S directly. The number of the linear solenoids is determined based on the conclusion of the analyses.

The operation of the prototype is controlled by a programmable logic controller (PLC). The prototype can switch the linear solenoids on and off instantly (i.e., less than 10 ms) by opening or closing the relays between the DC power source and the linear solenoids at the designated timing. Because it is connected to the laser displacement sensor, the PLC can measure the rotation angle of the output part. In the measurement of the rotation angle, the distance is measured between the laser displacement sensor fixed on the base (Fig. 6.12(b)) and the reflecting plate fixed on the C/S-S and moving in the circumferential direction with the output part rotation (Fig. 6.12(c)). The displacement of the reflecting plate from the initial position in the progressing direction of the laser is converted into the rotation angle.

6.5.2 Experiments

First, the qualitative experiments were conducted by observing the tooth engagement. In the proposed reduce-integrated motor, various input patterns of the linear solenoids, other than in Fig. 6.3, are feasible as demonstrated in Section 6.4. Concerning this characteristics, four input patterns were tested and compared. Fig. 6.14 illustrates the four input patterns and their observation results at each of the first four steps. The pictures show the tooth engagement between the F/S and C/S-S seen through a stereomicroscope. Pattern 1 is the same movement as in Fig. 6.3, where only one pair of linear solenoids is used at the same time. The observation results showed the prototype moved without any problem. In Pattern 2, the number of pairs of the working linear solenoids alternates between one and two. The pushing force when two pairs are operated is half of the power when one pair is operated. According to the observation, the rotation angle of the C/S-S seemed to delay in the steps when two pairs were operated. In Pattern 3, two pairs of the linear solenoids always push the F/S. The observation verified that the prototype rotated basically without any problem as well as Pattern 1 except small delay. In Pattern 4, two pairs of the linear solenoids are used at the odd-numbered steps and three pairs are used at the even-numbered steps. When three pairs are operated, the power of the pairs on the sides is half of that of the pair in the middle. The result of the observation was similar to that of Pattern 2: the rotation angles at the even-numbered steps were delayed. Fig. 6.14 picked up only the first four steps, but the motor prototype successfully continued rotating after them in every input pattern. From the above, though some rotational delays occur in certain situations, the proposed reducer-integrated motor achieved the tooth engagement between the F/S and C/Ss appropriately and output rotation as expected in

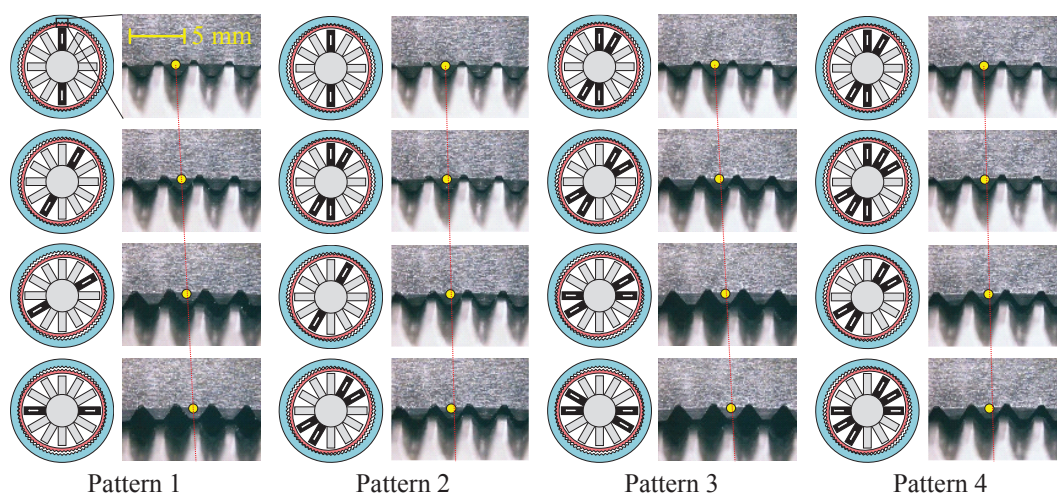


Fig. 6.14

Four input patterns and rotation of the C/S-S observed by the stereomicroscope. The yellow dot is marked on a certain tooth of the C/S-S. The red dashed line indicates the desired path of the yellow dot based on the position at the first step.

every input patterns. This result verifies the effectiveness of the design condition derived in Section 6.3.

To evaluate and compare each input pattern quantitatively, the output rotation was measured by using the laser displacement sensor. Fig. 6.15 shows the measured and theoretical rotation angle in each input patterns with 500-ms intervals of steps. At this time, the average rotational speed is $\bar{N} = 1/(12 \times 81 \times 0.5) \approx 2.06 \times 10^{-3} \text{ s}^{-1}$ according to Eq. (6.2). The graphs basically support the results given from the observation in Fig. 6.14. The measurement results, as well as the observation results, suggest that the input patterns where the constant number of the pairs of the linear solenoids are operated such as Pattern 1 or 3 are favorable. In Pattern 3, however, there were certain errors between the measured and theoretical rotation angle in Fig. 6.15. The cause of this error is thought to be mainly the error in the initial state due to a defect of the teeth contact or a backlash for excessive deformation, because the errors at the second step or later are almost constant. On the other hand, relatively large errors occurred in Pattern 2 and 4. This might be because hysteresis effects on the deformation of the F/S, friction effects between the teeth of the F/S and C/Ss, or the output force disturbance of the linear solenoids caused problem in the rotation of the C/S-S between the odd- and even-numbered steps, when the power distribution of the linear solenoids is changed.

Next, the experimental results are evaluated in another situation. Fig. 6.16 shows the graphs of the measurement results when the load torque $2.5 \text{ kg} \cdot \text{m}$ was applied to the output part of the prototype in a counter direction of the rotation. In Fig. 6.16, the measured rotation angle in Pattern 1 and 3 nearly agree with the theoretical values for the entire time. On the other hand, in Pattern 2 and 4, the measured angle deviated sharply from the theoretical values, especially in the even-numbered steps. These results basically match those in Fig. 6.14 and Fig. 6.15, when the load torque was not applied.

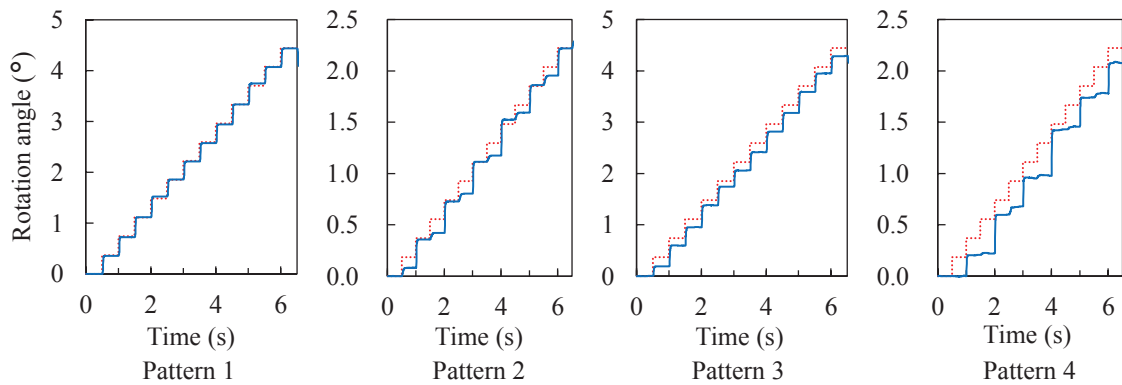


Fig. 6.15

Measured rotation angle for the experiments in Fig. 6.14. The blue solid line indicates the measured rotation angle and the red dashed line indicates the theoretical rotation angle.

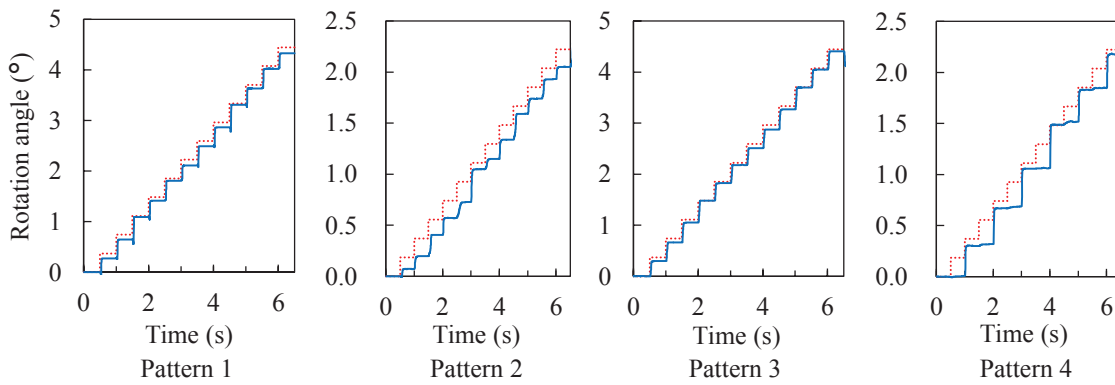


Fig. 6.16

Experimental results of the rotation when the load torque $2.5 \text{ kg} \cdot \text{m}$ is applied. The blue solid line indicates the measured rotation angle and the red dashed line indicates the theoretical rotation angle.

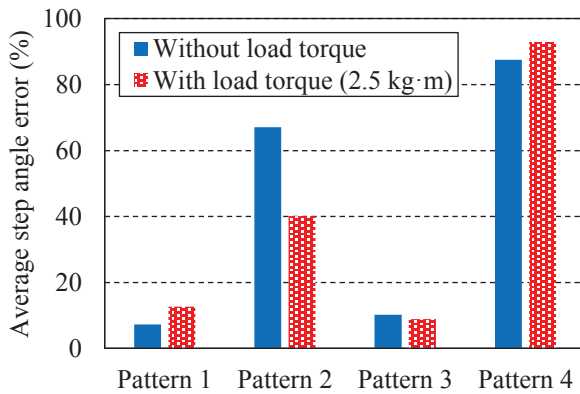


Fig. 6.17

Average step angle errors in the experimental results.

6.5.3 Discussion

Based on the numerical results, which of the four input patterns is optimal is discussed. Here, the average step angle errors are compared. The step angle error is the difference between the theoretical and measured values in the rotation angle of the C/S-S at each step. The theoretical step angle is 0.370° in Pattern 1 and 3, and 0.185° in Pattern 2 and 4. When the average value of the step angle errors in 130 steps to the theoretical step angles is taken in each pattern, the results are shown in Fig. 6.17. Obviously, the errors in Pattern 2 and 4 are larger than those in Pattern 1 and 3 as discussed above. Compared to Pattern 3, Pattern 1 causes a smaller error when the load torque is not applied, whereas larger error when the load torque is applied. The main reason is thought to be the torque omission. In Pattern 1, no linear solenoids may work on switching the linear solenoids, so that the F/S returns to the circular shape and teeth engagement between the F/S and C/Ss is no longer kept. Actually, in Fig. 6.16, the measured rotation angle reverses instantly on switching the linear solenoids because of the load torque. In contrast, the errors in Pattern 2 and 3 when the load

torque is applied are smaller than the errors without load torque. It might be because the load torque pushed back the linear solenoids and the excessive deformation of the F/S was resolved. From the above, it is concluded that the optimal input pattern in practical use would be Pattern 3.

6.6 Summary

For the purpose of downsizing of the motor system, a novel reducer-integrated motor using the principle of a strain wave gearing system was proposed. The proposed motor has the potential to gain small size, high load-supporting capacity, and high stiffness. The analyses and experiments were carried out and the following results were obtained.

- The proposed reducer-integrated motor can achieve downsizing by installing the linear actuators inside the reducer part. Using a strain wave gearing system and supporting the output part symmetrically contribute to increasing of load-supporting capacity and stiffness.
- The tooth interference in the F/S deformed by the linear actuators was analyzed. The simulation of the positional relations of the teeth derived the condition of the linear actuator arrangement to avoid the interference.
- The theoretical input-output relation in the proposed motor is derived with a focus on the elastic deformation of the F/S. The result gave the pushing force of the linear actuators for the motor to output an arbitrary rotation angle.
- The experiments were conducted by using the proposed motor prototype. Observations of the tooth engagement verified that the proposed motor actually worked as expected. The comparison of the four input patterns based on the measurement result suggested an optimal input pattern.

Chapter 7

Reducer-Integrated Motor Using Inscribed Planetary Gear Mechanism

7.1 Introduction

To realize the miniaturized motor system, a reducer-integrated motor using the principle of a strain wave gearing system was proposed in Chapter 6. The proposed motor can achieve downsizing because the actuators are all installed inside the reducer part. Additionally to this, it is also expected to have the high load-supporting capacity and high stiffness because the load torque applied to the output part is supported by two internal gears fixed on the base.

However, the strain wave gearing system used in the motor requires a special gear, the F/S, which is an elastic and flexible ringed metal external gear difficult to produce. To address this issue, a new reducer-integrated motor using only rigid gears is proposed to make it easier to produce in this chapter. The proposed motor utilizes the differential gear mechanism in which the gear pair with a small difference in teeth number and the gear pairs with no difference in teeth number engage simultaneously. This gear mechanism realizes a high reduction ratio and the motor achieves downsizing similarly to the existing reducer-integrated motors. At the same time, the internal gears fixed on the base support the load torque, and then the motor can gain high load-supporting capacity and high stiffness, as well as the previously proposed motor.

In this chapter, the structure and movement of the new proposed motor are explained first. Next, the geometrical conditions for the simultaneous engagement of two gear pairs are clarified. Through the discussion on the strength of the gears, a design method and algorithm is proposed. The experiment is conducted by using the prototype and verifies the effectiveness of the proposed motor.

7.2 New Reducer-Integrated Motor

7.2.1 Principle of an Inscribed Planetary Gear

The new reducer-integrated motor uses an inscribed planetary gear mechanism [48]–[51] as the reducer part. Fig. 7.1 illustrates the structure and motion of the gear mechanism. The inscribed planetary gear, or a KHV planetary gear, consists of an external and internal gear pair with one or a few differences in teeth number and a crankshaft revolving the external gear. When the motor rotates the crankshaft, the external gear revolves around the internal circumference of the internal gear with tooth engagement. The external gear rotates relative to the internal gear by the difference in teeth number per one revolution. By using this relative rotation as output, the reducer realizes a high reduction ratio in a single stage.

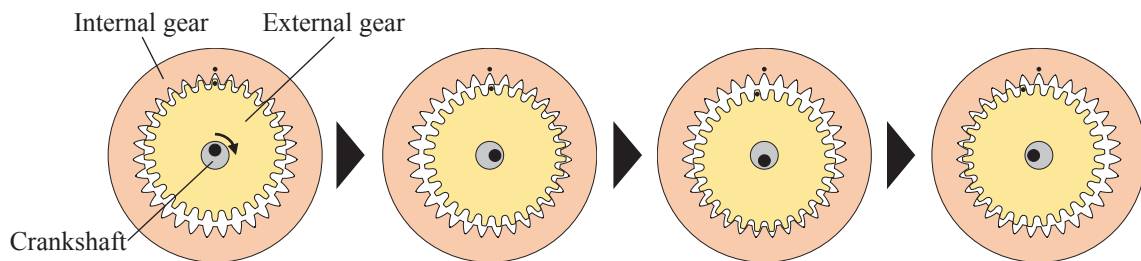


Fig. 7.1
Movement of an inscribed planetary gear mechanism.

7.2.2 Proposed New Reducer-Integrated Motor

Fig. 7.2(a) and (b) shows the structure of the proposed new reducer-integrated motor. The fundamental mechanism is the same as the motor proposed in Chapter 6: the external gear corresponds to the F/S, the internal A gear to the C/S-S, and the internal B gears to the C/S-Ds. However, the external gear in this motor is a ringed rigid gear, while the F/S is an elastic and flexible gear. As shown in Fig. 7.2(b), linear actuators are arranged radially at equal intervals and fixed on the base inside the external gear. The external gear can rotate and move in the radial direction freely. The internal A gear is supported to rotate around the central axis. The two internal B gears are fixed on the base and do not rotate.

Next, the movement of the proposed reducer-integrated motor is explained. When the linear actuators are operated and push the external gear, the external gear moves in the radial direction and engages with the internal A and B gears. Here, the situation where the neighboring two linear actuators are operated at the same time and switched clockwise is considered, as shown in Fig. 7.3. The teeth number of the external gear is the same as those of the internal B gears fixed on the base,

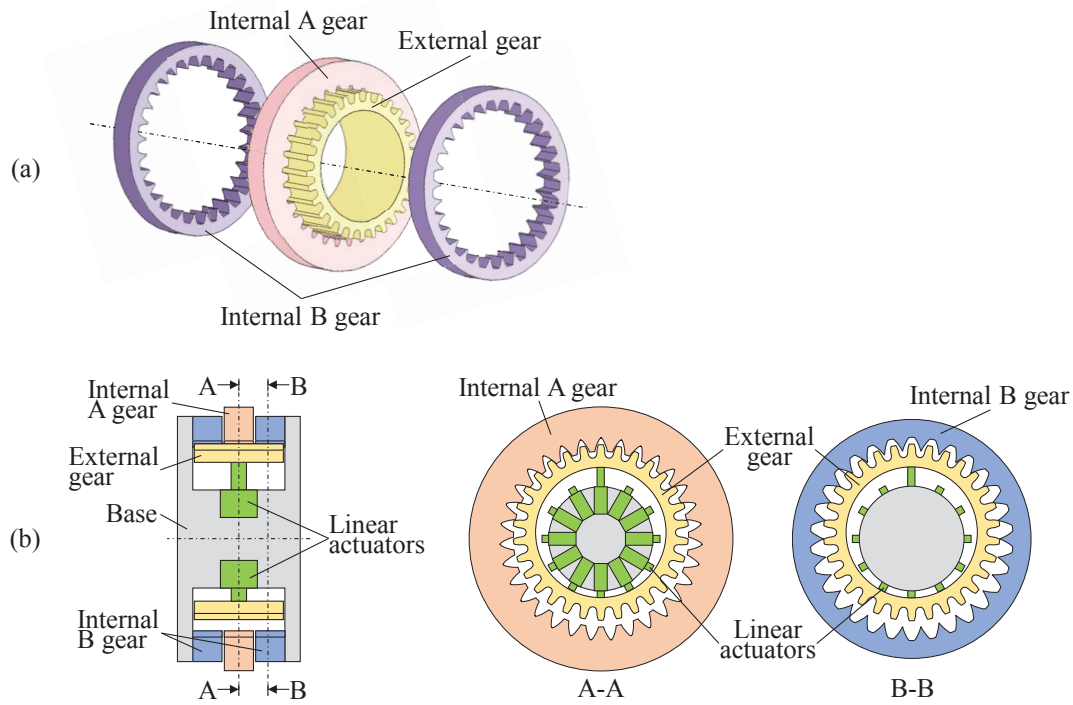


Fig. 7.2
Structure of the proposed new reducer-integrated motor. (a) One external gear, one internal A gear, and two internal B gears. (b) Sectional views of the motor.

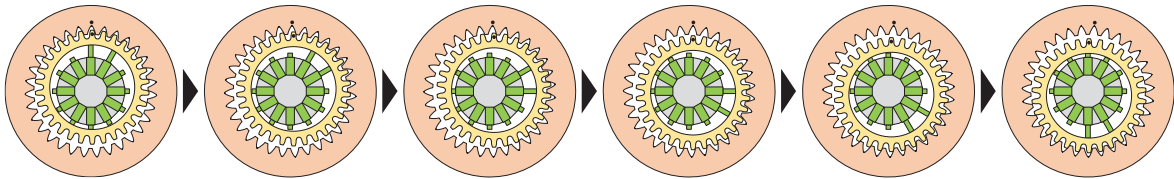


Fig. 7.3
Rotation of the internal A gear by the revolution of the external gear.

so that the external gear does not rotate but revolves around the internal circumferences of the internal gears. When the external gear revolves, the relative rotation to the internal A gear occurs according to the same principle of the inscribed planetary gear. Because the external gear does not rotate, only internal A gear rotates. Every clockwise revolution of the external gear, that is, every cycle of switching the operating linear actuators, causes rotation of internal A gear by the difference in teeth number from the external gear. The revolution of the external gear by switching linear actuators is set as the input and the rotation of the internal A gear is set as the output, and the reduction ratio R_s is represented by Eq. (7.1).

$$R_s = \frac{z_{2A}}{z_{2A} - z_1} \quad (7.1)$$

Here, z_1 is the teeth number of the external gear and the internal B gears, and z_{2A} is the teeth number of the internal A gear. The difference in teeth number $z_{2A} - z_1$ is a constant having a small

value (e.g., 1 or 2). For the constant $z_{2A} - z_1$, R_i becomes large in proportion to z_{2A} . This means that the proposed motor can directly output a sufficiently reduced rotation. The average rotational speed of the internal A gear is calculated by Eq. (6.2).

The proposed reducer-integrated motor has characteristics similar to the previous reducer-integrated motor. The entire motor can be miniaturized because the linear actuators are installed inside the reducer. The load torque applied to the internal A gear is supported by the internal B gears fixed on the base through the external gear, so that the motor can support high load torque and gain high stiffness. In the external gear, the load torque from the internal A gear and the reaction torque from the internal B gears become symmetrical in the axial direction, which is expected to suppress the tilt and torsion of the external gear and increase the stiffness of the motor. Hence, the proposed reducer-integrated motor can realize downsizing, high load-supporting capacity, and high stiffness.

Furthermore, the proposed reducer-integrated motor consists of rigid external and internal gears, as seen in common geared reducers. The motor does not need special gears, such as elastic gears or complicated transmission mechanisms, and so it is relatively easy to produce the reducer of the motor. This characteristic contributes to high design flexibility, as well as increased practicality and reliability.

7.3 Conditions for Engagement of Gear Pairs

The proposed reducer-integrated motor utilizes two kinds of external and internal gear pairs: one with a small difference in teeth number and the other with no difference in teeth number. Here, how the reducer of the motor is designed should be considered. Each gear pair has to satisfy a variety of geometrical conditions for tooth engagement. The proposed motor needs to meet the conditions that the gear pairs with a small difference in teeth number and those with no difference in teeth number share the external gear and its revolution. Thus, the gear pairs of the reducer must satisfy many conditions. Because it is not easy to design the gears, a design method is required. This section clarifies the geometrical conditions required for the gear pairs with small and no differences in teeth number to share the external gear. Based on the conditions, a design method for the reducer is constructed in the next section.

7.3.1 Engagement Equations for Two Kinds of External and Internal Gear Pairs

The engagement equations for the external gear and the internal A and B gears are shown. Here, the teeth of the external gear and internal A and B gears are assumed to have an involute profile. According to the theory on the engagement of the involute internal and external gears with a difference in teeth number [80], the engagement equation for the external gear and the internal A gear is represented as follows:

$$a_A = m_n \frac{z_{2A} - z_1}{2} \cdot \frac{\cos \alpha_0}{\cos \alpha_w} \quad (7.2)$$

$$\text{inv } \alpha_w = \text{inv } \alpha_0 + 2 \tan \alpha_0 \cdot \frac{x_{2A} - x_1}{z_{2A} - z_1} \quad (7.3)$$

where a_A denotes the center distance between the external gear and the internal A gear, m_n denotes the module of the gears, α_0 denotes the reference pressure angle, α_w denotes the working pressure angle, and x_1, x_{2A} denote the vertical profile shift coefficient of the external gear and the internal A gear, respectively. The lateral profile shift coefficient and the backlash are assumed to be zero for simplicity. The profile shift coefficient in the following discussions is the vertical coefficient. The function inv is the involute function defined by the following equation.

$$\text{inv } \gamma = \tan \gamma - \gamma \quad (7.4)$$

Next, the engagement equation for the external gear and the internal B gears is discussed. The theory on the engagement of the involute external and internal gear having no difference in teeth number gives the equation as follows [81]:

$$a_B = m_n (x_{2B} - x_1) \sin \alpha_0 \quad (7.5)$$

where a_B denotes the center distance between the external gear and the internal B gears, and x_{2B} denotes the profile shift coefficient of the internal B gears. The variables on the external gear are the same as those in the equations for the external gear and the internal A gear, because the external gear is shared.

In the proposed reducer-integrated motor, the external gear has to engage simultaneously with the internal A and B gears. When internal A and B gears are arranged coaxially, the engagement condition is given by

$$a_A = a_B \quad (7.6)$$

In the following discussion, $a_A = a_B = a_{AB}$ is set.

7.3.2 Conditions for Tooth Engagement of Each Gear Pair

Next, the conditions for the engagement in each gear pair of the external gear and the internal A and B gears are discussed. As the variables concerning the external gear, internal A gear and B gears, the reference circle diameters d_1, d_{2A}, d_{2B} , the tip circle diameters d_{a1}, d_{a2A}, d_{a2B} , the base circle diameters d_{b1}, d_{b2A}, d_{b2B} , and the tip pressure angles $\alpha_{a1}, \alpha_{a2A}, \alpha_{a2B}$ are given as follows, respectively:

$$d_1 = d_{2B} = z_1 m_n \quad (7.7)$$

$$d_{2A} = z_{2A} m_n \quad (7.8)$$

$$d_{a1} = m_n \{z_1 + 2(h_{kg1} + x_1)\} \quad (7.9)$$

$$d_{a2A} = m_n \{z_{2A} - 2(h_{kg2A} - x_{2A})\} \quad (7.10)$$

$$d_{a2B} = m_n \{z_1 - 2(h_{kg2B} - x_{2B})\} \quad (7.11)$$

$$d_{b1} = d_{b2B} = m_n z_1 \cos \alpha_0 \quad (7.12)$$

$$d_{b2A} = m_n z_{2A} \cos \alpha_0 \quad (7.13)$$

$$\alpha_{a1} = \cos^{-1} \left(\frac{d_{b1}}{d_{a1}} \right) \quad (7.14)$$

$$\alpha_{a2A} = \cos^{-1} \left(\frac{d_{b2A}}{d_{a2A}} \right) \quad (7.15)$$

$$\alpha_{a2B} = \cos^{-1} \left(\frac{d_{b2B}}{d_{a2B}} \right) . \quad (7.16)$$

Here, $h_{kg1}, h_{kg2A}, h_{kg2B}$ are the addendum coefficients, which are the values of the addendum divided by the module m_n .

Comparison between the tip circle diameter and the base circle diameter in the internal gears.

From Eqs. (7.10)–(7.13), the following conditions are required for $d_{a2A} > d_{b2A}$ and $d_{a2B} > d_{b2B}$.

$$z_{2A} > \frac{2(h_{kg2A} - x_{2A})}{1 - \cos \alpha_0} \quad (7.17)$$

$$z_1 > \frac{2(h_{kg2B} - x_{2B})}{1 - \cos \alpha_0} \quad (7.18)$$

Positional condition of the tooth tips in the gear pair with a difference in teeth number. In the engagement of the external gear and the internal A gear, in one area the teeth are in engagement

(e.g., the upper region in Fig. 7.2(b)) and in another area the teeth are separate from each other (e.g., the lower region in Fig. 7.2(b)). Therefore, the tooth tips of the external gear in the engagement area have to be outside the tooth tip of the internal A gear, and the tooth tip of the external gear in the separate area has to be inside the tooth tip of the internal A gear. These conditions are represented by the following equation.

$$\begin{aligned} d_{a1} + 2a_{AB} &> d_{a2A} > d_{a1} - 2a_{AB} \\ \Leftrightarrow \frac{2a_{AB}}{m_n} &> z_{2A} - z_1 - 2(h_{kg2A} + h_{kg1} - x_{2A} + x_1) > -\frac{2a_{AB}}{m_n} \end{aligned} \quad (7.19)$$

Avoidance of sharp tooth tips. The width of the top land of the external gear and the internal A and B gears has to be positive. The condition is given as follows [82]:

$$d_{a1} \left\{ \frac{\pi}{2z_1} + \frac{2x_1 \tan \alpha_0}{z_1} + (\text{inv } \alpha_0 - \text{inv } \alpha_{a1}) \right\} > 0 \quad (7.20)$$

$$d_{a2A} \left\{ \frac{\pi}{2z_{2A}} - \frac{2x_{2A} \tan \alpha_0}{z_{2A}} - (\text{inv } \alpha_0 - \text{inv } \alpha_{a2A}) \right\} > 0 \quad (7.21)$$

$$d_{a2B} \left\{ \frac{\pi}{2z_1} - \frac{2x_{2B} \tan \alpha_0}{z_1} - (\text{inv } \alpha_0 - \text{inv } \alpha_{a2B}) \right\} > 0 \quad (7.22)$$

Contact ratio higher than 1. To transmit the rotation of gears appropriately, the contact ratio has to be higher than 1. When the contact ratio between the external gear and the internal A gear is ε_A and that between the external gear and the internal B gears is ε_B , they are calculated by the following equations [51].

$$\varepsilon_A = \frac{\sqrt{d_{a1}^2 - d_{b1}^2} - \sqrt{d_{a2A}^2 - d_{b2A}^2} + 2a_{AB} \sin \alpha_w}{2\pi m_n \cos \alpha_0} > 1 \quad (7.23)$$

$$\varepsilon_B = \frac{1}{2\pi} \left\{ z_1 (\tan \alpha_{a1} - \tan \alpha_{a2B}) + \frac{2a_{AB}}{m_n \cos \alpha_0} \right\} > 1 \quad (7.24)$$

In general, it is preferable for the contact ratios to be larger with respect to the strength and the noise.

Avoidance of involute interference. The engagement of the external and internal gears might cause involute interference, in which the tooth root of the external gear interferes with the tooth tip of the internal gear and the rotation of the gears is interrupted. The condition that involute interference does not occur between the external gear and the internal A and B gears is represented by Eqs. (7.25) and (7.26), respectively [83], [81].

$$\frac{z_1}{z_{2A}} > 1 - \frac{\tan \alpha_{a2A}}{\tan \alpha_w} \quad (7.25)$$

$$\frac{z_1}{2} \cdot \frac{\tan \alpha_{a2B}}{\tan \alpha_0} > x_{2B} - x_1 \quad (7.26)$$

Avoidance of trochoid interference. The engagement of the external and internal gear might cause trochoid interference, in which the tooth tip of the external gear interferes with that of the internal gear when moving out of the tooth space and the rotation of the gears is interrupted. The condition that trochoid interference does not occur between the external gear and the internal A gear is as follows [51]:

$$\begin{aligned} \frac{z_1}{z_{2A}} \left\{ \cos^{-1} \left(\frac{d_{a2A}^2 - d_{a1}^2 - 4a_{AB}^2}{4a_{AB}d_{a1}} \right) + \text{inv } \alpha_{a1} - \text{inv } \alpha_w \right\} - (\text{inv } \alpha_{a2A} - \text{inv } \alpha_w) \\ > \cos^{-1} \left(\frac{d_{a2A}^2 - d_{a1}^2 + 4a_{AB}^2}{4a_{AB}d_{a2A}} \right) . \end{aligned} \quad (7.27)$$

For the external gear and the internal B gears with no difference in teeth number, trochoid interference never occurs [84].

Existence of clearance. When the external and internal gears are in engagement at the deepest part, clearance must exist between the tooth bottom of the external gear and the tooth tip of the internal gear, and between the tooth tip of the external gear and the tooth bottom of the internal gear. When the root circle diameters of the external gear and the internal A and B gears are given as d_{f1}, d_{f2A}, d_{f2B} , respectively, the conditions for clearance are as follows:

$$\begin{cases} \frac{d_{a2A} - d_{f1}}{2} - a_{AB} > 0 \\ \frac{d_{a2B} - d_{f1}}{2} - a_{AB} > 0 \end{cases} \Leftrightarrow d_{f1} < \min(d_{a2A}, d_{a2B}) - 2a_{AB} \quad (7.28)$$

$$\frac{d_{f2A} - d_{a1}}{2} - a_{AB} > 0 \Leftrightarrow d_{f2A} > d_{a1} + 2a_{AB} \quad (7.29)$$

$$\frac{d_{f2B} - d_{a1}}{2} - a_{AB} > 0 \Leftrightarrow d_{f2B} > d_{a1} + 2a_{AB} . \quad (7.30)$$

7.4 Design Method for the Reducer Part

In this section, as mentioned in the introduction of Section 7.3, a design method for the reducer to satisfy a variety of geometrical conditions clarified in the previous section is constructed. First, the basic policy of the design is decided based on the movement principle of the proposed motor by discussing which variables should be mainly treated in the design. Then, by using the characteristics of the set of design solutions satisfying the conditions, a method to acquire the available design solutions from graphs is proposed. Next, the strength of the gears corresponding to those design solutions is examined, and the design solutions realizing excellent strength are shown. Finally, the design algorithm for the reducer is constructed according to these discussions.

7.4.1 Design Policy

One of the indicators of the optimal design for the proposed reducer-integrated motor is to make the center distance between the external and internal gears as small as possible. When linear solenoids, which have a quick response and large force output, are used as the linear actuators of the proposed motor, the output force exponentially decreases with the strokes. Thus, it is preferable to minimize the strokes for efficient operation. The required strokes depend on the eccentricity between the external and internal gear, so that the strokes become small when the center distance is small. Not only with linear solenoids but also with other kinds of linear actuators, it thought to be better to keep the center distance small.

Let us discuss the parameters determining the center distance a_{AB} . From the engagement equations (7.2), (7.3), and (7.5), a_{AB} is determined by the parameters of the reference pressure angle α_0 , the difference in teeth number $z_{2A} - z_1$, the module m_n , and the differences in profile shift coefficients $x_{2A} - x_1$ and $x_{2B} - x_1$. The smaller that each parameter becomes, the smaller that a_{AB} becomes. Reference pressure angle α_0 has a certain degree of design freedom, but the value chosen in standard usage is limited and an extremely small value cannot be used. With respect to $z_{2A} - z_1$, the value is considered to be a small constant in this research. If $z_{2A} - z_1 = 1$ is chosen, for example, the value achieves the minimum. The value of m_n affects the size of the gears, so that the possible alternatives are limited. As discussed above, making α_0 , $z_{2A} - z_1$, m_n small is limited. On the other hand, $x_{2A} - x_1$ and $x_{2B} - x_1$ can be small as far as the engagement of each gear pair is established. It is favorable to set these parameters at the center of the discussion in the design. Therefore, it is mainly aim to make a_{AB} small by keeping the differences in the profile shift coefficient as small as possible. Note that $x_{2A} - x_1$ and $x_{2B} - x_1$ are not independent of each other, but one is determined dependently on the other from Eq. (7.5).

Based on the discussion above, the situation where the values of α_0 , m_n , z_1 , and z_{2A} are given and $x_{2A} - x_1$ is assumed to be a certain value ζ is considered. In this situation, Eqs. (7.17), (7.19), (7.23), (7.25), and (7.27) become inequalities using $h_{kg1} + x_1$ and $h_{kg1} + h_{kg2A}$ as variables. A graph where the horizontal axis is $h_{kg1} + x_1$ and the vertical axis is $h_{kg1} + h_{kg2A}$ is proposed in order to make the solution region for the conditions easy to understand. As an example, Fig. 7.4 shows the graph of the solution region when $\alpha_0 = 20^\circ$, $m_n = 2.0$, $z_1 = 49$, $z_{2A} = 50$, and $\zeta = 0.3$. By picking out a point inside the region on the graph, the solution for $h_{kg1} + x_1$ and $h_{kg1} + h_{kg2A}$ satisfies the conditions. Additionally, the value of ζ can be small as far as the available solution exists on the graph.

As the next step, the situation where the values of α_0 , m_n , z_1 , z_{2A} , ζ , $h_{kg1} + x_1$, and $h_{kg1} + h_{kg2A}$ are given is considered. As mentioned above, when $\zeta = x_{2A} - x_1$ is given, $x_{2B} - x_1$ is determined subordinately. If the values of h_{kg1} and h_{kg2B} are decided additionally to these values, the profile shift coefficients and addendum coefficients of all gears are determined. Here, a graph with the horizontal axis h_{kg1} and the vertical axis h_{kg2B} is proposed in order to visually show the solution region at this step. The conditions that constrain these variables are Eqs. (7.18), (7.20)–(7.22), (7.24), and (7.26), and then the solution region is given by describing these inequalities on the proposed graph. Fig. 7.5 shows the graph when $h_{kg1} + x_1 = -0.84$ and $h_{kg1} + h_{kg2A} = 0.96$ are selected from Fig. 7.4.

Once x_1 , x_{2A} , x_{2B} , h_{kg1} , h_{kg2A} , and h_{kg2B} are determined by using Figs. 5 and 6, the values of the root circle diameters d_{f1} , d_{f2A} , and d_{f2B} can be easily chosen, because Eqs. (7.28)–(7.30) give the conditions for these parameters explicitly.

From the above, the parameters of each gear satisfying all conditions clarified in Section 7.3 are determined, that is, the reducer is designed.

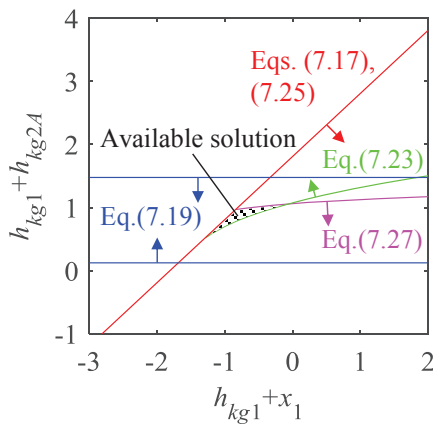


Fig. 7.4

Region for $h_{kg1} + x_1$ and $h_{kg1} + h_{kg2A}$ satisfying the conditions under a certain value of $x_{2A} - x_1$. The arrows indicate the direction in which each inequality is satisfied.

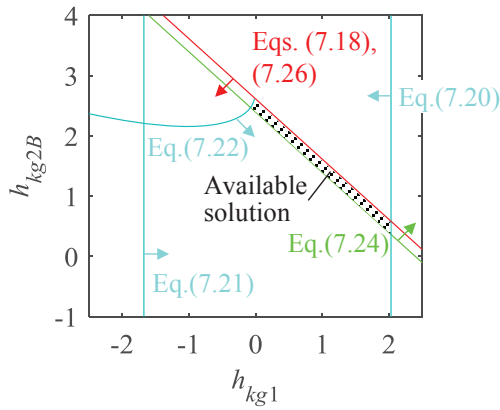


Fig. 7.5

Region for h_{kg1} and h_{kg2B} satisfying the conditions under a certain value of $x_{2A} - x_1, h_{kg1} + x_1, h_{kg1} + h_{kg2A}$. The arrows indicate the direction in which each inequality is satisfied.

7.4.2 Strength Analysis

In Section 7.4.1, the design method based on the geometrical conditions for the gears of the reducer is clarified. On the other hand, the strength of the gears is also an important indicator in the design. The gears used in the proposed motor especially tend to have special tooth shapes different from the shapes of regular gears, because of the conditions that the gear pairs with small and no differences in teeth number share the external gear. Therefore, the effect of the tooth shapes on the strength should be taken into consideration in the design. In this section, by examining the strength of the gears corresponding to the design solutions satisfying the geometrical conditions, the regions of the design solution that not only satisfy the geometrical conditions but also realize excellent strength are clarified.

A strength analysis is carried out with the K-H-V differential Gear Design System (AMTEC INC.). In this software, the tooth root bending stress σ_F is calculated based on the Japan Gear Manufacturers Association (JGMA) method (JGMA 401-01). The formula is as follows [85]:

$$\sigma_F = F_t \frac{Y_F Y_\epsilon Y_\beta}{m_n b_g} \left(\frac{K_V K_O}{K_L K_{FX}} \right) S_F \quad (7.31)$$

where F_t is the nominal circumferential force on the reference circle, Y_F is the tooth shape coefficient, Y_ϵ is the load distribution coefficient (the inverse of the contact ratio), Y_β is the helix angle coefficient, b_g is the facewidth, K_V is the dynamic load coefficient, K_O is the overload coefficient, K_L is the life coefficient, K_{FX} is the size coefficient to the tooth root stress, and S_F is the safety factor for tooth breakage. The analysis sets $Y_\beta = 1, K_V = 1, K_O = 1, K_L = 1, K_{FX} = 1, S_F = 1.2$. Other conditions are as follows: the torque applied to the external gear is 98 N·m, the

revolution speed is 100 min^{-1} , the facewidth of the external gear is 20 mm, the facewidth of the internal A gear is 10 mm, the total facewidth of the two internal B gears is 10 mm, the root circle diameter of the external gear is the maximum value in Eq. (7.28), and the root circle diameters of the internal A and B gears are the minimum values in Eqs. (7.29), (7.30), respectively. The material of the gears is assumed to be induction-hardened S45C steel with 263 HV as the core hardness and 660 HV as the surface hardness. The allowable bending stress is 245 MPa.

Not only the bending stress but also the contact stress, Hertzian stress, was calculated and taken into consideration (JGMA 402-01 [85]). As a result of the analysis, however, the bending stress always reached its maximum allowable value before the Hertzian stress did. Therefore, the bending strength of the teeth is focused on, with the aim to construct the design method.

Fig. 7.6 shows the result of the bending strength analysis for each gear corresponding to the solution region in Fig. 7.5. Fig. 7.6 indicates the ratio of the allowable bending stress to the tooth root bending stress, which becomes higher when the strength is higher. Note that the bending strength is calculated on each node of the region in Fig. 7.6, and the graphs are filled by linear interpolation among the nodes. The upper graphs in Fig. 7.6 show the strength of the engagement of the external gear and the internal A gear. The strength of the external gear becomes higher when h_{kg1} becomes smaller. This could be because the arm length of the moment applied to the tooth root by the force on the tooth tip becomes short when the addendum is small, and then the bending stress becomes small. The strength of the internal A gear is constant over the entire region. The reason is that the bending stress on the internal A gear becomes constant under the condition in

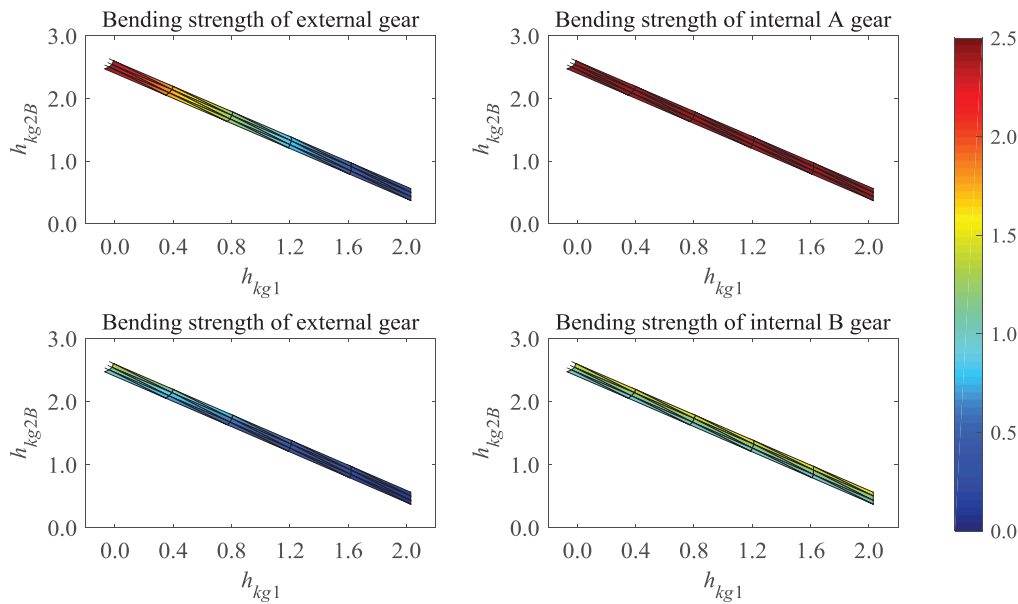


Fig. 7.6

Results of strength analysis on bending strength of the external gear and internal gears in Fig. 7.5. Upper graphs show the result of the engagement between the external gear and internal A gear. Lower graphs show the result of the engagement between the external gear and internal B gear.

Fig. 7.5 that the values of α_0 , m_n , z_1 , z_{2A} , ζ , $h_{kg1} + x_1$, and $h_{kg1} + h_{kg2A}$ are determined. For the result of the engagement of the external gear and the internal B gear in the lower graphs in Fig. 7.6, the strength of the internal B gears becomes higher in the upper-right region. This distribution is similar to that of the contact ratio given by Eq. (7.24), possibly because the bending stress is distributed on some teeth of the internal B gears under the higher contact ratio. The strength of the external gear becomes higher in the upper region. The following two points are thought as the reason: the moment on the tooth root becomes small in the left region because the addendum gets smaller, as in the engagement with the internal A gear, and the loads are distributed in the upper-right region with the increase of the contact ratio, as in the internal B gears. The analyses with other conditions give results similar to those in Fig. 7.6. From the above, in the design using Fig. 7.5, a point should be chosen from the upper-left of the solution region, especially from the point where h_{kg1} is small when the strength of the gear pairs including the internal A gear is prior, or from the point where h_{kg2B} is large when the strength of the gear pairs including the internal B gears is prior.

Next, the analysis result of the solution region in Fig. 7.4 is shown in Fig. 7.7. In the analysis, it is assumed that the point where h_{kg2B} becomes maximum is selected at the step of the design using Fig. 7.5. Fig. 7.7 shows that the strength of the external gear, and the internal A and B gears becomes higher in the upper-left region. This distribution is the same as that of the contact ratio given by Eq. (7.23), and so the load distribution in the higher contact ratio could contribute to increasing the strength. Under other conditions, the results are similar. Therefore, in the design

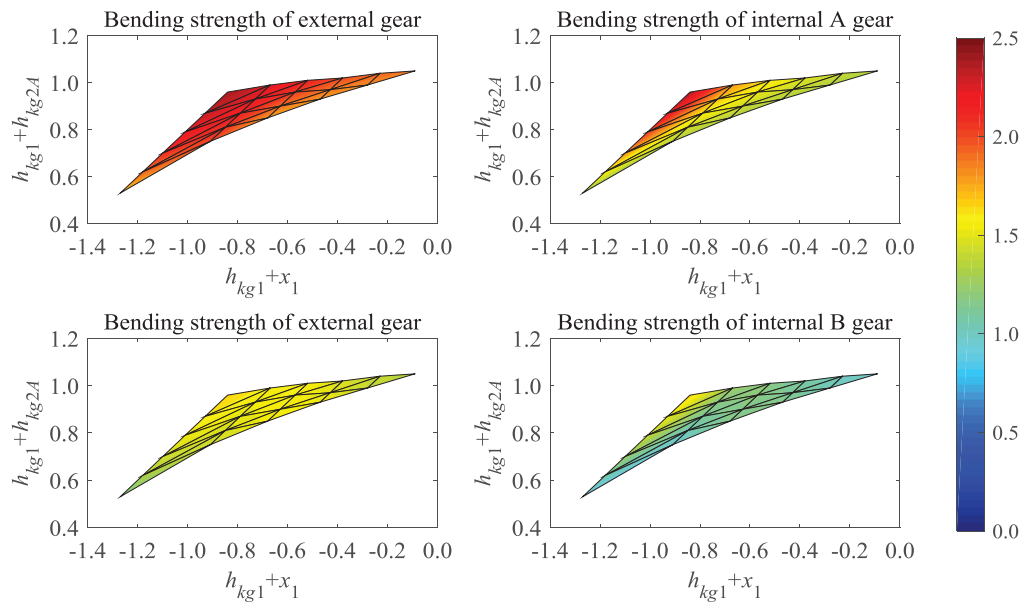


Fig. 7.7

Results of strength analysis on bending strength of the external gear and internal gears in Fig. 7.4. Upper graphs show the result of the engagement between the external gear and internal A gear. Lower graphs show the result of the engagement between the external gear and internal B gear.

using Fig. 7.4, the point for the highest contact ratio in the upper left of the solution region satisfying the geometrical conditions should be picked out.

In the way explained above, the best solution for the strength can be given in the design using Fig. 7.4 and Fig. 7.5.

7.4.3 Design Algorithm

Based on the discussions in Sections 7.4.1 and 7.4.2, a design process for the gear pairs used in the proposed reducer-integrated motor is next organized. Fig. 7.8 shows the flowchart of the entire design algorithm. First, the module m_n , the reference pressure angle α_0 , and the teeth numbers z_1 and z_{2A} are determined according to the requirements for the size of the motor or the reduction ratio. Then, the initial value of $\zeta = x_{2A} - x_1$ is set as 0 and draw the graph for the conditions using $h_{kg1} + x_1$ and $h_{kg1} + h_{kg2A}$ as variables, as shown in Fig. 7.4. We check whether the solution satisfying the conditions exists in this graph. If no solution exists, we increase the value of ζ and redraw the graph. After obtaining the solution region satisfying the conditions, we choose the upper-left point as the solution for $h_{kg1} + x_1$ and $h_{kg1} + h_{kg2A}$. By using these values, we draw the graph for the conditions using h_{kg1} and h_{kg2B} as variables, as in Fig. 7.5. Similarly to the previous step, we again increase ζ and select parameters if the solution for the conditions is not given. After obtaining the solution region satisfying the conditions, we pick out a point from the upper-left region as the solution for h_{kg1} and h_{kg2B} . Up to this point, six parameters of $x_{2A} - x_1$, $x_{2B} - x_1$, $h_{kg1} + x_1$, $h_{kg1} + h_{kg2A}$, h_{kg1} , and h_{kg2B} are determined. By substituting each parameter, the profile shift coefficients x_1 , x_{2A} , and x_{2B} , and the addendum coefficients h_{kg1} , h_{kg2A} , and h_{kg2B} are given. From Eqs. (7.28)–(7.30) assigning the given values, we determine the root circle diameters d_{f1} , d_{f2A} , and d_{f2B} . Thus, the variables concerning the external gear, the internal A and B gears satisfying the all conditions are decided. Next, the strength analysis on each selected gear pair is carried out by providing the condition of use, such as the effective facewidth and load torque. Here, we check whether the tooth root bending stress is smaller than the allowable bending stress in each gear; if the strength condition is not satisfied, we again increase ζ and repeat the above steps. From the above process, the design of the gear pairs for the reducer of the proposed motor is completed. However, in the loop of calculations with increasing ζ , it is impossible to increase ζ unlimitedly. Therefore, if the conditions on the selection are not satisfied even with the maximum ζ , no feasible gear pair exists with the designated module m_n , reference pressure angle α_0 , and teeth numbers z_1 and z_{2A} . In this case, it is necessary to reconsider the requirements given first (e.g., to increase m_n).

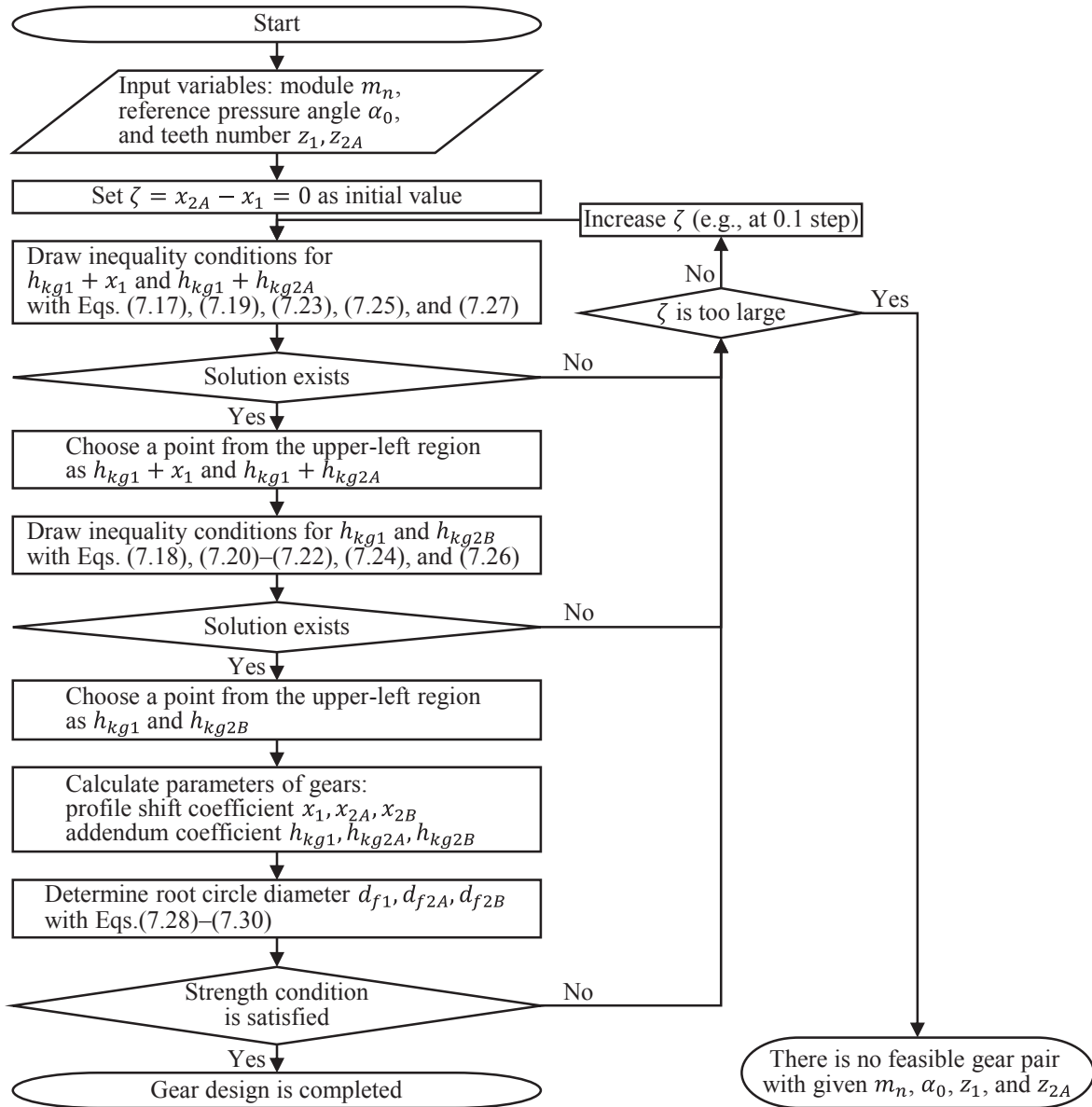


Fig. 7.8

Flow chart for designing the gears of the reducer used in the proposed reducer-integrated motor

7.5 Experiments

In order to show the proposed reducer-integrated motor performs as expected, experiments are conducted using the prototype.

7.5.1 Proposed Motor Prototype

The overview of the prototype is shown in Fig. 7.9. In fact, the motor developed in Chapter 6 is used as the fundamental structure of this motor. Then, twelve linear solenoids are installed, whose pushing force is transmitted to the external gear through the connecting parts and pushing parts as described in Fig. 6.13. The specifications of the produced gears are shown in Table 7.1. These gears are designed according to the method proposed in Section 7.4 and satisfy all the conditions clarified in Section 7.3. Fig. 7.10 shows the tooth shapes of the gears calculated by the abovementioned software with the chosen parameters. The gears are made of NAK55 steel and processed by wire cut.

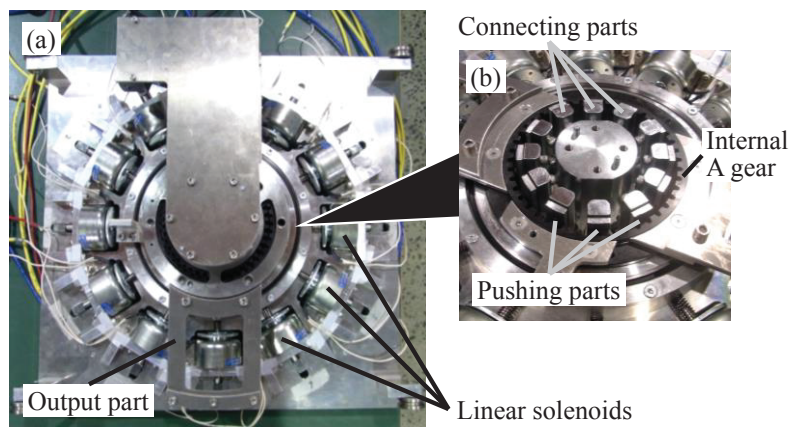


Fig. 7.9

Overview of the proposed reducer-integrated motor prototype. (a) Overview. (b) Internal structure, in which the internal A gear is exposed by removing one of the two internal B gears and the external gear.

Table 7.1

Specifications of the gears used in the proposed reducer-integrated motor prototype.

	External gear	Internal A gear	Internal B gear
Module	3		
Reference pressure angle (°)	20		
Teeth number	43	44	43
Profile shift coefficient	-1.21	-0.91	0.7687
Addendum coefficient	0.60	0.35	2.0
Root circle diameter (mm)	115.14	132	132
Facewidth (mm)	52	26	12.5 × 2
Inner diameter (mm)	110.4	-	-
Outer diameter (mm)	-	170	170

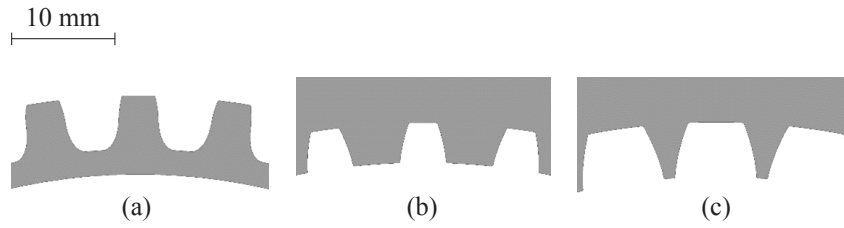


Fig. 7.10

Calculated result of the tooth shapes of the gears for the experiment. (a) External gear. (b) Internal A gear. (c) Internal B gear.

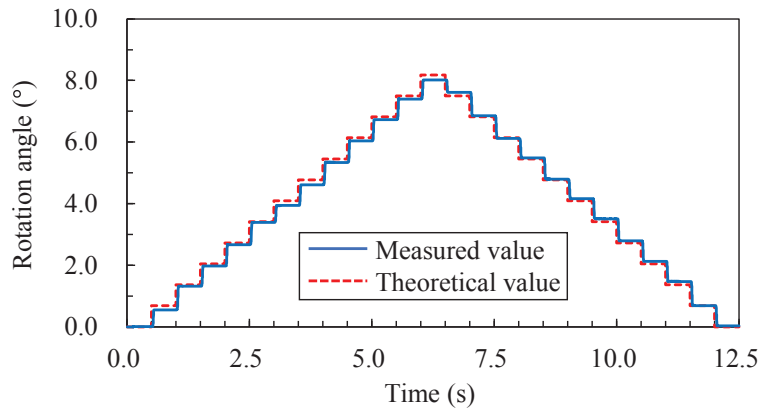


Fig. 7.11

Experimental result of output rotation angle over time.

7.5.2 Experimental Results and Discussion

In the experiment, the linear solenoids are operated in the same way as in Fig. 7.3 and the internal A gear is rotated. The switching of the operating linear solenoids makes one cycle in the forward direction (clockwise) and one cycle in the reverse direction at 500-ms intervals per step. By substituting $n_a = 12$, $R_s = 44$, and $T_a = 0.5$ s in Eq. (6.2), the theoretical value of the average rotational speed $\bar{N}$ is given as $\bar{N} \approx 3.8 \times 10^{-3} \text{ s}^{-1}$. According to visual observation of the movement of the output part, it was verified that the motor could rotate without any problem.

Fig. 7.11 shows the measurement result of the rotation angle of the output part. In the figure, the blue solid line indicates the measured rotation angle and the red dashed line indicates the theoretical rotation angle. Note that the results are offset so that the theoretical and measured values match at the initial time. The graph shows that the measured values nearly agree with the theoretical values for the entire time. This result verifies that the proposed reducer-integrated motor works as expected, and that the design method for the gears of the reducer proposed in Section 7.4 is effective fundamentally.

By taking a closer look at the graph in Fig. 7.11, small differences are seen between the theoretical and measured values. These differences are discussed by using the step angle errors.

The step angle error is the difference between the theoretical and measured values in the step angle, that is, in the rotation angle of the internal A gear at each step. The theoretical step angle is $360^\circ/n_a R_s \approx 0.682^\circ$. The same experiments shown in Fig. 7.11 were repeated ten times. Fig. 7.12 shows a box plot of the result of the step angle errors at each step. In this figure, the maximum length of the whiskers is restricted to 1.5 times the length of the boxes, and the data outside the regions are treated as the outliers, designated by + markers. The upper and lower fences show the maximum and minimum values of the data except for the outliers. The horizontal axis indicates the step of switching the operating linear solenoids. As in Fig. 7.11, the former 12 steps are in the forward rotation, and the latter 12 steps are in the reverse rotation.

The result shows that even though errors of outliers exist, the distributions of the errors in a certain step between the trials are smaller than the distributions of the errors in a certain trial between the steps. This means that almost the same errors occurred in ten trials of the experiments. This result suggests that the errors in the experiments are mainly caused by stationary factors, such as manufacturing errors or assembling errors. Additionally, in these experiments, the pair of two operating linear solenoids are the same at the i -th step in the forward rotation and the $(26 - i)$ -th step in the reverse rotation ($i = 1, 2, \dots, 12$). However, the distribution of the step angle errors in Fig. 7.12 does not show the correlation between the errors in the forward and reverse rotations, and so the errors might be caused by gear shape errors or pitch errors of the manufactured gears. These errors could be suppressed by confirming the precision of the processed gears.

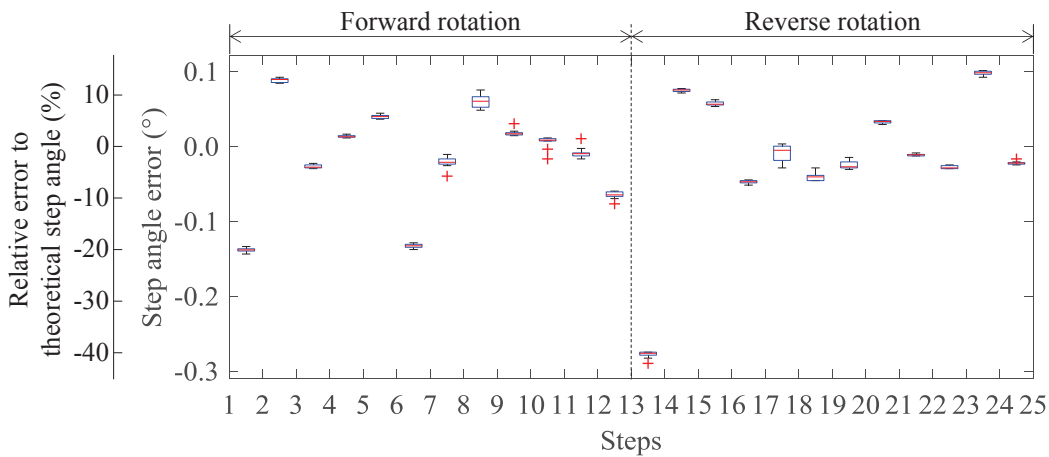


Fig. 7.12
Step angle errors in the experimental result.

7.5.3 Evaluation of the Load-Supporting Capacity

Based on the strength analysis procedure used in Section 7.4.2, the load-supporting capacity of the proposed motor is numerically evaluated in this section. The parameters of the analyzed gear pairs

are the same as those of the produced gear pairs shown in Table 7.1, but their material is assumed to be induction-hardened S45C instead of NAK55 steel because sufficient data on NAK55 steel was not obtained. The result of the analysis showed that the weakest point was in the engagement between the external gear and the internal B gear, and the maximum supportable torque is 457 N·m in terms of gear strength. In the proposed motor, the linear solenoids scarcely receive the circumferential force, so that the load-supporting capacity is thought to be determined by the strength of the gears. Therefore, the load-supporting capacity of the developed motor, i.e., the maximum supportable load torque applied to the output part, is calculated as $457 \times z_{2A}/z_1 \approx 467$ N·m.

If the linear solenoids need to support such load torque like the conventional reducer-integrated motors, the circumferential force applied to each linear solenoid is estimated to be $457/(12 \times d_{f1}/2000) \approx 662$ N. This is almost as large as the pushing force of the linear solenoids. Considering that linear solenoids are originally not designed to support lateral force, the estimated circumferential force should be beyond the capacity of the linear solenoids. In this way, the structure of the proposed motor leads to higher load-supporting capacity than that of the conventional reducer-integrated motors.

7.6 Summary

There is a concern that the reducer-integrated motor proposed in Chapter 6 has insufficient design flexibility because of the specialized gear in its reducer part. To solve this problem, a new reducer-integrated motor, in which two kinds of conventional gear pairs engage simultaneously, is proposed. The analyses and experiments were conducted, and the following results were obtained.

- A new reducer-integrated motor established not only downsizing, high load-supporting capacity, and high stiffness, but also high design flexibility due to the rigid gears easy to produce.
- The geometrical conditions for the gear pairs with small and no differences in teeth number sharing the external gear were clarified. The design method was proposed to acquire design solutions that satisfy the conditions from the graphs.
- The strength of the gears corresponding to the design solutions was calculated and graphed. Solution regions realizing excellent strength were derived.
- Based on the discussions on the geometrical conditions and the strength, a design algorithm for the gears of the reducer was constructed.

- The experiments were conducted by using the prototype of the proposed reducer-integrated motor. The results verified that the prototype worked as expected and that the proposed design method was effective fundamentally.

Chapter 8

Conclusions and Future Work

8.1 Conclusions

Locomotion devices with wheeled mechanisms are widely used due to high energy efficiency, high-speed runnability, and high stability resulting from simple structure and motion. In order to make their movement more efficient in time and space, omnidirectional mobility is now required. This thesis approached this problem by synthesizing a novel wheeled mechanism that brings omnidirectional mobility and introducing the technologies for another omnidirectional wheeled mechanism to get advanced in its structure and transmission mechanism.

Chapter 2 and 3 focused on the development of an unprecedented wheeled mechanism. First, through the analysis of the existing wheeled mechanisms, generalized kinematic conditions for omnidirectional wheeled mechanisms were derived. Based on the conditions, a novel omnidirectional wheeled mechanism using a passive sliding joint, a slidable wheel, was synthesized. The slidable wheel has simple structure thanks to a normal wheel and avoids the overconstraint-like situation that other mechanisms have encountered. Then, a mobile robot using three slidable wheels, which was named SWOM, was proposed. The kinematic analyses of SWOM were conducted and its characteristics necessary to the movement were clarified, such as the input-output relation, reachable region, and trajectory generation strategy. The research proceeded to construct the feedback control system of SWOM for further practicality. To handle its nonlinear system with nonholonomic constraints, feedback linearization method was adapted and a controller that stabilizes SWOM system was designed in taking account of unwanted singular configurations. Through the simulations and experiments, the effectiveness of the proposed mechanisms and theories was verified.

In Chapter 4, the possibilities of another wheeled mechanism to accomplish the objective were discussed. Here, the AOW was focused on, which is the omnidirectional wheel proposed in the past, and it was tried to improve the usefulness in suppressing vertical vibration and reducing the number of components. As a result, a new AOW design possessing a seamless periphery and simplified structure was developed. Through the analysis, the geometric condition of the AOW design and the characteristics of its kinematic behavior were clarified. Additionally, an omnidirectional 2WD vehicle using the proposed AOW was constructed. The prototype of the

AOW and the omnidirectional vehicle was developed and the effectiveness in its design objective was proved.

Chapter 5 tackled the reduction in the number of motors to drive the AOW because the omnidirectional mobile vehicles usually use a large number of motors. Then, the problem was defined as consistency with fewer motors and higher mobility, and proposed a transmission mechanism to meet both requirements by driving AOWs with switchable transmission paths. This mechanism allows 2WD vehicles with the AOW to move in both the longitudinal and lateral direction, as well as to turn, by using only one or two motors. The kinematic analysis of the vehicle with the proposed transmission mechanism brought the input-output relation and the establishment condition. Then, the 2WD vehicle was developed and it is confirmed that the prototype performed as expected in the experiments.

The most important elements of wheeled locomotion devices besides wheels will be motors. Actually, motors are now widely used not only in the wheeled locomotion devices but also in every kind of machine. Such motors need high precision, large torque, and small size. The conventional reducers can fulfill the high-precision positioning and large torque output, but make the whole motor system large in contrast with the requirement. Then, small-sized motors are awaited. This thesis addressed this problem by considering the integrated motor mechanism.

Chapter 6 and 7 were spent to develop reducer-integrated motors for the purpose of miniaturizing the whole motor system. The proposed motor has its actuators inside the reducer and outputs the reduced rotation directly, so that downsizing of the motor system is achieved. At the same time, simultaneous engagement of three gear pairs contributes to increasing the load-supporting capacity and stiffness. First, the reducer-integrated motor using the principle of a strain wave gearing system was developed. The tooth tip interference in the deformation of the gear is analyzed and the condition for the actuator arrangement to avoid the interference is derived. After that, another reducer-integrated motor is developed, which utilized the principle of an inscribed planetary gear mechanism, for the sake of higher design flexibility. The geometrical conditions for the tooth engagement were clarified and tooth strength analysis was carried out. Based on these discussions, a design algorithm for the gears of the proposed motor was constructed. The experiments were conducted by using each of the proposed motor prototypes. From the qualitative and quantitative evaluation, it was verified that the prototype worked as expected.

For a summary of this thesis, this research proposed novel mechanisms, such as the wheeled mechanism, transmission mechanism, and driving mechanism, to solve the limited mobility of the locomotion devices and unsuppressed size of the motors, designed these mechanisms based on the kinematic analyses, and evaluated their effectiveness through the experiments. The mechanisms brought by this work will be useful for the vehicles and robots working in the real world.

8.2 Future Work

In future research, the following perspectives should be explored:

- With respect to the slidable wheel and SWOM, a more robust and efficient controller should be constructed. In the experimental results, some deviation from the simulation was observed, whose main factor was considered to be dynamical disturbances, such as wheel slippage, sliding joint friction, drive mechanism viscosity, and changes in load distribution. A disturbance observer or a dynamics-based controller can address these factors and achieve a more stable system. Additionally, SWOM needs to balance both the movement of the main body and the relative movements of the sliding joints in control; if the balance is broken, the stability or efficiency in the movement will be lost. Regarding this issue, a feedback controller based on inequality constraints, such as a potential method, can be used.
- In the new AOW, it is planned to analyze and evaluate slippage between the transmission roller and outer roller for the sake of practicality, because the AOW utilizes friction drive mechanism. The AOWs, as well as the omni wheels, originally have anisotropy in the radii of the main wheel and outer rollers. It may affect the step-climbing capability, which should also be discussed considering the characteristics of the proposed mechanism. The proposed transmission mechanism with fewer motors actually worked as expected, but there remained a potential to make the vehicle even smaller. It is also planned to conduct experimental investigations such as the measurement of errors in following target paths under more practical conditions and evaluation of operability in the living environment.
- Regarding the reducer-integrated motor, further downsizing will be carried out so that the linear actuators are installed inside the F/S as sketched by using appropriate-sized linear solenoids. At the same time, a position/force control system for the linear actuators needs implementing, which may enable the motor to rotate not only discretely but also continuously. This will lead to an increase in the efficiency and usability of the proposed motor.

It is also important to generalize the research results; to construct the design methodology for the locomotion mechanism according to the purpose of use from the viewpoint of the industrial world, and also to arrange the rules to operate the novel mobility devices for society.

References

- [1] R. E. Andersen et al., “Integration of a Skill-Based Collaborative Mobile Robot in a Smart Cyber-Physical Environment,” *Procedia Manuf.*, vol. 11, pp. 114–123, 2017.
- [2] E. Cardarelli, V. Digani, L. Sabattini, C. Secchi, and C. Fantuzzi, “Cooperative Cloud Robotics Architecture for the Coordination of Multi-AGV Systems in Industrial Warehouses,” *Mechatronics*, vol. 45, pp. 1–13, 2017.
- [3] I. Nielsen, Q. D. Grzegorz, B. Zbigniew, and Z. Banaszak, “A Methodology for Implementation of Mobile Robot in Adaptive Manufacturing Environments,” *J. Intell. Manuf.*, vol. 28, pp. 1171–1188, 2017.
- [4] H. Masuzawa, J. Miura, and S. Oishi, “Development of a Mobile Robot for Harvest Support in Greenhouse Horticulture – Person Following and Mapping –,” in *Proceedings of IEEE/SICE International Symposium on System Integration*, 2017, pp. 541–546.
- [5] H. Sori, H. Inoue, H. Hatta, and Y. Ando, “Effect for a Paddy Weeding Robot in Wet Rice Culture,” *J. Robot. Mechatronics*, vol. 30, no. 2, pp. 198–205, 2018.
- [6] K. Tanaka et al., “A Study on Path Planning for Small Mobile Robot to Move in Forest Area,” in *Proceedings of IEEE International Conference on Robotics and Biomimetics*, 2017, pp. 2167–2172.
- [7] R. Wang and Y. Kawamura, “An Automated Sensing System for Steel Bridge Inspection Using GMR Sensor Array and Magnetic Wheels of Climbing Robot,” *J. Sensors*, vol. 2016, p. 8121678, 2016.
- [8] R. V. Espinoza, A. S. de Oliveira, L. V. R. de Arruda, and F. Neves Junior, “Navigation’s Stabilization System of a Magnetic Adherence-Based Climbing Robot,” *J. Intell. Robot. Syst.*, vol. 78, no. 1, pp. 65–81, 2015.
- [9] K. Iizuka, T. Sasaki, H. Hama, A. Nishitani, T. Kubota, and I. Nakatani, “Development of a Small, Lightweight Rover with Elastic Wheels for Lunar Exploration,” *J. Robot. Mechatronics*, vol. 24, no. 6, pp. 1031–1039, 2012.
- [10] H. Gross et al., “ROREAS : Robot Coach for Walking and Orientation Training in Clinical Post-Stroke Rehabilitation — Prototype Implementation and Evaluation in Field Trials,” *Auton. Robots*, vol. 41, no. 3, pp. 679–698, 2017.
- [11] S. Saegusa, Y. Yasuda, Y. Uratani, E. Tanaka, T. Makino, and J.-Y. Chang, “Development of a Guide-Dog Robot : Leading and Recognizing a Visually-Handicapped Person using a LRF,” *J. Adv. Mech. Des. Syst. Manuf.*, vol. 4, no. 1, pp. 194–205, 2010.
- [12] K. Tobita, K. Sagayama, M. Mori, and A. Tabuchi, “Structure and Examination of the Guidance Robot LIGHBOT for Visually Impaired and Elderly People,” *J. Robot. Mechatronics*, vol. 30, no. 1, pp. 86–92, 2018.

- [13] D. Fischinger *et al.*, “Hobbit, a Care Robot Supporting Independent Living at Home: First Prototype and Lessons Learned,” *Rob. Auton. Syst.*, vol. 75, pp. 60–78, 2016.
- [14] K. Hashimoto, F. Saito, T. Yamamoto, and K. Ikeda, “A Field Study of the Human Support Robot in the Home Environment,” in *Proceedings of IEEE Workshop on Advanced Robotics and its Social Impacts*, 2013, pp. 143–150.
- [15] D. Portugal, L. Santos, P. Alvito, J. Dias, G. Samaras, and E. Christodoulou, “SocialRobot: An Interactive Mobile Robot for Elderly Home Care,” in *Proceedings of IEEE/SICE International Symposium on System Integration*, 2015, pp. 811–816.
- [16] J. E. Bares and D. S. Wettergreen, “Dante II: Technical Description, Results, and Lessons Learned,” *Int. J. Rob. Res.*, vol. 18, no. 7, pp. 621–649, 1999.
- [17] M. Raibert, K. Blankespoor, G. Nelson, and R. Playter, “BigDog, the Rough-Terrain Quadruped Robot,” in *Proceedings of the 17th World Congress The International Federation of Automatic Control*, 2008, pp. 10822–10825.
- [18] K. Hashimoto *et al.*, “WAREC-1 - A Four-Limbed Robot Having High Locomotion Ability with Versatility in Locomotion Styles,” in *Proceedings of IEEE International Symposium on Safety, Security and Rescue Robotics*, 2017, pp. 172–178.
- [19] T. T. Topping, G. Kenneally, and D. E. Koditschek, “Quasi-Static and Dynamic Mismatch for Door Opening and Stair Climbing With a Legged Robot,” in *Proceedings of IEEE International Conference on Robotics and Automation*, 2017, pp. 1080–1087.
- [20] R. K. Das, A. Upadhyay, and R. K. Garg, “An Unmanned Tracked Vehicle for Snow Research Applications,” *Def. Sci. J.*, vol. 67, no. 1, pp. 74–79, 2017.
- [21] W. Wang, Z. Yan, and Z. Du, “Experimental Study of a Tracked Mobile Robot’s Mobility Performance,” *J. Terramechanics*, vol. 77, pp. 75–84, 2018.
- [22] M. Kamezaki *et al.*, “Design of Four-Arm Four-Crawler Disaster Response Robot OCTOPUS,” in *Proceedings of IEEE International Conference on Robotics and Automation*, 2016, pp. 2840–2845.
- [23] H. Miyanaka *et al.*, “Development of an Unit Type Robot ‘KOHGA2’ with Stuck Avoidance Ability,” in *Proceedings of IEEE International Conference on Robotics and Automation*, 2007, pp. 3877–3882.
- [24] K. Nagatani *et al.*, “Redesign of Rescue Mobile Robot Quince –Toward Emergency Response to the Nuclear Accident at Fukushima Daiichi Nuclear Power Station on March 2011–,” in *Proceedings of IEEE International Symposium on Safety, Security and Rescue Robotics*, 2011, pp. 13–18.
- [25] K. Miyasaka, G. Kawano, and H. Tsukagoshi, “Long-Mover: Flexible Tube In-pipe Inspection Robot for Long Distance and Complex Piping,” in *Proceedings of IEEE/ASME International Conference on Advanced Intelligent Mechatronics*, 2018, pp. 1075–1080.
- [26] D. Rollinson and H. Choset, “Pipe Network Locomotion with a Snake Robot,” *J. F. Robot.*, vol. 33, no. 3, pp. 322–336, 2016.

- [27] A. Ferworn, C. Wright, J. Tran, C. Li, and H. Choset, "Dog and Snake Marsupial Cooperation for Urban Search and Rescue Deployment," in *Proceedings of IEEE International Symposium on Safety, Security and Rescue Robotics*, 2012, pp. 1–5.
- [28] J. Whitman, N. Zevallos, M. Travers, and H. Choset, "Snake Robot Urban Search After the 2017 Mexico City Earthquake," in *Proceedings of IEEE International Symposium on Safety, Security, and Rescue Robotics*, 2018, pp. 1–6.
- [29] S. Hirose and H. Takeuchi, "Study on Roller-Walk (Basic Characteristics and its Control)," in *Proceedings of IEEE International Conference on Robotics and Automation*, 1996, pp. 3265–3270.
- [30] K. Tadakuma *et al.*, "Mechanical Design of the Wheel-Leg Hybrid Mobile Robot to Realize a Large Wheel Diameter," in *Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems*, 2010, pp. 3358–3365.
- [31] F. Zhou, X. Xu, H. Xu, and X. Zhang, "A Multimodal Hybrid Robot with Transformable Wheels," in *Proceedings of IEEE International Conference on Real-time Computing and Robotics*, 2017, pp. 139–144.
- [32] K. Nagatani, H. Kinoshita, K. Yoshida, K. Tadakuma, and E. Koyanagi, "Development of Leg-Track Hybrid Locomotion to Traverse Loose Slopes and Irregular Terrain," *J. F. Robot.*, vol. 28, no. 6, pp. 950–960, 2011.
- [33] F. Michaud *et al.*, "Multi-modal Locomotion Robotic Platform Using Leg-Track-Wheel Articulations," *Auton. Robots*, vol. 18, pp. 137–156, 2005.
- [34] L. Bruzzone and G. Quaglia, "Review Article: Locomotion Systems for Ground Mobile Robots in Unstructured Environments," *Mech. Sci.*, vol. 3, pp. 49–62, 2012.
- [35] J. C. L. Barreto S., A. G. S. Conceicao, C. E. T. Dorea, L. Martinez, and E. R. de Pieri, "Design and Implementation of Model-Predictive Control with Friction Compensation on an Omnidirectional Mobile Robot," *IEEE/ASME Trans. Mechatronics*, vol. 19, no. 2, pp. 467–476, 2014.
- [36] G. Campion and G. Bastin, "On Adaptive Linearizing Control of Omnidirectional Mobile Robots," in *Robust Control of Linear Systems and Nonlinear Control*, Birkhäuser Boston, 1990, pp. 531–538.
- [37] G. Indiveri, "Swedish Wheeled Omnidirectional Mobile robots: Kinematics Analysis and Control," *IEEE Trans. Robot.*, vol. 25, no. 1, pp. 164–171, 2009.
- [38] J. Santos, A. Conceição, T. Santos, and H. Araújo, "Remote Control of an Omnidirectional Mobile Robot with Time-Varying Delay and Noise Attenuation," *Mechatronics*, vol. 52, pp. 7–21, 2018.
- [39] S. Ok, A. Kodama, Y. Matsumura, and Y. Nakamura, "SO(2) and SO(3), Omni-directional Personal Mobility with Link-Driven Spherical Wheels," in *Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems*, 2011, pp. 268–273.
- [40] S. Ostrovskaya, J. Angeles, and R. Spiteri, "Dynamics of a Mobile Robot with Three Ball-Wheels," *Int. J. Rob. Res.*, vol. 19, no. 4, pp. 383–394, 2000.

- [41] M. Wada and H. Asada, "Design and Control of a Variable Footprint Mechanism for Holonomic Omnidirectional Vehicles and Its Application to Wheelchairs," *IEEE Trans. Robot. Autom.*, vol. 15, no. 6, pp. 978–989, 1999.
- [42] M. West and H. Asada, "Design and Control of Ball Wheel Omnidirectional Vehicles," in *Proceedings of IEEE International Conference on Robotics and Automation*, 1995, pp. 1931–1938.
- [43] R. Holmberg and O. Khatib, "Development and Control of a Holonomic Mobile Robot for Mobile Manipulation Tasks," *Int. J. Rob. Res.*, vol. 19, no. 11, pp. 1066–1074, 1999.
- [44] W. Jia, G. Yang, C. Zhang, and G. Chen, "Dynamic Modeling with Nonsqueezing Load Distribution for Omnidirectional Mobile Robots with Powered Caster Wheels," in *Proceedings of the 13th IEEE Conference on Industrial Electronics and Applications*, 2018, pp. 2327–2332.
- [45] J. Ma, H. Kharboutly, A. Benali, F. Ben Amar, and M. Bouzit, "Design of Omnidirectional Mobile Platform for Balance Analysis," *IEEE/ASME Trans. Mechatronics*, vol. 19, no. 6, pp. 1872–1881, 2014.
- [46] M. Wada, Y. Tominaga, and S. Mori, "Omnidirectional Holonomic Mobile Robot Using Nonholonomic Wheels," *Proc. IEEE/RSJ Int. Conf. Intell. Robot. Syst.*, vol. 3, pp. 446–453, 1995.
- [47] C. W. Musser, "Strain wave gearing," U.S. Patent 2 906 143, Sept. 29, 1959.
- [48] Kinki Gear Association, Ed., *Design and Manufacturing of Gears (I)*. TAIGA Publishing, 1971.
- [49] D. Yu, "KHV Planetary Gearing," *Gear Technol.*, vol. November/D, pp. 21–48, 1987.
- [50] D. Yu, "KHV Planetary Gearing - Part II," *Gear Technol.*, vol. January/Fe, pp. 28–48, 1988.
- [51] M. Morozumi, *The Theory and Design Calculation Method of Planetary Gears and Differential Gears*. The Nikkan Kogyo Shimbun, 1989.
- [52] H. Terada, "The Development of Gearless Reducers with Rolling Balls," *J. Mech. Sci. Technol.*, vol. 24, pp. 189–195, 2010.
- [53] I. Pantic and M. Blagojevic, "Kinematic Analysis of Single-Stage Cycloidal Speed Reducer," *Mach. Des.*, vol. 7, no. 4, pp. 113–118, 2015.
- [54] S. Guo, Y. Jin, S. Bao, and F.-F. Xi, "Accuracy Analysis of Omnidirectional Mobile Manipulator With Mecanum Wheels," *Adv. Manuf.*, vol. 4, no. 4, pp. 363–370, 2016.
- [55] P. F. Muir and C. P. Neuman, "Kinematic Modeling for Feedback Control of an Omnidirectional Wheeled Mobile Robot," in *Proceedings of IEEE International Conference on Robotics and Automation*, 1987, pp. 1772–1778.
- [56] S. K. Saha, J. Angeles, and J. Darcovich, "The Design of Kinematically Isotropic Rolling Robots with Omnidirectional Wheels," *Mech. Mach. Theory*, vol. 30, no. 8, pp. 1127–1137, 1995.
- [57] N. Yamada, H. Komura, G. Endo, H. Nabae, and K. Suzumori, "Spiral Mecanum Wheel Achieving Omnidirectional Locomotion in Step-Climbing," in *Proceedings of IEEE International Conference on Advanced Intelligent Mechatronics*, 2017, pp. 1285–1290.

- [58] G. Mouriaux, C. Novales, G. Poisson, and P. Vieyres, "Omni-directional Robot with Spherical Orthogonal Wheels: Concepts and Analyses," in *Proceedings of IEEE International Conference on Robotics and Automation*, 2006, pp. 3374–3379.
- [59] C. Ren and S. Ma, "Generalized Proportional Integral Observer Based Control of an Omnidirectional Mobile Robot," *Mechatronics*, vol. 26, pp. 36–44, 2015.
- [60] X. Wang, W. Cui, X. Xu, and C. Ye, "Research on an Omni-directional AGV with Differential Wheels," in *Proceedings of IEEE International Conference on Mechatronics and Automation*, 2016, pp. 1566–1571.
- [61] S. Yu, C. Ye, H. Liu, and J. Chen, "Development of an Omnidirectional Automated Guided Vehicle with MY3 Wheels," *Perspect. Sci.*, vol. 7, pp. 364–368, 2016.
- [62] M. Komori, K. Matsuda, T. Terakawa, F. Takeoka, H. Nishihara, and H. Ohashi, "Active Omni Wheel Capable of Active Motion in Arbitrary Direction and Omnidirectional Vehicle," *J. Adv. Mech. Des. Syst. Manuf.*, vol. 10, no. 6, p. JAMDSM0086, 2016.
- [63] K. Tadakuma, R. Tadakuma, and J. Berengueres, "Tetrahedral Mobile Robot with Spherical Omnidirectional Wheel," *J. Robot. Mechatronics*, vol. 20, no. 1, pp. 125–134, 2008.
- [64] D. Oetomo and M. H. Ang Jr., "Singularity-Free Joint Actuation in Omnidirectional Mobile Platforms With Powered Offset Caster Wheels," *J. Mech. Des.*, vol. 130, no. 5, p. 054501, 2008.
- [65] S. Ishida and H. Miyamoto, "Collision-Detecting Device for Omnidirectional Electric Wheelchair," *ISRN Robot.*, vol. 2013, 2013.
- [66] M. Wada, "A Synchro-caster Drive System for Holonomic and Omnidirectional Mobile Robots," in *Proceedings of the 26th Annual Conference of the IEEE Industrial Electronics Society.*, 2000, pp. 1937–1942.
- [67] Y. Inoue, T. Hiram, and M. Wada, "Design of Omnidirectional Mobile Robots with ACROBAT Wheel Mechanisms," in *Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems*, 2013, pp. 4852–4859.
- [68] G. Campion, G. Bastin, and B. D'andrea-Novel, "Structural Properties and Classification of Kinematic and Dynamic Models of Wheeled Mobile Robots," *IEEE Trans. Robot. Autom.*, vol. 12, no. 1, pp. 47–62, 1996.
- [69] R. Siegwart, I. R. Nourbakhsh, and D. Scaramuzza, *Introduction to Autonomous Mobile Robots*, 2nd ed. Cambridge, MA, USA: MIT Press, 2011.
- [70] B. d'Andrea-Novel, G. Bastin, and G. Campion, "Dynamic Feedback Linearization of Nonholonomic Wheeled Mobile Robots," in *Proceedings of the 1992 IEEE International Conference on Robotics and Automation*, 1992, pp. 2527–2532.
- [71] B. d'Andrea-Novel, G. Campion, and G. Bastin, "Control of Nonholonomic Wheeled Mobile Robots by State Feedback Linearization," *Int. J. Rob. Res.*, vol. 14, no. 6, pp. 543–559, 1995.
- [72] Society of Automotive Engineers. Vehicle Dynamics Committee, *Ride and Vibration Data Manual*. New York: SAE, 1965.

- [73] I. Hayashi, N. Iwatsuki, M. Kawai, J. Shibata, and T. Kitagawa, "Development of a Piezoelectric Cycloid Motor," *Mechatronics*, vol. 2, no. 5, pp. 433–444, 1992.
- [74] K. Suzumori and K. Hori, "Development of Pneumatic Wobble Motors," *JSME Int. J. Ser. C Mech. Syst. Mach. Elem. Manuf.*, vol. 42, no. 2, pp. 392–397, 1999.
- [75] *Harmonic Drive Reducer Catalog*, Rev 201806. Harmonic Drive Systems, Inc., 2018.
- [76] S. Ishikawa, "Wave Gear Drive Having High Ratcheting Torque Tooth Profile," U.S. Patent 2008/0060466, Mar. 13, 2008.
- [77] K. Kondo and J. Takada, "Study on Wave Gear Drives (1st Report, Kinematics and Geometry of Tooth Engagement)," *Trans. Japan Soc. Mech. Eng. Ser. C*, vol. 52, no. 481, pp. 2456–2462, 1986.
- [78] S. Ishikawa and N. Takizawa, "Strain Wave Gearing with Positive Deviated Profile," in *Proceedings of the Symposium on Motion and Power Transmission*, 2007, pp. 81–82.
- [79] The Japan Society of Mechanical Engineers, Ed., "Bending of Beams," in *JSME Mechanical Engineers' Handbook a3 Material Mechanics*, Maruzen Publishing, 2005, pp. 21–39.
- [80] K. Yaejima and A. Yano, "Designing of Generalized Profile Shifted Involute Internal Gears," *J. Japan Soc. Precis. Eng.*, vol. 57, no. 1, pp. 132–137, 1991.
- [81] M. Morozumi and K. Yaejima, "Study on Profile Shifted Involute Internal Gears Having No Difference in Number of Teeth Between Internal Gear and Pinion," *J. Fac. Eng. Shinshu Univ.*, vol. 27, pp. 157–170, 1969.
- [82] M. Semba, *Profile Shifting of Gears*. Kaihatsusha Inc, 1972.
- [83] O. Doi, T. Ukai, H. Ochiai, and H. Kudo, "Tooth Profile Interference and Minimum Teeth Number of Internal Involute Gear," *Bull. Fac. Eng. Hokkaido Univ.*, vol. 81, pp. 1–13, 1976.
- [84] S. Ogino, "A Study on the Involute Spur Gear of Different Center in Equal Speed," *Trans. Japan Soc. Mech. Eng.*, vol. 23, no. 126, pp. 88–93, 1957.
- [85] The Japan Society of Mechanical Engineers, Ed., *Technical Data Book: Design Data of Gear Strength*. 1979.
- [86] *CORONEX Series Reducers for Precision Control*. Nidec-Shimpo Corporation, 2017.

List of Publications

Chapter 2 consists of:

Tatsuro Terakawa, Masaharu Komori, Kippe Matsuda, and Shinji Mikami, “A Novel Omnidirectional Mobile Robot with Wheels Connected by Passive Sliding Joints”, *IEEE/ASME Transactions on Mechatronics*, vol. 23, no. 4, pp. 1716–1727, 2018. <DOI: 10.1109/TMECH.2018.2842259> © 2018 IEEE

Chapter 3 consists of:

Tatsuro Terakawa, Masaharu Komori, and Kenji Fujimoto, “Control of an Omnidirectional Mobile Robot with Wheels Connected by Passive Sliding Joints,” *Journal of Advanced Mechanical Design, Systems, and Manufacturing*, vol. 13, no. 1, p. JAMDSM0006, 2019. <DOI: 10.1299/jamdsm.2019jamdsm0006>

Chapter 4 consists of:

Tatsuro Terakawa, Masaharu Komori, Yuuta Yamaguchi, and Yugo Nishida, “Active Omni Wheel Possessing Seamless Periphery and Omnidirectional Vehicle Using It,” *Precision Engineering*, in press, <DOI: 10.1016/j.precisioneng.2019.02.003>

Chapter 5 consists of:

Tatsuro Terakawa, Masaharu Komori, Mitsuru Sakamoto, Yuji Kawato, Yuya Morita, and Yugo Nishida, “Two-wheel-drive Vehicle That Is Movable in the Longitudinal and Lateral Directions with a Small Number of Motors,” *Journal of Japan Society for Design Engineering*, vol. 54, no. 2, pp. 145–160, 2019. <DOI: 10.14953/jjsde.2018.2811>

Chapter 6 consists of:

Tatsuro Terakawa, Masaharu Komori, and Yuya Morita, “Reducer-Integrated Motor Using the Principle of Strain Wave Gearing System,” *Journal of Japan Society for Design Engineering*, vol. 52, no. 11, pp. 683–694, 2017. <DOI: 10.14953/jjsde.2017.2737> (in Japanese)

Tatsuro Terakawa, Masaharu Komori, and Yuya Morita, “Motion Analysis and Experiments of Reducer-integrated Motor Using Strain Wave Gearing System,” in *Proceedings of the ASME 2018 International Design Engineering Technical Conference & Computers and Information in Engineering Conference (IDETC/CIE 2018)*, Quebec, Canada, August 2018. <DOI: 10.1115/DETC2018-85299> © 2018 ASME

Chapter 7 consists of:

Tatsuro Terakawa, Masaharu Komori, Shouta Tamura, Ryo Kataoka and Yuya Morita, “Reducer-Integrated Motor Using Simultaneous Engagement of Gear Pairs with Small and No Differences in Teeth Number,” *Journal of Advanced Mechanical Design, Systems, and Manufacturing*, vol. 12, no. 1, p. JAMDSM0014, 2018. <DOI: 10.1299/jamdsm.2018jamdsm0014>

In reference to IEEE copyrighted material which is used with permission in this thesis, the IEEE does not endorse any of Kyoto University's products or services. Internal or personal use of this material is permitted. If interested in reprinting/republishing IEEE copyrighted material for advertising or promotional purposes or for creating new collective works for resale or redistribution, please go to

http://www.ieee.org/publications_standards/publications/rights/rights_link.html

to learn how to obtain a License from RightsLink.