

Doctoral Dissertation

Hyperons in nuclear matter based on hypernuclear spectroscopy and SU(3) chiral effective field theory

(ハイパー核分光とSU(3)カイラル有効場
理論に基づく核物質中のハイペロン)

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Abstract

Nuclear matter plays a central role across a wide range of physics, including the interiors of heavy nuclei, heavy-ion collisions, core-collapse supernovae, and neutron stars. In the high-density nuclear matter present in neutron-star interiors, hyperons are expected to appear due to the rise of the neutron Fermi surface. Although many phenomenological models in the 20th century predict such an appearance, most models with hyperons fail to account for the existence of the massive neutron stars first reported in 2010, owing to insufficient pressure. This contradiction between the nuclear phenomenology and the neutron-star observation is called the hyperon puzzle. A key ingredient for discussing the appearance of hyperons is the single-particle potentials, in particular the one of the lightest hyperon Λ .

In this work, we investigate the hyperon single-particle potentials in nuclear matter based on the hypernuclear spectroscopy and baryon interactions within chiral effective field theory (EFT). For the hypernuclear study, the Skyrme-Hartree-Fock method is employed to compare the Λ single-particle potential models with experimental data. To investigate to what extent the Λ single-particle potential is constrained based on the experimental binding energies, we have generated variety of the single-particle potentials and have extracted those that are consistent with experimental data. An important progress is that we determined the range of the Λ effective mass, which is an important input for the neutron stars and heavy-ion collision simulations. The density dependence of the Λ potential below the saturation density is constrained to some extent, while the high-density part is scattered. Then, we still need further study to pin down the Λ single-particle potentials at high densities for resolving the hyperon puzzle.

Next, we use the baryon interactions based on chiral EFT. With a two-body force up to next-to-leading order (NLO) and a three-body force with decuplet-baryon insertion, we newly focus on the Σ single-particle potential besides the Λ one. We choose the family of three-body forces making the Λ single-particle potentials enough repulsive to solve the hyperon puzzle. By using the Brueckner theory, we found that the three-body force can be constructed to be consistent with phenomenological values on Σ at the saturation density and with constraints on Λ by hypernuclear spectroscopy.

Finally, we present an analysis using the up-to-date two-body force up to next-to-next-to-leading order (N^2 LO), constructed in 2023 as extension of previous works up to NLO. It is found that the Λ single-particle potential is comparable to the results achieved with the NLO interaction from 2019. The resulting Σ potential becomes more attractive compared to the NLO results due to the constraint from the ΣN differential cross sections recently measured by the J-PARC E40 experiment. An estimate of the theoretical uncertainty in terms of the chiral perturbation exhibits similar trend to that of N^2 LO.

Contents

Acknowledgments	i
Abstract	iii
Contents	iv
1 Introduction	1
1.1 Dense matter equation of state	1
1.2 Hyperons in neutron stars	4
1.3 Hyperon single-particle potentials	9
1.4 Structure of this thesis	13
2 Hypernuclei and repulsive Lambda single-particle potential	15
2.1 Introduction	15
2.2 Skyrme-Hartree-Fock method for Lambda hypernuclei	19
2.3 Fitting to chiral effective field theory results	24
2.4 Results	28
2.5 Short summary	30
3 Uncertainty estimate using Lambda hypernuclei	31
3.1 Uncertainty estimate for Lambda single-particle potentials	31
3.1.1 Procedure	31
3.1.2 Results	33
3.2 Lambda admixture in neutron star matter	36
3.3 Short summary	38
4 Baryon forces within SU(3) flavor chiral EFT	40
4.1 A short review of baryon interaction studies	40
4.2 Quantum chromodynamics and chiral symmetry	42
4.3 Chiral effective field theory	44
4.3.1 Construction of Lagrangian	45
4.3.2 Power counting	47
4.3.3 Feynman diagrams	48
4.4 Heavy-baryon formalism	49
4.5 Regularization of potentials	51
4.6 Baryon forces including hyperons	52

5	Brueckner theory for hyperons in medium	54
5.1	Introduction	54
5.2	Foundation	56
5.3	Detailed expressions for hyperons in matter	58
5.4	Numerical details	61
6	Chiral two- and three-body forces up to NLO in matter	64
6.1	Introduction	64
6.2	Leading three-baryon force via decuplet saturation	65
6.3	Numerical details	68
6.4	Hyperon single-particle potentials with two- and three-body forces at NLO	70
6.5	Short summary	72
7	Chiral two-body forces up to N^2LO in matter	74
7.1	Introduction	74
7.2	Theoretical components	76
7.3	Results	77
7.3.1	Nuclear matter	78
7.3.2	Lambda in symmetric nuclear matter	79
7.3.3	Sigma in symmetric nuclear matter	86
7.3.4	Lambda and Sigma in pure neutron matter	91
7.4	Uncertainty estimate	93
7.5	Short summary	98
8	Summary and outlook	100
A	Conventions	105
	Bibliography	107

Chapter 1

Introduction

This thesis investigates the property of hyperons in nuclear matter, especially the corresponding single-particle potentials. The Λ hypernuclear spectroscopy is employed as a boundary condition to the single-particle potential around saturation density. We elaborate also on the baryon forces within chiral effective field theory (EFT), providing a guide to extend above the saturation density. This Chapter addresses the motivation for studying hyperon properties in dense matter and highlights the importance of hyperon single-particle potentials for advancing our understanding of dense matter in the universe.

1.1 Dense matter equation of state

One of the ultimate goals in nuclear physics is to understand the properties of dense matter formed under wide range of environments. The nuclear matter plays a central role in various physics including (hyper) nuclear structure and reaction, heavy-ion collisions, early universe, core-collapse supernovae, and neutron star structure and their mergers. The property ranges wide interest: finite density and/or temperature in equilibrium, transport properties, phase structure like whether quarks are deconfined or not.

From the theoretical point of view, nuclear matter is described by quantum chromodynamics (QCD), the theory of the strong interaction. If one can simulate exactly based on QCD, all of the properties of the matter can be revealed. However, it is well-known that a first-principle lattice QCD calculation cannot be performed in the finite density region (especially at zero temperature) due to the notorious sign problem. Although there have been many attempts (see Ref. [1] and references therein) to mitigate the problem, it is still hard to derive exact consequences. Then, it is currently the popular way that first construct equation of state (EOS) models,

$$P = P(\epsilon; \alpha), \tag{1.1}$$

where ϵ is the energy density and α corresponds to labels like temperature, particle composition, etc. Thereafter, the constructed EOSs can be compared with available observations and experiments via some model calculations, constraining the properties of EOS as a feed back of comparison.

The property of the QCD matter EOS is embedded in its phase diagram as a function of the temperature and the baryonic chemical potential or the baryon number density. A

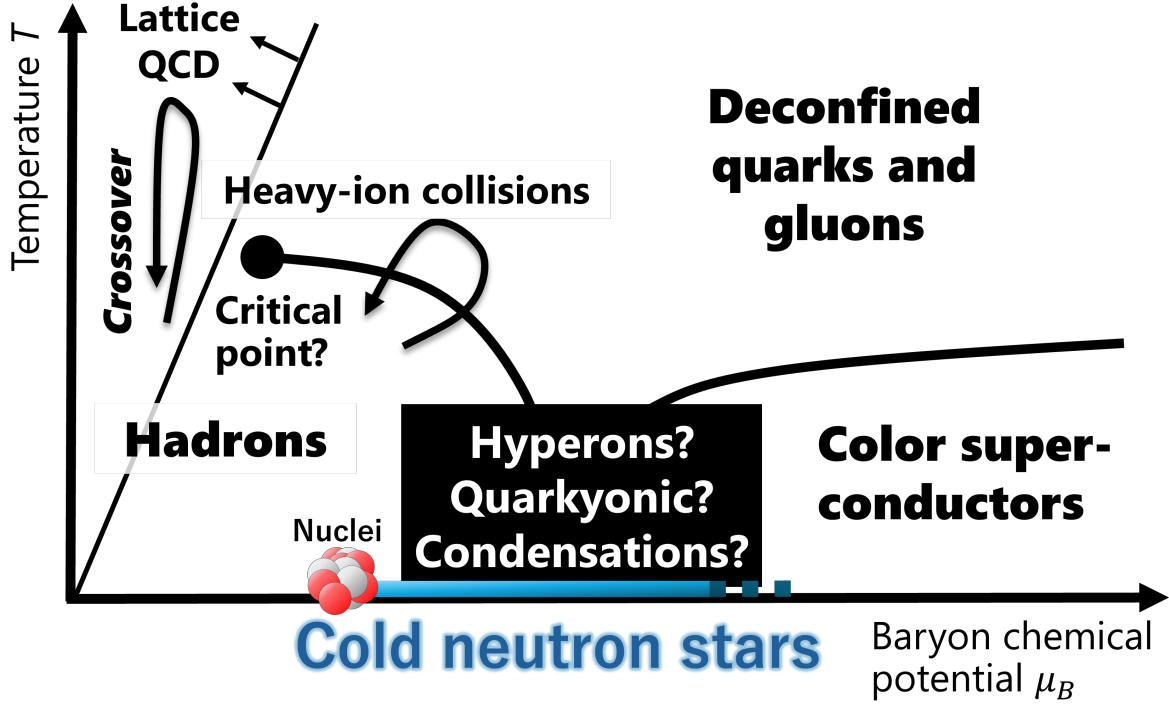


Figure 1.1: A schematic view on the hypothetical QCD phase diagram.

current assumed diagram is shown in Fig. 1.1. Some also considers the isospin chemical potential, which accounts for the isospin-imbalanced environment like neutron stars. At finite temperature and zero baryon number density, it is well-known that there is no phase transition as studied by the first-principle lattice QCD calculations [2–5]. The ultra high-energy heavy-ion collisions at few TeV are corresponding to this region. At finite temperature but small zero baryon number density, $\mu_B/T < 1-2$, the region can be accessed via some sort of the perturbation technique, see Refs. [6–8] for recent reviews.

In this thesis, we focus on a basic environment of the uniform nuclear matter in finite density and at zero temperature, where now a frontier of the phase structure and intensively discussed based on QCD phenomenology, neutron star mass-radius observations, gravitational waves from binary neutron star mergers, etc. The low-density part is well-described solely by the nucleon degree of freedom. There are a large number of the nucleon experiments, well constraining the nucleonic EOS. In the left panel of Fig. 1.2, an example of the constrained parameter regarding the symmetry energy, associated with the energy difference between the isospin symmetric and asymmetric matter. A number of experiments including heavy-ion collisions (HIC), giant dipole resonance (GDR), etc. constrain the Taylor-expansion parameters around the saturation density being $\rho_0 \simeq 0.16 \text{ fm}^{-3}$. Furthermore, chiral EFT provides a systematic way to construct the baryon interactions, allowing some sort of the error estimation. In the right panel of Fig. 1.2, the chiral EFT calculations for the nuclear matter EOS of pure neutron matter (PNM) are shown. Although they are applicable for a lower density region, say $\rho < 2\rho_0$. The error band that is estimated by the energy scale in the perturbation series provides a basis to extrapolate to the higher density region.

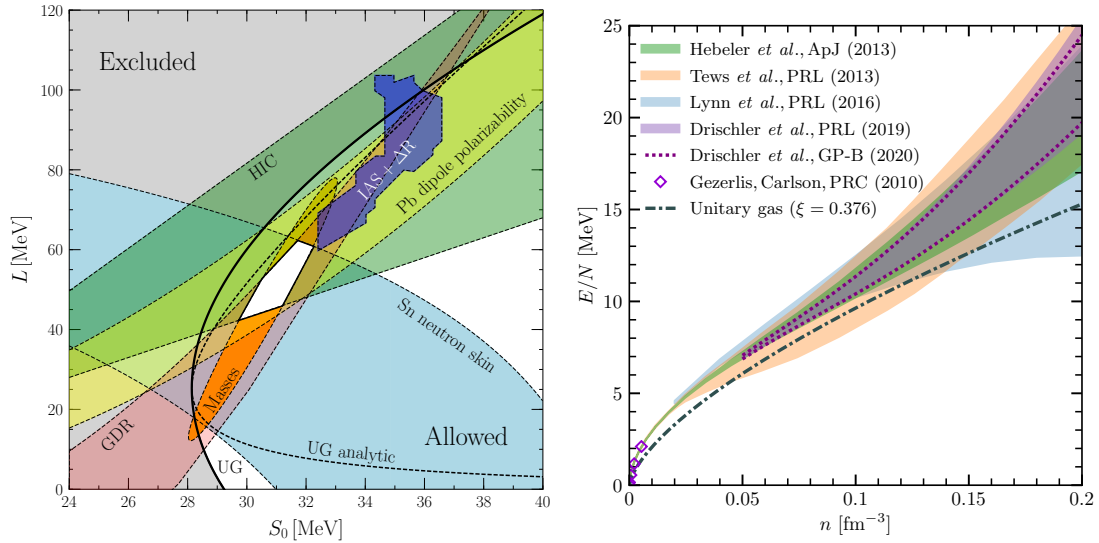


Figure 1.2: Empirical constraints on the property of the nuclear matter EOS. (Left panel) Experimental constraints on the symmetry energy parameters. Figure is taken from Ref. [9]. (Right panel) Energy per particle E/N of PNM as a function of density from various many-body calculations with chiral EFT interactions. Figure is taken from Ref. [10].

Understanding matter at high densities remains a major challenge. The theoretical description is not settled in this region. Then, many attempt to construct some EOS models and compare them with e.g. the neutron-star observations and the heavy-ion collision data via some simulations. The inner structure of neutron stars is one of the important subjects in astrophysics and nuclear physics. For the neutron star matter case, the mass-radius relation is one of the strongest constraints on the dense matter EOS. The EOS models have one-to-one correspondence to the mass-radius relation via the so-called Tolman-Oppenheimer-Volkoff equation [11, 12]

$$\frac{dP(r)}{dr} = -G_N \frac{\rho(r) + \frac{P(r)}{c^2}}{r \left(r - \frac{2G_N M(r)}{c^2} \right)} \left(M(r) + 4\pi r^3 \frac{P(r)}{c^2} \right). \quad (1.2)$$

Here, G_N is Newton's gravitational constant, $\rho(r)$ is the mass density at the distance r from the neutron star center, P is the pressure of neutron-star matter, and $M(r)$ is the enclosed mass within radius r . One can relate the mass-radius relation of a neutron star to a given equation of state (EOS). The validity of a nuclear-matter EOS can thus be examined by checking whether it reproduces the observed mass-radius data. In recent years, several observations such as the discovery of a two-solar-mass neutron star in 2010 [13] and precise mass-radius measurements by NICER [14–16] have placed strict constraints on the mass-radius relation, providing valuable clues on the dense matter EOS.

The heavy-ion collision reaction can be performed on the earth and provides versatile information on dense matter EOSs. The reaction collides heavy nuclei like Au and Pb and measure the final momentum of the produced particles. Especially for the low to

intermediate energy region, say $\sqrt{s_{NN}} \approx 3 - 5$ GeV, the colliding nucleons stop at the initial collision point by the strong repulsive core of nucleons. There, dense matter is created with not short lifetime [17]. The pressure gradient of the formed matter produces the nontrivial dependence of the final momentum distribution on the EOS and the mean-field potentials. An observable expected to be sensitive to the dense matter EOS is the anisotropic collective flow v_n [18, 19]

$$v_n = \langle \cos n\phi \rangle, \quad (1.3)$$

where ϕ is the azimuthal angle measured from the reaction plane. See Ref. [20] for recent review of investigation of the dense matter property via the collective flows.

A unified approach combining as much as information has been done by many collaborations for obtaining well-constrained EOS. For lower density region, they employ the modern two- and three-body force within chiral EFT or well-constrained mean-field models. For higher density region, they [21–28] use the neutron star observations and someone [29, 30] additionally uses heavy-ion collision observables. These approaches are successful for directly constraining $P = P(\epsilon, T = 0)$, useful for investigating the neutron star internal structure.

Toward more comprehensive understanding of the EOSs, one has to explore the microscopic description of EOS. Such a description is useful to extend the EOS description for finite temperature cases, increasing its importance by the observation of the gravitational waves from the binary neutron-star mergers. One must construct a nice microscopic picture of the EOSs by many methods. An important degree of freedom is, we will see in the next section, the hyperon.

1.2 Hyperons in neutron stars

The hyperons are baryons that contain at least one strange quark, and their presence reflects the extension of hadronic structure beyond the non-strange nucleonic sector. Historically, hyperons were first identified in cosmic-ray emulsions in the early 1950s through the observation of unusually long-lived, highly ionizing tracks. This discovery marked the beginning of strangeness physics. Within the ground-state baryon octet, the low-lying strange baryons are the Λ , Σ , and Ξ hyperons, whose vacuum masses are

$$m_\Lambda = 1115 \text{ MeV}, \quad m_\Sigma \approx 1190 \text{ MeV}, \quad m_\Xi \approx 1317 \text{ MeV}.$$

A single hyperon is characterized by the strangeness quantum number $S = -1$ for the Λ and Σ hyperons, and $S = -2$ for the Ξ hyperon. Their quark composition consists of the two light u and d quarks and one strange quark, and naïve quark counting may suggest that the hyperon-nucleon interaction strength is roughly $2/3$ of the nucleon-nucleon interaction. However, the mass of the strange quark is not negligible, and the symmetry structure becomes complex for the hyperon case. These differences become particularly significant in dense environments such as neutron stars, where the appearance of hyperons at high baryon density is expected.

In the following, we summarize the historical development of hyperon physics in neutron star matter.

Early period: Ideal Fermi gas

The existence of hyperon in nuclear matter is first discussed in 1960 by Ambertsumyan and Saakyan [31] based on the free Fermi gas model. Considering the hyperon was first found as the 'V particle' in In right of that, let us here illustrate how hyperon appears inside dense matter with the simplest model including only non-interacting neutrons and Λ 's. The energy density and pressure at baryon density ρ are given as

$$\mathcal{E}_{\text{free}}(\rho, \rho_n, \rho_\Lambda, \alpha) = g \sum_{i=n, \Lambda} \left(\int_{|\mathbf{k}| < k_{Fi}} \frac{d^3 \mathbf{k}}{(2\pi)^3} \sqrt{m_i^2 + \mathbf{k}^2} \right), \quad (1.4)$$

$$P(\rho) = \rho^2 \frac{\partial}{\partial \rho} \left(\frac{\mathcal{E}_{\text{free}}}{\rho} \right), \quad (1.5)$$

where $g = 2$ is the spin degeneracy, $k_{Fi} = (6\pi^2 \rho_i / g)^{1/3}$ for $i = n, \Lambda$ is the Fermi momentum for baryon species, m_i is the baryon mass. The condition of minimum $\mathcal{E}$ under the variation in ρ_i gives the chemical potential

$$\mu_i = \frac{\partial \mathcal{E}}{\partial \rho_i} = \sqrt{m_i^2 + k_{Fi}^2}. \quad (1.6)$$

The chemical-equilibrium condition can be obtained as¹

$$\beta\text{-equilibrium condition: } \mu_n = \mu_\Lambda. \quad (1.7)$$

The particle fraction at a specific density ρ is evaluated by solving the β -equilibrium condition under fixed $\rho = \rho_n + \rho_\Lambda$. The particle fraction at a certain baryon density ρ is determined from the equations

$$\sqrt{m_n^2 + k_{Fn}^2} = \sqrt{m_\Lambda^2 + k_{F\Lambda}^2}, \quad (1.8)$$

$$\rho = \rho_n + \rho_\Lambda. \quad (1.9)$$

The chemical potentials and particle fraction for the free-Fermi gas model are shown in Fig. 1.3. For lower density region, there is only neutrons due to mass splitting between neutron and Λ . The Λ hyperons appear when the neutron Fermi energy becomes enough large. Thereafter, the hyperons populate quickly.

An important consequence when hyperon appears inside neutron stars is the *softening* of the equation of states. This is clearly illustrated in Fig. 1.4, where the pressures as functions of the energy density are shown. The pure neutron matter (PNM) case is also shown for comparison. The pressure clearly decreases by the appearance of hyperon, as a consequence of the degenerated pressure of neutrons is converted to the mass excess of Λ . This results in the reduction of the pressure, or the softening of the equation of states.

¹Strictly speaking, the β -equilibrium condition of neutron star matter can be obtained only if there are protons and electrons, since baryons undergo the processes $n \rightarrow p + e^- + \bar{\nu}_e$, $p + e^- \rightarrow \Lambda + \nu_e$.

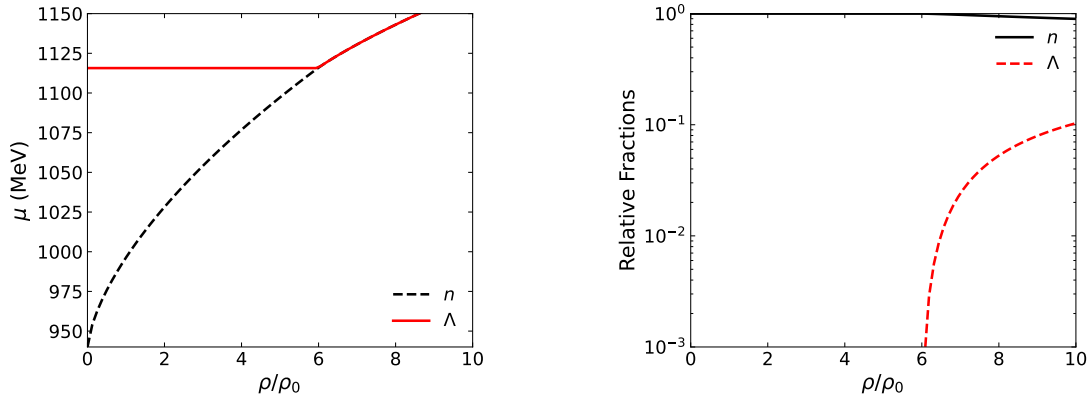


Figure 1.3: Chemical potentials (left panel) and particle fractions (right panel) in the $n\Lambda$ matter by the free Fermi gas model.

1980–2010: Phenomenological models

A realistic study requires, of course, the interactions. One needs experimental information on hyperons. The nuclear experiment including hyperon is far more difficult than the normal nuclear case due to its short lifetime by weak-decay processes, e.g. $\Lambda \rightarrow N + \pi$ with life time $\approx 2.6 \times 10^{-10}$ sec. Nevertheless, the advent of intense K^- beam at CERN in the late 1960s and later at Brookhaven National Laboratory (BNL) enables the hyperon experiments with statistics. The hyperon-nucleon scattering data provides a guide to construct phenomenological hyperon-nucleon forces. Based on the interaction in vacuum, the Brueckner G -matrix calculation [32, 33] has been performed to construct in-medium hyperon-nucleon forces and to evaluate neutron star EOSs. The Λ hypernuclear experiment can also be done with the strong bound state, which is enough long to form the bound state compared to the weak decay rate. A detailed review of hypernuclear spectroscopy will be given in Chapter 2. These data are used as a "boundary condition" around ρ_0 to construct mean-field models [34–36] with hyperons.

Most all of such studies predict that hyperons are admixed in the neutron star matter at the onset density of $2\text{--}4\rho_0$. There they also observe the admixture of the hyperons soften EOS of neutron star matter and significantly reduces the maximum allowed mass of neutron stars.

It is worth mentioning that already in 2002, the necessity to include the three-body force is claimed according to Nishizaki *et al.* [37]. There has been observed the $1.44M_\odot$ neutron star, the Hulse-Taylor pulsar PSR1913+16 [38]. They employ the G -matrix calculation with Nijmegen D hard-core BB interaction [39–41] and additional three-body forces. As shown in Fig. 1.5, the hyperon degree of freedom decreases the maximum mass, even below the $1.44M_\odot$ neutron stars with $M_\odot$ being the solar mass, but the hyperonic three-body forces stiffen the EOS.

More exotic state of the neutron stars have been proposed at this age. Witten wrote a famous paper conjecturing the self-bound strange star [42], followed by more phenomenological considerations [43–45]. Some [46] discussed the possibility of stable quark matter. Some condensate in neutron stars have been discussed like pions [47] and kaons [48]. Tam-

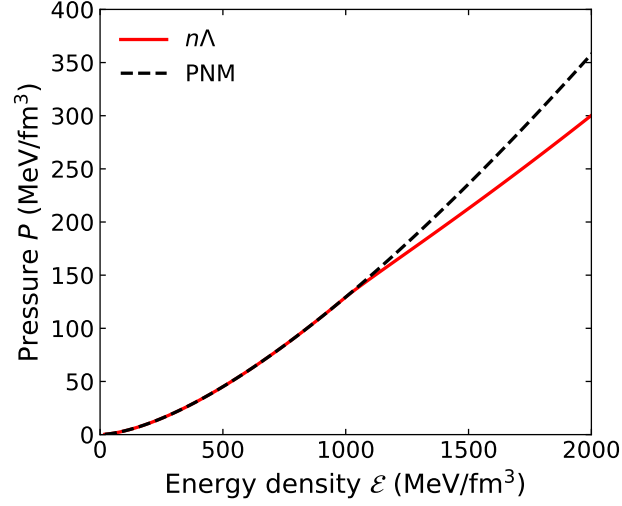


Figure 1.4: Pressures as functions of energy density. The red line is for the $n\Lambda$ matter, while the black dashed line corresponds to the PNM case.

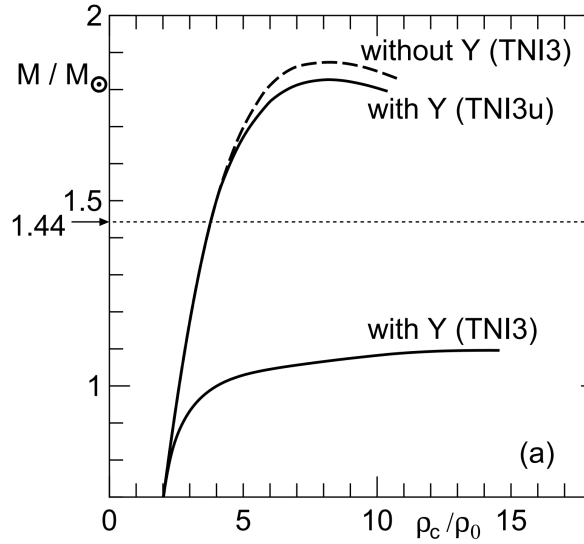


Figure 1.5: Mass of a neutron star in units of the solar mass $M_\odot$ as functions of the central baryon density ρ_c . All models uses the nucleon three-body interactions (TNI). The TNI3u model employs the universal three-body interaction including one and more hyperons. Figure is taken from Ref. [37].

agaki proposed [49] the hypothetical matter formed by the H -dibaryon, the $uuddss$ -quark singlet state particle.

2010–: Hyperon puzzle

The softening problem by the appearance of hyperons becomes more serious by the finding of the massive neutron stars with $M \gtrsim 2M_{\odot}$ first in 2010 [13]. Thereafter, more and more massive neutron stars are found [16, 50–52]. This problem of inconsistency between the hyperon phenomenology and the neutron star observation is called the *hyperon puzzle*; see for the recent reviews [53–56]. A main cause of the controversial situation is the uncertainty in the property of hyperons in dense matter. The hypernuclear spectroscopy gives information only near the saturation density. The hyperon-nucleon scattering data constrains the two-body force. Then, the strength of the three-body force, which is important in dense matter, remains unclear. Then, naïve construction of the phenomenological EOSs with hyperons did a significant extrapolation to higher density region. This extrapolated region gives a basis to propose the solutions for resolving the puzzle.

A straightforward approach to resolving the so-called hyperon puzzle is to incorporate repulsive interactions acting on hyperons at high densities. In particular, three-body interactions that produce further repulsion have been extensively discussed [37, 57, 58, 58–65]. These effects can be modeled either through effective density-dependent two-body forces or via density-dependent couplings in mean-field models. Another way is to address the uncertainty in the hyperon-hyperon (YY) interaction. Since double Λ -hypernuclear data remain limited to only light systems, substantial model uncertainties remains there. Studies [66–68] suggest that sufficiently strong YY repulsion could support massive neutron stars.

A different class of solutions is the crossover transition from hadronic matter to deconfined quark matter. The smooth transition without a specific phase transition can avoid the rapid softening of the EOS. Furthermore, if such a transition takes place before hyperons become abundant, the softening associated with rapid hyperon population growth may be avoided [69–71]. This scenario is often discussed in the context of quarkyonic matter [72, 73].

Although many theoretical solutions have been put forward, none has been firmly confirmed, largely due to the limited experimental constraints on hyperon properties in dense matter. This situation highlights the importance of improving empirical knowledge of hyperons in nuclear matter. Constraining the hyperon single-particle potential remains one of the key steps toward understanding the behavior of hyperons in neutron-star matter, as will be discussed in the next section.

1.3 Hyperon single-particle potentials

As seen above, one of the key issues for hyperons in neutron stars is whether hyperons appear in neutron stars: when hyperons appear in neutron stars, hyperon population increases rapidly, resulting in the softening of EOSs. The hyperon with lightest mass Λ appears in neutron stars when the neutron Fermi energy exceeds the in-medium hyperon

mass as

$$\epsilon_{n,F} \geq m_\Lambda + U_\Lambda(k=0), \quad (1.10)$$

where $U_\Lambda(k=0)$ is the Λ single-particle potential at zero momentum. There are two important quantity for determining the onset density of hyperon: The EOS of normal nuclear matter, especially the symmetry energy determines the density dependence of the neutron Fermi energy. The other important quantity is the hyperon single-particle potential U_Y . It is naïvely the averaged baryon interactions that a single hyperon feels inside nuclear matter. Also, U_Y can be accessible by many experimental data and theoretical development. Here, we summarize briefly the current experimental constraints on U_Y with selected important theoretical works.

The hypernuclear spectroscopy directly constrains U_Y near ρ_0 . The hypernuclei are the nuclei with at least one hyperon. Since hyperon is not Pauli-blocked inside nuclei because of its strange-quark content, the hyperon especially in the S -wave state is well bounded inside nuclei. If the mass number of the nuclei is enough large, they are described by the mean-field models. The most of the internal nuclei show the density region with ρ_0 except for the surface region. Then, the Λ separation energies from the hypernuclei provide the information on U_Y at and possibly around ρ_0 .

The Λ hypernuclear spectroscopy have rich amount of data. For example, Millener *et al.* [74] in 1988 performed the fitting to the observed Λ hypernuclear spectra with various functional dependencies with $U_\Lambda(\rho; a, b, \gamma) = a\rho + v\rho^\gamma$ by a Hartree-Fock type calculation. They choose $\gamma = 5/3, 2$, and 3 as a test case and performed a best fitting of the parameters a and b . All of these functional forms reproduce the widely confirmed constraint (e.g. Ref. [75])

$$U_\Lambda(\rho_0, k=0) \approx -30 \text{ MeV}. \quad (1.11)$$

Contrary, for higher density region, they exhibit considerably different values in the higher density region. Note that the central density of neutron stars are assumed around $5\text{--}10\rho_0$, very far from the hypernuclear region.

The Λ single-particle potential is also well examined via the baryon interactions in vacuum. The Brueckner G -matrix calculation is widely used to calculate the in-medium two-body force via that in vacuum. The hyperon single-particle potentials are calculated during evaluating the G matrix. The meson-exchange models by the Nijmegen [58, 76] and Jülich [77–79] groups, constituent-quark models [80–82], lattice QCD [83], and chiral EFT [59, 84–86] have been used to simulate the hyperon single-particle potentials. The three-body forces are sometimes added phenomenologically as e.g. the density-dependence two-body force to the models. The strength of such forces is frequently fixed to reproduce the Λ hypernuclear constraint (1.11).

An important work on U_Λ concerning the hyperon puzzle was reported by Gerstung *et al.* [59]. They employ the two- and three-body forces within chiral EFT. The YNN force is implemented as the density-dependent two-body force, whose low-energy constants (LECs) are adjusted to reproduce $U_\Lambda(\rho_0) = -30 \text{ MeV}$. The Brueckner theory is employed to simulate the hyperon single-particle potentials. They showed that the LECs of the YNN force can be chosen to suppress Λ 's in neutron star matter as shown in Fig. 1.7: The hyperon's energy at $k=0$ $\mu_\Lambda(\rho) = m_\Lambda + U_\Lambda(\rho, k=0)$ is always larger than the neutron

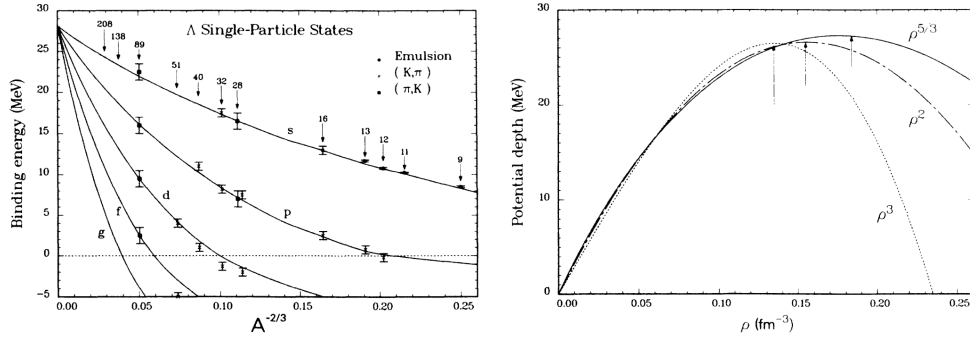


Figure 1.6: (Left panel) Λ hypernuclear spectra as a function of $A^{-2/3}$ with A being the mass number of the core nucleus. The solid line represents the Hartree-Fock calculation with the functional form of the Λ single-particle potential $U_\Lambda(\rho; a, b) = a\rho + b\rho^{4/3}$. (Right panel) U_Λ as functions of the density ρ for various density dependencies. Figures are taken from Ref. [74].

chemical potential μ_n in PNM up to high densities of $5\rho_0$. We note that this work does not point out that the two- and three-body force is unique but they only claim there exists the possible parameter sets to suppress Λ 's in PNM. One must verify the interactions by adopting to other physical observables and by comparing with experimental values.

We have performed a study [88] to verify the above U_Λ strongly repulsive at higher densities by adopting it to the heavy-ion collision simulation. We consider the rapidity dependence of the Λ directed flow in heavy-ion collisions, the first coefficient of the collective flow,

$$v_1 = \langle \cos \phi \rangle = \left\langle \frac{p_x}{\sqrt{p_x^2 + p_y^2}} \right\rangle, \quad (1.12)$$

where ϕ is the azimuthal angle measured from the reaction plane and p_x and p_y are the transverse momenta of a particle. We use the Lorentz vector version of the relativistic quantum molecular dynamics (RQMDv) model [89] implemented in the JAM2 transport code². The different Λ single-particle potentials are fitted by Lorentz-vector form. The results of v_1 of Λ in mid-central Au + Au collisions at $\sqrt{s_{NN}} = 4.5$ GeV are shown on the right panel of Fig. 1.8 and compared with the data [90] reported by the STAR collaboration. One can see that both Chi3momSoft and LY-IVmomSoft reproduce v_1 of Λ with equal accuracy, which implies that v_1 of Λ is not so sensitive to the density dependence of the Λ single-particle potential. On the other hand, Chi3momHard underestimates v_1 of Λ , which indicates that v_1 of Λ is sensitive to the momentum dependence of the Λ single-particle potential. Then, to investigate the density dependence of the Λ single-particle potentials, which is important for solving the hyperon puzzle, the momentum dependence should be constrained first.

There is another hyperon in the strangeness $S = -1$ sector, the isospin $I = 1$ Σ hyperons. This has close mass to the Λ hyperon of $m_\Sigma - m_\Lambda \approx 75$ MeV, highlighting the importance of the $\Lambda N - \Sigma N$ coupled channel. It is widely believed that the Σ hyperon feels repulsion in nuclear matter as considered from the fact that the Σ hypernuclei are

²<https://gitlab.com/transportmodel/jam2>

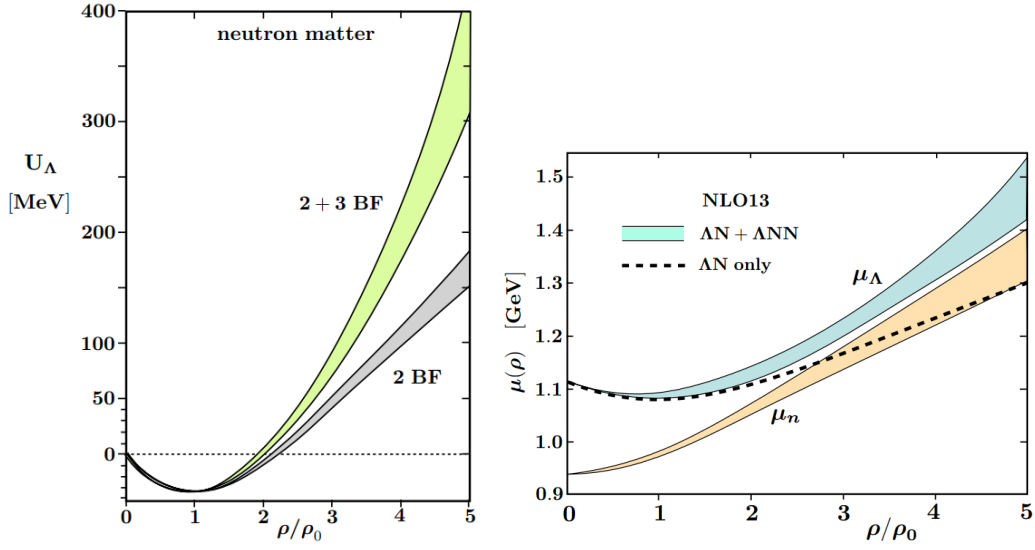


Figure 1.7: Hyperon single-particle potentials (left) and the chemical potentials (right) in PNM. The chiral two- and three-body forces are employed. Here, the Λ chemical potential means $\mu_\Lambda(\rho) = m_\Lambda + U_\Lambda(\rho, k = 0)$. The neutron chemical potential μ_n [87] is estimated by using the chiral SU(2) nucleon-meson field theory combined with functional renormalization group methods. Figure is taken from Ref. [59].

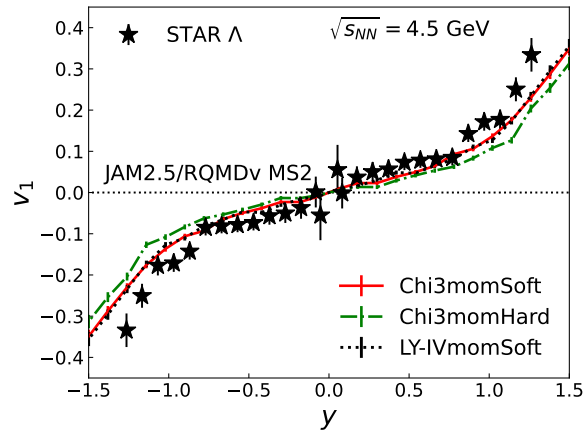


Figure 1.8: Directed flow of Λ in mid-central Au+Au collisions at $\sqrt{s_{NN}} = 4.5$ GeV. The STAR data are taken from Ref. [90]. The figure is updated from Ref. [88] by using the updated version of JAM2.

not observed except for the special case ${}^4_2\text{He}$ [91]. Even though the direct measurement of the property of Σ in nuclear matter is difficult, its information can be inferred from the measurement of the Σ atom, or the bound state governed by the electric force, and the (π, K) inclusive spectra. A phenomenological value based on such data is suggested in the review by Gal, Hungerford, and Millener [75],

$$U_{\Sigma}(\rho_0, k=0) = 30 \pm 20 \text{ MeV}, \quad (1.13)$$

in symmetric nuclear matter (SNM). In the particle basis, there are three Σ state, Σ^0 , and $\Sigma^{\pm}$. Corresponding conditions that the Σ states appear in neutron star matter at zero temperature are given as

$$\epsilon_{F,n} = m_{\Sigma^0} + U_{\Sigma^0}(k=0), \quad (1.14)$$

$$\epsilon_{F,n} + \epsilon_{F,e^-} = m_{\Sigma^-} + U_{\Sigma^-}(k=0), \quad (1.15)$$

$$\epsilon_{F,n} = \epsilon_{F,e^-} + m_{\Sigma^+} + U_{\Sigma^+}(k=0), \quad (1.16)$$

where $\epsilon_{F,i}$ represents the Fermi energy of the particle specie i . Then, the Σ^- hyperon is likely first appeared due to the electron Fermi energy. However, due to the repulsive nature of the Σ single-particle potential, it is currently believed that the Σ hyperon is not favored in the neutron star matter.

For completeness, we briefly mention studies on the strangeness $S = -2$ and isospin $I = 1/2$ hyperon, Ξ in nuclear matter. Detailed overview is given in Refs. [92, 93]. The inclusive (K^-, K^+) spectra measured at KEK and at BNL on ${}^{12}\text{C}$ [94, 95] suggest the the well depth $U_{\Xi} \approx -15$ MeV analyzed by the distorted wave impulse approximation, but the reliability of the attraction is ambiguous according to the further analysis [96]. Thereafter in 2015, evidence for bound state by the strong interaction is reported [92] by emulsion experiments, suggesting at least the attractive Ξ -nucleus interaction. The ongoing J-PARC E70 experiment [97, 98] is performing the missing-mass spectroscopy of ${}^{12}_{\Xi}\text{Be}$ for aiming the high-precision reaction spectroscopy by obtaining sufficient statistics.

1.4 Structure of this thesis

In this thesis, we investigate the Λ and Σ single-particle potentials from various theoretical techniques. We aim to obtain an up-to-date constraint on the single-particle potentials toward well-constrained extrapolation to reach the central density of neutron stars. The illustration of our strategy is found in Fig. 1.9.

In Chapter 2, we employ the currently available Λ hypernuclear spectroscopy to firmly fix the boundary condition on U_{Λ} at and near ρ_0 . We will verify whether the repulsive Λ single-particle potentials by Gerstung *et al.* shown in Fig. 1.7 reproduce measured Λ binding energies by using the Skyrme-Hartree-Fock method. Thereafter in Chapter 3, we generate considerable U_{Λ} and estimate current uncertainty of U_{Λ} . We also constrain the effective mass, or the momentum dependence, which is important for the heavy-ion collision simulation input. Discussions in these Chapters are based on Ref. [99].

The following Chapters 4 and 5 serve as preparations for our works based on baryon interactions within chiral EFT. Chapter 4 introduces the basic concepts of chiral EFT,

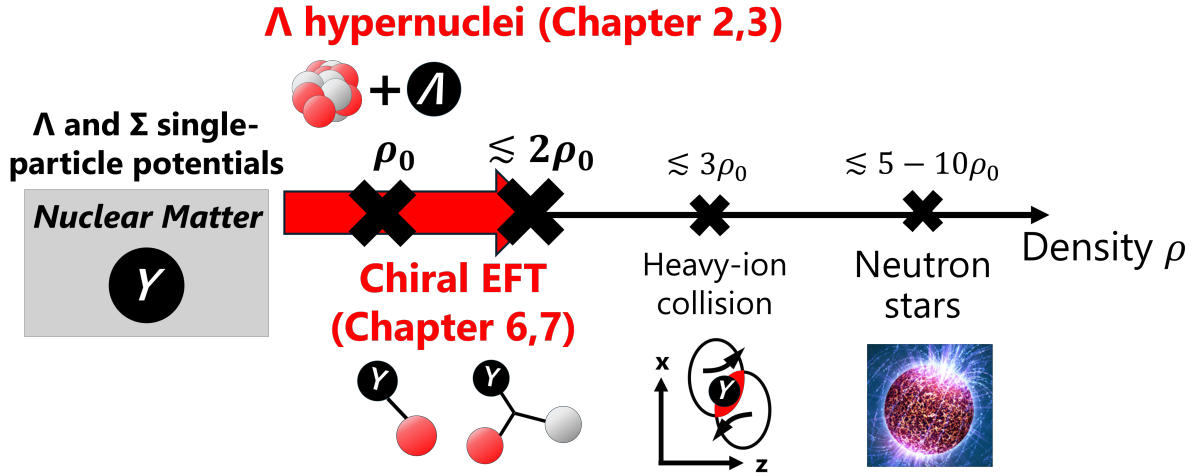


Figure 1.9: Illustration of our strategy to constrain the hyperon single-particle potentials.

including how to build the baryon interactions. To calculate the hyperon single-particle potentials in nuclear matter based on chiral EFT, we employ the conventional Brueckner theory introduced in Chapter 5.

With these preparations, we discuss in Chapter 6 the hyperon single-particle potentials using the two-body force and the three-body forces constructed via the decuplet baryon insertion. The set up is similar to that by Gerstung *et al.* [59], but here we newly focus on whether Σ single-particle potentials and the momentum dependence of Λ fulfill the current phenomenological value and our hypernuclear constraint, respectively. Discussions in this Chapter are based on Ref. [100].

Chapter 7 employs the up-to-date chiral two-body forces at next-to-next-to-leading order (N^2LO) presented in 2023 [101]. This force has several extensions from the conventional NLO potentials [102, 103]. This work reveals that the situation of the hyperon single-particle potentials has now changed especially for that of Σ hyperon, because of new experimental constraint by the ΣN differential cross sections [104–106]. Discussions in this Chapter are based on Ref. [107]. Chapter 8 summarizes our findings and provides an outlook.

Chapter 2

Hypernuclei and repulsive Lambda single-particle potential

In this Chapter, we calculate the Λ separation energy from Λ hypernuclei to verify the reliability of the Λ single-particle potential repulsive at high densities that includes the effects of the YNN three-body force within chiral effective field theory (EFT). The Skyrme-Hartree-Fock method is employed to compare the Λ mean-field potential with experimental data. We show that the Λ single-particle potential is consistent with the measured medium-to-heavy hypernuclear spectroscopy. As a result, we reproduced the experimental data of Λ binding energies, implying that the behavior of the single-particle potential in the low-density and low-momentum region is verified. We also find that Λ single-particle potential more attractive at higher density region, which causes the hyperon puzzle in neutron stars, reproduces the measured Λ separation energies at the same level of accuracy as the above-mentioned repulsive model. Then, we confirmed that the hypernuclear spectroscopic data is not as strong as to solve the hyperon puzzle of neutron stars.

2.1 Introduction

The Λ hypernuclei $^A_\Lambda Z$ are nuclei in which a single Λ hyperon is bounded by the strong interaction to a nucleus ^{A-1}Z . Their existence was first confirmed in 1953 through emulsion experiments exposed to cosmic rays [108]. The advent of the K beam enables the beam line experiments to obtain statistic to some extent. Such early time review can be found in Ref. [109]. One of the useful data extracted from hypernuclear experiments is the Λ binding energy B_Λ . The emulsion experiments directly see the tracks of particles and infer the Λ binding energy by reconstructing their momenta through the length of the tracks. The missing mass method is widely used for the beamline experiments, relatively easy to accumulate statistics. As an example, we illustrate the missing mass measurements to one of the measure process $^A Z(\pi^+, K^+)^A_\Lambda Z$. By applying the energy-momentum conservation $p_{\pi^+}^\mu + p_A^\mu = p_{\text{hyp}}^\mu + p_{K^+}^\mu$, the mass of the hypernucleus M_{hyp} can be related to

the meson momenta as

$$M_{\text{hyp}} = \sqrt{(p_A^\mu + p_{\pi^+}^\mu - p_{K^+}^\mu)^2} \\ = \sqrt{(M_A + E_{\pi^+} - E_{K^+})^2 - (\vec{p}_{\pi^+}^2 + \vec{p}_{K^+}^2 - 2|\vec{p}_{\pi^+}||\vec{p}_{K^+}|\cos\theta)}, \quad (2.1)$$

where E_{π^+} and E_{K^+} are the zeroth components of the π^+ and K^+ momenta, respectively. The symbol θ is the scattering angle of K^+ measured from the π^+ direction. The mass of the target nucleus is denoted as M_A . Therefore, by measuring the momentum magnitudes of the incident π^+ and the produced K^+ , one can calculate the mass of the hypernucleus M_{hyp} . Note that for the scattering angle θ , the average is taken within the range where the detector is placed. The Λ binding energy

$$B_\Lambda = M_{A-1} + M_\Lambda - M_{\text{hyp}} \quad (2.2)$$

can be obtained with M_Λ being the Λ mass. As an example of the measured experimental data, Fig. 2.1 shows the scattering cross section in the $^{12}\text{C}(\pi^+, K^+)_{\Lambda}^{12}\text{C}$ reaction, plotted as a function of B_Λ . The spectra show the clear splitting peaks that can be regarded as the single-particle levels of Λ .

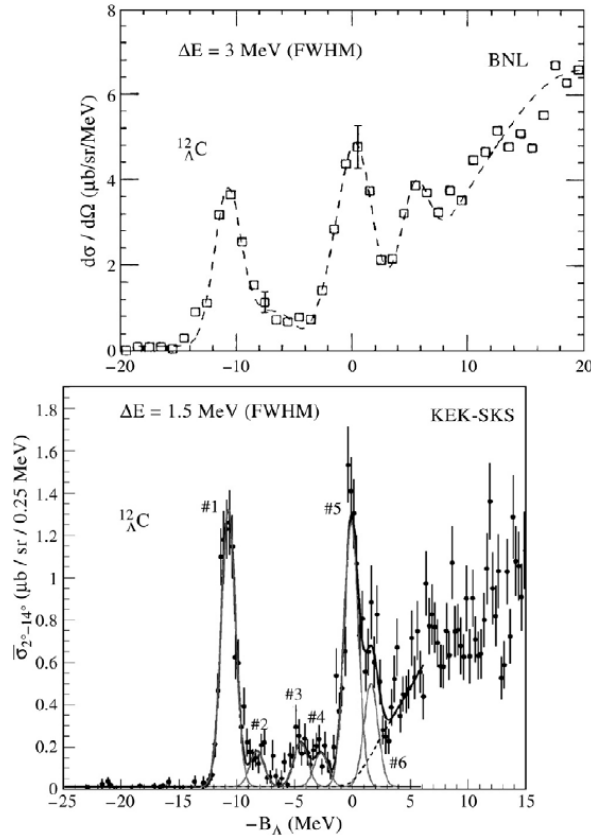


Figure 2.1: Spectrum of the Λ binding energy in the $^{12}\text{C}(\pi^+, K^+)_{\Lambda}^{12}\text{C}$ reaction. The upper panel shows measurements from BNL AGS [110], and the lower panel shows measurements from KEK [111]. Figure is taken from Ref. [112].

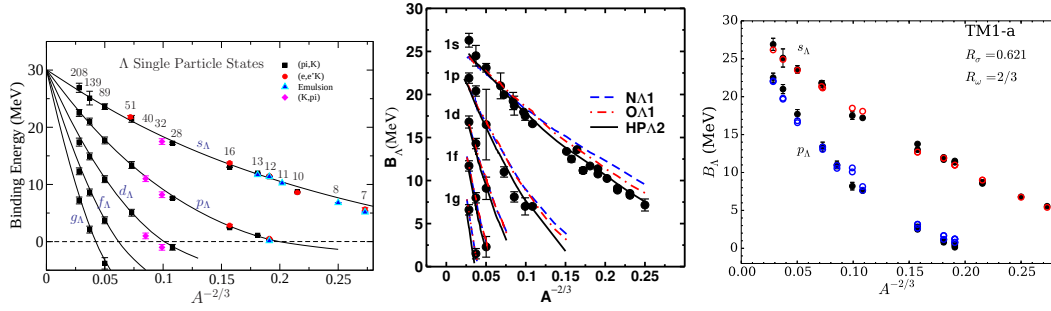


Figure 2.2: Mean-field studies on the Λ hypernuclear spectroscopy. Each panel uses the Woods-Saxon potential (left), the Skyrme-Hartree-Fock method (middle), and the relativistic mean-field model (right). Figures are taken from Refs. [68, 75, 126] from left to right, respectively.

Various spectra from few to heavy Λ hypernuclei have been measured using both emulsion and detector experiments with an accuracy enough to resolve the single-particle spectra. Such Λ hypernuclear spectra are measured by various processes, including e.g. the counter experiments of the (K^-, π^-) reaction at CERN and BNL, the (π^+, K^+) reaction at BNL and KEK, and the $(e, e'K^+)$ reaction at JLab. See reviews [112, 113] for the summary of experiments until early 2000.

Theoretically, thanks to the rich spectroscopic data many works in from various perspectives has been done for extracting information on the in-medium hyperonic force and the in-medium property of Λ . The few-body hypernuclei have been worked with the no-core shell model (NCSM) [114–117] and the Gaussian expansion method [118, 119]. applied for studying the few-body hypernuclei. See review [120] for the recent NCSM study with the baryon forces within chiral EFT. To explain the Λ binding energies of Λ hypernuclei from middle to large mass numbers, mean-field calculations have been employed, including the naïve Woods-Saxon potential [75], the Skyrme-Hartree-Fock methods [121–126] and the relativistic mean-field models [68, 127–129]. They mostly perform the best fit of the parameters therein to the existing experimental Λ binding energies. The G -matrix calculations [58, 65] based on the phenomenological YN force tuned by the YN scattering data and the additional YNN forces to reproduce the Λ hypernuclear spectra.

Chiral effective field theory (chiral EFT) provides a systematic and model-independent approximation of QCD at low densities [130]. The three-body forces were found to cause repulsive Λ single-particle potential at high densities in chiral EFT [59, 64, 65, 86] with the decuplet saturation model for the three-baryon interactions [131]. This Λ single-particle potential, $U_{\Lambda}^{\text{chi3}}$, can prevent Λ from appearing in neutron stars [59] so that we can avoid the hyperon puzzle.

The proposed Λ single-particle potential $U_{\Lambda}^{\text{chi3}}$ should be validated by experiments and observations. In particular, the density dependence [59, 86] and the momentum dependence [86] of the Λ single-particle potential $U_{\Lambda}^{\text{chi3}}$ can be tested with different types of experimental data, such as the observables from heavy-ion collisions and the Λ binding energies of hypernuclei. The up-to-date data of the Λ binding energies in hypernuclei provide useful constraints. The density dependence of the Λ single-particle potential at lower densities $\rho < \rho_0$ can be constrained by the Λ binding energies [74] while the momen-

tum dependence is sensitive to the energy difference between orbitals of the Λ binding energy through the effective mass [132]. The Λ single-particle potentials LY-IV [124] and HPA2 [126] reproduce the Λ binding energy for hypernuclei in a wide range of mass number. Millner *et al.* [74] pointed out that the density dependence at higher densities determined by the best fitting to the Λ binding energy largely depends on the fitting form while the density dependence at lower densities is well constrained. However, the Λ single-particle potential $U_{\Lambda}^{\text{chi3}}$ has a distinct shape from LY-IV and HPA2: it is more attractive at $\rho < \rho_0$ and has a larger value of the effective mass than LY-IV and HPA2. Thus, it should be tested whether $U_{\Lambda}^{\text{chi3}}$ can reproduce the Λ binding energy data. In Ref. [133], the Λ single-particle potential based on the next-to-leading order (NLO) chiral EFT [102, 103] that includes only two-body interaction, has been tested by the G -matrix calculation. This two-body interaction only partially reproduces the observed binding energies for hypernuclei with mass number $A > 12$.

In this work, we verify the Λ single-particle potential repulsive at high densities from chiral EFT by using the Λ binding energy data. The Skyrme-Hartree-Fock method is employed to implement the chiral EFT results.

This chapter is organized as follows. In Sec. 2.2, we introduce the Skyrme-Hartree-Fock method for the Λ hypernuclei. The following Sec. 2.3 performs fitting the Skyrme parameters of Λ to the results based on the baryon forces within chiral EFT. Results of the Λ binding energies are shown in Sec. 2.4, with discussions on the level of reproduction of experimental data with other existing models. A summary is given in Sec. 2.5.

2.2 Skyrme-Hartree-Fock method for Lambda hypernuclei

In this section, we illustrate the Skyrme-Hartree-Fock method for the single- Λ hypernuclei. The wave function of the Λ hypernuclei ${}^A_{\Lambda}Z$ with baryon number A and proton number Z is taken as [121]

$$\Phi_{\text{hyp}} = \phi_{\Lambda} \Phi_{\text{core}}, \quad (2.3)$$

where $\phi_{\Lambda}(\mathbf{r}, \sigma)$ is the single-particle wave function of Λ , with $\mathbf{r}$ and $\sigma = \pm 1/2$ being the spatial and spin coordinates, respectively. The Slater determinant Φ_{core} is constructed from $A - 1$ nucleon single-particle wave functions $\phi_i(\mathbf{r}, q, \sigma)$ with $q = \pm 1/2$ being the isospin. The Skyrme-type baryon-baryon interaction with one Λ hyperon is expressed as

$$V = V^{NN} + V^{\Lambda N} = \sum_{i < j}^{A-1} v^{NN}(\mathbf{r}_i, \mathbf{r}_j) + \sum_{i=1}^{A-1} v^{\Lambda N}(\mathbf{r}_{\Lambda}, \mathbf{r}_i), \quad (2.4)$$

where $\mathbf{r}_i$ and $\mathbf{r}_{\Lambda}$ are the spatial coordinates of the i -th nucleon and Λ , respectively. We use the SLy4 parametrization [134] for the nucleon-nucleon interaction v^{NN} . The following

Λ -nucleon interaction $v^{\Lambda N}$ is assumed as in Refs. [122, 124, 135]:

$$\begin{aligned}
 v^{\Lambda N}(\mathbf{r}_\Lambda, \mathbf{r}_N) = & t_0^\Lambda (1 + x_0^\Lambda P_\sigma) \delta(\mathbf{r}_\Lambda - \mathbf{r}_N) \\
 & + \frac{1}{2} t_1^\Lambda \left[\overleftarrow{\mathbf{k}}^2 \delta(\mathbf{r}_\Lambda - \mathbf{r}_N) + \delta(\mathbf{r}_\Lambda - \mathbf{r}_N) \overrightarrow{\mathbf{k}}^2 \right] \\
 & + t_2^\Lambda \overleftarrow{\mathbf{k}} \delta(\mathbf{r}_\Lambda - \mathbf{r}_N) \cdot \overrightarrow{\mathbf{k}} \\
 & + \frac{3}{8} t_{3,1}^\Lambda (1 + x_{3,1}^\Lambda P_\sigma) \delta(\mathbf{r}_\Lambda - \mathbf{r}_N) \rho_N^{\gamma_1} \left(\frac{\mathbf{r}_N + \mathbf{r}_\Lambda}{2} \right) \\
 & + \frac{3}{8} t_{3,2}^\Lambda (1 + x_{3,2}^\Lambda P_\sigma) \delta(\mathbf{r}_\Lambda - \mathbf{r}_N) \rho_N^{\gamma_2} \left(\frac{\mathbf{r}_N + \mathbf{r}_\Lambda}{2} \right), \quad (2.5)
 \end{aligned}$$

where t_k^Λ , $t_{k,l}^\Lambda$, x_k^Λ , and $x_{k,l}^\Lambda$ are the Skyrme potential parameters. The spin-exchange operator is given by $P_\sigma = (1 + \boldsymbol{\sigma}_\Lambda \cdot \boldsymbol{\sigma}_N)/2$ with $\boldsymbol{\sigma}_\Lambda$ and $\boldsymbol{\sigma}_N$ being the Pauli matrices acting on the spin wave functions of Λ and the nucleon, respectively. The derivatives, $\overleftarrow{\mathbf{k}} = -(\overleftarrow{\nabla}_\Lambda - \overleftarrow{\nabla}_N)/2i$ and $\overrightarrow{\mathbf{k}} = (\overrightarrow{\nabla}_\Lambda - \overrightarrow{\nabla}_N)/2i$, operate on the left- and right-hand sides, respectively, where $\nabla_\Lambda = \partial/\partial\mathbf{r}_\Lambda$, and $\nabla_N = \partial/\partial\mathbf{r}_N$. The symbol $\rho_N(\mathbf{r}_N)$ denotes the nucleon density specified later. The $\rho_N^{\gamma_1}$ term with $\gamma_1 = 1/3$, which is motivated by the expansion in the Fermi momentum ($\propto \rho_N^{1/3}$), is needed for the Skyrme-type Λ single-particle potential to reproduce the G -matrix result of the Λ single-particle potential [124]. The $\rho_N^{\gamma_2}$ term with $\gamma_2 = 2/3$ is added to reproduce the results from chiral EFT [59]. The spin-orbit force is neglected in this work because it is expected to be small from the experimental data [136, 137]. The pairing correlation and charge-symmetry breaking effect are not considered either.

The expectation value of the total energy for the Λ hypernuclei $\mathcal{E}_{\text{hyp}}$ is obtained as

$$\begin{aligned}
 \mathcal{E}_{\text{hyp}} &= \langle \Phi_{\text{hyp}} | T + V | \Phi_{\text{hyp}} \rangle - \mathcal{E}_{\text{c.m.}} \\
 &= \mathcal{E}_N + \mathcal{E}_\Lambda - \mathcal{E}_{\text{c.m.}}, \quad (2.6)
 \end{aligned}$$

where

$$T = T_N + T_\Lambda = - \sum_{i=1}^{A-1} \frac{\hbar^2 \nabla_i^2}{2m_i} - \frac{\hbar^2 \nabla_\Lambda^2}{2m_\Lambda} \quad (2.7)$$

is the kinetic energy operator, and m_i and m_Λ are the masses of the nucleons and Λ , respectively. The energies of $\mathcal{E}_N$ and $\mathcal{E}_\Lambda$ are the summation of contributions from $T_N + V^{NN}$ and $T_\Lambda + V^{\Lambda N}$, respectively. The total energy of the single Λ takes the following form

$$\begin{aligned}
 \mathcal{E}_\Lambda &= \int d^3r H_\Lambda, \quad (2.8) \\
 H_\Lambda &= \frac{\hbar^2}{2m_\Lambda} \tau_\Lambda + a_1^\Lambda \rho_N \rho_\Lambda \\
 &\quad + a_2^\Lambda (\tau_\Lambda \rho_N + \tau_N \rho_\Lambda) - a_3^\Lambda (\rho_\Lambda \cdot \Delta \rho_N) \\
 &\quad + a_4^\Lambda \rho_N^{1+\gamma_1} \rho_\Lambda + a_5^\Lambda \rho_N^{1+\gamma_2} \rho_\Lambda, \quad (2.9)
 \end{aligned}$$

where H_Λ is the energy density of Λ , and the coefficients a_i^Λ are related to the parameters

in the Λ -nucleon interaction $v^{\Lambda N}$ as

$$\begin{aligned} a_1^\Lambda &= t_0^\Lambda \left(1 + \frac{1}{2} x_0^\Lambda \right), & a_2^\Lambda &= \frac{1}{4} (t_1^\Lambda + t_2^\Lambda), & a_3^\Lambda &= \frac{1}{8} (3t_1^\Lambda - t_2^\Lambda), \\ a_4^\Lambda &= \frac{3}{8} t_{3,1}^\Lambda \left(1 + \frac{1}{2} x_{3,1}^\Lambda \right), & a_5^\Lambda &= \frac{3}{8} t_{3,2}^\Lambda \left(1 + \frac{1}{2} x_{3,2}^\Lambda \right). \end{aligned}$$

The densities ρ_N , τ_N , ρ_Λ , and τ_Λ are defined as

$$\rho_N = \sum_{i=1}^{A-1} |\phi_i|^2, \quad \tau_N = \sum_{i=1}^{A-1} |\nabla \phi_i|^2, \quad (2.10)$$

$$\rho_\Lambda = |\phi_\Lambda|^2, \quad \tau_\Lambda = |\nabla \phi_\Lambda|^2. \quad (2.11)$$

The center-of-mass energy $\mathcal{E}_{\text{c.m.}}$ in Eq. (2.6) is approximated by the average of the center-of-mass kinetic operator neglecting the cross terms [121],

$$\mathcal{E}_{\text{c.m.}} \simeq \int d^3r \frac{\hbar^2 (\tau_N + \tau_\Lambda)}{2 [Zm_p + (A - Z - 1)m_n + m_\Lambda]}. \quad (2.12)$$

The Hartree-Fock equations for single-particle wave functions ϕ_i ($i = 1, 2, \dots, A-1, \Lambda$) are derived from the variational equation:

$$\frac{\delta}{\delta \phi_i} \left(\mathcal{E}_{\text{hyp}} - \sum_{i=1}^{A-1} \epsilon_i \int d^3r |\phi_i|^2 - \epsilon_\Lambda \int d^3r |\phi_\Lambda|^2 \right) = 0, \quad (2.13)$$

where ϵ_i is the single-particle energy. Equation (2.13) combined with Eq. (2.6) yields the equation for the single-particle wave function of the i th particle with the baryon type $B_i = p, n, \Lambda$,

$$\left[-\nabla \cdot \left(\frac{\hbar^2}{2m_{B_i}^*} \nabla \right) + V_{B_i}(\mathbf{r}) - i \mathbf{W}_{B_i}(\mathbf{r}) \cdot (\nabla \times \boldsymbol{\sigma}) \right] \phi_i = \epsilon_i \phi_i. \quad (2.14)$$

The first term is the kinetic energy including the effective mass, the second is the single-particle potential, and the third is the spin-orbit potential. The expressions for those terms are specified below.

In this work, we assume spherical symmetry of the hypernuclei as done in Refs. [121–124, 126, 135]. We assign to each i the principle quantum number n , the orbital angular momentum ℓ , the total angular momentum j , and the magnetic quantum number m_j . The single-particle wave function for the i th nucleon with the isospin q is expressed as

$$\phi_i(\mathbf{r}, q) = \frac{R_\alpha(r)}{r} \mathcal{Y}_{ljm_j}(\hat{\mathbf{r}}) \chi_q, \quad \alpha = \{nljq\}, \quad (2.15)$$

where $r = |\mathbf{r}|$, $\hat{\mathbf{r}} = \mathbf{r}/|\mathbf{r}|$, and

$$\mathcal{Y}_{ljm_j}(\hat{\mathbf{r}}) = \sum_{m_l m_s} \langle lm_l 1/2 m_s | jm_j \rangle Y_{lm_l}(\hat{\mathbf{r}}) \chi_{m_s}. \quad (2.16)$$

The symbols χ_q and χ_{m_s} denote the isospin and spin wave functions, respectively, and Y_{lm_l} is the spherical harmonics. The Λ single-particle wave function is similarly written as

$$\phi_\Lambda(\mathbf{r}) = \frac{R_\alpha(r)}{r} \mathcal{Y}_{l_j m_j}(\hat{\mathbf{r}}). \quad (2.17)$$

The Skyrme-Hartree-Fock equation (2.14) is reduced to the equation for the radial wave function R_α :

$$\begin{aligned} -\frac{\hbar^2}{2m_{B_\alpha}^*} R_\alpha''(r) - \frac{d}{dr} \left(\frac{\hbar^2}{2m_{B_\alpha}^*} \right) R_\alpha'(r) + \left[\frac{\hbar^2}{2m_{B_\alpha}^*} \frac{l_\alpha(l_\alpha + 1)}{r^2} + V_{B_\alpha}(r) + \frac{1}{r} \frac{d}{dr} \left(\frac{\hbar^2}{2m_{B_\alpha}^*} \right) \right. \\ \left. + \frac{W_{B_\alpha}}{r} \left(j_\alpha(j_\alpha + 1) - l_\alpha(l_\alpha + 1) - \frac{3}{4} \right) \right] R_\alpha(r) = \epsilon_\alpha R_\alpha(r). \end{aligned} \quad (2.18)$$

Instead of solving this equation, it is enough to solve the differential equation without the first-order derivative by applying the transformation $R_\alpha \rightarrow (\hbar^2/2m_{B_\alpha}^*)^{-1/2} R_\alpha$. This is practically advantageous for applying the Numerov method, making the computation more precise.

Table 2.1: SLy4 [134] parameter set

Parameter	Value
t_0 (MeV fm ³)	-2488.91
t_1 (MeV fm ⁵)	486.82
t_2 (MeV fm ⁵)	-546.39
t_3 (MeV fm ^{3+3γ})	13777.0
x_0	0.834
x_1	-0.344
x_2	-1.000
x_3	1.354
γ	1/6
W_0 (MeV fm ⁵)	123.0

The explicit forms of $m_{B_\alpha}^*$, V_{B_α} , and W_{B_α} for nucleons are expressed as [134]

$$\frac{\hbar^2}{2m_q^*} = \frac{\hbar^2}{2m_q} + \frac{1}{8} [t_1(2 + x_1)t_2(2 + x_2)] + \frac{1}{8} [t_1(2x_1 + 1) - t_1(2x_1 + 1)] \rho_q + a_2^\Lambda \rho_\Lambda. \quad (2.19)$$

$$V_q = V_q^N + V_q^\Lambda + \delta_{q,p} V_{\text{coul}}, \quad (2.20)$$

$$W_q = \frac{1}{2} W_0 \frac{d}{dr} [\rho_N + \rho_q] - \frac{1}{8} (t_1 x_1 + t_2 x_2) J_N(r) + \frac{1}{8} (t_1 - t_2) J_q(r), \quad (2.21)$$

respectively, where

$$\begin{aligned}
 V_q^N = & \frac{1}{2}t_0(2+x_0)\rho_N + \frac{1}{2}t_0(2x_0+1)\rho_q \\
 & + (\gamma+2)\frac{1}{24}t_3(2+x_3)\rho_N^{\gamma+1} \\
 & + \frac{1}{24}t_3(2x_3+1)\left[\gamma\rho_N^{\gamma-1}(\rho_p^2+\rho_n^2)+2\rho_N^\gamma\rho_q\right] \\
 & + \frac{1}{8}(t_1(2+x_1)t_2(2+x_2))\tau_N \\
 & + \frac{1}{8}(t_1(2x_1+1)-t_1(2x_1+1))\tau_q \\
 & - \frac{1}{16}(3t_1(2+x_1)-t_2(2+x_2))\Delta\rho_N \\
 & - \frac{1}{16}(3t_1(2x_1+1)+t_2(2x_2+1))\Delta\rho_q \\
 & - \frac{1}{2}W_0[\nabla\cdot\mathbf{J}_N+\nabla\cdot\mathbf{J}_q], \tag{2.22}
 \end{aligned}$$

$$\begin{aligned}
 V_q^\Lambda = & a_1^\Lambda\rho_\Lambda + a_2^\Lambda\tau_\Lambda - a_3^\Lambda\Delta\rho_\Lambda \\
 & + (1+\gamma_1)a_4^\Lambda\rho_N^{\gamma_1}\rho_\Lambda + (1+\gamma_2)a_5^\Lambda\rho_N^{\gamma_2}\rho_\Lambda, \tag{2.23}
 \end{aligned}$$

$$V_{\text{coul}} = e^2 \int d^3r' \frac{\rho_p(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} - e^2 \left(\frac{3\rho_p}{\pi}\right)^{1/3}. \tag{2.24}$$

The values of the parameters t_i , x_i , γ , and W_0 are given by the SLy4 parameter set shown in Table 2.1. The isospins $q = \pm 1/2$ specify the neutron and proton, respectively. The density ρ_q and the kinetic density τ_q are given by Eq. (2.10) while the spin density $\mathbf{J}_q$ is written as

$$\mathbf{J}_q(\mathbf{r}) = -i \sum_i \phi_i^*(\mathbf{r}, q) \nabla \phi_i(\mathbf{r}, q) \times \langle \sigma' | \boldsymbol{\sigma} | \sigma \rangle. \tag{2.25}$$

The nucleon density, the kinetic density, and the spin density are defined as $\rho_N = \rho_p + \rho_n$, $\tau_N = \tau_p + \tau_n$, and $\mathbf{J}_N = \mathbf{J}_p + \mathbf{J}_n$, respectively.

For Λ , the effective mass, single-particle potential, and the spin-orbit potential are expressed as

$$\frac{\hbar^2}{2m_\Lambda^*} = \frac{\hbar^2}{2m_\Lambda} + a_2^\Lambda\rho_N, \tag{2.26}$$

$$V_\Lambda = a_1^\Lambda\rho_N + a_2^\Lambda\tau_N - a_3^\Lambda\Delta\rho_N + a_4^\Lambda\rho_N^{1+\gamma_1} + a_5^\Lambda\rho_N^{1+\gamma_2}. \tag{2.27}$$

In the case of the open-shell nuclei, we employ the filling approximation [138]: when there are m nucleons in the open shell, they are filled in the highest j states at the same occupation probability $m/(2j+1)$. Similarly, Λ occupies each state of j with an occupation probability $1/(2j+1)$. Then, the Λ density is calculated as

$$\rho_\Lambda(r) = \frac{1}{2j+1} \sum_{m_j} \left| \frac{R_\alpha(r)}{r} \mathcal{Y}_{ljm_j} \right|^2 = \frac{R_\alpha^2(r)}{4\pi r^2}. \tag{2.28}$$

After solving Eq. (2.14) self-consistently, the Λ binding energy is obtained as

$$B_\Lambda = -(\mathcal{E}_{\text{hyp}} - \mathcal{E}_{\text{core}}), \quad (2.29)$$

where the total energy of the core nucleus $\mathcal{E}_{\text{core}}$ is independently calculated by solving the self-consistent equation for the Slater determinant of the core nucleus.

The Λ binding energy data that compared with the calculated values in this study are summarized in Table 2.2. The data measured in the (π^+, K^+) experiments listed in Ref. [112] (${}^{16}_\Lambda\text{O}$, ${}^{28}_\Lambda\text{Si}$, ${}^{51}_\Lambda\text{V}$, ${}^{139}_\Lambda\text{La}$, ${}^{208}_\Lambda\text{Pb}$) are here modified by 0.5 MeV according to Gogami *et al.* [139]. We note that the spectra corresponding to the $1d$ orbital of ${}^{51}_\Lambda\text{V}$ has no single peak but the double-humped shape, and we take the mean value of the two peaks.

Table 2.2: Experimental data of Λ binding energy for various hypernuclei used in this work.

Hypernuclei	Λ binding energy (MeV)				
	$1s$	$1p$	$1d$	$1f$	$1g$
${}^{16}_\Lambda\text{O}$ [112]	12.92 ± 0.35	2.35 ± 0.05			
${}^{28}_\Lambda\text{Si}$ [112]	17.1 ± 0.2	7.5 ± 0.2			
${}^{32}_\Lambda\text{S}$ [140]	17.5 ± 0.5				
${}^{40}_\Lambda\text{Ca}$ [110]	18.7 ± 1.1				
${}^{51}_\Lambda\text{V}$ [111, 112]	20.47 ± 0.13	11.77 ± 0.16	3.05 ± 0.13		
${}^{56}_\Lambda\text{Fe}$ [140]	21.0 ± 1.0				
${}^{89}_\Lambda\text{Y}$ [112]	23.6 ± 0.5	17.0 ± 1.0	9.6 ± 1.3	2.8 ± 1.2	
${}^{139}_\Lambda\text{La}$ [112]	25.0 ± 1.2	20.9 ± 0.6	14.8 ± 0.6	8.5 ± 0.6	2.0 ± 0.6
${}^{208}_\Lambda\text{Pb}$ [112]	26.8 ± 0.8	22.4 ± 0.6	17.3 ± 0.7	12.2 ± 0.6	7.1 ± 0.6

2.3 Fitting to chiral effective field theory results

In this section, we parametrize the Skyrme-type Λ single-particle potential based on the baryon interactions within chiral EFT. In uniform matter, the single-particle wave function of Λ is represented by the plane wave $\propto e^{ik_\Lambda \mathbf{r}}$ with the Λ momentum k_Λ . Its kinetic density τ_Λ reads

$$\tau_\Lambda = |\nabla \phi_\Lambda|^2 = k_\Lambda^2 |\phi_\Lambda|^2 = k_\Lambda^2 \rho_\Lambda. \quad (2.30)$$

Then, the Λ single-particle potential is obtained as

$$\begin{aligned} U_\Lambda(\rho_N, k_\Lambda) &= \frac{\delta}{\delta \rho_\Lambda} \left[H_\Lambda - \frac{\hbar^2}{2m_\Lambda} \tau_\Lambda \right] \\ &= a_1^\Lambda \rho_N + a_2^\Lambda (k_\Lambda^2 \rho_N + \tau_N) + a_4^\Lambda \rho_N^{1+\gamma_1} + a_5^\Lambda \rho_N^{1+\gamma_2}, \end{aligned} \quad (2.31)$$

with

$$\tau_N = \tau_p + \tau_n, \quad (2.32)$$

$$\tau_q = \frac{3}{5} (3\pi^2)^{2/3} \rho_q^{5/3}, \quad q = p, n, \quad (2.33)$$

$$\gamma_1 = 1/3, \quad \gamma_2 = 2/3. \quad (2.34)$$

The Skyrme potential parameters of Λ , a_i^Λ , are determined by the fitting to the G -matrix results from chiral baryon interactions by Gerstung, Kaiser, and Weise (GKW) [59] and by Kohno [86].

We first fix the momentum-dependent part of the Λ single-particle potential, proportional to a_2^Λ , at the saturation density $\rho_0 = 0.16 \text{ fm}^{-3}$ by using the results by Kohno [86]. Figure 2.3 shows the momentum dependence of the Λ single-particle potential. Kohno3 in Fig. 2.3 is obtained by including the YNN three-body forces using the decuplet saturation model [131].¹ Kohno2 in Fig. 2.3 is evaluated using only the two-body force. Because the depth of the Λ single-particle potential at ρ_0 differs between GKW [59] and Kohno [86], we subtract the value at $k_\Lambda = 0 \text{ fm}^{-1}$ from the Λ single-particle potential as $U_\Lambda(k_\Lambda) - U_\Lambda(k_\Lambda = 0 \text{ fm}^{-1})$ and use the value for fitting the momentum dependence of the Λ single-particle potential. Chi2 and Chi3 in Fig. 2.3 are the fitting results of Kohno2 and Kohno3 at $k_\Lambda < 1.5 \text{ fm}^{-1}$, respectively.

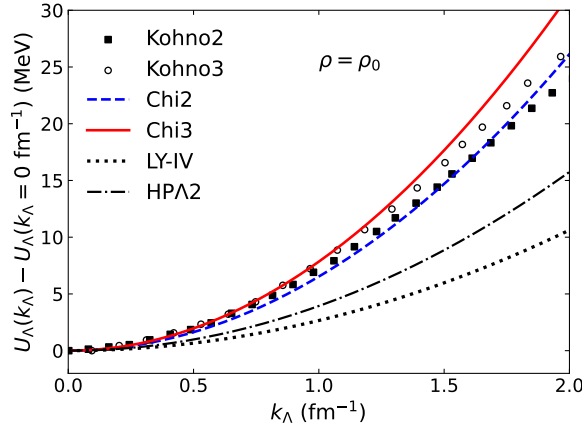


Figure 2.3: Momentum dependence of the Λ single-particle potentials in symmetric nuclear matter at the saturation density with its value at zero momentum subtracted. Kohno2 and Kohno3 represent the results of the Λ single-particle potential with only two-body interactions and two- and three-body interactions [86] from chiral EFT [86], respectively. Solid and dashed lines represent the fitting results to Kohno2 and Kohno3, respectively. The dotted and dash-dotted lines correspond to the Λ single-particle potentials LY-IV [124] and HPA2 [126], respectively.

For the Skyrme-potential parameters associated with the density dependence ($a_1^\Lambda, a_4^\Lambda, a_5^\Lambda$), we consider the density dependence of the Λ single-particle potential obtained from chiral EFT [59]. In Fig. 2.4, GKW2 and GKW3 are the results from chiral EFT with the YN and $YN + YNN$ forces, respectively. The YNN three-body interaction in GKW3 is calculated by the decuplet saturation model [131]. The parameters a_1^Λ , a_4^Λ , and a_5^Λ of Chi2 (Chi3) are obtained by fitting Eq. (2.31) to the upper and lower lines of GKW2 (GKW3) and then by taking their average.

Unlike our previous work [88] on heavy-ion collisions, where we fitted the data in the region $\rho/\rho_0 < 3.5$, the fitting region in the present study is limited to $\rho/\rho_0 < 1.5$ in order to reproduce the GKW results in the low-density region more accurately. The resultant

¹The decuplet-saturated three-body force will be elaborated in Chapter 6

Skyrme parameters a_1^Λ , a_2^Λ , a_4^Λ , and a_5^Λ from the fitting are listed in Table 2.3. The fitting results accurately reproduce the chiral EFT results at $\rho/\rho_0 \lesssim 2.0$, which is relevant for calculating the Λ hypernuclei.

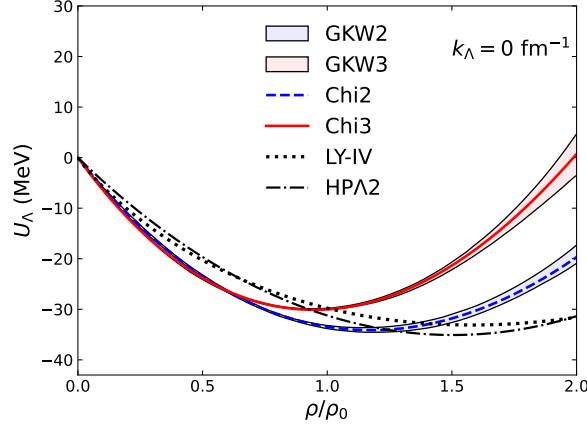


Figure 2.4: Normalized baryon density dependence of the single-particle potentials for Λ in symmetric nuclear matter. GKW2 and GKW3 represent the results of the Λ single-particle potential with only two-body interactions and with two- and three-body interactions obtained from the chiral EFT [59], respectively. The solid and dashed lines represent the fitting results to GKW2 and GKW3, respectively. The dotted and dash-dotted lines correspond to the Λ single-particle potentials LY-IV [124] and HPA2 [126], respectively.

For comparison, the Λ single-particle potentials that are used to explain the Λ binding energy data (LY-IV [124] and HPA2 [126]) are shown in Figs. 2.3 and 2.4. The LY-IV and HPA2 Λ single-particle potentials exhibit different characteristics compared to the single-particle potentials by chiral EFT. Specifically, they display enhanced repulsion in low-density regions, increased attraction in high-density regions, and a weaker momentum dependence. Such a attractive behavior in the density dependence result in the appearance of the Λ hyperons for lower density region in neutron star matter, causing the hyperon puzzle.

The Skyrme parameter a_3^Λ associated with the derivative of the density cannot be determined by fitting the results in the uniform matter. We choose a_3^Λ as a fitting parameter to reproduce the experimental value of the Λ binding energy of $^{13}_\Lambda\text{C}$, 11.88 MeV. The experimental value is taken from Ref. [112] with a correction of 0.5 MeV, which is pointed out in Ref. [139]. There are two reasons for choosing $^{13}_\Lambda\text{C}$: First, it has a larger surface-energy effect compared with a heavier nucleus. Second, the spherical Skyrme-Hartree-Fock method is expected to provide a relatively good description of $^{13}_\Lambda\text{C}$ because it has even numbers of protons and neutrons.

We show in Table 2.3 the Taylor coefficients and the normalized effective mass at ρ_0 ,

Table 2.3: The sets of Λ Skyrme-potential parameters. Chi2 and Chi3 are the fitting results to the chiral EFT calculations [59, 86]. LY-IV [124] and HPA2 [126] are the Λ single-particle potentials, which can explain the Λ binding energy data. We also list the values that characterize potentials: the Taylor coefficients (J_Λ , L_Λ , K_Λ) and the normalized effective mass m_Λ^*/m_Λ at ρ_0 defined by Eqs. (2.35)–(2.38). The mean squared deviation of the calculated Λ binding energy from the experimental data ΔB_Λ is defined by Eq. (2.39).

	Chi2	Chi3	LY-IV	HPA2
t_0^Λ (MeV fm ³)	−352.2	−388.3	−542.5	−399.9
t_1^Λ (MeV fm ⁵)	143.7	120.4	56.0	83.4
t_2^Λ (MeV fm ⁵)	13.7	68.7	8.0	11.5
$t_{3,1}^\Lambda$ (MeV fm ⁴)	−951.9	−1081.8	1387.9	2046.8
$t_{3,2}^\Lambda$ (MeV fm ⁵)	2669	3351	0	0
x_0^Λ	0	0	−0.153	−0.486
$x_{3,1}^\Lambda$	0	0	0.107	−0.660
$x_{3,2}^\Lambda$	0	0	0	0
γ_1	1/3	1/3	1/3	1
γ_2	2/3	2/3	0	0
J_Λ (MeV)	−33.5	−30.0	−29.8	−31.2
L_Λ (MeV)	−23.5	9.3	−36.2	−46.1
K_Λ (MeV)	415	532	218	277
m_Λ^*/m_Λ	0.73	0.70	0.87	0.82
ΔB_Λ (MeV)	1.55	0.72	0.71	0.78

which characterize the Λ single-particle potential:

$$J_\Lambda = U_\Lambda(\rho_N = \rho_0, k_\Lambda = 0), \quad (2.35)$$

$$L_\Lambda = 3\rho_N \left. \frac{\partial U_\Lambda}{\partial \rho_N} \right|_{\rho_N = \rho_0, k_\Lambda = 0}, \quad (2.36)$$

$$K_\Lambda = 9\rho_N^2 \left. \frac{\partial^2 U_\Lambda}{\partial \rho_N^2} \right|_{\rho_N = \rho_0, k_\Lambda = 0}, \quad (2.37)$$

$$\left. \frac{m_\Lambda^*}{m_\Lambda} \right|_{\rho_N = \rho_0} = \frac{1}{1 + \frac{2m_\Lambda}{\hbar^2} a_2^\Lambda \rho_0}. \quad (2.38)$$

We note that the value of the curvature K_Λ around ρ has no relation with the so-called incompressibility of the nuclear matter EOS. The Λ single-particle potential corresponds to the single-hyperon property in nuclear matter, but the EOS of nuclear matter is the property of the matter itself.

2.4 Results

We now present the results of the Skyrme-Hartree-Fock calculations for Λ hypernuclei using the Λ Skyrme parameters discussed in the previous section. Figure 2.5 shows the Λ single-particle potential (2.27) for hypernucleus ${}^{208}_{\Lambda}\text{Pb}$. At a distance $r < 4$ fm where the nucleon density ρ_N is close to the saturation density ρ_0 , both Chi3 and LY-IV have the potential depth of -30 MeV while Chi2 has a slightly greater depth of -33 MeV. Those values reflect J_Λ , the Λ -potential depth at ρ_0 (see Table 2.3). In the surface region around $r = 6$ fm, LY-IV is more attractive compared to the Chi3 potential, reflecting the behavior of the density dependence of U_Λ at $\rho < \rho_0$.

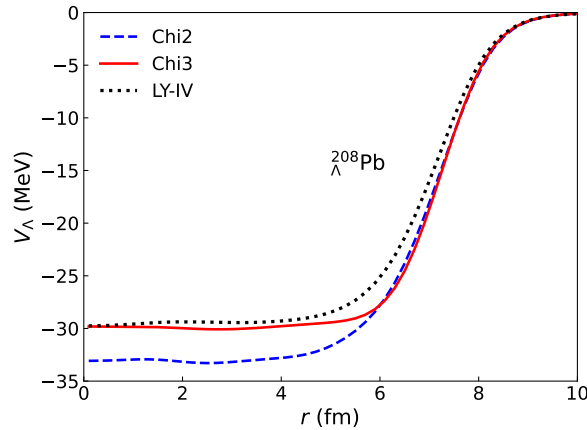


Figure 2.5: Λ single-particle potential (2.27) for hypernucleus ${}^{208}_{\Lambda}\text{Pb}$ in the coordinate space. The dashed and solid lines show the results from the Λ single-particle potentials Chi2 and Chi3, respectively. The dotted line corresponds to the result from the LY-IV parameter sets.

Figure 2.6 compares the Λ binding energies within mass number $A = 13$ – 208 in $1s$, $1p$, $1d$, $1f$, and $1g$ orbitals, calculated with various Λ single-particle potentials. Chi3,

which includes the YNN three-body force, reproduces the data. This implies that the strong repulsive Λ single-particle potential, which is sufficient to suppress the presence of Λ hyperons in dense nuclear matter, is consistent with the observed Λ hypernuclear data. Contrary, Chi2, which includes only the YN two-body force, predicts the overbinding of the data in the $1s$ orbital. This is because its $U_\Lambda(\rho_0, k=0)$ is as deep as approximately -33 MeV. We note that Chi2 and Chi3 have almost the same effective mass. These results indicate the necessity of three-baryon interaction to reproduce the binding energy of Λ , as claimed in other non-relativistic models [58, 141, 142].

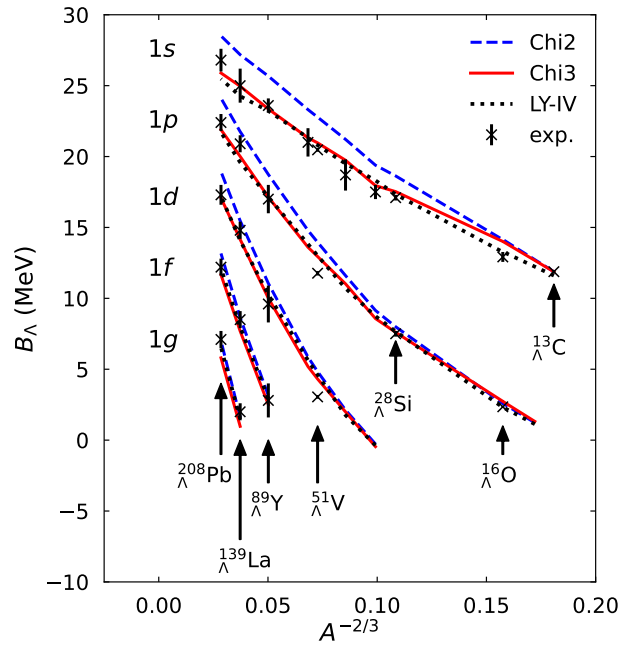


Figure 2.6: Calculated Λ binding energies of $1s$, $1p$, $1d$, $1f$, and $1g$ orbitals as a function of $A^{-2/3}$, where A is the mass number of hypernuclei. Chi2 and Chi3 represent the results using the Λ single-particle potential with only two-body and with two- and three-body interactions from chiral EFT, respectively. The dotted line corresponds to the result using the LY-IV Λ single-particle potential. The crosses indicate the experimental data.

The last row of Table 2.3 shows the root-mean-square deviation of the model calculation from the experimental data,

$$\Delta B_\Lambda = \sqrt{\frac{1}{N_{\text{data}}} \sum_i^{N_{\text{data}}} (B_{\Lambda,i}^{\text{exp}} - B_{\Lambda,i}^{\text{cal}})^2}, \quad (2.39)$$

where N_{data} is the number of the experimental data. The Λ binding energies $B_{\Lambda,i}^{\text{exp}}$ ($B_{\Lambda,i}^{\text{cal}}$) of nuclide i correspond to the experimental value and calculated values, respectively. The experimental data $B_{\Lambda,i}^{\text{exp}}$ are listed in Table 2.2. We avoid using the chi squared because it is hard to quantify the systematic error of our mean-field calculation. In addition, neglecting

the model error in the chi-square would cause an excessive fitting to lighter hypernuclei, for which the experimental data has a smaller error, while the mean-field calculation would have a larger error. Chi3 is found to be consistent with the experimental data at the same level of accuracy as the Λ single-particle potentials LY-IV, with the value of $\Delta B_\Lambda \approx 0.7$ MeV. We note that the HPA2 potential has a larger $\Delta B_\Lambda = 0.78$ MeV because it is parametrized by chi-square minimization using the data without the correction of 0.5 MeV [139].

2.5 Short summary

We have examined the Λ single-particle potentials using the binding energies of Λ in hypernuclei within the Skyrme-Hartree-Fock method with spherical symmetry. It is found that the Λ single-particle potential from chiral EFT [59, 86] with two- and three-body force reproduces the experimental Λ binding energy data at the same level of accuracy as LY-IV [124] potential while chiral EFT with only two-body force overestimates the Λ binding energy. Thus, the chiral EFT Λ single-particle potential that suppresses Λ in dense neutron star matter is consistent with the Λ binding energy. Taken together with our previous work [88], we conclude that the Λ -suppressed scenario is consistent with the Λ directed flow data of heavy-ion collisions and the Λ binding energy data of hypernuclei.

To precisely determine the Λ single-particle potential at high densities, it would also be important to investigate other experimental data in future works. For example, the observables in heavy-ion collisions may be sensitive to the Λ single-particle potential at high densities: the elliptic flow including the centrality dependence and nuclear cluster production can be affected by the Λ single-particle potential. Also, the hypertriton ${}^3_\Lambda\text{H}$ directed flow, which was recently measured by the STAR Collaboration [143], would be a promising future work. For another example, the value of K_Λ could be constrained by the excitation spectra of the breathing mode for Λ hypernuclei [144] in the same way as the incompressibility of nuclear matter, K . The Λ hypernuclei with large surface, such as neutron-rich hypernuclei, might also give constraints on the Λ single-particle potential.

Chapter 3

Uncertainty estimate using Lambda hypernuclei

We have confirmed that both repulsive and attractive type Λ single-particle potentials are consistent with experimental data. In this Chapter, we examine the uncertainty range of the Λ single-particle potential by using the Λ binding energy data, for imposing a boundary condition around ρ_0 . By assigning Skyrme parameters in a systematic way and calculating Λ binding energies from medium to heavy hypernuclei, we extract the Λ single-particle potentials well reproducing the experimental Λ binding energies. Our analysis shows that the degree of attraction above the saturation density is constrained. Furthermore, the momentum dependence of the Λ single-particle potential, or the Λ effective mass, is also constrained. These constraints are expected to be important for applications to dynamical systems, such as heavy-ion collisions. We also apply the extracted Λ single-particle potentials to neutron star matter.

3.1 Uncertainty estimate for Lambda single-particle potentials

In the previous Chapter, we showed that both the repulsive and attractive Λ single-particle potentials at high densities, Chi3 and LY-IV, reproduce the Λ binding energy data. This is because the Λ binding energy is mostly determined by the potential values at low densities. However, another attractive potential, HPA2, does not reproduce the data at the same level as Chi3 and LY-IV, which suggests that the details of the potential, including the values of the Taylor coefficients are important for the Λ binding energy. In this section, we will investigate the parameter space of the Λ potential that can reproduce the binding energy of the Λ hypernuclear data. From this analysis, we expect constraints on the relations among the Taylor coefficients (2.35)–(2.37) and the normalized effective mass (2.38).

3.1.1 Procedure

The symmetry-energy parameters of nuclear matter are used to examine the nuclear matter EOS at high densities using the behavior around ρ_0 [9]. Similarly, to investigate

the parameter space of the Λ single-particle potential, we parametrize the Skyrme-type Λ single-particle potential (2.31) by the Taylor coefficients (2.35)–(2.37) and the normalized effective mass m_Λ^*/m_Λ at the saturation density. We generate $13 \times 25 \times 21 \times 21 = 143325$ parameter sets of $(J_\Lambda, L_\Lambda, K_\Lambda, m_\Lambda^*/m_\Lambda)$ as combinations of the following parameter points:

$$J_\Lambda = -33, -31.5, -32, \dots, -27 \text{ MeV}, \quad (3.1a)$$

$$L_\Lambda = -50, -45, -40, \dots, 70 \text{ MeV}, \quad (3.1b)$$

$$K_\Lambda = 0, 50, 100, \dots, 1000 \text{ MeV}, \quad (3.1c)$$

$$\frac{m_\Lambda^*}{m_\Lambda} = 0.5, 0.525, 0.55, \dots, 1.00. \quad (3.1d)$$

The range of J_Λ is chosen to be consistent with the Λ binding energy data of the $1s$ orbital: J_Λ below the range overestimates the data, and vice versa. The effective mass m_Λ^*/m_Λ has upper and lower limits because it is sensitive to the separation of the Λ binding energies between different orbitals. The second derivative K_Λ has a lower limit of 0 MeV because the Λ single-particle potential should become repulsive at high densities. We note that these parameters in the existing models shown in Table 2.3 fall within the above parameter ranges.

For each parameter set of $(J_\Lambda, L_\Lambda, K_\Lambda, m_\Lambda^*/m_\Lambda)$, the Skyrme potential parameters a_i^Λ are determined in the following procedure. First, a_2^Λ is determined from its relation to the effective mass (2.38):

$$a_2^\Lambda = \frac{1}{\rho_0} \frac{\hbar^2}{2m_\Lambda} \left(\frac{1}{m_\Lambda^*/m_\Lambda} - 1 \right). \quad (3.2)$$

The potential parameters a_1^Λ , a_4^Λ , and a_5^Λ are determined by solving the following relations obtained from Eqs. (2.35)–(2.37) with $\gamma_1 = 1/3$ and $\gamma_2 = 2/3$:

$$J_\Lambda = a_1^\Lambda \rho_0 + a_2^\Lambda \tau_0 + a_4^\Lambda \rho_0^{4/3} + a_5^\Lambda \rho_0^{5/3}, \quad (3.3a)$$

$$L_\Lambda = 3a_1^\Lambda \rho_0 + 5a_2^\Lambda \tau_0 + 4a_4^\Lambda \rho_0^{4/3} + 5a_5^\Lambda \rho_0^{5/3}, \quad (3.3b)$$

$$K_\Lambda = 10a_2^\Lambda \tau_0 + 4a_4^\Lambda \rho_0^{4/3} + 10a_5^\Lambda \rho_0^{5/3}, \quad (3.3c)$$

where $\tau_0 = 3/5(3\pi^2/2)^{2/3}\rho_0^{5/3}$. The remaining parameter a_3^Λ is determined to reproduce the experimental value of the Λ binding energy of ^{13}C , 11.88 MeV, which is taken from Ref. [112] with a correction of 0.5 MeV [139].

Using the determined potential parameters a_i^Λ for each parameter set, we calculate the Λ binding energy using the Skyrme-Hartree-Fock method as explained in Sec. 2.2. We evaluate the root-mean-square deviation ΔB_Λ (2.39) and select the parameter sets that satisfy $\Delta B_\Lambda < 0.75$ MeV. We hereafter call the Λ single-particle potentials with the selected parameter sets *selected* Λ single-particle potentials and the others *rejected* Λ single-particle potentials.

3.1.2 Results

We show the density dependence of the Λ single-particle potentials in the upper panel of Fig. 3.1, and the momentum dependence of the Λ single-particle potentials subtracting

their values at $k_\Lambda = 0 \text{ fm}^{-1}$ in the lower panel of Fig. 3.1. The bold red lines indicate the selected Λ single-particle potentials. We found that the density dependence at high densities $\rho > \rho_0$ spreads more widely compared to the low-density region $\rho < \rho_0$. Namely, the Λ binding energy data constrain the Λ single-particle potential in the low-density region better than in the high-density region. We note that the spread of our results at $\rho < \rho_0$ is larger than that observed in Ref. [74]. This is because we considered the finite size of the uncertainty range of the Λ single-particle potential parameters, while Ref. [74] only shows the best fitting results by several functional forms.

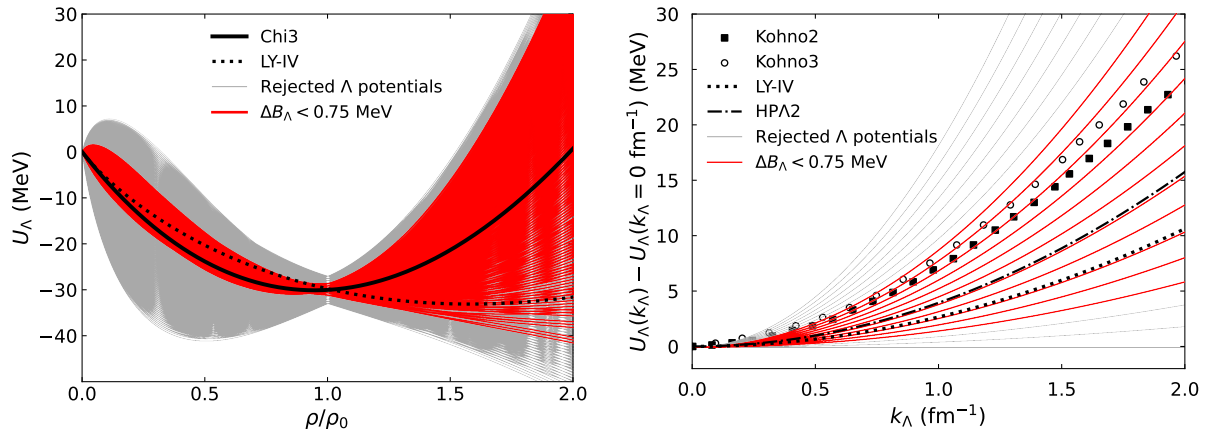


Figure 3.1: The left panel shows the density dependence in symmetric nuclear matter, while the right panel shows the momentum dependence of the Λ single-particle potentials. The thin gray lines represent all generated Λ single-particle potentials, while the bold red lines are the selected Λ single-particle potentials with $\Delta B_\Lambda < 0.75 \text{ MeV}$. Chi2 and Chi3 represent the result using the Λ single-particle potential with only two-body interactions and with two- and three-body interactions from the chiral EFT, respectively. The dotted line corresponds to the result using the LY-IV Λ single-particle potential.

In contrast to the density dependence of the Λ single-particle potentials, the magnitude of the momentum dependence has upper and lower limits, as shown in the lower panel of Fig. 3.1. The LY-IV and HPA2 potentials lie around the lower limit while the Chi3 potentials are close to the upper limit. The sensitivity of the energy separation between orbitals to the effective mass [132] constrains the momentum dependence of the Λ single-particle potential.

Let us now examine the correlations among the parameters of the selected Λ single-particle potentials. For this purpose, we choose two parameters from $(J_\Lambda, L_\Lambda, K_\Lambda, m_\Lambda^*/m_\Lambda)$ and plot ΔB_Λ as a function of the chosen parameters. We consider several choices of the two parameters as shown in Fig. 3.2. In calculating ΔB_Λ , the other parameters are optimized to minimize ΔB_Λ by using the golden-section search [145] within the ranges in Eqs. (3.1). For example, for the J_Λ - m_Λ^*/m_Λ plot in the top left panel of Fig. 3.2, the remaining parameters, L_Λ and K_Λ , are optimized for each point of $(J_\Lambda, m_\Lambda^*/m_\Lambda)$.

The top left panel of Fig. 3.2 shows ΔB_Λ as functions of J_Λ and m_Λ^*/m_Λ . The selected parameter sets are within the ranges of $-31.5 < J_\Lambda < -28 \text{ MeV}$ and $0.65 < m_\Lambda^*/m_\Lambda < 0.95$. The potential depth J_Λ is consistent with the well-known results by the Woods-Saxon

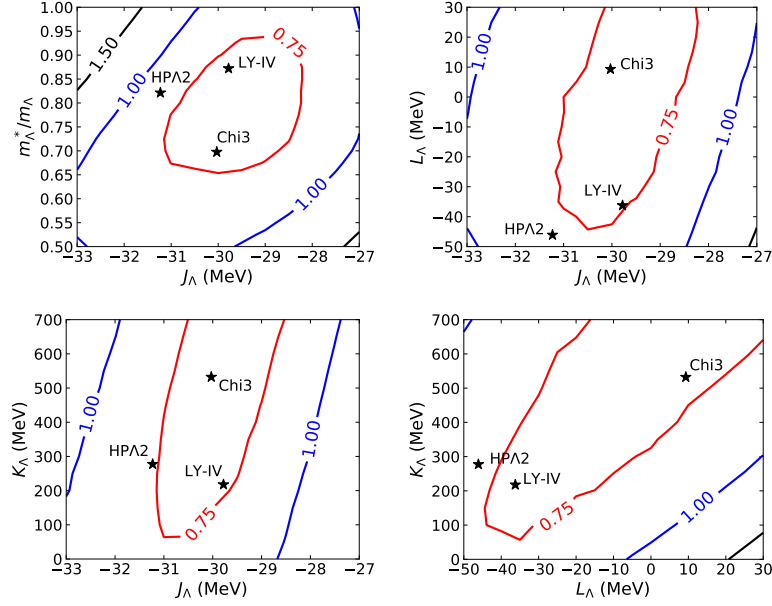


Figure 3.2: Contour plot of the root-mean-square deviation ΔB_Λ of the calculated Λ binding energy from the experimental data. The stars indicate the points corresponding to the two parameters of Chi3, LY-IV, and HPA2. Note that the other parameters of those specific models do not necessarily match the optimized ones. Top left panel: ΔB_Λ as a function of the depth of the Λ single-particle potential at the saturation density, J_Λ , and the normalized effective mass m_Λ^*/m_Λ . The first and second derivatives, L_Λ and K_Λ , are optimized for minimizing ΔB_Λ for each point of $(J_\Lambda, m_\Lambda^*/m_\Lambda)$ by using the golden-section search. Top right panel: ΔB_Λ as a function of the depth of the Λ single-particle potential, J_Λ , and the first derivative L_Λ , where $(K_\Lambda, m_\Lambda^*/m_\Lambda)$ are optimized. Bottom left panel: ΔB_Λ as a function of the depth of Λ single-particle potential J_Λ and the second derivative K_Λ , where $(L_\Lambda, m_\Lambda^*/m_\Lambda)$ are optimized. Bottom right panel: ΔB_Λ as a function of the first derivative L_Λ and the second derivative K_Λ , where $(J_\Lambda, m_\Lambda^*/m_\Lambda)$ are optimized.

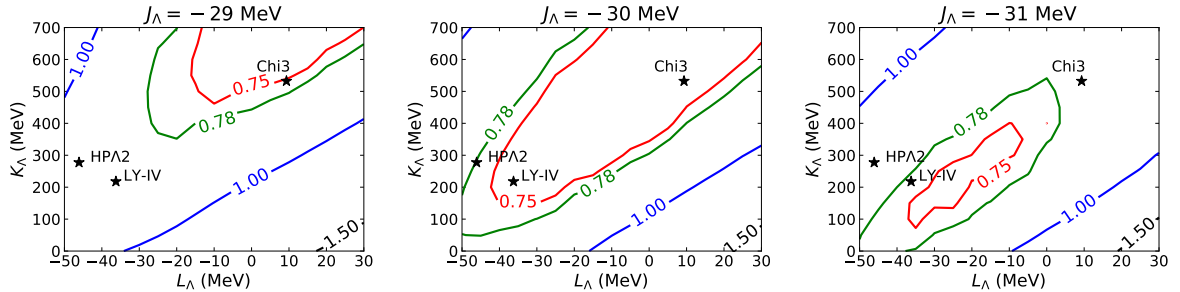


Figure 3.3: Same as the bottom right panel of Fig. 3.2, but for three different fixed values of J_Λ . The normalized effective mass m_Λ^*/m_Λ is optimized for minimizing ΔB_Λ for each $(J_\Lambda, L_\Lambda, K_\Lambda)$ by using the golden-section search.

potential [74, 75]. A positive correlation between J_Λ and m_Λ^*/m_Λ is found because the contribution of the kinetic-energy term $-\hbar^2\nabla_\Lambda^2/2m_\Lambda^*$ increases as the normalized effective mass m_Λ^*/m_Λ decreases. The HPA2 potential is located outside of the region of the selected Λ single-particle potentials because HPA2 was originally constructed through the chi-square minimization using the data without the 0.5 MeV correction [139]. In the top right panel of Fig. 3.2, we show ΔB_Λ as functions of J_Λ and L_Λ . The region of the selected potentials is bounded from below, $L_\Lambda \geq -45$ MeV because the experimental data of the Λ binding energy cannot be reproduced with the Λ single-particle potential that is too shallow at $\rho < \rho_0$. In the bottom left panel of Fig. 3.2, we show ΔB_Λ as functions of J_Λ and K_Λ . The condition $\Delta B_\Lambda < 0.75$ MeV is satisfied for $K_\Lambda \geq 50$ MeV. There is no upper limit in K_Λ , which implies that the Λ single-particle potential can be even more repulsive than the ones covered in this parameter search. In the bottom right panel of Fig. 3.2, we show ΔB_Λ as functions of L_Λ and K_Λ . For the region with $\Delta B_\Lambda < 0.75$ MeV, there is a positive correlation between L_Λ and K_Λ so that the effects of the two parameters compensate for each other at low densities at which the Λ single-particle potential is constrained. Larger values of L_Λ make the potential deeper at $\rho < \rho_0$ while larger values of K_Λ make it shallower. Nevertheless, the uncertainty region with $\Delta B_\Lambda < 0.75$ MeV is not small enough, i.e., the sizes of the region are about 40 and 400 MeV for L_Λ and K_Λ , respectively. This reflects the fact that the Λ single-particle potential at $\rho < \rho_0$ is not sufficiently limited to discriminate Chi3 from LY-IV.

It should be noted that the ΔB_Λ values of the specific models (e.g., LY-IV) in Table. 2.3 do not necessarily match ΔB_Λ at their locations in Figs. 3.2 and 3.3. This is because the other parameters (i.e., optimized parameters and a_3^Λ) are different from the parameters in the specific model. The ΔB_Λ value depends on $(J_\Lambda, L_\Lambda, K_\Lambda, m_\Lambda^*/m_\Lambda, a_3^\Lambda)$, but only two of them are the same with the specific models in each panel of Figs. 3.2 and 3.3. For example, in the top left panel of Fig. 3.2, the other parameters $(K_\Lambda, L_\Lambda, a_3^\Lambda)$ are determined for each pair of $(J_\Lambda, m_\Lambda^*/m_\Lambda)$ so that they minimize ΔB_Λ while keeping the Λ binding energy of $^{13}_\Lambda\text{C}$, $B_\Lambda(^{13}_\Lambda\text{C})$. However, such a set of parameters does not necessarily match the parameters of the specific models, which are determined by different criteria. Specifically, we use the up-to-date value of $B_\Lambda(^{13}_\Lambda\text{C}) = 11.88$ MeV in our work to fix the value of a_3^Λ , while $B_\Lambda(^{13}_\Lambda\text{C}) = 11.69$ MeV is used in LY-IV [124]. It should also be noted that the ΔB_Λ value at the location of a specific model (e.g., LY-IV) differs for each panel because the optimized parameter sets at the LY-IV locations are different for different panels. Only two parameters are matched to LY-IV in each panel, and the combination of the two parameters differs for each panel.

We show ΔB_Λ for three different values of J_Λ as functions of L_Λ and K_Λ in Fig. 3.3, where the parameter m_Λ^*/m_Λ is optimized to minimize ΔB_Λ for each $(J_\Lambda, L_\Lambda, K_\Lambda)$ employing the golden-section search. For $J_\Lambda = -29$ MeV, we see that the parameter region of the Λ single-particle potentials with small ΔB_Λ have larger L_Λ and K_Λ compared to the case of unconstrained J_Λ (the bottom right panel of Fig. 3.2). This is because to explain the Λ binding energy with a shallow potential, the Λ single-particle potential at $\rho \lesssim \rho_0$ has to be deeper by taking large L_Λ . For $J_\Lambda = -31$ MeV, the parameter region of $\Delta B_\Lambda < 0.78$ MeV has upper limits at $L_\Lambda = 0$ MeV and $K_\Lambda = 550$ MeV. In contrast to the case of $J_\Lambda = -29$ MeV, the Λ single-particle potential at $\rho \lesssim \rho_0$ has to be shallower by taking small L_Λ . Thus, measuring J_Λ within the error of 1 MeV enables us to

constrain the range of L_Λ and then K_Λ through the positive correlation. This helps to discuss whether the Λ single-particle potential is repulsive or attractive at high densities. For $J_\Lambda = -30$ MeV, the parameter region within $\Delta B_\Lambda < 0.75$ MeV includes various K_Λ values that are the same as those in the bottom right panel of Fig. 3.2. Therefore, if J_Λ is determined as -30 MeV by high-resolution data of the Λ binding energy, further constraint at $\rho < \rho_0$ is needed to determine L_Λ and K_Λ .

3.2 Lambda admixture in neutron star matter

In the previous section, we examined the parameter space of Λ single-particle potentials consistent with the Λ binding energy data. In this section, we discuss the admixture of Λ in neutron stars by the chemical potentials and investigate the parameter region where Λ 's do not appear in neutron stars. The Λ hyperons appear in neutron star matter at the baryon density ρ when the minimal chemical potential of Λ ,

$$\mu_\Lambda^0(\rho) = m_\Lambda c^2 + U_\Lambda(\rho, k_\Lambda = 0 \text{ fm}^{-1}), \quad (3.4)$$

exceeds the neutron chemical potential.

We determine the neutron chemical potential in neutron star ($npe\mu$) matter by solving the β -equilibrium conditions,

$$\mu_n = \mu_p + \mu_e, \quad \mu_e = \mu_\mu, \quad (3.5)$$

together with the baryon number conservation and the charge neutrality condition,

$$\rho = \rho_p + \rho_n, \quad \rho_p = \rho_e + \rho_\mu, \quad (3.6)$$

where

$$\mu_i = \left(\frac{\partial \tilde{\mathcal{E}}}{\partial \rho_i} \right)_{\rho_j \neq \rho_i}, \quad i, j = n, p, e, \mu, \quad (3.7)$$

are the chemical potentials of the matter constituents, and ρ_i ($i = n, p, e, \mu$) are the corresponding densities. The total energy density $\tilde{\mathcal{E}}$ is given as

$$\tilde{\mathcal{E}}(\rho_n, \rho_p, \rho_e, \rho_\mu) = \tilde{\mathcal{E}}_N(\rho_n, \rho_p) + m_n c^2 \rho_n + m_p c^2 \rho_p + \tilde{\mathcal{E}}_e(\rho_e) + \tilde{\mathcal{E}}_\mu(\rho_\mu), \quad (3.8)$$

where $\tilde{\mathcal{E}}_N$ is the nucleon energy density, and c is the speed of light. The energy densities of electrons $\tilde{\mathcal{E}}_e$ and muons $\tilde{\mathcal{E}}_\mu$ are assumed to be those of the Fermi gas. For the energy density of nucleons $\tilde{\mathcal{E}}_N$, we use the SLy4 [134] and BSk24 [146] parameter sets. Both are in good agreement with the pure neutron matter EOS from the chiral EFT up to the next-to-next-to-next-to-leading order (N3LO) [54, 147]. SLy4 is a softer EOS with the maximum neutron star mass of $2.06 M_\odot$ compared to BSk24 with $2.28 M_\odot$.

In Fig. 3.4, we compare the minimal chemical potential of Λ , μ_Λ^0 , and μ_n in the neutron star matter as a function of the normalized baryon density ρ/ρ_0 . In the top panel, we confirm that the minimal Λ chemical potential of Chi3 is larger than the neutron chemical potential at $\rho/\rho_0 \leq 5$ as reported in Ref. [59]. On the other hand, by using the Λ single-particle potentials of LY-IV and HPA2, Λ hyperons are found to admix in neutron stars

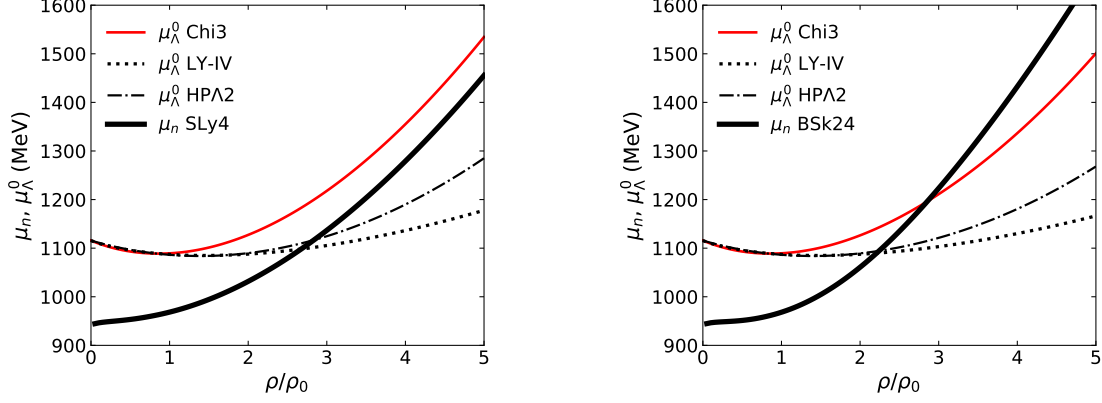


Figure 3.4: Λ chemical potentials at zero momentum and neutron chemical potential in neutron star ($npe\mu$) matter are depicted as a function of the normalized baryon density. The solid, dotted, and dash-dotted lines correspond to the Λ chemical potential at zero momentum for Chi3, LY-IV, and HPA2, respectively. The bold solid line represents the neutron chemical potential calculated by using the SLy4 (upper panel) and BSk24 (lower panel) parameter sets.

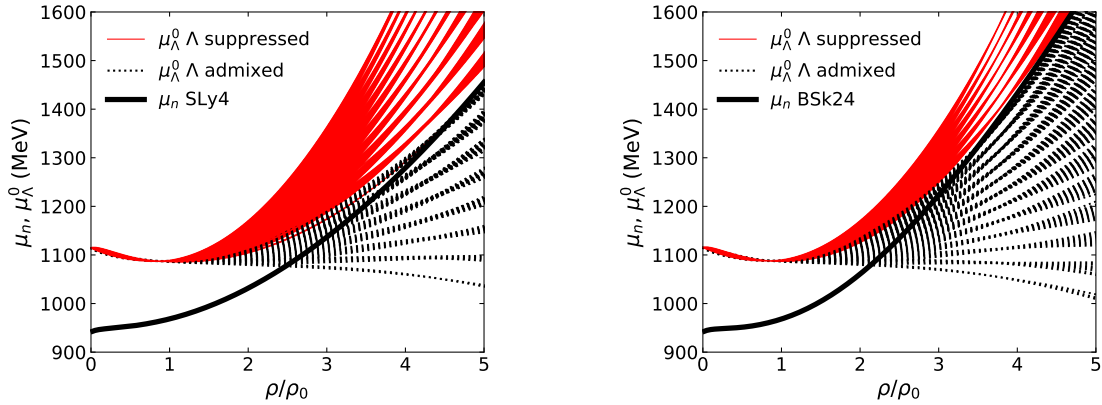


Figure 3.5: Same as Fig. 3.4, but the Λ chemical potentials at zero momentum using the density dependence of the selected Λ single-particle potentials in Fig. 3.1 are shown. The solid red lines are the Λ single-particle potentials with $\mu_\Lambda^0 > \mu_n$ at $\rho/\rho_0 \leq 5$, which suppress the Λ hyperons in neutron star matter, while the dotted lines are those which fulfill the condition $\mu_\Lambda^0 \leq \mu_n$ at some densities so that Λ hyperons are admixed.

in the density range $2\text{--}3\rho_0$, as found in phenomenological models with hyperons [32, 33, 35, 36, 148] causing the softening of the EOS. In the bottom panel, all the Λ -potential models are found to exhibit the appearance of Λ 's in neutron stars, representing that the appearance of Λ 's depends on the model of the nucleonic EOS.

In Fig. 3.5, the minimal Λ chemical potentials calculated from the selected Λ single-particle potentials shown in Fig. 3.1 are compared with the neutron chemical potential in neutron star matter. The solid red lines correspond to the minimum Λ chemical potentials that suppress the appearance of Λ in neutron matter, i.e., $\mu_\Lambda^0 > \mu_n$ at $\rho/\rho_0 \leq 5$. We confirmed that the suppression of Λ appearance hardly changes even when we check it up to the central density of a maximum-mass neutron star ($\rho \leq \rho_c^{\text{max}}$) instead of $\rho/\rho_0 \leq 5$: the number of Λ single-particle potentials with no Λ appearance decreases only by two for SLy4 with $\rho_c^{\text{max}} = 7.5\rho_0$ and is unchanged for BSk24 with $\rho_c^{\text{max}} = 9.4\rho_0$, where ρ_c^{max} is taken from the CompOSE database [149–151] for each EOS. For $\rho > 3.5\rho_0$, the minimum Λ chemical potentials are split into 19 groups corresponding to $K_\Lambda = 100, 150, 200, \dots$, and 1000 MeV. We note that the gaps between the 19 groups would be filled by considering more points in K_Λ . We also note that several parameter sets with different L_Λ 's and J_Λ 's are degenerate in each group, which means that the high-density part of the Λ chemical potential is mostly determined by K_Λ . We argue that the onset of Λ hyperons in neutron stars can be judged using the value of K_Λ : with the SLy4 EOS Λ 's do not appear in neutron stars if $K_\Lambda \geq 500$ MeV, while with the BSk24 EOS Λ 's do not appear if $K_\Lambda \geq 700$ MeV. Therefore, the second derivative of the Λ single-particle potential, K_Λ , would be the important parameter in discussing the admixture of Λ 's in neutron star matter.

3.3 Short summary

We have examined uncertainty of the Λ single-particle potentials by the current Λ hypernuclear data. We systematically generate various Λ single-particle potentials by varying the Taylor coefficients and the effective mass at the saturation density. The root-mean-square deviation ΔB_Λ is used to evaluate the consistency between the calculated Λ binding energies and experimental data. It is shown that the depth of the Λ single-particle potential J_Λ is constrained from the Λ binding energy data within the accuracy of $-31.5 < J_\Lambda < -28$ MeV. These values are consistent with the well-known Woods-Saxon results [75]. Furthermore, there are lower and upper limits for the normalized effective mass: $0.65 < m_\Lambda^*/m_\Lambda < 0.95$. This reflects the fact that the energy splitting between orbitals is sensitive to the effective mass. This is important in studying the

A positive correlation between the first- and second-order Taylor coefficients of the Λ single-particle potential, L_Λ and K_Λ , is found, which reflects the fact that the Λ single-particle potential at $\rho < \rho_0$ is constrained. It is shown that K_Λ can be well constrained by determining J_Λ within the accuracy of 1 MeV, i.e., $K_\Lambda > 350$ MeV is favored for $J_\Lambda = -29$ MeV, while $K_\Lambda < 550$ MeV is favored for $J_\Lambda = -31$ MeV. In the future, the value of J_Λ is expected to be determined more precisely through high-resolution data obtained at the Japan Proton Accelerator Research Complex (J-PARC) [152]. These data can be used to constrain the second-order Taylor coefficient K_Λ . Also, the Λ single-particle potentials with $K_\Lambda \geq 500$ MeV are found to suppress Λ s in β -stable neutron star matter

for SLy4, while the Λ single-particle potentials with $K_\Lambda \geq 700$ MeV for BSk24 suppress Λ . Therefore, the determination of J_Λ may help discriminate the Λ -avoiding scenario from the Λ -admiring scenario in neutron stars.

Chapter 4

Baryon forces within SU(3) flavor chiral EFT

Chiral effective field theory (EFT) is a powerful method to construct the baryon interactions. This section introduces the basic concept of chiral EFT, including how to construct the baryon interactions practically. This section is based on mainly Refs. [153–155].

4.1 A short review of baryon interaction studies

The baryon interactions in vacuum provide a natural starting point for investigating the hyperon single-particle potential in nuclear matter, besides starting from the mean-field level. These interactions are constrained by available hyperon-nucleon (YN) scattering observables, mainly the elastic and total cross sections [156–161], and are then implemented in many-body calculations to study in-medium properties.

One of the traditional approaches for constructing baryon interactions is the meson-exchange models. The Nijmegen [39–41, 58, 76] and Jülich [77, 162] interactions are widely known. For the Nijmegen model, many-body forces can be added phenomenologically as a form of the multi-Pomeron exchange [58] in such a way to reproduce the Λ hypernuclear data. An alternative microscopic approach is the quark models, such as the FSS model [163, 164] and its extended version including vector-meson exchanges, fss2 [81, 165, 166]. These models are founded on the quark-cluster picture developed by Oka and Yazaki [167–169], in which baryons are treated as composite systems of constituent quarks. The resulting two-body potentials are derived using the resonating-group method (RGM). These models, however, inevitably suffer from model dependence and limited predictive power.

The lattice QCD calculations also offer a different standpoint, providing an *ab initio* approach based on QCD. After the successful reconstruction of the repulsive core in the NN force by this method [170], significant progress has been made, e.g. in the strangeness $S = -2$ sector [171]. However, calculations for lighter-mass systems suffer from large statistical noise. As a consequence, the $S = -1$ YN interaction is currently not yet well established. Even if the $\Lambda N/\Sigma N$ force is constructed, constructing the three-body force, which plays a crucial role in the neutron stars, would require further efforts.

Chiral effective field theory, which will be illustrated in detail below, provides a most efficient way for studying the matter properties even above the saturation density. The perturbation technique called a power counting, initially suggested by Weinberg for application to the nucleon-nucleon (NN) interaction, allows to improve the results order by order. The perturbative expansion enables an estimation of the theoretical uncertainty, unlike in the above-mentioned methods. Furthermore, such an expansion classifies not only the two-body force but also the many-body forces, important for predicting the higher density region. Although the theory cannot be applied to the higher energy scale, currently the theory is the only method to reliably make a prediction above the saturation density.

Let us mention the so-called femtoscopy technique in high-energy collisions, intensively studied recently. There, the momentum correlation functions are measured in high-energy collisions for investigating the baryon-baryon interactions that are difficult to measure in usual scattering experiments, such as $p\Lambda$ [172–174], $p\Sigma^0$ [175], $\Lambda\Lambda$ [173, 176–178], $p\Omega^-$ [179, 180], $p\Xi^-$ [178, 180, 181], $\Lambda\Xi$ [182], and $\Xi\Xi$ [178] systems. Furthermore, the momentum correlation for the baryon number $A \geq 3$ systems comes to be studied to access the many-body interactions both experimentally (p -deuteron (d) [183], and $pp\Lambda$ [184]) and theoretically (pd [185], Λd [186], and Ξd [187]). We have performed a study on the Λ - ^4He [188] and Ξ - ^4He [189] correlation functions for obtaining further information on the baryon interactions.

4.2 Quantum chromodynamics and chiral symmetry

Let us introduce basic concepts of QCD that guide us to construct chiral EFT. The QCD Lagrangian reads

$$\mathcal{L}_{\text{QCD}} = \bar{q}(i\not{D} - m)q - \frac{1}{2} \text{tr} G_{\mu\nu} G^{\mu\nu}, \quad (4.1)$$

where q is the quark field. The covariant derivative $D^\mu = \partial^\mu + igA^\mu$ is defined using the gauge coupling g and the gauge field A^μ . The field strength tensor includes the self-interactions due to the non-Abelian nature of the gauge field A^μ : $G^{\mu\nu} = \frac{1}{ig} [D^\mu, D^\nu] = \partial^\mu A^\nu - \partial^\nu A^\mu + ig[A^\mu, A^\nu]$. The gluon self-interactions lead to an energy scale Q dependence of the running gauge coupling at the one-loop level [190, 191]

$$\alpha_s(Q) = \frac{g^2}{4\pi} \simeq \frac{12\pi}{33 - 2N_f} \frac{1}{\ln(Q^2/\Lambda_{\text{QCD}}^2)}, \quad (4.2)$$

where $\Lambda_{\text{QCD}} \simeq 200$ MeV is the empirically determined QCD scale parameter, and N_f is the number of quark flavors. This coupling implicates an important characteristic in QCD, the asymptotic freedom, which states that the coupling approaches to zero for high energy scale, while the coupling becomes large for low energy scale, requiring some non-perturbative analysis or low-energy effective theory.

The quark mass spectrum offers insight into which degrees of freedom are relevant in the low energy-scale region. The current quark masses listed in the 2024 version of Particle Data Group [192] read

$$m_u = 2.16 \pm 0.07 \text{ MeV}, \quad m_d = 4.70 \pm 0.07 \text{ MeV}, \quad m_s = 93.5 \pm 0.8 \text{ MeV}, \quad (4.3)$$

$$m_c = 1273.0 \pm 4.6 \text{ MeV}, \quad m_b = 4183 \pm 7 \text{ MeV}, \quad m_t \approx 170 \text{ GeV}, \quad (4.4)$$

where the u -, d -, and s -quarks masses are the $\overline{\text{MS}}$ masses at the scale $\mu = 2 \text{ GeV}$. The c - and b -quark masses also utilizes the $\overline{\text{MS}}$ masses but renormalized at the $\overline{\text{MS}}$ mass, $\mu = \bar{m}$. The t -quark mass is extracted from event kinematic analyses of high-energy collisions. One can find a separation in mass scales,

$$m_u, m_d, m_s \ll 1 \text{ GeV} < m_c, m_b, m_t. \quad (4.5)$$

This separation in scales suggests that for enough low-energy scale, the dynamics governed by strong interaction can be well described with only the up, down, and strange degree of freedom. There, heavier flavors cannot be resolved. From this point, we focus on the three-flavor case, $N_f = 3$.

In the following, we will introduce the chiral symmetry, opening a pathway to construct a low-energy effective theory. The quark fields can be decomposed into *left* and *right* components as

$$q_L = P_L q_f, \quad q_R = P_R q_f, \quad (4.6)$$

using the left- and right-handed projection operators

$$P_L = \frac{1}{2}(1 - \gamma_5), \quad P_R = \frac{1}{2}(1 + \gamma_5), \quad (4.7)$$

with the chirality matrix γ_5 . The QCD Lagrangian in the chiral limit, the case $m \rightarrow 0$, can be decomposed as

$$\mathcal{L}_{\text{QCD}}^0 = \bar{q}_R i \not{D} q_R + \bar{q}_L i \not{D} q_L - \frac{1}{2} \text{tr} G_{\mu\nu} G^{\mu\nu}, \quad (4.8)$$

showing that the massless QCD is invariant under the chiral transformation

$$q_L \rightarrow L q_L, \quad q_R \rightarrow R q_R, \quad (4.9)$$

with independent unitary matrices L and R belonging to $U(3)$

$$L = \exp \left(-i \sum_{a=1}^8 \theta_a^L \frac{\lambda_a}{2} \right) \exp(-i\theta^L), \quad R = \exp \left(-i \sum_{a=1}^8 \theta_a^R \frac{\lambda_a}{2} \right) \exp(-i\theta^R), \quad (4.10)$$

with real parameters θ and generators λ_a of the Lie algebra $\mathfrak{su}(3)$. This means that the massless QCD Lagrangian possesses a global $SU(3)_L \times SU(3)_R \times U(1)_L \times U(1)_R$ symmetry (at the classical level). The Noether's theorem implies the following conserved currents

$$J_L^{\mu,a} = \bar{q}_L \gamma^\mu \frac{\lambda_a}{2} q_L, \quad J_R^{\mu,a} = \bar{q}_R \gamma^\mu \frac{\lambda_a}{2} q_R, \quad (4.11)$$

$$J_L^\mu = \bar{q}_L \gamma^\mu q_L, \quad J_R^\mu = \bar{q}_R \gamma^\mu q_R. \quad (4.12)$$

It is useful to introduce the vector and axial-vector currents

$$J_V^{\mu,a} = J_R^{\mu,a} + J_L^{\mu,a} = \bar{q} \gamma^\mu \frac{\lambda_a}{2} q, \quad J_A^{\mu,a} = J_R^{\mu,a} - J_L^{\mu,a} = \bar{q} \gamma^\mu \gamma_5 \frac{\lambda_a}{2} q, \quad (4.13)$$

$$J_V^\mu = J_R^\mu + J_L^\mu = \bar{q} \gamma^\mu q, \quad J_A^\mu = J_R^\mu - J_L^\mu = \bar{q} \gamma^\mu \gamma_5 q. \quad (4.14)$$

After quantization of the Dirac fermions, the conservation of the flavor-singlet axial-vector current J_A^μ is broken due to the Adler-Bell-Jackiew anomaly [193, 194]. The Vafa-Witten theorem [195] states that the vector symmetry $SU(3)_V \times U(1)_V$ is not spontaneously broken.

A crucial aspect of QCD is the spontaneous breaking of chiral symmetry, $SU(3)_L \times SU(3)_R \rightarrow SU(3)_V$, in the low-energy scale. The Nambu-Goldstone theorem [196–198] states that if a symmetry is spontaneously broken, a particle with zero mass called Nambu-Goldstone (NG) bosons appears for each broken generator (within relativistic theories¹). As a consequence, pion mass is small of $m_\pi \approx 138$ MeV compared to the value by a naïve quark counting, two times the constituent quark mass $m_u, m_d \approx 330$ MeV. Its small mass comes from the explicit chiral-symmetry breaking by the current quark mass. In the three-flavor case, the spontaneous breaking pattern is $SU(3)_L \times SU(3)_R \rightarrow SU(3)_V$. The number of broken generators is 8 and there appears corresponding pseudoscalar mesons known as $\pi(\pi^+, \pi^-, \pi^0)$, $K(K^+, K^-, K_0, \bar{K}_0)$, and η .

The NG-boson masses are small compared to low-lying vector meson, ρ meson with mass $m_\rho \approx 770$ MeV. Below this scale, the dynamics can be well-described by only considering the NG boson degree of freedom. There, the higher energy scale components are “integrated out”, incorporated in the parameters called low-energy constants (LECs). This is the most basis of constructing the low-energy effective field theory.

For later usage, let us introduce the so-called external field method. One can encode the chiral Ward identities in the generating functional

$$Z[v, a, s, p] = \int \mathcal{D}A \mathcal{D}q \mathcal{D}\bar{q} \exp \left[i \int d^4x \mathcal{L}(v, a, s, p) \right], \quad (4.15)$$

where

$$\mathcal{L} = \mathcal{L}_{\text{QCD}}^0 + \mathcal{L}_{\text{ext}}, \quad (4.16)$$

$$\mathcal{L}_{\text{ext}}(v, a, s, p) = \bar{q} \gamma^\mu (v_\mu + \gamma_5 a_\mu) q - \bar{q} (s - i \gamma_5 p) q, \quad (4.17)$$

where v, a, s, p corresponds to the vector, axial-vector, scalar, and pseudoscalar external fields. The transformation properties of the external fields are given by requiring the local $SU(3)_L \times SU(3)_R$ invariance as

$$v_\mu + a_\mu \rightarrow R(v_\mu + a_\mu)R^\dagger + iR\partial_\mu R^\dagger, \quad (4.18)$$

$$v_\mu - a_\mu \rightarrow L(v_\mu - a_\mu)L^\dagger + iL\partial_\mu L^\dagger, \quad (4.19)$$

$$s + ip \rightarrow R(s + ip)L^\dagger, \quad (4.20)$$

$$s - ip \rightarrow L(s - ip)R^\dagger. \quad (4.21)$$

The mass matrix can be represented by the scalar external field. Within the external field method, the mass matrix can be transformed by the chiral rotation. Then, the Lagrangian now possesses the spurious $SU(3)_L \times SU(3)_R$ symmetry. After constructing the Lagrangian, if the external field is set to be the constant, e.g. the mass matrix, the resulting EFT Lagrangian explicitly breaks the symmetry as the original QCD Lagrangian.

¹For non-relativistic theories, the Lagrangian is not so strongly constrained as in the relativistic case, resulting in some additional terms like single time-derivative term. The number of NG bosons is not necessarily equals to the number of broken generators. The formula for the correct number of NG bosons have been proved by Watanabe and Murayama [199] and by Hidaka [200] using different methods.

4.3 Chiral effective field theory

The effective field theory is based on the spontaneous symmetry breaking pattern. The philosophy of the effective field theory is well written in the famous Weinberg's paper [201]

If one writes down the most general possible Lagrangian, including *all* terms consistent with assumed symmetry principles, and then calculates matrix elements with this Lagrangian to any given order of perturbation theory, the result will simply be the most general possible S -matrix consistent with perturbative unitarity analyticity, cluster decomposition, and the assumed symmetry properties.

This “theorem” guides to construct the effective field theory [202]:

1. Identify the soft and hard scales, and the degrees of freedom appropriate for (low-energy) nuclear physics. (**Discussed below**)
2. Identify the relevant symmetries of low-energy QCD and investigate if and how they are broken. (**Here, $SU(3)_L \times SU(3)_R \rightarrow SU(3)_V$.**)
3. Construct the most general Lagrangian consistent with those symmetries and symmetry breakings. (**See Sec. 4.3.1**)
4. Design an organizational scheme that can distinguish between more and less important contributions: a low-momentum expansion. (**See Sec. 4.3.2 and See Sec. 4.4**)
5. Guided by the expansion, calculate Feynman diagrams for the process under consideration to the desired accuracy. (**See Sec. 4.3.3**)

In addition to these, the baryon interaction study practically requires the following procedure:

6. Choose the regularization scheme to prevent the low-energy observables from being disturbed by unphysical ultra-violet behaviors. (**See Sec. 4.5**)

Regarding the first point, the soft scale is identified as the NG boson mass scale, while the hard scale can be the rho meson mass $m_\rho \approx 770$ MeV or the chiral symmetry breaking scale $\approx 4\pi f_\pi \approx 1$ GeV with the pion decay constant $f_\pi \approx 92$ MeV. Since the Kaon and eta meson masses are $m_K \approx 495$ MeV and $m_\eta \approx 547$ MeV respectively, the scale separation is not as well in the flavor $SU(3)$ case as the $SU(2)$ case. Nevertheless, the $SU(3)$ chiral EFT is widely used thanks to its prediction power realized by the systematic technique to choose important diagrams as we will see later. Also, the required accuracy for the strange nuclear physics is not as high as the nuclear physics. The effective degrees of freedom is the pseudoscalar mesons π, K, η . Other probes like baryon fields and electric fields can be added as a forms of matter fields and external fields.

Hereafter, we illustrate one-by-one how to construct the baryon interactions based on the above-mentioned EFT philosophy. The explanation is, however, just a sketch to catch the concepts. Further details can be found in the references in the beginning of this chapter.

4.3.1 Construction of Lagrangian

To construct the EFT Lagrangian, first one has to identify the transformation properties under the chiral transformation $SU(3)_L \times SU(3)_R$, including how to realize the transformation.

The pseudoscalar meson field is represented by the 3×3 $SU(3)$ matrix

$$\phi = \sum_{a=1}^8 \lambda_a \phi_a = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}, \quad (4.22)$$

where the $\mathfrak{su}(3)$ generator λ_a is now the Gell-Mann matrices. The way to realize the chiral transformation to the meson field is arbitrary, i.e. both the linear and non-linear transformations can be applied. In any realization, if one calculates the S matrix to the infinite order of the chiral perturbation, same result should be obtained. Nevertheless, the following non-linear realization [203, 204] under $SU(3)_L \times SU(3)_R$ transformation is usually employed for the hadron interaction studies:

$$U = \exp[i\phi/f_0], \quad U \rightarrow RUL^\dagger, \quad (4.23)$$

where the f_0 is called the decay constant of the pseudoscalar mesons in the chiral limit. By using the matrix U as a building block to construct the EFT Lagrangian, the vertices with any number of pions can be contracted with a single term. This is efficient to describe, for example, the two baryons-one pion and two baryons-two pions matrices simultaneously with only single interaction term.

The octet baryon field is represented as

$$B = \sum_{a=1}^8 B_a \lambda_a = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}, \quad (4.24)$$

belonging to the adjoint representation of $SU(3)_L \times SU(3)_R$:

$$B \rightarrow K(L, R, U)BK^\dagger(L, R, U). \quad (4.25)$$

The $SU(3)$ -valued K is defined to fulfill the transformation property by $V \in SU(3)_V$,

$$K(V, V, U) = V. \quad (4.26)$$

We here want to derive the interaction term among baryons and mesons. Since the Baryon is transformed by using the matrix K , one should construct the building block $X(\phi, v, a, r, l)$ transformed as $X \rightarrow KXK^\dagger$. Also, to realize the local chiral symmetry, or invariant under $K(L(x), R(x), U(x))$, one must construct the covariant derivative associated with the baryon kinetic term. It is convenient to introduce the square-root matrix $\xi = \sqrt{\phi}$. From the transformation property $\phi \rightarrow R\phi L^\dagger$, one can insert the matrix K as

$$\xi \rightarrow \xi' = R\xi K^\dagger = K\xi L^\dagger = \sqrt{LUR^\dagger}. \quad (4.27)$$

The covariant derivative under the chiral transformation realized by $K(L, R, U)$ is defined as

$$D_\mu A = \partial_\mu A + [\Gamma_\mu, A], \quad (4.28)$$

$$\Gamma_\mu = \frac{1}{2} [u^\dagger (\partial_\mu - ir_\mu) u + u (\partial_\mu - il_\mu) u^\dagger]. \quad (4.29)$$

The Lorentz-vector block can also be defined as

$$u_\mu = i [u^\dagger (\partial_\mu - ir_\mu) u - u (\partial_\mu - il_\mu) u^\dagger]. \quad (4.30)$$

One can check that both Γ_μ and u_μ transform as $X \rightarrow K X K^\dagger$. We note that the r^μ and l^μ are required even when they are zero, or trivial gauge fields, to realize the transformation.

The chiral Lagrangian can be ordered in the number of baryon fields:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_\phi + \mathcal{L}_B + \mathcal{L}_{BB} \cdots, \quad (4.31)$$

where $\mathcal{L}_\phi$ denotes the purely mesonic part. The leading-order meson-baryon Lagrangian $\mathcal{L}_B^{(1)}$ (without external fields) is at $\mathcal{O}(q^1)$ and is constructed to be invariant under the $SU(3)_L \times SU(3)_R$ symmetry,

$$\mathcal{L}_B^{(1)} = \text{tr} [\bar{B} (i\not{D} - m) B] + \frac{D}{2} \text{tr} [\bar{B} \gamma^\mu \gamma_5 \{u_\mu, B\}] + \frac{F}{2} \text{tr} [\bar{B} \gamma^\mu \gamma_5 [u_\mu, B]], \quad (4.32)$$

where the octet-baryon mass m is in the chiral limit. The constants D and F may be determined by fitting the semi-leptonic decays $B \rightarrow B' + e^- + \bar{\nu}_e$ at tree level [205] as $D \approx 0.80$ and $F \approx 0.50$.

The contact term is also included to take into account the unresolved short range behavior. Technically, this term serves as a counter term to realize the order-by-order convergence. For example, in the flavor $SU(2)$ case, the contact term at $\mathcal{O}(q^0)$ reads

$$\mathcal{L}_{\text{ct}}^{(0)} = \sum_{i=1}^5 C_i (\bar{N} \Gamma_i N) (\bar{N} \Gamma_i N), \quad (4.33)$$

where $\Gamma_i = (1, \gamma_5, \gamma^\mu, \gamma^\mu \gamma_5, \sigma^{\mu\nu})$. In this case, the LECs C_i must be determined by tuning to e.g. the BB scattering observables. The full leading $SU(3)$ contact Lagrangian is given by Polinder *et al.* [206]. The next-to-leading order $SU(3)$ contact Lagrangian at $\mathcal{O}(q^2)$ is given by Petschauer and Kaiser [207].

4.3.2 Power counting

The chiral Lagrangian contains infinite terms. Since it is hard to employ all the terms for calculating observables, one has to choose important terms and resulting Feynman diagrams, to describe the low-energy dynamics interested in. One clue is to focus on the dimension ν of the soft momentum scale over the hard scale $(q/\Lambda_\chi)^\nu$, where the q represents the momentum involved in or the NG boson mass and $\Lambda_\chi \approx 1$ GeV is the chiral symmetry breaking scale.

The power counting technique, which is introduced by Weinberg [201], gives us a direction to formulate the efficient perturbative calculation from finite terms. There, the meson propagator is counted as $\mathcal{O}(q^{-2})$. The baryon propagator $i/(\not{p} - m + i\epsilon)$ is usually given at $\mathcal{O}(q^{-1})$. By using the momentum convention $p = mv + q$ with $v = (1, 0, 0, 0)$ and q as the soft momentum, one can see that the order is exact in the non-relativistic baryon limit, [208]

$$i \frac{\not{p} + m}{(mv + q)^2 - m^2 + i\epsilon} = i \frac{\not{p} + m}{2mq^0 + \mathcal{O}(q^2) + i\epsilon} = \frac{A}{q^0 + \mathcal{O}(q^2/m) + i\epsilon}, \quad (4.34)$$

where A is the matrix $A = i(\not{p} + m)(2m) = \mathcal{O}(q^0)$. The four-dimensional loop integral yields $\mathcal{O}(q^4)$. Each spatial derivative from interaction vertices yields the soft momentum at $\mathcal{O}(q^1)$.

After applying some topological identities and tuning baryon-number dependence [154], the standard power of a Feynman diagram is written as

$$v = -4 + 2B + 2L + \sum_i v_i(\Delta_i), \quad \Delta_i = d_i + \frac{1}{2}b_i - 2, \quad (4.35)$$

where B is the number of external baryons, L represents the number of loops. The number of vertices with index Δ_i is denoted by v_i . The symbol d_i is number of derivatives or pseudoscalar-meson mass insertions at the vertex and b_i is the number of internal baryon lines at the considered vertex.

4.3.3 Feynman diagrams

The Feynman diagrams for baryon interactions that derived from the power counting (4.35) are illustrated in Fig. 4.1. Here, the irreducible graphs are considered. In the actual calculation, one has to calculate the Lippman-Schwinger type equation to take into account all contributions including the reducible graphs. The lowest order (LO) contribution has the one S -wave contact term and the one-pion exchange. The next-to-leading order (NLO) at $\mathcal{O}(q^2)$ emerges the additional contact term, required to be determined by fitting to e.g. the baryon scattering observables. The two-pion exchange potentials appear at NLO and next-to-next-to-leading order (N²LO).

A remarkable feature in the chiral expansion is the systematic construction of the many-body forces. The three-body forces first appear at N²LO, including the six-baryon contact term, the one meson-four baryon contact term, and the two-meson exchange term. We will use these forces in Chapter 6.

The LECs in the contact terms must be determined by fitting to experimental data. In the nucleonic sector, the NN force up to N⁴LO and including a part of N⁵LO contribution has been constructed [209] thanks to the rich NN phase shift data and the precise few-body nuclear data. For the hyperon sector, there are not so much experimental information: Mainly total scattering cross YN sections have been measured. Using these information, the LO YN interaction is constructed in 2006 [206]. The NLO YN interaction is constructed in a same way in 2013 [102]. Other NLO version in 2019 [103] modifies several LECs with reference to the NN LECs. Recently, the Jülich-Bonn group establishes the N²LO YN interaction [101]. This force uses the new data of ΣN differential cross sections measured by the J-PARC E40 experiment [104–106].

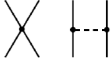
		BB force	3B force	4B force
$\mathcal{O}(q^0)$	LO			
$\mathcal{O}(q^2)$	NLO			
$\mathcal{O}(q^3)$	N ² LO			

Figure 4.1: Baryon interactions derived within the power counting 4.35. From top to below, lowest order (LO), next-to-leading order (NLO), and next-to-next-to-leading order (N²LO) contributions are listed. The small dot, large dot, and square symbols represent the index $\Delta_i = 0, 1$, and 2 vertices, respectively.

4.4 Heavy-baryon formalism

The original relativistic chiral Lagrangian in deed suffers from a consistent power counting problem due to the large baryon mass comparable to the hard scale ~ 1 GeV. One troublesome thing arises in the one-loop correction in the nucleon mass. Although the corresponding diagrams have the order $\mathcal{O}(q^3)$, the dimensional regularization combined with the $\overline{\text{MS}}$ scheme gives the $\mathcal{O}(q^2)$ contribution. Although this correction can be absorbed to the redefinition of the counter term at $\mathcal{O}(q^2)$, a desired one-to-one correspondence between the expansion in NG boson loops and the expansion in terms of small external momenta and meson masses is destroyed.

One solution to the problem is the heavy-baryon formalism [210, 211], which is a commonly used approach for the baryon-baryon interaction studies within chiral EFT. A pedagogical introduction can be found in Ref. [212]. The formalism is motivated by the heavy-baryon limit of the baryon propagator (4.34)

$$i \frac{\not{p} + m}{(mv + q)^2 - im^2\epsilon} \xrightarrow{m \rightarrow \infty} i \frac{1 + \not{p}}{2q \cdot v + i\epsilon}, \quad (4.36)$$

including the operator $P^+ = (1 + \not{p})/2$, extracting the heavy component of the Dirac spinor as illustrated below. The Dirac spinors have large and small components:

$$\Psi(x) = e^{-ip \cdot x} u(\vec{p}, s), \quad u(\vec{p}, s) = \sqrt{\frac{E + m}{2m}} \begin{pmatrix} I \\ \frac{\vec{\sigma} \cdot \vec{p}}{E + m} \end{pmatrix} \chi_s, \quad (4.37)$$

where I is the two-dimensional identity matrix, $p^0 = E = \sqrt{\vec{p}^2 + M^2}$, and χ_s is a Pauli spinor representing the spin state. The lower component is suppressed by the factor $|\vec{p}|/m$, corresponding to the light component. The baryon spinor field with the four-velocity $v^\mu = p^\mu/m$ can be represented as

$$\Psi(x) = e^{-imv \cdot x} (N + h), \quad (4.38)$$

where

$$N_p = e^{imv \cdot x} P_v^+ u(\vec{p}, s), \quad h_p = e^{imv \cdot x} P_v^- u(\vec{p}, s), \quad (4.39)$$

$$P_v^\pm = \frac{1 \pm \not{v}}{2}. \quad (4.40)$$

Assuming the static baryon limit, or $v^\mu \rightarrow (1, 0, 0, 0)$, N and h correspond to the large and small components:

$$N_p \rightarrow e^{imt} \sqrt{\frac{E+m}{2m}} \begin{pmatrix} I \\ 0 \end{pmatrix} \chi_s \sim \mathcal{O}(1), \quad (4.41)$$

$$h_p \rightarrow e^{imt} \sqrt{\frac{E+m}{2m}} \begin{pmatrix} 0 \\ \frac{\vec{\sigma} \cdot \vec{p}}{E+m} \end{pmatrix} \chi_s \sim \mathcal{O}(1/m), \quad (4.42)$$

$$P^+ \rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad P^- \rightarrow \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad (4.43)$$

where the order notation regards with the $1/m$ expansion.

Following utilizes the general momentum case with $p = mv + q$ with $v^2 = 1$. Using the decomposition (4.38), the Dirac Lagrangian is then decomposed into

$$\bar{\Psi}(i\not{D} - m)\Psi = \bar{N}iv \cdot DN + \bar{h}(iv \cdot D - 2m)h + \bar{N}i\not{D}h + \bar{h}i\not{D}N. \quad (4.44)$$

One can see from the expressions that the N field corresponds to the soft component, while the h field has the mass $2m$. By using the equation of motion for h

$$(iv \cdot D - 2m)h + i\not{D}N = 0 \quad (4.45)$$

and inserting this to the Lagrangian (4.44), one can expand the Lagrangian in terms of $1/m$ as

$$\bar{\Psi}(i\not{D} - m)\Psi = \bar{N}iv \cdot DN + \mathcal{O}(1/m). \quad (4.46)$$

If one takes the lowest order Lagrangian at $\mathcal{O}((1/m)^0)$, the troublesome heavy baryon-mass m disappears, ensuring the consistent power counting.

As an additional merit of this formalism, the number of LECs is reduced. For example, in the relativistic lowest-order contact Lagrangian, in the SU(2) case (4.33)

$$\mathcal{L} = \sum_{i=1}^5 C_i (\bar{N}\Gamma_i N)(\bar{N}\Gamma_i N), \quad (4.47)$$

there are the five LECs corresponding to the gamma-matrix structures $\Gamma_i = (1, \gamma_5, \gamma^\mu, \gamma^\mu \gamma_5, \sigma^{\mu\nu})$. By extracting the large component N , the structure is reduced to only two as

$$\mathcal{L} = -\frac{1}{2}C_S(\bar{N}N)^2 - \frac{1}{2}C_T(\bar{N}\vec{\sigma}N) \cdot (\bar{N}\vec{\sigma}N). \quad (4.48)$$

This additional feature makes the construction of the baryon forces efficient.

4.5 Regularization of potentials

Regularization to cut higher momentum region is necessary to prohibit the low-energy observables to be affected by the unphysical higher energy behaviors. For example, if we use the lowest order two-body interactions with some regulator in the non-relativistic formalism,

$$V^{1\pi}(\vec{p}, \vec{p}') = \frac{\alpha}{\vec{q}^2 + M_\pi^2}, \quad (4.49)$$

$$V^{\text{contact}}(\vec{p}, \vec{p}') = C, \quad (4.50)$$

where $\vec{q} = \vec{p}' - \vec{p}$ is the momentum transfer with $\vec{p}$ and $\vec{p}'$ begin the initial and final momentum, respectively. The parameters α and C denote the spin-momentum-isospin structure and low-energy constant (LEC), respectively. According to the diagrammatic expansion with $1\pi, 2\pi$, etc. exchanges, the 1π exchange should be valid only larger than the 2π exchange range. The expression in the above Eq. (4.50) is, however, applicable for all phase space including the short-range/high-momentum region. Then, we must cut the short range behavior of the $V^{1\pi}$ and tune the contact LEC appropriately to absorb the unresolved short range behavior as

$$V_\Lambda^{1\pi}(\vec{p}, \vec{p}') = \frac{\alpha}{q^2 + M_\pi^2} f_\Lambda^{1\pi}(\vec{p}, \vec{p}'), \quad (4.51)$$

$$V_\Lambda^{\text{contact}}(\vec{p}, \vec{p}') = C f_\Lambda^{\text{contact}}(\vec{p}, \vec{p}'), \quad (4.52)$$

where smooth regulator function f_Λ satisfy the condition $f_\Lambda \rightarrow 1$ under the $\Lambda \rightarrow \infty$ limit.

There are many methods of regularization. The so-called non-local (in the coordinate space) is

$$f_\Lambda(\vec{p}, \vec{p}') = \exp\left[-\frac{p^n + p'^n}{\Lambda^n}\right], \quad (4.53)$$

where n is the integer. This is very simple and utilized from early time [213] for all contributions including the pion-exchange and contact interactions. This, however, results in a large off-shell behavior [209], makes the potentials not perturbative and prohibits the construction of the well-behaved functions. This is because the non-local regularization disturbs the physical long-range part of the pion-exchange interaction.

The semi-local regularization schemes are introduced in this context. It is a hybrid implementation of the regularization: the pion exchange is regularized locally (in coordinate space), while the contact term is regularized non-locally to keep its angle dependence for simpler partial-wave decompositions. The SMS regularization is introduced in this context. Its concept is similar to the so-called semilocal coordinate-space regularized (SCS) potentials developed by Epelbaum, Krebs, and Meißner [214, 215]:

$$V_\pi(\vec{r}) \rightarrow V_{\pi,R}(\vec{r}) = V_\pi(\vec{r}) [1 - \exp(-r^2/R^2)]^n, \quad (4.54)$$

where the exponent n was set $n = 6$. SMS interaction is introduced to obtain more easily the analytical expressions in the momentum space. The SMS potential respects the physical structure of each contribution and facilitates a more rapid convergence already at low chiral orders. These developments contribute to construct the fifth order chiral potential [209].

4.6 Baryon forces including hyperons

Below we briefly summarize the development of the chiral YN interactions. The LO interaction is first constructed in 2006 [206]. Thereafter, the NLO force is first constructed in 2013 [102] and 2019 [103], developed by the Jülich-Bonn-Munich and Jülich-Bonn groups. All the interactions utilize the non-local regularization.

Recently, the Jülich-Bonn group has established a YN potential for the strangeness $S = -1$ sector (ΛN , ΣN) up to $N^2\text{LO}$ in the chiral expansion [101]. The contributions from pseudoscalar-meson exchanges (π , η , K) are, in principle, fixed by the assumed $SU(3)$ flavor symmetry. In contrast, the contact terms represent the unresolved short-distance dynamics, and the associated LECs are essentially free parameters. In practice, they are fitted to low-energy YN scattering data and additionally constrained by the hypertriton ($^3_\Lambda\text{H}$) binding energy. At the order considered the contact terms yield contributions to the S - and P -waves of the potential.

Although $SU(3)$ flavor symmetry is imposed for constructing the interaction, the explicit $SU(3)$ symmetry breaking by the physical masses of the pseudoscalar mesons (π , K , η) is taken into account when evaluating the potential. The mass differences induce also a possible $SU(3)$ breaking in the leading-order contact terms [102, 207] which is likewise considered in the YN potential of Ref. [101]. A novel regularization scheme, the so-called semilocal momentum space regularization, has been employed. The cutoff masses entering the regulator function considered in Ref. [101] are in the range of 500–600 MeV. For comparison, the cutoffs employed in the SMS NN interaction are in the range 350–550 MeV [216].

With the NLO and $N^2\text{LO}$ SMS YN potentials an excellent description of the low-energy Λp , $\Sigma^- p$ and $\Sigma^+ p$ scattering cross sections could be achieved, resulting in a total χ^2 of 15–16 for the commonly considered 36 data points [102]. Clearly, those data are primarily sensitive to the YN interaction in the S -waves and, accordingly, only the LECs contributing to the S -wave potential could be reliably fixed. Nonetheless, in addition, new measurements of angular distributions for the ΣN channels from J-PARC [104–106] have been analyzed in an attempt to determine also the strength of the contact interactions in the P -waves.

It is also noteworthy to mention that the SMS YN potentials have been utilized in calculations of light Λ -hypernuclei, based on the Faddeev-Yakubovsky approach and the no-core shell model (so far up to $A \approx 8$). The results, reported in Refs. [120, 217], show that the separation energies based on the two-body interaction alone are already fairly close to the experimental values [218]. When chiral (ΛNN , ΣNN) three-body forces are included, which arise at $N^2\text{LO}$ in the chiral expansion [219], a quantitative agreement with the experimental separation energies is achieved.

We will use the non-locally regularized NLO interactions in Chapter 6 with additional chiral three-body forces. The SMS $N^2\text{LO}$ interaction is employed in Chapter 7 for analyzing the in-medium property of hyperons.

Chapter 5

Brueckner theory for hyperons in medium

This chapter explains the Brueckner theory, an efficient method to calculate the single-hyperon property in nuclear medium using the baryon interactions in vacuum. The theory was formulated in old time of 1950s, and is still active and leads to compatible results of nuclear matter energy density to *ab-initio* type calculations. The key quantity to be calculated in this thesis is the G matrix, the in-medium version of the scattering T matrix in the Lippman-Schwinger equation. We first introduce the general foundation on the Brueckner theory, and then the detailed expressions of the G matrix and related quantities are given.

5.1 Introduction

To simulate the nuclear matter properties, there are mainly two types of methods: the perturbative methods like Brueckner theory and the non-perturbative *ab-initio* methods. Although the *ab-initio* methods become to be widely used over a decade thanks to the developments in the computational resources, the Brueckner theory is still a useful method to simulate the hyperon single-particle potentials in matter. As we illustrated later, the single-particle potentials are usually introduced as the Hartree-Fock form, enabling a reasonable comparison with other mean-field models. Furthermore, the Brueckner theory can yield comparable results compared to other methods as discussed by Baldo *et al.* [220]. Figure 5.1 shows a comparison of the energy per particle in pure neutron matter (PNM) with different many-body calculations. The lowest order Brueckner calculation, the Brueckner-Hartree-Fock method yields the compatible results to other methods with 10 MeV up to the density $0.5 \text{ fm}^{-3} \approx 4\rho_0$. Since the required precision for the hyperon system is not as high as the normal nuclear matter, the Brueckner theory is still a theory sufficient to discuss the in-medium property of hyperons.

We will employ the Brueckner theory to calculate the single-particle potential of hyperons in nuclear matter using the two-body forces and/or the density-dependent two-body force in vacuum. We briefly summarize the method in the following sections. A pedagogical introduction is given by Day [221]. Detailed description including hyperons is discussed in Ref. [80] and Refs. [32, 76, 78, 131, 222, 223]. Here, we focus on the non-

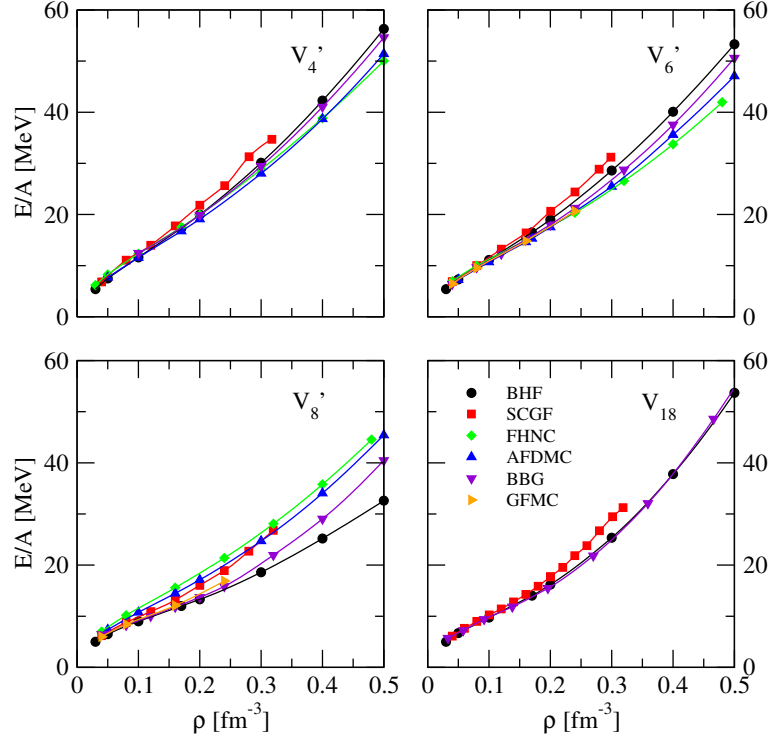


Figure 5.1: Energy per particle of pure neutron matter as a function of density calculated with several many-body approaches for different Argonne potentials: V_4' (upper left), V_6' (upper right), V_8' (lower left), and V_{18} (lower right). The abbreviations stand for Brueckner-Hartree-Fock method (BHF), Self-Consistent Green's Function method (SCGF), Fermi Hyper Netted Chain (FHNC), Auxiliary Field Diffusion Monte Carlo (AFDMC), the Brueckner-Bethe-Goldstone approach (BBG) up to three hole-line expansion, and Green's Function Monte Carlo (GFMC), respectively. Figure is taken from Ref. [220].

relativistic formalism. The work with the relativistic formalism including hyperons can be found in Refs [224–227].

5.2 Foundation

Let us consider calculating the energy density of the A -identical fermion system with the Hamiltonian

$$H = T + V = \sum_{i=1}^A t_i + \sum_{i<j} v_{ij}. \quad (5.1)$$

One can insert the arbitral single-particle potential U as

$$H = T + U + (V - U) \equiv H_0 + H_1. \quad (5.2)$$

The basic concept of the mean-field theory is to approximate the system by only the one-body operator $H_0 + U$ and the residual interaction H_1 can be taken as perturbative. The

single-particle potential U should be chosen to make the perturbative expansion converge faster. Choices of U will be discussed later in this section and Sec. 5.4.

The unperturbed Schrödinger equation is written as

$$H_0 \phi_p = \epsilon_p \phi_p, \quad (5.3)$$

where the index p denotes the p -th single-particle state. The unperturbed ground-state wave function and energy are expressed as

$$\Psi_0 = \frac{1}{\sqrt{A!}} \mathcal{A}(\phi_1 \phi_2 \cdots \phi_A), \quad E_0 = \sum_{i=1}^A \epsilon_i. \quad (5.4)$$

The energy is calculated by the perturbation series obtained from the exact Schrödinger equation $H\Psi = E\Psi$,

$$\begin{aligned} E = E_0 &+ \langle \Psi_0 | H_1 | \Psi_0 \rangle + \langle \Psi_0 | H_1 (E_0 - H_0)^{-1} P H_1 | \Psi_0 \rangle + \\ &\langle \Psi_0 | H_1 (E_0 - H_0)^{-1} P H_1 (E_0 - H_0)^{-1} P H_1 | \Psi_0 \rangle - \\ &\langle \Psi_0 | H_1 | \Psi_0 \rangle \langle \Psi_0 | H_1 (E_0 - H_0)^{-2} P H_1 | \Psi_0 \rangle + \mathcal{O}((H_1)^4), \end{aligned} \quad (5.5)$$

where the projection operator $P = 1 - |\Psi_0\rangle\langle\Psi_0|$ prohibits the ground state in the intermediate state.

To evaluate the perturbative series explicitly, it is convenient to work with the second-quantization form:

$$H_1 = \sum_{pqrs} \langle pq | v | rs \rangle a_p^\dagger a_q^\dagger a_s a_r - \sum_{pq} \langle p | U | q \rangle a_p^\dagger a_q, \quad (5.6)$$

$$\{a_r, a_s\} = 0, \quad \{a_r^\dagger, a_s^\dagger\} = 0, \quad \{a_r, a_s^\dagger\} = \delta_{rs}, \quad (5.7)$$

where the brackets represent the anti commutator $\{A, B\} = AB + BA$. The terms up to $\mathcal{O}(H_1)$ in the perturbation series (5.5) are expressed as

$$E_0 = \sum_{n=1}^A \langle n | T + U | n \rangle, \quad (5.8)$$

$$\langle \Psi_0 | H_1 | \Psi_0 \rangle = \frac{1}{2} \sum_{1 \leq m, n \leq A} (\langle mn | v | mn \rangle - \langle mn | v | nm \rangle) - \sum_{n=1}^A \langle n | U | n \rangle. \quad (5.9)$$

Since $\langle n | U | n \rangle$ is canceled out, the perturbation series (5.5) becomes

$$E = \sum_{n=1}^A \langle n | T | n \rangle + \frac{1}{2} \sum_{1 \leq m, n \leq A} (\langle mn | v | mn \rangle - \langle mn | v | nm \rangle) + \mathcal{O}(H_1^2). \quad (5.10)$$

However, the realistic nuclear potential has repulsive core. Each term in the power of H_1 then becomes too large, making the series diverge!

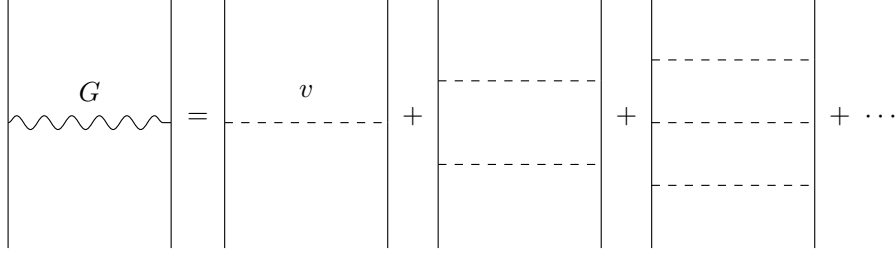


Figure 5.2: Schematic picture of the Bethe-Goldstone equation (5.11).

The G -matrix is introduced to overcome this issue. The G matrix is the sum of the ladder diagrams appearing in higher order terms, see Fig. 5.2 for an schematic picture. The Bethe-Goldstone equation is given as

$$G = v + v \frac{Q}{e} G, \quad (5.11)$$

where the Pauli operator Q ensures the intermediate state has momenta higher than the Fermi sea. The energy denominator e includes the single-particle energy including the modification by the single-particle potential. This equation can be written as $G = (1 - vQ/e)^{-1}v$. The divergent contribution from the interaction v is compensated between the denominator and numerator, mitigating the divergence problem. The perturbation series in the G matrix has been proven to be given in the number of hole lines and is called the hole-line expansion [228, 229]. The Brueckner-Hartree-Fock approximation in the energy only includes the two hole-line contribution:

$$E^{\text{HF}} = \sum_{n=1}^A \langle n | T | n \rangle + \frac{1}{2} \sum_{n=1}^A U_N(k_n), \quad (5.12)$$

where k_n is the momentum of the n -th nucleon.

The form of the single-particle potential can be chosen arbitrarily. Nevertheless, the Hartree-Fock one

$$U_{B_1}^{(B_2)} = \sum_{b \in \text{Fermi sea of } B_2} \langle B_1 b | G | B_1 b \rangle - \delta_{B_1, B_2} \langle B_1 b | G | b B_1 \rangle \quad (5.13)$$

is usually employed since a number of higher-order diagrams cancel with each other according to the Bethe-Brandow-Petschek theorem [230]. This enables one to compare the Brueckner-Hartree-Fock results of U with ones by other Hartree-Fock type models. The Bethe-Goldstone equation (5.11) must be solved self-consistently in terms of U . The single-particle potential U is calculated solely by the G -matrix and is independent of the perturbative expansion.

The convergence of the hole-line expansion is non-trivial. For lower density region, there is a naïve discussion [231]. Let c be the typical radius of the repulsive core. The average radius r_0 between particles are given by the density as $4\pi r_0^3/3 = 1/\rho$. The probability that two-particles have strong correlation is calculated from the proportion of

the volume as $(c/r_0)^3$. On the other hand, the probability for the three-body correlation is $(c/r_0)^6$. The four- and more-body correlation follow so. Since the number of the hole line in diagram represents the number of particles correlated, the n hole-line diagram contributes $(c/r_0)^3$ times the $n-1$ hole-line contribution. Then, the hole-line expansion is expected to be converged if the density is enough small. Baldo *et al.* [232–234] show that the three hole-line contribution has only a few percent below $2\rho_0$ for the continuous choice of the single-particle potential (see Sec. 5.4 for details) the Argonne [235] two-nucleon forces.

The three-body force is usually introduced as the density-dependent two-body force. The genuine three-body force v_{ijk} requires one to solve the so-called Bethe-Faddeev equation, the in-medium version of the Faddeev equation. This is extraordinary hard to be solved and requires huge numerical sources, spoiling the efficiency of the Brueckner theory. Then, usual treatment [131, 236–238] is to introduce them as the density-dependent two-body force by integrating out one of the fermion line up to its Fermi sea.

5.3 Detailed expressions for hyperons in matter

We now present the detailed expressions of the G matrix with hyperon-nucleon two-body force v . Here we rewrite the Bethe-Goldstone equation (5.11)

$$G(\omega) = v + v \frac{Q}{e(\omega) + i\epsilon} G(\omega). \quad (5.14)$$

The energy denominator $e(\omega)$ will be given later and is the difference between the so-called starting energy ω and the total energy of the intermediate state including the baryon single-particle potentials above the fermi sea. The infinitesimal value ϵ is added in the denominator to regularize the pole singularity, appearing typically in the ΛN – ΣN coupled channel.

The G -matrix can be explicitly expressed by the matrix element of the two-body state $|\mathbf{k}; \mathbf{K}, B_1, B_2, \alpha\rangle$, where B_1 and $B_2 \in (p, n, \Lambda, \Sigma^\pm, \Sigma^0)$ are the baryons, $\mathbf{k}$ and $\mathbf{K}$ correspond to the relative and center-of-mass momenta respectively, and α represents the quantum numbers of the two-body state. We introduce the following abbreviation of the initial, final, and intermediate states for convenience:

$$\begin{aligned} |\mathbf{k}; \mathbf{K}, i\rangle &= |\mathbf{k}; \mathbf{K}, B_1^i, B_2^i, \alpha^i\rangle, & |\mathbf{k}; \mathbf{K}, f\rangle &= |\mathbf{k}; \mathbf{K}, B_1^f, B_2^f, \alpha^f\rangle, \\ |\mathbf{k}; \mathbf{K}, m\rangle &= |\mathbf{k}; \mathbf{K}, b_1, b_2, \beta\rangle. \end{aligned}$$

Using the above conventions, the Bethe-Goldstone equation (5.14) is expressed as

$$\begin{aligned} \langle \mathbf{k}^f; \mathbf{K}, f | G(\omega) | \mathbf{k}^i; \mathbf{K}, i \rangle &= \langle \mathbf{k}^f; \mathbf{K}, f | v | \mathbf{k}^i; \mathbf{K}, i \rangle \\ &+ \sum_m \int \frac{d^3k}{(2\pi)^3} \langle \mathbf{k}^f; \mathbf{K}, f | v | \mathbf{k}; \mathbf{K}, m \rangle \frac{Q(\mathbf{k}; \mathbf{K}, m, \omega)}{e(\mathbf{k}; \mathbf{K}, m, \omega) + i\epsilon} |\mathbf{k}; \mathbf{K}, m\rangle G(\omega) | \mathbf{k}^i; \mathbf{K}, i \rangle, \end{aligned} \quad (5.15)$$

where $m = (b_1, b_2, \beta)$ runs over all possible species and quantum states. The momenta

have a relation with the those of the single baryons $\mathbf{k}_1$ and $\mathbf{k}_2$ with masses M_1 and M_2 as

$$\mathbf{K} = \mathbf{k}_1 + \mathbf{k}_2, \quad \mathbf{k} = \frac{\xi_{12}\mathbf{k}_1 - \mathbf{k}_2}{1 + \xi_{12}}, \quad \xi_{12} = \frac{M_2}{M_1}, \quad (5.16)$$

$$\Leftrightarrow \quad \mathbf{k}_1 = \frac{1}{1 + \xi_{12}}\mathbf{K} + \mathbf{k}, \quad \mathbf{k}_2 = \frac{\xi_{12}}{1 + \xi_{12}}\mathbf{K} - \mathbf{k}. \quad (5.17)$$

The original G matrix (5.15) involves Q operator, which depends on the angle between $\mathbf{k}$ and $\mathbf{K}$ of the intermediate YN state. This makes the sum $\mathbf{J} = \mathbf{L} + \mathbf{S}$ of spin $\mathbf{S}$ and relative angular momentum $\mathbf{L}$ not conserved. In order to facilitate a partial wave decomposition, the angle between $\mathbf{k}$ and $\mathbf{K}$ are usually used from the Brueckner's early study [239]. The angle-averaged Bethe-Goldstone equation is given as

$$\begin{aligned} \langle k^f; K, f | G(\omega) | k^i; K, i \rangle &= \langle k^f; K, f | v | k^i; K, i \rangle \\ &+ \sum_m \int \frac{k^2 dk}{(2\pi)^3} \langle k^f; K, f | v | k; K, m \rangle \frac{\bar{Q}(k; K, m)}{\bar{e}(k; K, m, \omega) + i\epsilon} \langle k; K, m | G(\omega) | k^i; K, i \rangle, \end{aligned} \quad (5.18)$$

where $\bar{Q}$ is given as [85],

$$\bar{Q}(k; K, m) = \frac{1}{2} \int_{-1}^1 d\cos\theta \Theta(|\mathbf{k}_1| - k_F^{(1)}) \Theta(|\mathbf{k}_2| - k_F^{(2)}) \quad (5.19)$$

$$= \left[0 \left| \frac{[-1|z_1|1] + [-1|z_2|1]}{2} \right| 1 \right], \quad (5.20)$$

when averaged over the angle θ between $\mathbf{K}$ and $\mathbf{k}$, with the shorthand notation $[a|b|c] \equiv \max(a, \min(b, c))$ introduced in Ref. [222], and the arguments

$$z_1 = \frac{1 + \xi_{12}}{2kK} \left\{ \left(\frac{1}{1 + \xi_{12}} K \right)^2 + k^2 - \left(k_F^{(1)} \right)^2 \right\}, \quad (5.21)$$

$$z_2 = \frac{1 + 1/\xi_{12}}{2kK} \left\{ \left(\frac{\xi_{12}}{1 + \xi_{12}} K \right)^2 + k^2 - \left(k_F^{(2)} \right)^2 \right\}. \quad (5.22)$$

The angle-averaged energy denominator is given by

$$\bar{e}(k; K, m, \omega) = \omega - \frac{K^2}{2M_m} - \frac{k^2}{2\mu_m} - M - \text{Re } U_{B_1}(\bar{k}_1) - \text{Re } U_{B_2}(\bar{k}_2), \quad (5.23)$$

with total and reduced masses, $M_m = M_1 + M_2$ and $\mu_m = M_1 M_2 / (M_1 + M_2)$. The angle-average is done for the arguments of the single particle potentials U_{B_i} of the intermediate baryons:

$$\bar{k}_1 = \left(\frac{1}{(1 + \xi_{12})^2} K^2 + k^2 + 2 \frac{1}{1 + \xi_{12}} K k \cos\theta \right)^{1/2}, \quad (5.24)$$

$$\bar{k}_2 = \left(\frac{\xi_{12}^2}{(1 + \xi_{12})^2} K^2 + k^2 - 2 \frac{\xi_{12}}{1 + \xi_{12}} K k \cos\theta \right)^{1/2}, \quad (5.25)$$

with the mean directional cosine

$$\overline{\cos \theta} = \frac{\int_{-1}^1 d \cos \theta \cos \theta Q(\mathbf{K}, \mathbf{k})}{\int_{-1}^1 d \cos \theta Q(\mathbf{K}, \mathbf{k})} = \frac{1}{2}([-1|z_2|1] - [-1|z_1|1]), \quad (5.26)$$

where $Q(\mathbf{K}, \mathbf{k})$ is the exact Pauli blocking operator. If two nucleons are involved, the previous expression for $\overline{\cos \theta}$ would vanish (in symmetric nuclear matter), because of the almost equal masses,

$$z_1 \approx z_2 \approx z_0 = \frac{1}{2kK} \left(\frac{1}{4} K^2 + k^2 + K k \overline{\cos \theta} \right) \quad \text{for} \quad \xi_{12} \approx 1. \quad (5.27)$$

Then, the alternative angular average

$$\overline{\cos \theta} \equiv \sqrt{\overline{\cos^2 \theta}} = \sqrt{\frac{\int_{-1}^1 d \cos \theta \cos^2 \theta Q(\mathbf{K}, \mathbf{k})}{\int_{-1}^1 d \cos \theta Q(\mathbf{K}, \mathbf{k})}} = \frac{1}{\sqrt{3}}[0|z_0|1] \quad (5.28)$$

is often used and applied in this thesis.

The single-particle potential of baryon specie B_1 in the medium of B_2 is given before angle average as

$$U_{B_1}^{(B_2)}(\bar{k}_1) = \sum_{J,L,S} (1 + \delta_{B_1 B_2} (-1)^{L+S}) (2J+1) \times \int \frac{d^3 k_2}{(2\pi)^3} \langle \mathbf{k}; \mathbf{K}, B_1, B_2, S, L, J | G_{\omega_{o.s.}} | \mathbf{k}; \mathbf{K}, B_1, B_2, S, L, J \rangle. \quad (5.29)$$

It is common practice to introduce a further angle average to utilize the angle-averaged G matrix. The squared momenta $K^2 = K^2(\mathbf{k}_1, \mathbf{k})$ and $k_2^2 = k_2^2(\mathbf{k}_1, \mathbf{k})$ entering the Bethe-Goldstone equation are replaced by their angle averages as

$$\begin{aligned} \bar{K}^2(k_1, k) &= \frac{\int_{\mathbf{k}_2 \leq k_F^{(2)}} d \cos \vartheta K^2(k_1, k, \cos \vartheta)}{\int_{\mathbf{k}_2 \leq k_F^{(2)}} d \cos \vartheta} \\ &= (1 + \xi_{12})^2 [k_1^2 + k^2 - k_1 k (1 + [-1|x_0|1])], \end{aligned} \quad (5.30)$$

$$\bar{k}_2^2(k_1, k) = \frac{\xi_{12}}{1 + \xi_{12}} \bar{K}^2(k_1, k) + (1 + \xi_{12}) k^2 - \xi_{12} k_1^2, \quad (5.31)$$

where ϑ is the angle between $\mathbf{k}_1$ and $\mathbf{k}$, and x_0 means

$$x_0 = \frac{\xi_{12}^2 k_1^2 + (1 + \xi_{12})^2 k^2 - (f_F^{(2)})^2}{2\xi_{12}(1 + \xi_{12})k_1 k}. \quad (5.32)$$

Changing the integral variable in Eq. (5.29) to the relative momentum $\mathbf{k}$ and applying the above angle average, the expression of the single-particle potential is given as

$$U_{B_1}^{(B_2)}(k_1) = \sum_{J,L,S} (1 + \delta_{B_1 B_2} (-1)^{L+S}) \frac{(1 + \xi_{12})^3}{2} \times (2J+1) \int_{k_{\min}}^{k_{\max}} \frac{dk}{(2\pi)^3} W(k_1, k) \langle B_1 B_2(k \bar{K}) S L J | G(\omega_{o.s.}) | B_1 B_2(k \bar{K}) S L J \rangle, \quad (5.33)$$

Table 5.1: Coupled channels involved for calculating the nucleon, Λ , and Σ single-particle potentials.

Strangeness S	Charge Q	Coupled states
0	0	nn
0	+1	np
0	+2	pp
-1	-1	$\Sigma^- n$
-1	0	$\Lambda n, \Sigma^0 n, \Sigma^- p$
-1	+1	$\Lambda p, \Sigma^+ n, \Sigma^0 p$
-1	+2	$\Sigma^+ p$

where the weight function $W(k_1, k)$ is defined as

$$W(k_1, k) = \frac{1}{2} \int_{|\mathbf{k}_2| \leq k_F^{(2)}} d\cos\vartheta = \frac{1}{2} (1 - [-1|x_0|1]). \quad (5.34)$$

The lower $k_{\min}$ and upper $k_{\max}$ integration boundaries are determined from the condition $W(k_1, k) = 0$ as

$$k_{\min} = \max \left(0, \frac{-k_F^{(2)} + \xi_{12} k_1}{1 + \xi_{12}} \right), \quad k_{\max} = \frac{k_F^{(2)} + \xi_{12} k_1}{1 + \xi_{12}}. \quad (5.35)$$

The starting energy ω is chosen as the on-shell one

$$\omega_{\text{o.s.}} = \frac{K^2}{2M} + \frac{k^2}{2\mu} + M + \text{Re } U_{B_1}(k_1) + \text{Re } U_{B_2}(\bar{k}_2). \quad (5.36)$$

as in the usual calculations to compensate large number of higher-order diagrams [221] according to the Bethe-Brandow-Petschek theorem [230]. The single-particle potential $U(B_2)_{B_1}$ is then calculated self-consistently by iterating the processes calculating $G = (1 - vQ/e)^{-1}v$ and the resulting single-particle potential evaluated by Eq. (5.33).

5.4 Numerical details

There are the coupled channels as shown in Table 5.1. The nucleon single-particle potentials are first calculated by the NN states and are employed their results for calculating U_Λ and U_Σ . For symmetric nuclear matter (SNM), the isospin symmetry implies that all three Σ single-particle potentials in the particle basis are equal, i.e. $U_{\Sigma^-} = U_{\Sigma^0} = U_{\Sigma^+}$. Then, it is not unnecessary to compute all the coupled channels, and U_{Σ^0} is chosen to be the representative of U_Σ . In pure neutron matter (PNM), the Σ^- can be calculated by only the single channel $\Sigma^- n$. In our procedure, U_Λ and U_Σ^0 are calculated by the charge $Q = 0$ coupled channel. If needed, U_Σ^+ is additionally evaluated by the charge $Q = +1$ coupled channel.

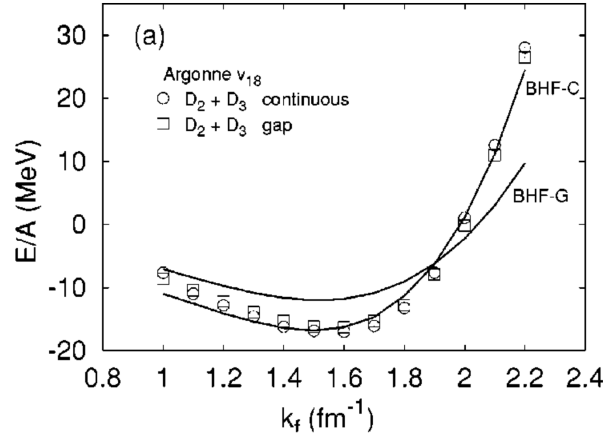


Figure 5.3: The equation of state of symmetric nuclear matter for the Argonne v_{18} potential [235]. The solid lines indicate the result at the Brueckner two-hole-line level of approximation, for the gap (BHC-G) and the continuous (BHF-C) choices. The full EOS including the three-hole-line contribution is labeled by the open squares and the open circles for the gap and continuous choices, respectively. Figure is taken from Ref. [234].

The definition of the U_{B_i} in Eq. (5.33) applies only to occupied states within the Fermi sea. The single-particle potential above the Fermi sea can be chosen arbitrary and should be chosen to achieve faster convergence in the hole-line expansion. There are commonly two choices of U_{B_i} : the gap choice which omits U above the Fermi sea, and the continuous choice which uses U as defined in Eq. (5.33) also above the Fermi sea. There are calculations that include higher-order contributions in the hole-line expansion [232, 232, 234, 240, 241], and those observe a faster convergence by the continuous choice over the gap choice, e.g. as shown in Fig. 5.3, which shows that the two hole-line calculation within continuous choice for the nuclear matter EOS is almost similar as the three-hole line calculations.

The pole singularity in the Bethe-Goldstone equation (5.11) is treated with the $\epsilon \rightarrow 0$ limit throughout this thesis by using the formula

$$\lim_{\epsilon \rightarrow 0^+} \frac{1}{x \pm i\epsilon} = \mp i\pi\delta(x) + \mathcal{P}\left(\frac{1}{x}\right), \quad (5.37)$$

where δ is the delta function and $\mathcal{P}$ denotes the principal value integral. Detailed expression can be found in the Appendix B of Ref. [59]. We note that the pilot study in the N²LO YN paper [101] utilizes finite ϵ value.

Chapter 6

Chiral two- and three-body forces up to NLO in matter

We are now in the position to discuss the in-medium behavior of the hyperons with the baryon forces within chiral EFT. First, as done in the previous work by Gerstung *et al.* [242], we use the next-to-leading order interaction and three-body forces whose low-energy constants are estimated via decuplet-baryon inserted diagrams. We newly focus on the Σ single-particle potential. The calculated Σ single-particle potential is found consistent with a phenomenological value and simultaneously the Λ single-particle potential fulfills the condition to suppress Λ 's in dense nuclear matter.

6.1 Introduction

The three-body forces are given in the mode to adjusted in such a way that the Λ single-particle potential is sufficiently repulsive at high density so that the appearance of Λ hyperons in neutron stars is suppressed. This is possible for different combinations of the low-energy constants (LECs), H_1 and H_2 , that characterize the strength of the 3BF (see the solid lines in Fig. 6 of Gerstung *et al.* [59]) However, it remains unclear how those 3BFs affect the corresponding Σ potential. The effective two-body forces resulting from the 3BFs considered in Ref. [59] contribute not only to the ΛN and ΣN channels but also to the ΛN - ΣN transition potential. A presently accepted constraint on the Σ potential is $U_\Sigma(\rho_0) = 30 \pm 20$ MeV [75], which is inferred using data on Σ^- atoms and on (π^+, K^+) inclusive spectra. This constraint is fairly well met by the original chiral YN potentials from 2013 and 2019 without 3BF [85, 103].

We evaluate the Σ potential in the same way as Gerstung *et al.* [59] have done for the Λ single-particle potential. The Brueckner-Hartree-Fock method with a continuous choice [85] is employed to calculate the hyperon single-particle potential; see Sec. 5.4 for details. Regarding the nuclear forces, the N³LO NN potential from Ref. [213] is used, while the NNN 3BF at N²LO is taken into account via a density-dependent two-body force. The cutoff in the regulator function [213] is chosen as 500 MeV. The nuclear saturation properties are reproduced by the nucleon forces [242].

6.2 Leading three-baryon force via decuplet saturation

The three-body force first appears at N²LO in the usual chiral perturbation within octet baryons and pseudoscalar mesons. Their diagrams are shown in Fig. 6.1. Especially in the SU(3)-flavor case, there involves many low-energy constants (LECs) in the six-baryon and meson-four-baryon contact vertices if one tries to determine them solely by using e.g. few-body hypernuclear spectroscopy like ${}^3_\Lambda\text{H}$, ${}^4_\Lambda\text{H}$, ${}^4_\Lambda\text{He}$, etc. Since the YNN forces play an essential role in dense matter, it is desirable to implement them by estimating the LECs reasonably.

A pathway is to include the diagrams with intermediate decuplet baryons. In the nucleonic sector, it is known that some of the LECs in the NNN force at N²LO becomes too large compare to the naïve order of magnitude expected from the perturbative series. This originates from the fact that the πN has a large coupling strength to the $\Delta(1232)$ resonance. One of the reasons lies in their rather small mass difference, $m_\Delta - m_\pi - m_N \simeq 150$ MeV. Therefore, it is natural to include the $\Delta(1232)$ resonance as an explicit degree of freedom to the chiral Lagrangian (cf. [243–245]) for a better power counting.

The SU(3) case can be discussed in a similar manner. The $\Sigma^*(1385)$ resonance is the low-lying hyperon resonance, which strongly couples to the $\pi\Lambda$ channel due to mass difference $m_{\Sigma^*} - m_\pi - m_\Lambda \simeq 130$ MeV. It should be noted that the mass difference is smaller than the $\pi N \rightarrow \Delta$ case, implying the importance of the $YN \rightarrow \Sigma^*$ channel. The two-meson exchange diagram has been known as Fujita-Miyazawa type three-body force [246]. Then, it is the good starting point to estimate the YNN LECs with only taking into account the decuplet-inserted diagrams. This assumption is called the decuplet saturation. As we show later, this assumption reduces the number of LECs to three. This kind of the three-body force has been employed widely [59, 65, 86, 217, 247, 248].

The 3BF diagrams with intermediate decuplet baryons are shown in Fig. 6.1. The vertex at N²LO that has the order $\Delta_i = 1$ is replaced by the decuplet baryon propagator. They involve only two-types of vertices as shown in Fig. 6.2. First one is the three-octet and one-decuplet $BBBB^*$ vertex, which appears when estimating LECs in both the six-baryon contact and four-octet and one-meson vertices. The $BBBB^*$ vertex is described by the non-relativistic Lagrangian with a minimal set of terms [131]

$$\mathcal{L} = H_1 \sum_{a,b,c,d,e,f=1}^3 \epsilon_{abc} \left[(\bar{T}_{ade} \vec{S}^\dagger B_{bd}) \cdot (\bar{B}_{fc} \vec{\sigma} B_{ef}) + (\bar{B}_{bd} \vec{S} T_{ade}) \cdot (\bar{B}_{fe} \vec{\sigma} B_{cf}) \right] \quad (6.1)$$

$$+ H_2 \sum_{a,b,c,d,e,f=1}^3 \epsilon_{abc} \left[(\bar{T}_{ade} \vec{S}^\dagger B_{fb}) \cdot (\bar{B}_{dc} \vec{\sigma} B_{ef}) + (\bar{B}_{bf} \vec{S} T_{ade}) \cdot (\bar{B}_{fe} \vec{\sigma} B_{cd}) \right], \quad (6.2)$$

where the decuplet baryons are represented by the totally symmetric three-index tensor T . The spin $\frac{1}{2}$ to $\frac{3}{2}$ transition operators $\vec{S}$ connect the two-component spinors of octet baryons with the four-component spinors of decuplet baryons. Conventions of T and $\vec{S}$ are given in the Appendix A. The $BBBB^*$ contact vertex includes only two LECs, H_1, H_2 , significantly reducing the number of parameters to be estimated.

The number of LECs in the $BBBB^*$ vertices also can be understood from the group

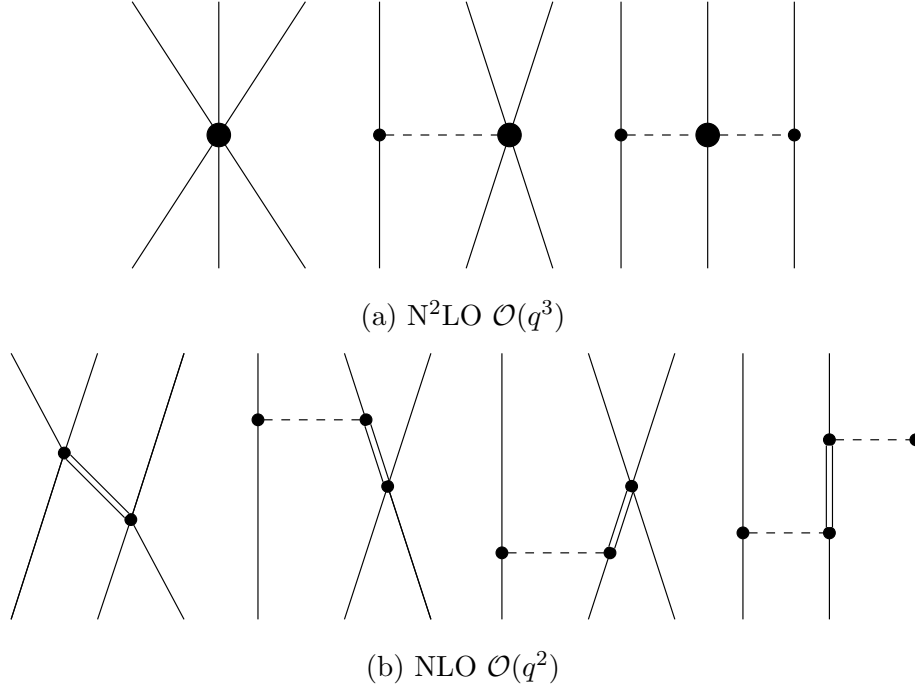


Figure 6.1: Three-baryon forces with different degree of freedoms. Decuplet baryons are represented by the double lines. The dot and large dot represent the vertex with orders $\Delta_i = 0$ and $\Delta_i = 1$, respectively, see Eq. (4.35) for definition. (a) Diagrams that appears at N²LO in a usual power counting. (b) Diagrams within decuplet-full EFT. The diagrams are promoted to NLO by the additional decuplet propagator.

theory viewpoint. The initial octet-octet BB state presents the flavor and spin representations

$$\begin{aligned} \text{Flavor of } BB: \quad & \mathbf{8} \otimes \mathbf{8} = \mathbf{27} \oplus \mathbf{8}_s \oplus \mathbf{1} \oplus \mathbf{10} \oplus \mathbf{10}^* \oplus \mathbf{8}_a, \\ \text{Spin of } BB: \quad & \mathbf{2} \otimes \mathbf{2} = \mathbf{1}_a \oplus \mathbf{3}_s. \end{aligned}$$

The first three representations $\mathbf{27}, \mathbf{8}_s, \mathbf{1}$ are symmetric under the exchange of the two baryons, while the others $\mathbf{10}, \mathbf{10}^*, \mathbf{8}_a$ belong to the anti-symmetric representations. The final octet-decuplet BB^* state is decomposed into

$$\begin{aligned} \text{Flavor of } BB^*: \quad & \mathbf{10} \otimes \mathbf{8} = \mathbf{35} \oplus \mathbf{27} \oplus \mathbf{10} \oplus \mathbf{8}, \\ \text{Spin of } BB^*: \quad & \mathbf{4} \otimes \mathbf{2} = \mathbf{3} \oplus \mathbf{5}. \end{aligned}$$

Since we are working on the SU(3)-flavor symmetric contact interactions, the initial BB and final BB^* representations must be identical. The spin state is then chosen as $\mathbf{3}$. One also has to take into account the Fermi statistics between the initial BB . In other words, the tensor product of the flavor and spin states has to be anti-symmetric. Therefore, the flavor state in the BB state must be totally anti-symmetric. The flavor representation is finally selected as $\mathbf{10}$ and $\mathbf{8}_a$, implying that the number of LECs are only two.

An important point is that some part of the spin structures within the original baryon contact vertices of the N²LO 3BFs are spoiled within the decuplet saturation. The above

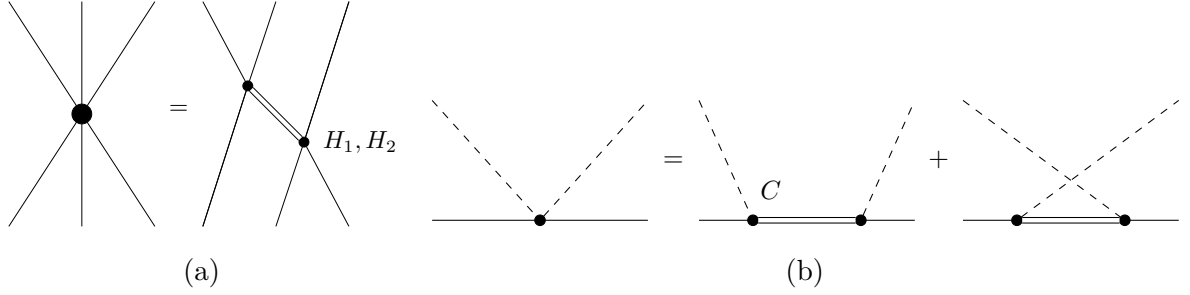


Figure 6.2: Vertices involved in the decuplet inserted diagrams: (a) Three-octet and one-decuplet vertex involved in six-baryon contact vertex. (b) Meson-baryon-decuplet vertex.

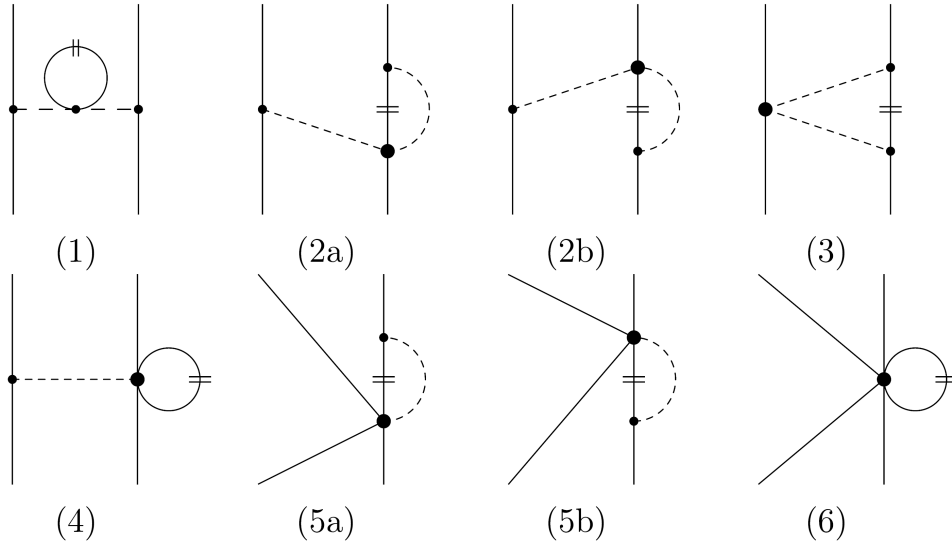


Figure 6.3: In-medium two-body forces.

group theory consideration tells us that the spin-singlet representation $\mathbf{1}_a$ in the BB disappears within the decuplet saturation. It has been known that the triton binding energy cannot be reproduced. Then, it is only the approximated way. Nevertheless, the spoiled spin structure is less for the decuplet case.

Next, we discuss the pseudoscalar meson-octet-decuplet vertex, shown as (b) in Fig. 6.1. The interaction Lagrangian is given as (e.g. Ref. [249])

$$\mathcal{L} = \frac{C}{f_0} \sum_{a,b,c,d,e=1}^3 \epsilon_{abc} \left(\bar{T}_{ade} \vec{S}^\dagger \cdot (\vec{\nabla} \phi_{db}) B_{ec} + \bar{B}_{ce} \vec{S} \cdot (\vec{\nabla} \phi_{bd}) T_{ade} \right), \quad (6.3)$$

with one LEC C . The value of C is typically filled with the large- N_c value $C = \frac{3}{4}g_A \approx 1$ is used. This value results in a decay width $\Gamma(\Delta \rightarrow \pi N) = 110.6$ MeV that is in good agreement with the empirical value of $\Gamma(\Delta \rightarrow \pi N) = (115 \pm 5)$ MeV [244].

The 3BFs are usually reduced to in-medium effective 2BFs for the Brueckner G -matrix calculation, since the three-body G -matrix calculation so-called Bethe-Faddeev equations is hard to solve and requires an enormous amount of computational resources.

Table 6.1: Considered combinations of LECs of the YNN force that reproduce $U_\Lambda(\rho_0) = -30$ MeV for NLO13 with 500 MeV of cutoff and simultaneously fulfill a "no- Λ in neutron star" constraint, $\mu_\Lambda(3\rho_0) \geq 80$ MeV in pure neutron matter. The values are in units of the inverse squared pion-decay constant with $f \approx 92$ MeV, and correspond to the left line in Fig. 6 of Ref. [59]. The values are taken from Gerstung's PhD thesis [242].

NLO13(500)	
H_1 (f^{-2})	H_2 (f^{-2})
-2.650	0.100
-2.200	0.000
-1.800	-0.100
-1.350	-0.200
-0.900	-0.300

The effective 2BFs are constructed by integrating out one external nucleon up to its Fermi momentum k_f^B as

$$V_{12} = \sum_{B=n,p} \text{tr}_{\sigma_3} \int_{|\vec{k}| \leq k_f^B} \frac{d^3 k}{(2\pi)^3} V_{123}. \quad (6.4)$$

Their diagrams can be found in Fig. 6.3. Explicit expressions of density-dependent YN forces are given fully in Gerstung's PhD thesis [242] and partly by Petschauer *et al.* [131].

6.3 Numerical details

The Brueckner calculation explained in Chapter 5 is employed to calculate the hyperon single-particle potentials. For the hyperonic force, the YN potential NLO13(500) [102] is employed. The YNN force is implemented as an effective density-dependent YN two-body force. The number of LECs involved in the YNN force is reduced by assuming decuplet dominance approximation [131]. Then, there are only three LECs: one related to the meson-octet-decuplet baryon vertex, and two denoted by H_1 and H_2 characterizing the strengths of the contact vertices with three-octet and one-decuplet baryons. The meson-octet-decuplet coupling is constrained by the decay width $\Gamma(\Delta \rightarrow N\pi)$, and its large- N_c value is employed [59]. The two LECs of the contact terms are fixed by requiring the reproduction of the empirical value of the Λ potential [75, 99],

$$U_\Lambda(\rho_0, k = 0) \simeq -30 \text{ MeV}, \quad (6.5)$$

inferred by using the Λ hypernuclear spectroscopy, and a strongly repulsive U_Λ at high density, sufficient to resolve the hyperon puzzle [59]. Some combinations of the contact LECs that fulfill these requirements are listed in table 6.1.

For nucleon force, we employ the chiral NN force to N³LO with 500 MeV cutoff by Entem and Machleidt [213] and supplemented three-body force tuned to reproduce some few-body nuclear observables [250]. This is same setting as Gerstung *et al.* [242]. There,

however, some sort of oscillatory behavior of U_N was found. This phenomena is also found in Ref. [86] employing the $N^3\text{LO}$ NN potential by Epelbaum *et al.* from 2005 [251]. The above calculations an additional cutoff regularization has been introduced and applied to the single-particle potential in the G -matrix equation to obtain numerically stable results. We follow the same prescription and we apply the additional cutoff as

$$U(k) \rightarrow U(k) \exp \left[- \left(\frac{k}{\Lambda_{\text{spp}}} \right)^6 \right], \quad (6.6)$$

where $\Lambda_{\text{spp}} = 700$ MeV is adopted in our calculation.

We will calculate the hyperon single-particle potentials up to $3\rho_0$, although in many nuclear matter calculation, chiral EFT is considered to brake down at about $2\rho_0$, corresponding to neutron Fermi momentum of $k_F = 400$ MeV in pure neutron matter (PNM). It should be noted that the applicable density region would be higher for hyperon single-particle potentials than the nucleon case. For the nucleon case, both two nucleons should be higher than the nucleon Fermi momentum, while the hyperon has no restriction on its momentum. For example, at $2\rho_0$ in PNM, both intermediate nucleon should have momentum $k_1, k_2 > 400$ MeV. For the hyperon-nucleon intermediate state, the hyperon's momentum can be $k \geq 0$ MeV and one nucleon should be higher than $k > 400$ MeV, about half of the involved momentum compared to the nucleon case. The scale for the hyperon single-particle potentials is then around 200 MeV in this case, well below the typical cutoff scale 500–600 MeV for the chiral YN interactions.

6.4 Hyperon single-particle potentials with two- and three-body forces at NLO

We first discuss the energy per nucleon shown in Fig. 6.4, calculated via the Brueckner-Hartree-Fock approximation as

$$E = \sum_{m \in \text{Fermi sea}} \left[\langle m | T | m \rangle + \frac{1}{2} U_N(k_m) \right]. \quad (6.7)$$

For the $NN + NNN$ force case, the energy per nucleon in SNM is consistent with the empirical saturation point, $E/A = -16$ MeV at ρ_0 . It must be emphasized that the NNN LECs are solely determined to reproduce the few-body observables [250]. In PNM, the chiral EFT $NN + NNN$ result is consistent with the well-known Akmal-Pandharipande-Ravenhall [47] EOS.

In Fig. 6.5, we show the density dependence of the single-particle potentials. One can see that certain sets of the YNN LECs reproduce the phenomenologically estimated value $U_\Sigma(\rho_0) = 30 \pm 20$ MeV [75]. Interestingly, those YNN forces also yield the Λ single-particle potentials that are strongly repulsive at high densities, as needed to suppress the Λ hyperons in neutron stars [242].

We also show in Fig. 6.6 the momentum dependence of the single-particle potentials. The variation in the YNN LECs gives similar momentum dependence. In the case with only YN , the shape of the momentum dependence resembles the two- and three-body

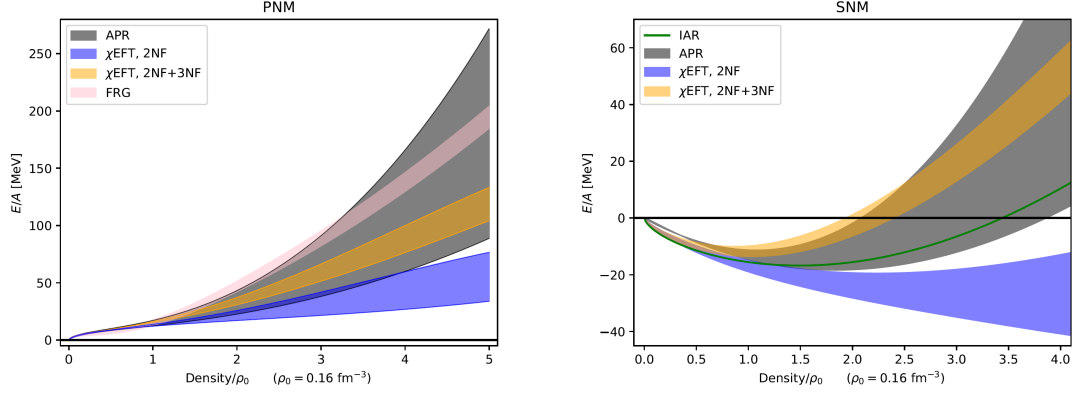


Figure 6.4: Energy per nucleon in PNM (left panel) and SNM (right panel). The chiral N³LO NN force calculations are shown in blue. The band represents a cutoff variation of 414, 450, and 500 MeV. The orange band additionally incorporates N²LO three-nucleon forces [250] tuned by few-body calculations. For comparison, Akmal-Pandharipande-Ravenhall EOS (APR, black) [47], functional-renormalization-group approach [252] (FRG, pink), and a Brueckner-Hartree-Fock approach by Isaule, Arellano, and Rios (IAR, green) [253] results are given. The figures are taken from the Gerstung's PhD thesis [242].

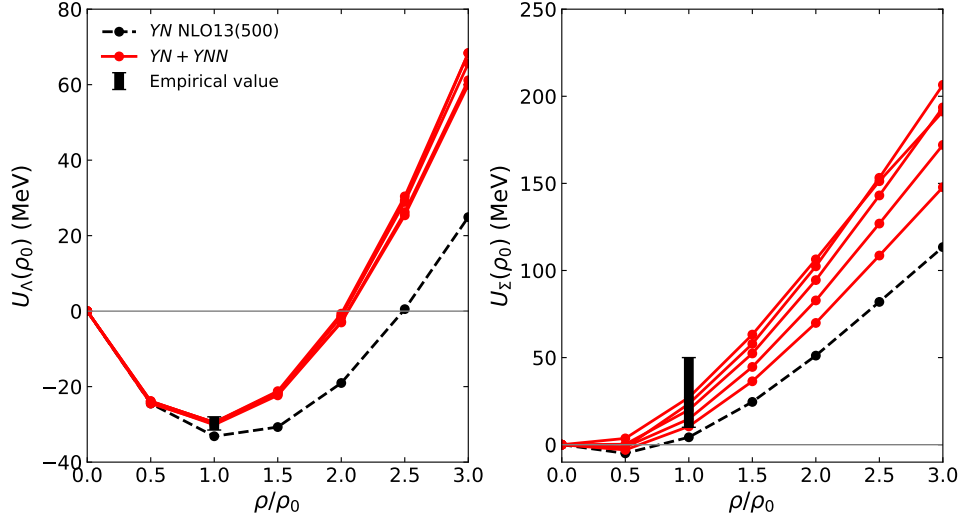


Figure 6.5: Density dependence of the Λ (left panel) and Σ (right panel) potentials in symmetric nuclear matter. The red solid lines are calculated by using the 3BF LECs in table 6.1. The only two-body case with the NLO13(500) parameter set is represented by the dashed line. The black bands show the empirical values of the Λ potential, $-31.5 < U_{\Lambda}(\rho_0) < -28.0$ MeV [99], and the Σ potential, $U_{\Sigma}(\rho_0) = 30 \pm 20$ MeV [75].

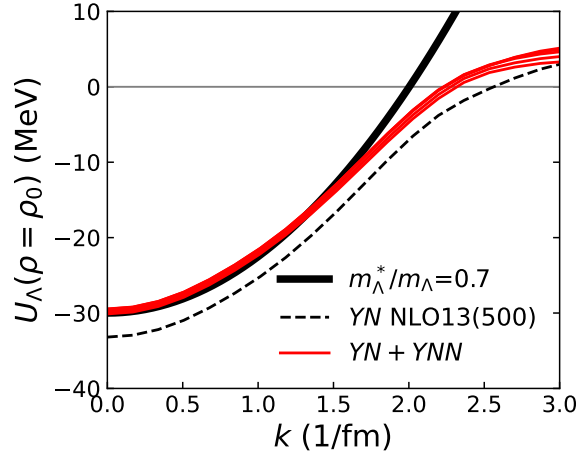


Figure 6.6: Momentum dependence of the Λ single-particle potential at ρ_0 . The result by NLO13 two-body force with 500 MeV cutoff is represented as the dashed line, while the results by the two- and three-body forces correspond to the thin solid lines. The quadratic form (6.8) with the effective mass $m_\Lambda^*/m_\Lambda = 0.7$ is also shown for comparison.

force cases, showing almost a constant shift at $p \leq 2.5 \text{ fm}^{-1}$. To illustrate the effective mass, the quadratic form of U_Λ with the effective mass $m_\Lambda^*/m_\Lambda = 0.7$

$$U_\Lambda(k) = U_\Lambda(k=0) + \frac{k^2}{2m_\Lambda} \left(\frac{m_\Lambda}{m_\Lambda^*} - 1 \right), \quad (6.8)$$

with $U_\Lambda(\rho_0, k=0) = -30 \text{ MeV}$ is shown. For lower momentum region, $k \leq 1.5 \text{ fm}^{-1}$, the above quadratic form can well reproduce the two- and three-body force results. It should be emphasized that the value of the effective mass $m_\Lambda^*/m_\Lambda = 0.7$ is in line with the hypernuclear constraint of $0.65 < m_\Lambda^*/m_\Lambda < 0.95$, see Fig. 3.2. Thus, the chiral YNN force can be chosen to solve the hyperon puzzle of neutron stars, reproduce the phenomenological value of U_Σ , and be consistent with the hypernuclear constraint.

6.5 Short summary

We have investigated the hyperon single-particle potentials based on the chiral two- and three-body forces. The YN force is chosen as a version of the NLO force constructed in 2013 [102] with 500 MeV of cutoff. The YNN LECs are estimated via the decuplet saturation assumption. They are determined to fulfill the hypernuclear constraint $U_\Lambda(\rho_0) = -30 \text{ MeV}$ and make $U_\Lambda(3\rho_0)$ enough repulsive to suppress Λ 's in pure neutron matter, avoiding the hyperon puzzle of neutron stars. The Brueckner theory with the continuous choice of the single-particle potential is employed to evaluate the hyperon potential in nuclear matter.

The calculated Σ single-particle potentials are found consistent the phenomenological value at ρ_0 , $U_\Sigma(\rho_0) = 30 \pm 20 \text{ MeV}$ [75]. They also fulfill the hypernuclear constraint on the effective mass elaborated in Chapter 2. Therefore, the chiral two- and three-

body force can be constructed to be consistent with the hypernuclear constraint, the Σ phenomenological analysis, and the hyperon puzzle of neutron stars.

Although it seems the two- and three-body forces are successful, one has to keep in mind that this work only shows the possibility to be consistent with the above data. There are still some points to be discussed more. First, the employed YN force NLO is an old-fashioned one. In 2023, the up-to-date chiral YN force at $N^2\text{LO}$ [101] has been constructed. There, some extensions from the old-type NLO force have been made, resulting in some qualitatively different behaviors in the hyperon single-particle potentials as we will discuss in the next Chapter. Additionally, we have considered only one cutoff of 500 MeV, which should be much more uncertainty related to the cutoff variation, usually around 500–600 MeV. Furthermore, it has been known in the nuclear matter studies (e.g. [254]) that the cutoff variation is not sufficient to estimate the uncertainty. One must at least perform uncertainty estimate of the truncation error in the chiral expansion. We will also perform such a more quantitative analysis of the hyperon single-particle potential in the next Chapter.

Chapter 7

Chiral two-body forces up to N²LO in matter

The Λ and Σ single-particle potentials in infinite nuclear matter are analyzed within a recently established chiral hyperon-nucleon (YN) interaction up to next-to-next-to-leading order (N²LO) in combination with an nucleon-nucleon interaction derived in the same scheme. The self-consistent Brueckner-Hartree-Fock method with the continuous choice of the single-particle potential is employed. It is found that the Λ single-particle potential is comparable to the results achieved with the next-to-leading order (NLO) YN interaction from 2019. The resulting Σ potential becomes more attractive compared to the previous NLO results due to the constraint from the recent ΣN differential cross section data measured in the J-PARC E40 experiment. An estimate of the theoretical uncertainty of the single-particle potentials is provided in terms of the truncation error in the chiral expansion.

7.1 Introduction

For many years phenomenological YN potentials have been used as starting point for such microscopic calculations. However, this has changed in recent years with the arrival of YN potentials derived within chiral effective field theory (EFT). That approach with an inherent power counting, initially suggested by Weinberg for application to the nucleon-nucleon (NN) interaction, allows to improve the results order by order and, equally importantly, enables also an estimation of the theoretical uncertainty. This is not only valid for the elementary YN interaction but applies also for few- and many-body studies where chiral YN interactions are employed.

Recently, the YN N²LO interaction has been introduced [101] with several extensions. The N²LO force is fitted to the Σ^+p and Σ^-p differential cross section data measured by the J-PARC E40 experiment [104–106]. Specifically, the backward scattering cross section of Σ^+p constrains the strength of the 3S_1 channel, resulting in less repulsive Σ potential compared to those by NLO13 and 19 potentials. We should note that the NLO19 can be tuned to the J-PARC E40 data by modifying LECs. The new N²LO interaction employs a noble regularization scheme called the semi-local momentum space regularization, which is shown to work for constructing NN force [216], while the non-

local regularization [213] is used for the conventional NLO13 and NLO19 interactions. By these extensions, one can expect that the qualitative feature on the hyperon single-particle potentials is different between the $N^2\text{LO}$ and the conventional NLO13 and NLO19 forces. The in-medium property of hyperons by the $N^2\text{LO}$ force partly worked in the original paper [101]. Working thoroughly on the $N^2\text{LO}$ YN force should provide a current understanding on the in-medium property of hyperons and on the unknown many-body force contributions toward the hyperon puzzle of neutron stars.

In this Chapter, we investigate the in-medium properties of the Λ and Σ hyperons employing chiral YN interactions. The study is performed within the conventional Brueckner theory [221, 228, 255] at first order in the hole-line expansion, i.e. the so-called Brueckner-Hartree-Fock (BHF) approximation. The starting point is the new YN potential published recently by the Jülich-Bonn group [101], based on $SU(3)$ chiral effective field theory up to next-to-next-to-leading order ($N^2\text{LO}$). Besides extending the chiral order with respect to our earlier next-to-leading order (NLO) YN potentials from 2013 (NLO13) [102] and 2019 (NLO19) [103], the new YN interaction builds also on a novel regularization scheme, the so-called semi-local momentum space regularization, which has been shown to work rather well in the nucleon-nucleon (NN) sector [216]. Furthermore, the potential incorporates new constraints from Σ^-p and Σ^+p differential cross sections measured in the J-PARC E40 experiment [104–106]. The new semi-local momentum space regularized (SMS) interactions are also the first YN potentials where the separation energies of light Λ hypernuclei can be rather well described [217].

Our study has two goals. One of them is to provide a general overview of the in-medium properties that result from the new SMS YN potentials. The other one, and may be the more important one, is to explore the convergence pattern of the chiral expansion and the regulator dependence of the properties of hyperons in nuclear matter. Specifically, we provide a first uncertainty estimate for the Λ and Σ single-particle potentials, following corresponding efforts for nucleons in infinite nuclear matter [256, 257]. So far in nuclear matter calculations with chiral YN potentials only the cutoff dependence has been used to estimate the uncertainty [84, 85, 227, 258].

An additional aspect we would like to address is the influence of YN P waves on the matter properties. Since experimental information on Λp differential cross sections at low momenta is rather limited [157, 158], the YN interaction in the P -waves cannot be determined reliably. In case of NLO13 (NLO19) [102, 103] the smallness of the Λp cross section above the ΣN threshold, i.e. at around $p_{lab} \approx 800$ MeV/ c was used as guideline to fix the strength of the P -wave interactions, together with constraints from the corresponding NN P -wave interactions provided by the underlying $SU(3)$ flavor symmetry. Agreement with the Λp data could be only achieved if the individual contributions from all the P -waves (1P_1 , 3P_0 , 3P_1 , 3P_2) are kept small, which means in turn that the corresponding phase shifts have to be small. As already mentioned, in the SMS YN potential Σ^-p and Σ^+p differential cross sections, measured at J-PARC [104–106], have been used to constrain the P -waves. However, also in this case a unique determination of all P -wave interactions was not possible [101].

We restrict our study to densities up to twice the nuclear matter saturation density $\rho_0 = 0.16$ fm $^{-3}$, i.e. to Fermi momenta below $k_F = 2.15$ fm $^{-1}$ (≈ 400 MeV) in pure neutron matter (PNM). This limit on the density is suggested by the breakdown scale

of the chiral expansion [214, 215] and by the cutoff values applied in the regulators of the chiral potentials [101–103], which are both in the order of 500 ± 100 MeV. See also the corresponding comments in Refs. [86, 256]. Moreover, since the presence of the Pauli operator in the G -matrix equation suppresses the contributions from lower momenta in nuclear matter calculations one is automatically more sensitive to the high-momentum part and that means to regulator artifacts. Extrapolations to higher density are, of course, always possible, but one should keep in mind that they are predominantly of phenomenological nature.

This Chapter is structured in the following way: Sec. 7.2 briefly introduces the baryon-baryon interactions employed in this work and then the Brueckner theory. In Sec. 7.3 we provide results for the single-particle potentials of the Λ and the Σ in symmetric nuclear matter (SNM) and in PNM. We focus on the results at nuclear matter saturation density, but we also discuss the density and the momentum dependence of the single-particle potentials. In Sec. 7.4 we perform an estimate of the theoretical uncertainty of our results for the hyperon single-particle potentials due to the truncation in the chiral expansion, the first of its kind. This Chapter ends with a brief summary.

7.2 Theoretical components

The YN forces within chiral effective field theory up to N^2LO are investigated in this work. The detailed description of the SMS YN forces are provided in the original paper [101]. The cutoff value are realized with the small χ^2 value when comparing with the YN scattering data. The SMS NLO and N^2LO interactions are applied cutoff that realizes the small χ^2 value when fitting to the YN scattering data. The cutoff values of 500–600 MeV are considered for the NLO and N^2LO forces, while the somewhat larger value of 700 MeV can only be applied [101] to the LO force. We note that the LO results are used solely for the uncertainty estimate and we do not discuss the results in detail. The N^2LO force with the 550 MeV cutoff has two variations. The conventional YN forces, NLO13 [102] and NLO19 [103], are also employed for comparison. The NN force is adopted the up-to-date N^4LO force with some contact NN force at N^5LO developed by Reinert *et al.* [216]. This force is regularized in the same manner as the N^2LO YN force. The NN force is then called the SMS N^4LO^+ NN force.

We utilize conventional Brueckner theory to calculate the single-particle potential of hyperons in nuclear matter using the YN interactions in free space. The details are introduced in Chapter 5. In the actual calculations partial waves up to a total angular momentum of $J = 5$ are taken into account. Note that due to the local regulator used for the meson-exchange contribution the potential drops off more slowly for large momenta which requires an appropriate mesh points with large maximum momentum for calculating the G -matrix. We employ the continuous choice of the single-particle potentials unless otherwise stated.

In Ref. [86], some sort of oscillatory behavior of U_N was found in calculations with the N^3LO NN potential by Epelbaum *et al.* from 2005 [251] and a density-dependent NN force [259]. A similar behavior was also observed by Gerstung [242] for the N^3LO NN potential by Entem and Machleidt [213]. In both calculations an additional cutoff regularization has been introduced and applied to the single-particle potential in the G -

matrix equation to obtain numerically stable results. We do not observe such artificial oscillations for the SMS NN potentials up to $\rho = 2\rho_0$ and, therefore, no additional cutoff is required. In fact, also for the semilocal coordinate-space regularized (SCS) NN potentials employed in the studies of Hu *et al.* [256,257] no cutoff artifacts were found [260].

7.3 Results

In this section we show and discuss results for the hyperon single-particle potentials. In particular, we document the momentum and density dependence of the Λ and Σ single-particle potentials predicted by the SMS YN interactions. We start with some results for nuclear matter based on the SMS NN potentials [216]. Specifically we show the predicted nucleon single-particle potential U_N . This quantity enters as input in the calculation of the in-medium properties of the hyperons.

7.3.1 Nuclear matter

Since to the best of our knowledge nuclear matter results for the SMS NN potentials of Reinert *et al.* [216] have so far not been reported, we present such results in this subsection. We focus on the potentials of highest order in the chiral expansion, namely $N^4\text{LO}^+$, which we use for our calculation of the hyperon properties. For comparison we present also results for the $N^3\text{LO}$ potentials by Entem and Machleidt (EM) [213], which have been used by Gerstung *et al.* [59]. It should be said that the actual calculations in that reference include also an additional effective density-dependent NN interaction, derived from $N^2\text{LO}$ NNN forces following the prescription of Holt *et al.* [236,237].

Nucleon single-particle potentials in symmetric nuclear matter (SNM) and in pure neutron matter (PNM) at nuclear matter saturation density are shown in Fig. 7.1, for the EM potentials [213] with cutoffs 450 and 500 MeV and for the SMS $N^4\text{LO}^+$ potentials with cutoffs 400–500 MeV [216]. Remarkably the momentum dependence of U_N resulting from the SMS NN potentials exhibits basically no variation with the cutoff, testifying that the residual regulator dependence at that order of the chiral expansion is indeed very small. In case of the EM $N^3\text{LO}$ potential the cutoff variation leads to some spread in the value of U_N , however, also here the actual momentum dependence is very similar.

In the BHF approximation the total energy is given by

$$E = \sum_{m \in \text{Fermi sea}} \left[\langle m | T | m \rangle + \frac{1}{2} U_N(k_m) \right], \quad (7.1)$$

where T is the kinetic energy operator. In Fig. 7.2 results for the total energy per particle in both SNM and in PNM are shown up to $2\rho_0$, corresponding to the Fermi momentum $k_F \leq 1.70 \text{ fm}^{-1}$ for SNM and $k_F \leq 2.15 \text{ fm}^{-1}$ for PNM. The Fermi momentum is related to the density via $\rho = 2k_F^3/(3\pi^2)$ for SNM and $\rho = k_F^3/(3\pi^2)$ for PNM, respectively. Both the EM $N^3\text{LO}$ and the SMS $N^4\text{LO}^+$ NN potentials yield similar results, expectedly because the underlying single-particle potentials are also very similar. The SNM results for $N^4\text{LO}^+$ are slightly too attractive and lie below the empirical value at the saturation point. In case of the EM potentials there is a larger variation with the cutoff.

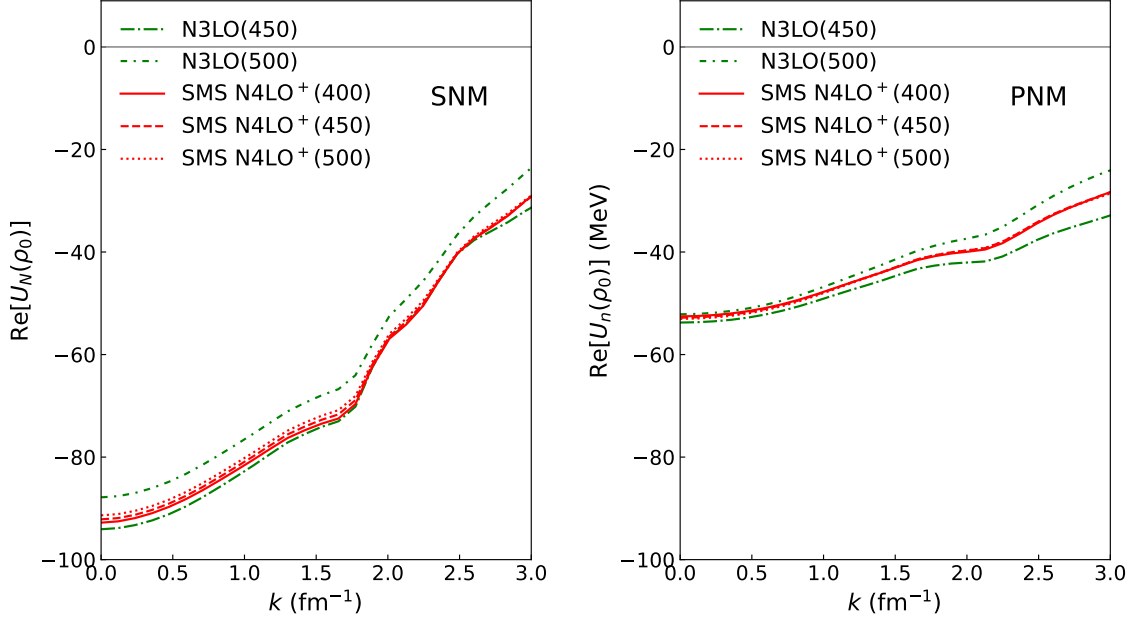


Figure 7.1: Nucleon single-particle potentials in symmetric nuclear matter (top) and pure neutron matter (bottom) at ρ_0 . $N^3\text{LO}(450)$ and $N^3\text{LO}(500)$ refer to the $N^3\text{LO}$ NN potential by Entem and Machleidt [213] with corresponding cutoff, while the SMS $N^4\text{LO}^+$ potential is by Reinert *et al.* [216].

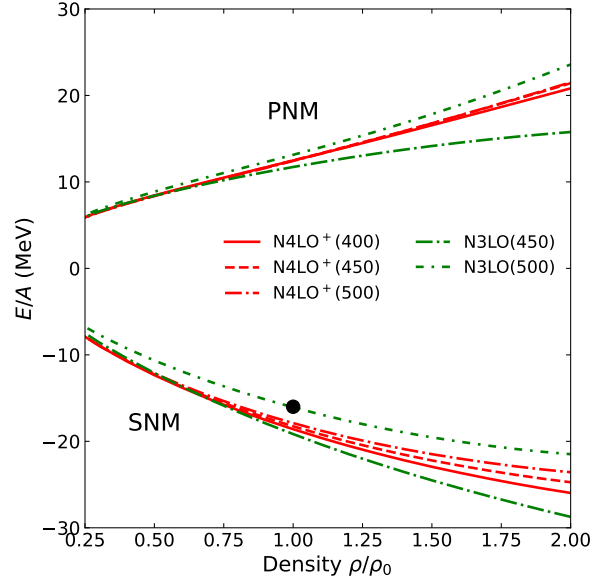


Figure 7.2: Total energy per particle in both symmetric nuclear matter and pure neutron matter. Results are shown for the $N^3\text{LO}$ [213] and SMS $N^4\text{LO}^+$ [216] NN potentials. The empirical value at the nuclear matter saturation point, $E/A = -16$ MeV, is indicated by a circle.

Table 7.1: Partial-wave contributions to $U_\Lambda(k=0)$ (in MeV) at $k_F = 1.35 \text{ fm}^{-1}$. Results are presented for the SMS YN interactions at different chiral orders and with different cutoffs [101] and, in addition, for the NLO13(500) [102] and NLO19(500) [103] interactions. For reference selected results based on the gap choice are included too.

	1S_0	$^3S_1 + ^3D_1$	3P_0	1P_1	3P_1	$^3P_2 + ^3F_2$	Total
SMS LO(700)	-13.9	-28.2	-1.0	0.7	1.0	-0.8	-42.4
SMS NLO(500)	-15.7	-25.6	0.4	4.5	2.0	-3.2	-37.7
SMS NLO(550)	-15.0	-24.5	0.5	2.5	1.5	-4.1	-39.3
SMS NLO(600)	-12.5	-20.2	0.4	1.0	1.5	-4.0	-34.0
SMS N ² LO(500)	-16.2	-24.8	0.3	2.9	0.6	-3.2	-41.2
SMS N ² LO(550) ^a	-15.2	-26.9	0.1	1.2	0.9	-4.9	-45.8
SMS N ² LO(550) ^b	-15.1	-27.3	0.4	1.5	1.6	-3.4	-43.4
SMS N ² LO(600)	-15.3	-25.4	0.2	1.6	1.0	-5.0	-44.1
NLO13(500)	-15.3	-18.9	0.9	0.2	1.6	-1.3	-33.1
NLO19(500)	-13.5	-28.7	0.9	0.3	1.6	-1.2	-40.9
SMS LO(700) gap	-12.1	-24.8	-0.9	0.7	1.0	-0.7	-37.0
SMS NLO(500) gap	-15.2	-22.2	0.5	5.0	2.1	-3.0	-32.9
SMS N ² LO(500) gap	-15.6	-20.8	0.4	3.3	0.6	-3.0	-35.9
NLO13(500) gap	-14.7	-13.7	1.0	0.3	1.6	-1.2	-27.0
NLO19(500) gap	-12.0	-27.1	1.0	0.3	1.7	-1.2	-37.5

7.3.2 Lambda in symmetric nuclear matter

We now proceed to the in-medium properties of the hyperons. The inputted U_N is calculated using the SMS N⁴LO⁺ NN force with the 450 MeV cutoff. First, we focus on the Λ single-particle potential at nuclear matter saturation density and for zero momentum, which can be compared with quasi-empirical values inferred from hypernuclear experiments. Table 7.1 summarizes the results for $U_\Lambda(k=0)$ and its partial-wave decomposition. We adopt the notation from Ref. [101] and specify the various potentials by their chiral order and by the employed cutoff mass. The values of $U_\Lambda(k=0)$ predicted by the SMS N²LO potentials range from -46 to -41 MeV, those of the NLO potentials are between -39 and -34 MeV. In comparison, the result for NLO19 (-41 MeV) is right in the middle of the SMS results while the one for NLO13 (-33 MeV) is at the upper edge. The usually cited quasi-empirical value, i.e. the well depths of a Λ -nucleus Woods-Saxon potential fixed in an analysis of hypernuclear data, amounts to $U_\Lambda \approx -30$ MeV [75]. A recent phenomenological analysis [142] based on hypernuclei estimates that the YN two-body contribution to $U_\Lambda(k=0)$ is -38.6 ± 0.8 MeV and attributes a contribution of 11.3 ± 1.4 MeV to repulsive three-body forces. It is interesting to see that the theoretical predictions from chiral YN interactions and the two-body result from the phenomenological analysis match so well. Nonetheless, one should keep in mind that contributions from three-body forces are in general scheme-dependent. However, within the Weinberg

scheme they can be made consistent with the two-body forces using gradient flow regularization [261]. Such work is underway by the LENPIC collaboration.

For testing purposes, we performed selected calculations with the gap choice. With that choice for the energy in the intermediate state, the single-particle potential U_Λ at ρ_0 is 3 to 7 MeV less attractive, well in line with earlier observations [76]. Also, our results with the gap choice agree within 1.2 MeV with the ones reported in Ref. [101], which utilized a parameterization of U_N results for the Argonne AV18 NN potential [235], provided in Ref. [253]. On the other hand, our result for NLO13(500) with continuous choice is clearly different from that in Ref. [85]. However, in that work only an NLO NN potential has been used. Potentials up to that order of the chiral expansion tend to be too repulsive for higher momenta/energies [85]. Still, the comparison illustrates that the nucleon single-particle potential can affect $U_Y(k=0)$. That said, when using the $N^3\text{LO}$ NN potentials from EM or the AV18 NN , we obtain results for U_Y very similar to the calculation with the SMS $N^4\text{LO}^+$ NN interaction.

We want to remind the reader that at LO only a very basic description of the YN interaction can be obtained [101]. We include the corresponding results primarily for completeness and because they enter into the uncertainty estimate.

Let us now discuss the results in Table 7.1 in detail. Obviously, there is a sizable variation in the contribution from the $^3S_1 - ^3D_1$ partial wave, within the SMS potentials but also in comparison to NLO13 and NLO19. It is in the order of 25 %. The 1S_0 contributions vary in the order of 10 %. There are also noticeable variations in the P -wave contributions. As explained in the Introduction, the P -waves have been determined differently in the SMS YN potentials and in NLO13 and NLO19. This is clearly reflected in the results in Table 7.1. While there is some variation in the P -wave contributions for the SMS potentials, there are more drastic differences to those for NLO13 and NLO19. As one can see, they differ by a factor 2-3 (3P_0 , 3P_2) or even by a factor 5-10 or more (1P_1). In this context, let us mention that calculations with a YN potential derived within covariant chiral EFT [227] predicts even larger (and repulsive) P -wave contributions at ρ_0 . Note, however, that the study is performed within the Dirac Brueckner-Hartree-Fock approach. Finally we want to remark that the P -wave interactions of the NLO13 and NLO19 potentials are identical. In contrast, in case of SMS $N^2\text{LO}(550)^a$ and $N^2\text{LO}(550)^b$ the P -waves are different while the S -wave interactions are the same. The two variants are the result of different fits to data on the Σ^+p differential cross section [101] and reflect remaining ambiguities in the P -wave contributions.

In Table 7.2, we list again the partial-wave contributions, however, from a different perspective, namely as a function of the density. We restrict ourselves to the SMS potentials with 550 MeV cutoff and NLO13 and NLO19. One can see that the total S -wave contribution clearly dominates U_Λ up to $2\rho_0$. The difference between NLO and $N^2\text{LO}$ is fairly moderate. The SMS results for the S -waves are also similar to that of NLO19.

With regard to the P -wave contributions, those are overall small but they become more relevant with increasing density. Their smallness is partly due to significant cancellations between the individual contributions. Comparing the individual P -wave results for SMS $N^2\text{LO}$ and NLO with those from NLO13 and NLO19 reveals that there is no clear tendency. This reflects the fact that the ΛN P -wave interactions are only poorly constrained by the existing experimental data. Specifically instructive are the results for the potentials

Table 7.2: Partial-wave contributions to $U_\Lambda(k=0)$ (in MeV) in symmetric nuclear matter for $\rho/\rho_0 = (0.5, 1.0, 2.0)$, corresponding to Fermi momenta $k_F = (1.07, 1.35, 1.7)$ fm $^{-1}$.

	ρ/ρ_0	1S_0	$^3S_1 + ^3D_1$	3P_0	1P_1	3P_1	$^3P_2 + ^3F_2$	S -waves	P -waves	D -waves	Total
SMS LO(700)	0.5	-8.1	-16.9	-0.4	0.2	0.3	-0.3	-25.0	-0.2	-0.1	-25.2
	1.0	-13.9	-28.2	-1.0	0.7	1.0	-0.8	-42.1	-0.2	-0.2	-42.4
	2.0	-23.3	-44.6	-2.4	1.8	2.9	-1.9	-68.2	0.3	-0.9	-68.6
SMS NLO(550)	0.5	-9.1	-17.0	0.1	0.7	0.5	-1.4	-26.1	-0.1	-0.1	-26.2
	1.0	-15.0	-24.5	0.5	2.5	1.5	-4.1	-39.6	0.4	-0.2	-39.3
	2.0	-24.1	-26.6	1.8	8.4	4.7	-11.3	-50.8	3.6	-0.8	-48.0
SMS N2LO(550) ^a	0.5	-9.2	-18.2	-0.0	0.3	0.2	-1.8	-27.4	-1.3	-0.3	-28.9
	1.0	-15.2	-26.9	0.1	1.2	0.9	-4.9	-42.0	-2.6	-1.1	-45.8
	2.0	-24.2	-31.8	0.6	4.9	3.4	-12.3	-55.5	-3.3	-4.4	-63.9
SMS N2LO(550) ^b	0.5	-9.2	-18.3	0.1	0.4	0.5	-1.3	-27.4	-0.4	-0.3	-28.0
	1.0	-15.1	-27.3	0.4	1.5	1.6	-3.4	-42.4	0.1	-1.1	-43.4
	2.0	-24.0	-33.0	1.6	5.7	5.7	-8.5	-56.7	4.7	-4.4	-56.9
NLO13(500)	0.5	-9.3	-15.3	0.2	0.1	0.5	-0.5	-24.6	0.3	-0.1	-24.3
	1.0	-15.3	-18.9	0.9	0.2	1.6	-1.3	-34.3	1.5	-0.2	-33.1
	2.0	-22.8	0.3	3.5	0.8	4.9	-3.5	-22.9	5.7	-0.8	-17.9
NLO19(500)	0.5	-8.7	-18.3	0.2	0.1	0.5	-0.5	-27.0	0.4	-0.1	-26.6
	1.0	-13.5	-28.7	0.9	0.3	1.6	-1.2	-42.3	1.5	-0.2	-40.9
	2.0	-15.6	-36.5	3.6	0.8	5.0	-3.3	-52.5	6.1	-0.8	-47.2

$N^2\text{LO}(550)^a$ and $N^2\text{LO}(550)^b$ established by alternative fits to the Σ^-p differential cross section, as already mentioned above. In this case, even the signs of the total P -wave contribution differ at ρ_0 and $2\rho_0$. A discussion of the impact of the P -wave ambiguities on U_Λ in the context of the YN potentials of the Nijmegen Group can be found in Ref. [76].

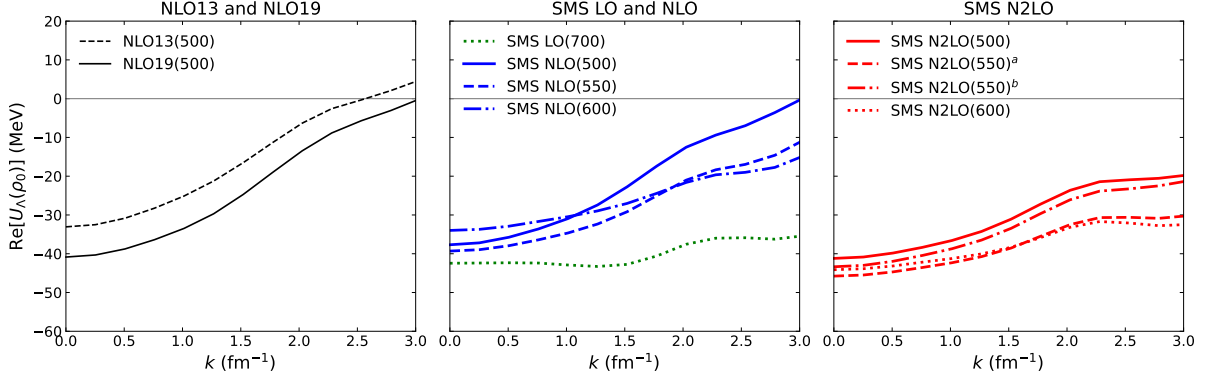


Figure 7.3: Momentum dependence of the Λ single-particle potentials in symmetric nuclear matter at ρ_0 .

Table 7.2 reveals that there are also sizable contributions from the D -waves, specifically for the $N^2\text{LO}$ potentials at $2\rho_0$. In order to understand that one has to recall that the contributions at $N^2\text{LO}$ which arise from two-meson exchange with subleading meson-baryon vertices are relatively large [251]. However, unlike in S - and P -waves, there are no contact terms in the D -waves yet which would counterbalance their effect and/or allow to reduce possible artifacts from the employed regulator. Such contact terms arise first at $N^3\text{LO}$ in the chiral expansion. In fact, in this context it is very instructive to look at the corresponding NN results by Hu *et al.* [256], see their Fig. 1. In the nuclear matter case, the $N^2\text{LO}$ result clearly deviates from the others and becomes exceptionally attractive with increasing density. When going to $N^3\text{LO}$ the general trend is restored. It is likely that the same will happen also in case of the in-medium properties of hyperons. Unfortunately, due to the lack of appropriate scattering data there is no way to extend the YN interaction to $N^3\text{LO}$ in the foreseeable future.

In Fig. 7.3 predictions for the momentum dependence of U_Λ at ρ_0 are shown. The momentum dependence of U_Λ from microscopic calculations is often used as guide line for establishing mean-field models. For example in Ref. [262], where it is argued that the momentum dependence can be a key to solve the hyperon puzzle of neutron stars. Furthermore, the momentum dependence has an impact on the hyperon dynamics in heavy-ion collisions [88].

In case of the NLO13 and NLO19 potentials, the resulting $U_\Lambda(k)$ rise monotonically with increasing momentum, with very similar trend. The SMS potentials exhibit a noticeably weaker momentum dependence, except for SMS NLO(500), where the behavior is comparable to that for NLO19(500). Overall, the momentum dependence predicted for the $N^2\text{LO}$ potentials is more moderate than those for the NLO interactions. The variation of $U_\Lambda(k)$ with the cutoff is of similar magnitude for the SMS NLO and $N^2\text{LO}$ potentials. Also, in both cases there is a slight increase with increasing momentum.

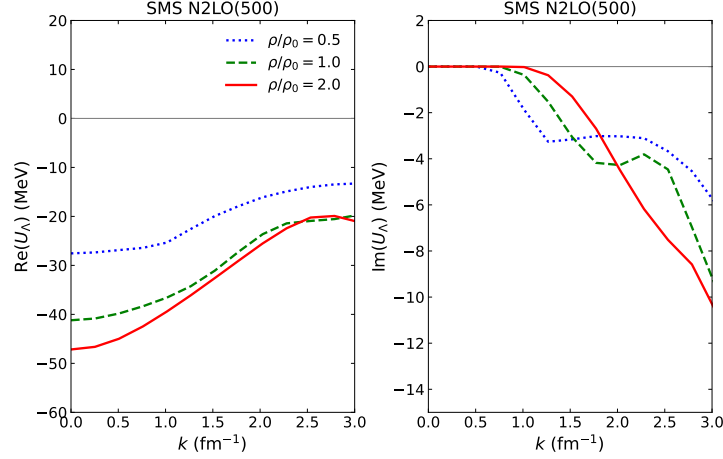


Figure 7.4: Momentum dependence of U_Λ in symmetric nuclear matter at for $\rho/\rho_0 = (0.5, 1.0, 2.0)$, corresponding to the Fermi momenta $k_F = (1.07, 1.35, 1.7) \text{ fm}^{-1}$. The SMS N²LO(500) YN potential is employed.

It is instructive to compare the variation of the momentum dependence with the cutoff to the one observed for U_N based on the N⁴LO⁺ potentials, see Fig. 7.1. It shows what one can expect in a case where the chiral expansion is already well converged and the residual regulator dependence is minimal.

An important quantity related to the momentum dependence is the effective mass, which is sensitive to the energy difference between orbitals of the Λ as considered from mean-field models [132]. The definition of the effective mass of Λ is usually given as

$$\frac{m_\Lambda^*}{m_\Lambda} = \left[1 + 2m_\Lambda \left. \frac{\partial \text{Re } U_\Lambda(k)}{\partial k^2} \right|_{k=0} \right]^{-1}. \quad (7.2)$$

Since for the considered YN potentials all U_Λ are in line with a k^2 dependence below $k = 1 \text{ fm}^{-1}$, we evaluate the effective mass as [80]

$$\frac{m_\Lambda^*}{m_\Lambda} = \left[1 + \frac{2m_\Lambda}{k^2} (U_\Lambda(k) - U_\Lambda(0)) \right]^{-1}, \quad (7.3)$$

with $k = 1 \text{ fm}^{-1}$. The calculated values are listed in Table 7.3. The values for the SMS N²LO (NLO) potentials are around $0.79 - 0.86$ ($0.73 - 0.83$), while those from NLO13 and NLO19 are around 0.7. For the SMS LO potential, $U_\Lambda(k)$ is practically constant for small k and, accordingly, the effective mass is close to 1.0. The SMS NLO and N²LO values are within the hypernuclear constraint, $0.65 < m_\Lambda^*/m_\Lambda < 0.95$ [99].

The variation in the momentum dependence with density is presented in Fig. 7.4, exemplary for the SMS N²LO(500) potential. We consider densities $\rho = 0.5\rho_0, \rho_0, 2\rho_0$, corresponding to the Fermi momenta $k_F = (1.07, 1.35, 1.7) \text{ fm}^{-1}$. For the Λ single-particle potential, the curvature around $k = 0 \text{ fm}^{-1}$ remains essentially unchanged with increasing density. This behavior means that the Λ effective mass decreases as the density increases, similar to the NLO13 results [85]. The imaginary part of U_Λ is also shown in Fig. 7.4. As one can see the actual momentum dependence changes noticeably with increasing density.

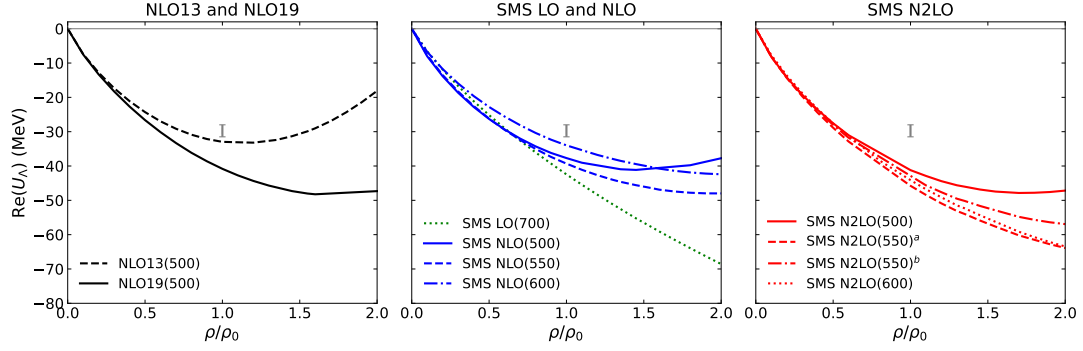


Figure 7.5: Density dependence of the Λ single-particle potential in symmetric nuclear matter. The bar symbolizes the quasi-empirical value [75].

Table 7.3: Normalized effective mass m_Λ^*/m_Λ in symmetric nuclear matter at ρ_0 .

	m_Λ^*/m_Λ
SMS LO(700)	1.03
SMS NLO(500)	0.73
SMS NLO(550)	0.79
SMS NLO(600)	0.83
SMS $N^2\text{LO}$ (500)	0.79
SMS $N^2\text{LO}$ (550) ^a	0.84
SMS $N^2\text{LO}$ (550) ^b	0.79
SMS $N^2\text{LO}$ (600)	0.86
NLO13(500)	0.69
NLO19(500)	0.71

Finally, in Fig. 7.5 we show the density dependence of U_Λ for SNM. For the SMS interactions with a cutoff of 500 MeV, and also for NLO13 and NLO19, U_Λ reaches a minimum in the considered density range. In the case of NLO13 there is even a pronounced rise from around $1.5\rho_0$ onwards, while a slower variation with density is seen for the other interactions. As known from the work of Gerstung *et al.*, U_Λ for NLO13 eventually changes sign and becomes repulsive around $2.5\rho_0$ [59]. The origin of the strikingly different behavior of the U_Λ results for NLO13 and NLO19 has been thoroughly discussed in Ref. [103]. It is a consequence of noticeable differences in the strength of the ΛN - ΣN transition potential between the two sets of YN interactions. Broadly speaking, a larger (smaller) channel-coupling strength leads to less attractive (more attractive) result for U_Λ , even when the corresponding YN scattering results, including the ΛN - ΣN transition cross sections, are identical. The sensitivity of U_Λ to the ΛN - ΣN transition strength is known for a long time [263–265]. For a complementary demonstration of coupled-channel effects on the in-medium properties of the Λ in the context of the NLO13 YN potential, we refer the reader to the work of Kohn [86]. As likewise discussed in Ref. [103], the actual strength of the transition potential is closely interrelated with possible contributions from three-body forces. Anyway, regarding the SMS NLO and N²LO interactions with larger cutoffs, their trends are still similar to that of the potentials with 500 MeV but, of course, the minimum of U_Λ will be reached at somewhat higher density.

Table 7.4: Partial-wave contributions to the real part of $U_\Sigma(k=0)$ (in MeV) at ρ_0 .

	1S_0	$^3S_1 + ^3D_1$	3P_0	1P_1	3P_1	$^3P_2 + ^3F_2$	Total
SMS LO(700)	−3.3	8.0	−1.0	−0.3	−0.4	−1.8	1.0
SMS NLO(500)	−3.4	−7.4	0.8	0.4	1.7	−1.5	−9.8
SMS NLO(550)	−2.7	−4.6	0.8	−0.5	0.6	−4.5	−11.3
SMS NLO(600)	−2.4	−3.1	0.7	−1.0	0.5	−5.2	−10.9
SMS N ² LO(500)	−5.9	−3.2	1.0	0.6	0.8	0.3	−7.8
SMS N ² LO(550) ^a	−3.4	−4.9	0.5	−0.8	1.0	−1.6	−10.3
SMS N ² LO(550) ^b	−3.5	−4.9	0.8	−3.3	1.5	−0.4	−11.0
SMS N ² LO(600)	−3.2	−5.3	0.6	−2.7	1.2	−2.0	−12.7
NLO13(500)	−4.9	6.4	1.3	0.2	1.2	0.2	3.7
NLO19(500)	−4.4	13.1	1.3	0.2	1.2	0.2	11.2
SMS LO(700) gap	−2.3	14.1	−1.0	−0.2	−0.4	−1.6	8.3
SMS NLO(500) gap	−2.7	−1.3	0.8	0.5	1.7	−1.2	−2.4
SMS N ² LO(500) gap	−5.4	6.2	1.0	0.6	0.7	0.6	2.8
NLO13(500) gap	−4.3	16.5	1.4	−0.0	1.1	0.2	14.6
NLO19(500) gap	−3.8	21.2	1.3	−0.1	1.1	0.1	19.7

Table 7.5: Partial-wave contributions to $\text{Re} U_\Sigma(k = 0)$ (in MeV). Total results for the isoscalar (U_Σ^0) and isovector (U_Σ^1) decomposition according to Eq. (7.4) are given as well. The column on the very right is the conversion width in nuclear matter, $\Gamma_\Sigma = -2 \text{Im} U_\Sigma(k = 0)$. Same description of interactions as in Table 7.1.

	Isospin $I = 1/2$			Isospin $I = 3/2$			$\text{Re} U_\Sigma^0$	$\text{Re} U_\Sigma^1$	Γ_Σ
	1S_0	$^3S_1 + ^3D_1$	P	1S_0	$^3S_1 + ^3D_1$	P			
SMS LO(700)	7.1	-16.7	-1.9	-10.4	24.7	-1.5	1.0	38.3	21.3
SMS NLO(550)	8.0	-25.0	-0.3	-10.7	20.3	-3.4	-11.3	43.1	30.1
SMS $N^2\text{LO}(550)^a$	7.5	-24.9	3.4	-11.0	20.0	-4.4	-10.3	35.0	34.0
SMS $N^2\text{LO}(550)^b$	7.5	-24.8	3.6	-11.0	20.0	-5.0	-11.0	33.6	30.4
NLO13(500)	6.2	-26.2	3.7	-11.1	32.5	-0.8	3.7	55.8	30.6
NLO19(500)	6.0	-19.5	3.8	-10.4	32.7	-0.8	11.2	43.2	22.3

7.3.3 Sigma in symmetric nuclear matter

Results for the total value and the partial-wave decomposition of the Σ single-particle potential at ρ_0 are listed in Table 7.4. As one can see, only the SMS LO interaction predicts a small repulsion, while the SMS NLO and $N^2\text{LO}$ potentials yield an attractive Σ potential at ρ_0 . The continuous choice results are around 10 MeV more attractive compared to those for the gap choice. Also, our own gap choice results are slightly more attractive compared to the initial results reported in Ref. [101]. But one has to keep in mind that the calculation in Ref. [101] is based on a finite ϵ in the Bethe-Goldstone equation, while now U_Σ is calculated exactly by taking the $\epsilon \rightarrow 0$ limit.

NLO13 as well as NLO19 yield a repulsive U_Σ at ρ_0 . The primary reason for the difference is that the contribution from the $^3S_1 - ^3D_1$ partial wave with isospin $I = 3/2$ is noticeably less repulsive for the SMS YN potentials. This can be more clearly seen from Table 7.5, where the isospin decomposition is listed. A reduction of the interaction strength in that channel has become necessary due to constraints from recent J-PARC E40 data on Σ^+p scattering [106]. The YN potentials NLO13 and NLO19 overshoot those data [101]. In fact, other YN potentials, like the FSS and fss2 interactions derived within the constituent-quark model by Fujiwara *et al.* [81], which predict a repulsive U_Σ at ρ_0 , likewise overestimate the cross section of the J-PARC experiment [106]. In this context let us mention that preliminary results on the Σ^+p correlation function by the ALICE Collaboration point also to a somewhat weaker Σ^+p interaction [266].

At the moment, it remains unclear to us how the constraints provided by the J-PARC data for the Σ^+p interaction can be reconciled with a strongly repulsive U_Σ in the order of 10 – 50 MeV as advocated in Ref. [75]. Certainly, there can be additional and repulsive contributions from three-body forces, of similar magnitude as expected for U_Λ . Indeed, a corresponding calculation, based on the ΛNN and ΣNN three-body forces introduced in Ref. [59] as possible solution of the hyperon puzzle, suggests that those contributions are repulsive and could be in the order of 10 to 20 MeV [100]. Then one would come already close to or would be even in agreement with the values inferred from phenomenological potentials. On the other hand, the large spread in the values deduced from various

Table 7.6: Partial-wave contributions to $\text{Re } U_{\Sigma}(k=0)$ (in MeV) in SNM for $\rho/\rho_0 = (0.5, 1.0, 2.0)$, corresponding to the Fermi momenta $k_F = (1.07, 1.35, 1.7) \text{ fm}^{-1}$. Same description of interactions as in Table 7.1.

	ρ/ρ_0	1S_0	$^3S_1 + ^3D_1$	3P_0	1P_1	3P_1	$^3P_2 + ^3F_2$	S -waves	P -waves	D -waves	Total
SMS LO(700)	0.5	-2.4	0.6	-0.5	-0.1	-0.3	-0.7	-1.8	-1.5	-0.1	-3.4
	1.0	-3.3	8.0	-1.0	-0.3	-0.4	-1.8	4.8	-3.5	-0.2	1.0
	2.0	-3.3	28.8	-2.0	-0.8	-0.2	-4.2	25.6	-7.3	-0.6	17.4
SMS NLO(550)	0.5	-2.6	-5.7	0.2	-0.2	0.1	-1.6	-8.2	-1.6	-0.1	-9.9
	1.0	-2.7	-4.6	0.8	-0.5	0.6	-4.5	-7.3	-3.6	-0.2	-11.3
	2.0	-1.1	4.8	3.0	-0.9	2.4	-12.1	3.8	-7.6	-0.8	-4.7
SMS N2LO(550) ^a	0.5	-3.0	-6.2	0.1	-0.4	0.2	-0.7	-9.1	-0.8	-0.2	-10.2
	1.0	-3.4	-4.9	0.5	-0.8	1.0	-1.6	-8.2	-0.9	-0.9	-10.3
	2.0	-2.0	1.8	2.1	-1.6	4.0	-2.1	0.2	2.4	-3.5	-1.5
SMS N2LO(550) ^b	0.5	-3.0	-6.1	0.2	-1.2	0.3	-0.3	-9.1	-1.0	-0.2	-10.4
	1.0	-3.5	-4.9	0.8	-3.3	1.5	-0.4	-8.2	-1.4	-0.9	-11.0
	2.0	-2.1	3.1	3.1	-8.2	5.6	1.0	1.4	1.5	-3.6	-1.1
NLO13(500)	0.5	-3.7	-2.4	0.3	0.0	0.3	-0.1	-6.1	0.5	-0.1	-5.6
	1.0	-4.9	6.4	1.3	0.2	1.2	0.2	1.6	2.8	-0.2	3.7
	2.0	-4.3	43.2	5.2	1.1	4.2	2.1	39.2	12.5	-0.7	50.9
NLO19(500)	0.5	-3.6	0.6	0.3	0.0	0.3	-0.1	-3.0	0.6	-0.1	-2.5
	1.0	-4.4	13.1	1.3	0.2	1.2	0.2	8.8	2.9	-0.2	11.2
	2.0	-2.6	63.1	5.3	1.3	4.2	2.6	60.6	13.4	-0.5	73.1

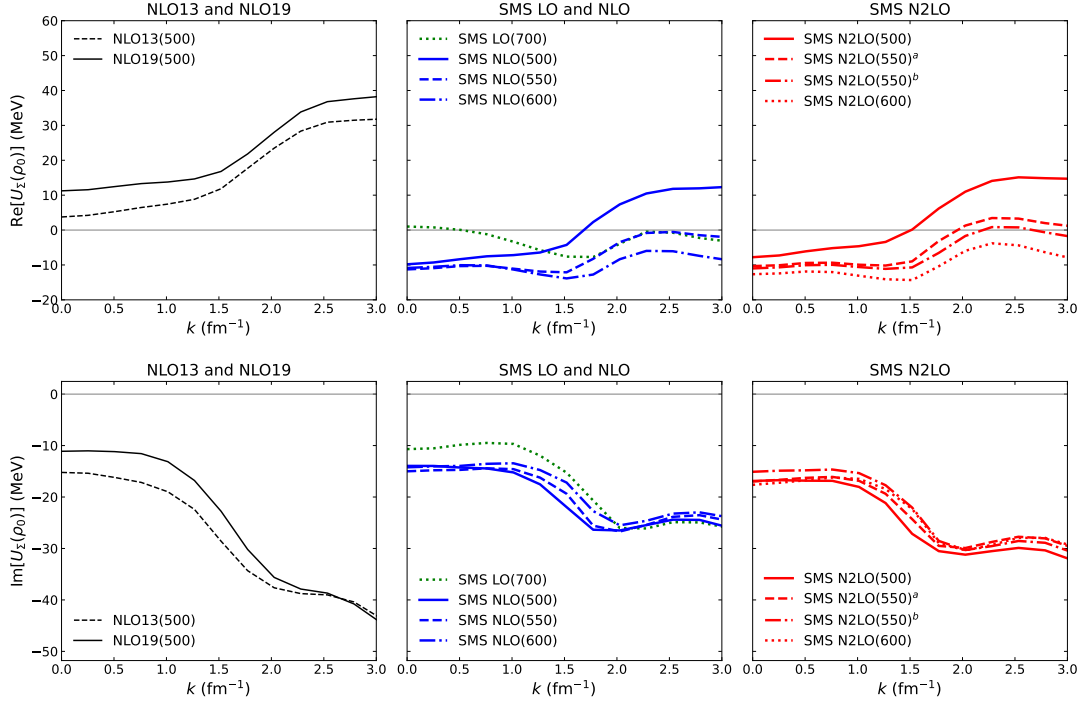


Figure 7.6: Momentum dependence of the Σ single-particle potential in symmetric nuclear matter at ρ_0 . Both real and imaginary parts are shown.

phenomenological analyses of data on Σ^- atoms and $(\pi^-, K^+)\Sigma$ spectra [267–271] discloses that there is a sizable model dependence in such studies. Therefore, one might question whether those potential parameters should be straightforwardly compared with results for U_Σ from a G -matrix calculation. After all, due to the complicated spin-isospin structure of the ΣN interaction, where some of the relevant S -waves are attractive and others repulsive, and the coupling to ΛN , the situation is rather complex and it remains unclear to which extent that can be captured by a simple Woods-Saxon form.

For completeness we include in Table 7.5 the isoscalar and vector decomposition of U_Σ [75]:

$$U_\Sigma = \left(U_0^\Sigma + \frac{1}{A} U_1^\Sigma \mathbf{T}_A \cdot \mathbf{t}_\Sigma \right), \quad (7.4)$$

where $\mathbf{t}_\Sigma$ is the Σ isospin operator and $\mathbf{T}_A$ is the nuclear isospin operator with z projection $(Z - N)/2$. The isoscalar U_0^Σ and isovector U_1^Σ components are evaluated by $U_\Sigma^0 = U_{\frac{3}{2}} + U_{\frac{1}{2}}$ and $U_\Sigma^1 = U_{\frac{3}{2}} - 2U_{\frac{1}{2}}$. We note that the isoscalar component is identical to the total value of U_Σ in SNM. The isovector part is smaller than the phenomenological value listed in Refs. [75, 272]. However, also here one has to be cautious with a direct comparison, see the discussion above.

The partial-wave breakdown for different densities is presented in Table 7.6. The SMS NLO and $N^2\text{LO}$ potentials yield similar S -wave contributions to U_Σ . They differ considerably from those by NLO13 and NLO19, due to the already mentioned difference in the $I = 3/2$ ${}^3S_1 - {}^3D_1$ channel. For P -waves, there are again drastic differences in

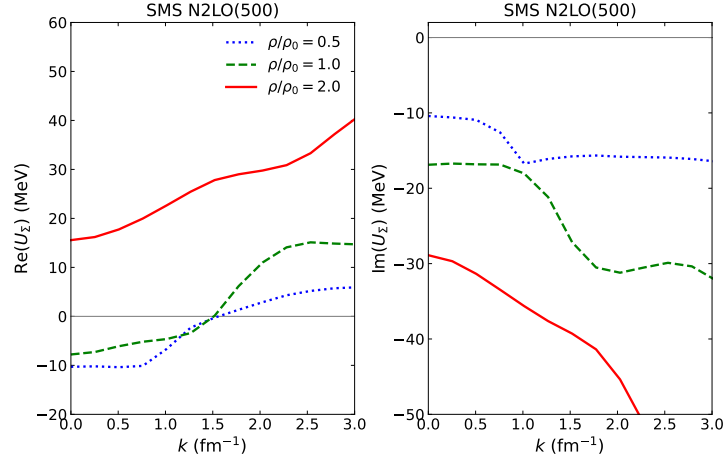


Figure 7.7: Momentum dependence of U_Σ in symmetric nuclear matter for $\rho/\rho_0 = (0.5, 1.0, 2.0)$, corresponding to the Fermi momenta $k_F = (1.07, 1.35, 1.7) \text{ fm}^{-1}$. The SMS N²LO(00) YN potential is employed.

the results for individual partial waves by the different potentials. In some cases the contributions are fairly large like those in the $^3P_2 - ^3F_2$ by the SMS NLO potential or in the 1P_1 by the N²LO(550)^b potential. Again, like in case of U_Λ , there are cancellations between the individual P -wave contributions. Due to that, for the two N²LO potentials the total P -wave contributions are almost the same, but they differ noticeably from those for SMS NLO and for the NLO13 and NLO19 potentials. Finally, there is a dramatic increase of the D -wave contribution to U_Σ at $2\rho_0$ in case of the SMS N²LO potentials. The reasons for that are the same as already discussed for U_Λ in the preceding subsection.

Figure 7.6 provides the momentum dependence of the Σ single-particle potential in SNM at ρ_0 . The real and imaginary part of U_Σ is shown. All SMS potentials predict a quite moderate momentum dependence for low k values. A more noticeable variation occurs only for momenta around $k = 1.5 \text{ fm}^{-1}$. Overall the results for the SMS NLO and N²LO potentials are very similar. Also the variation with the cutoff is very similar.

Results of $U_\Sigma(k)$ for different densities are shown in Fig. 7.7, exemplary for the SMS N²LO(500) potential. Overall, the Σ single-particle potential becomes more repulsive with increasing density. Also, the momentum dependence near $k = 0 \text{ fm}^{-1}$ gets more pronounced as density increases while its behavior above $k = 1 \text{ fm}^{-1}$ varies strongly with the density. The imaginary part becomes noticeably larger as density increases, indicating that Σ hyperons are more likely to convert into Λ hyperons at higher density and momenta. The imaginary part of U_Σ is related to the conversion (or spreading) width inside nuclear matter, $\Gamma_\Sigma = -2\text{Im} U_\Sigma(k = 0)$. Our result for SMS N²LO(500), shown in Fig. 7.7, corresponds to $\Gamma_\Sigma = 34 \text{ MeV}$ at ρ_0 . This value is consistent with the ones reported for other YN potentials [82, 222]. Some results for other chiral YN potentials are listed in Table 7.5.

Figure 7.8 shows the density dependence of the Σ single-particle potential in SNM. The SMS NLO and N²LO potentials predict an attractive U_Σ throughout, except for the potentials with a cutoff of 500 MeV where U_Σ turns to repulsion at around $1.5\rho_0$. The results for NLO13 and NLO19 are radically different. Here U_Σ becomes repulsive already

at lower densities, and then the repulsion increases rapidly with density. As already discussed above, the different behavior is a consequence of the constraints by the J-PARC Σ^+p data [106] on the ΣN interaction in the $I = 3/2$ channel that have been taken into account in the SMS YN potentials.

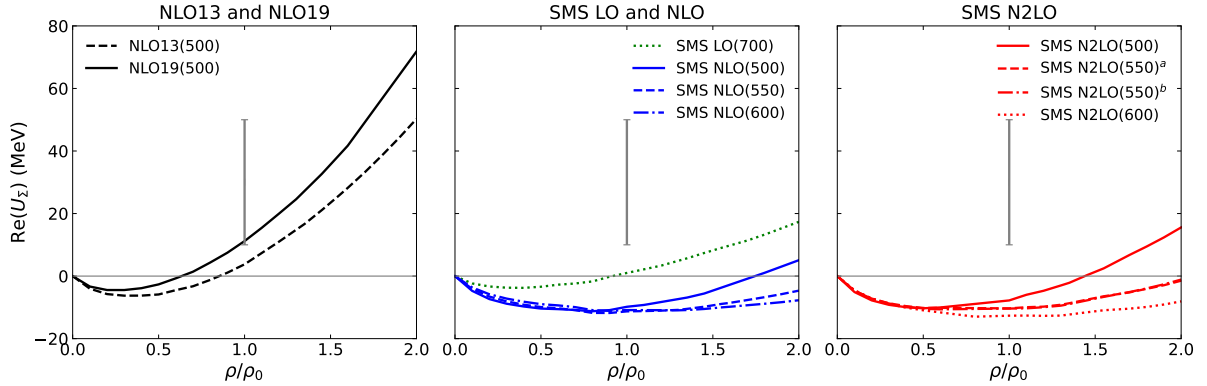


Figure 7.8: Density dependence of the Σ single-particle potential in symmetric nuclear matter. The bar indicates results from phenomenological analyses [75].

7.3.4 Lambda and Sigma in pure neutron matter

The dependence of U_Y on the momentum for PNM is presented in Fig. 7.9, for Λ , Σ^+ , Σ^0 , and Σ^- , using the SMS $N^2\text{LO}(500)$ potential. For all cases, U_Y exhibits a fairly weak momentum dependence, similar to that observed in the SNM case. Remarkably, U_{Σ^+} is larger than U_{Σ^-} , for $\rho \geq \rho_0$, in contrast to the NLO13 results [85, 86]. This is the consequence of the different isospin dependence of the SMS ΣN potential, i.e. the weaker $I = 3/2$ repulsion imposed by the J-PARC E40 data [106], as already seen in the SNM calculation, cf. Table 7.5.

In Fig. 7.10 the density dependence of the hyperon single-particle potentials in PNM is presented. With regard to the Λ , the SMS NLO and $N^2\text{LO}$ results exhibit a steady increase of the attraction with density whereas those from NLO13 and NLO19 become noticeably less attractive from around ρ_0 onward. Nevertheless the difference between the SNM and PNM results are small for the Λ , since it is an isoscalar particle.

In contrast, the isospin $I = 1$ hyperon Σ is strongly affected by the relative fraction of protons and neutrons in nuclear matter. The Σ^-n system has isospin $I = 3/2$. The corresponding interaction is identical to the one for Σ^+p when isospin-breaking effects are disregarded and predominantly repulsive. However, in case of the SMS YN potentials the repulsion is significantly reduced in comparison to NLO13 and NLO19, due to their adjustment to the new Σ^+p data, as already mentioned. As a result, the corresponding Σ^- single-particle potential is not strongly repulsive anymore.

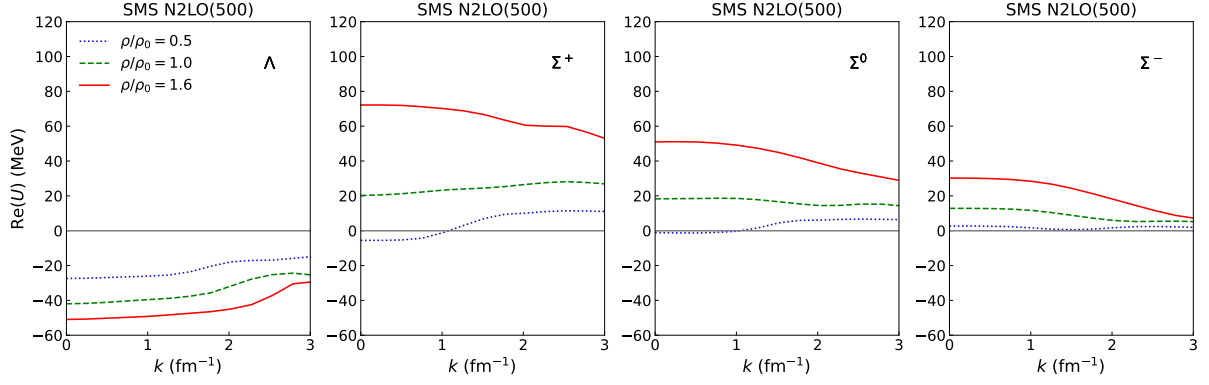


Figure 7.9: Momentum dependence of the real part of the single-particle potentials for hyperons in pure neutron matter for different densities $\rho = (0.5, 1.0, 1.6)$, corresponding to the Fermi momenta $k_F = (1.35, 1.7, 2.0) \text{ fm}^{-1}$.

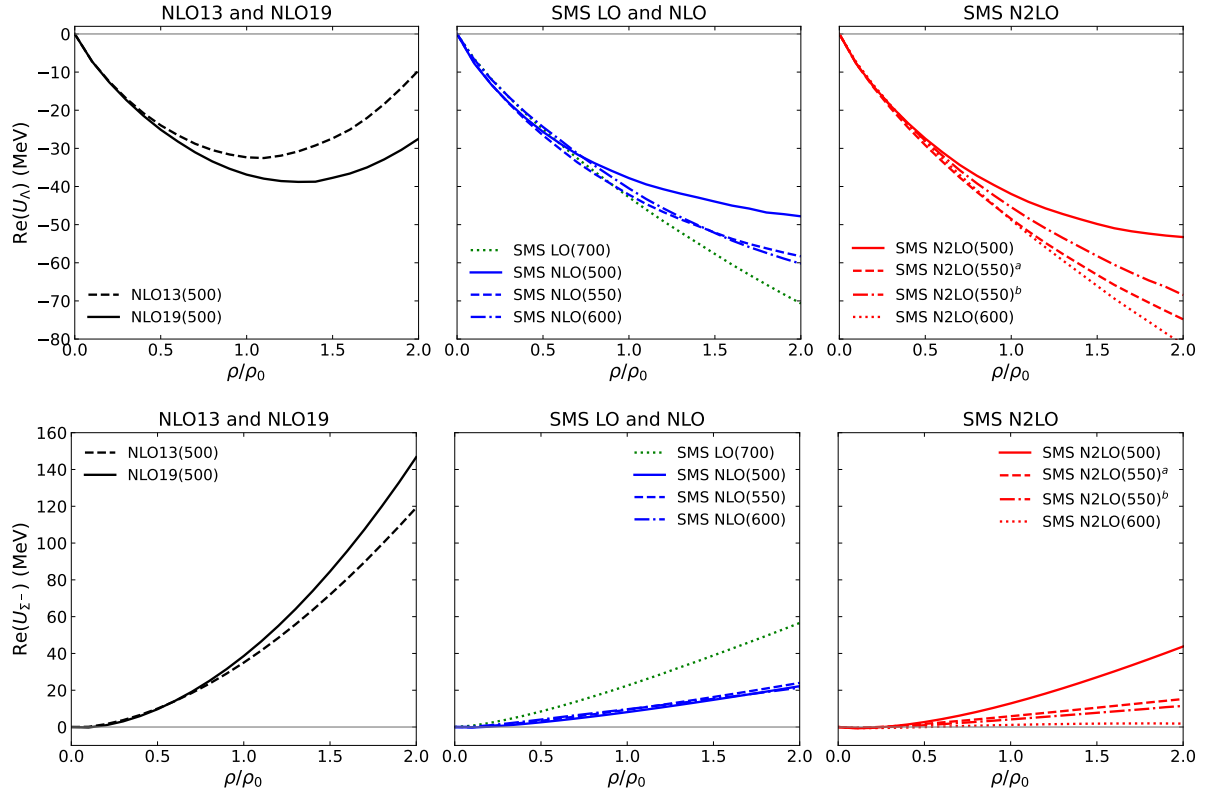


Figure 7.10: Density dependence of the Λ (upper panels) and Σ^- (lower panels) single-particle potentials in pure neutron matter.

7.4 Uncertainty estimate

In this section, we provide an uncertainty estimate for the hyperon single-particle potentials at $k = 0$, i.e. for the quantity that is associated with the binding energy of hyperons in infinite nuclear matter. In particular, we estimate and discuss the truncation error in the chiral expansion, following the method proposed by Epelbaum, Krebs, and Meißner (EKM) [214, 273]. It combines information about the expected size and actual size of higher-order corrections. It can be applied to any observable X which has been evaluated up to a specific order i in the chiral expansion, $X^{(i)}$. The concrete expressions for the corresponding uncertainty $\delta X^{(i)}$ are [215],

$$\begin{aligned}\delta X^{\text{LO}} &= Q^2 |X^{\text{LO}}|, \\ \delta X^{\text{NLO}} &= \max\left(Q^3 |X^{\text{LO}}|, Q |X^{\text{NLO}} - X^{\text{LO}}|\right), \\ \delta X^{\text{N}^2\text{LO}} &= \max\left(Q^4 |X^{\text{LO}}|, Q^2 |X^{\text{NLO}} - X^{\text{LO}}|, Q |X^{\text{N}^2\text{LO}} - X^{\text{NLO}}|\right),\end{aligned}\quad (7.5)$$

with the additional constraint for the theoretical uncertainties at LO and NLO to have at least the size of the actual higher-order contributions [214]. Here, Q represents the expansion scale in the chiral expansion, which is given by $Q \in \{p/\Lambda_b, M_\pi/\Lambda_b\}$, where p is the typical momentum of the baryons, M_π is the pion mass, and Λ_b is the breakdown scale of the chiral expansion. This method has been already applied to nuclear matter properties in the work by Hu *et al.* [256]. Thereby, the momentum scale p has been identified with the Fermi momentum k_F . We adopt the same prescription in our work.

For the breakdown scale we use $\Lambda_b = 480$ and 600 MeV for SNM and PNM, respectively. This choice was guided by the Bayesian analysis of nuclear matter properties by Hu *et al.* [257], which suggested a breakdown scale of 480 MeV for SNM and 660 – 720 MeV for PNM. In the initial study by Hu *et al.* [256] with the EKM method, $\Lambda_b = 600$ MeV has been used. The breakdown scale in the original work by EKM was estimated to be around $\Lambda_b = 400$ – 600 MeV [215], based on results for NN phase shifts and scattering observables. The typical values of Q are 0.55 and 0.69 at ρ_0 and $2\rho_0$ in SNM, respectively. For comparison, in the study of the uncertainty for light Λ hypernuclei a scale of $Q = 0.4$ was found appropriate [117]. As pointed out in Ref. [215], such a simple estimation of the theoretical uncertainty does not provide a statistical interpretation. Nonetheless, the procedure can be interpreted in a Bayesian sense [274].

In Fig. 7.11, the uncertainty estimate for the density dependence of $U_\Lambda(k = 0)$ in SNM is shown. We selected the SMS LO(700), SMS NLO(550), and SMS $N^2\text{LO}(550)^b$ YN potentials as exemplary set. The upper limits of the LO and NLO uncertainty bands at ρ_0 touch the quasi-empirical value of $U_\Lambda \approx -30$ MeV. In contrast, the $N^2\text{LO}$ band underestimates it. The actual estimates at ρ_0 are

$$\begin{aligned}U_\Lambda^{\text{NLO}}(\rho_0) &= -39.3 \pm 7.0 \text{ MeV}, \\ U_\Lambda^{\text{N}^2\text{LO}}(\rho_0) &= -43.5 \pm 3.8 \text{ MeV}.\end{aligned}\quad (7.6)$$

Obviously, the uncertainty is of an order which brings the SMS NLO results (and those of the NLO13 and NLO19 YN potentials) practically in line with the quasi-empirical

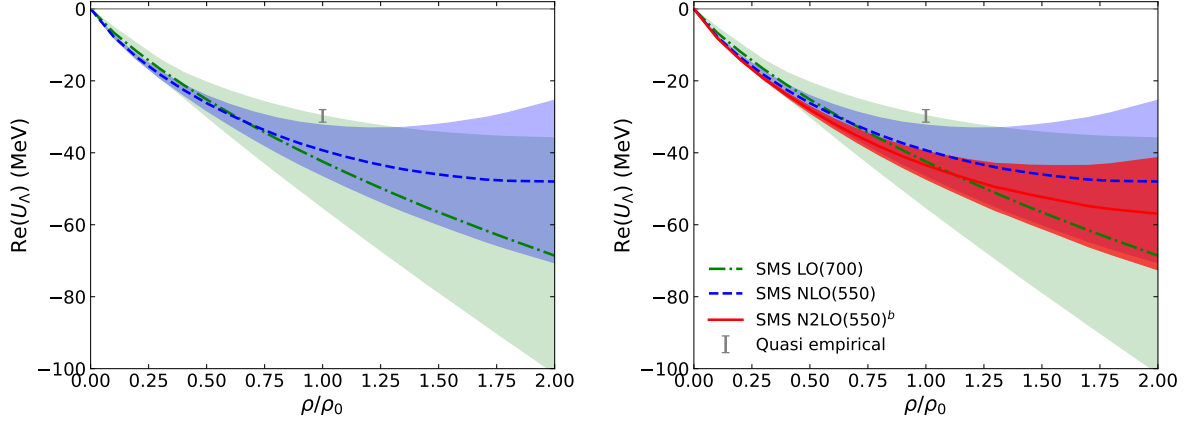


Figure 7.11: Uncertainty estimate of the Λ single-particle potentials in symmetric nuclear matter. The SMS NLO and N²LO interactions with cutoff 550 MeV are employed, while for SMS LO the potential with 700 MeV cutoff is used. The bar symbolizes the quasi-empirical value [75].

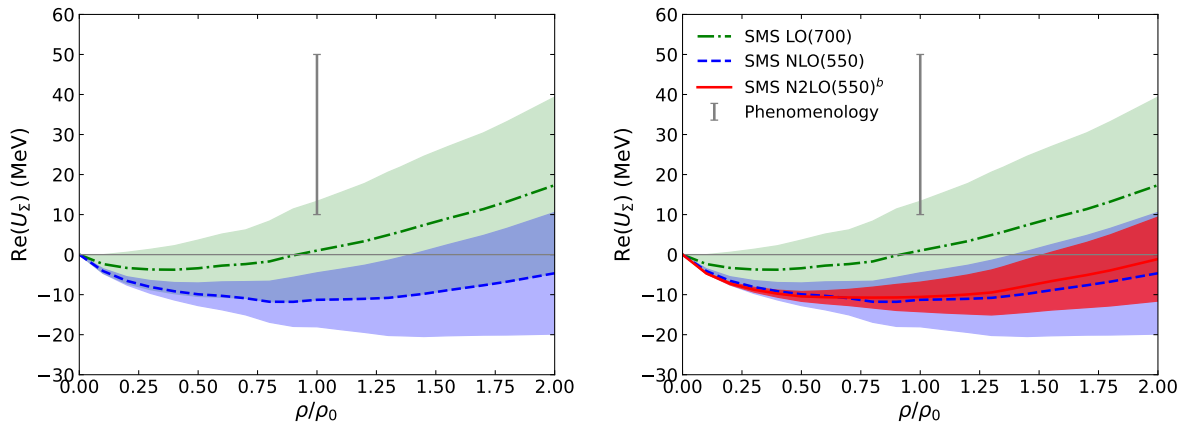


Figure 7.12: Same as Fig. 7.11, but for Σ . The bar indicates results from phenomenological analyses, taken from Ref. [75].

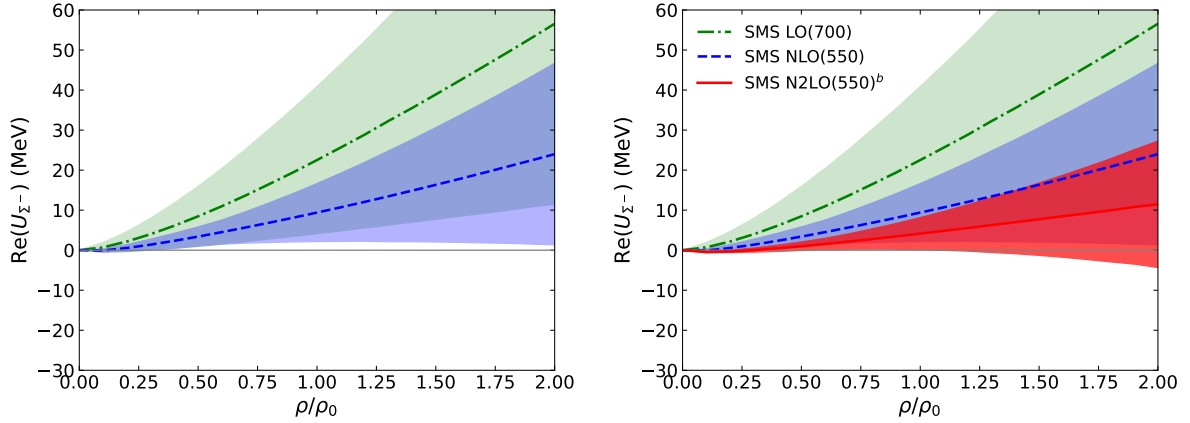


Figure 7.13: Uncertainty estimate of the Σ^- single-particle potentials in pure neutron matter.

value of U_Λ . An interesting aspect is, of course, that the uncertainty of the NLO result provides also a glimpse on the size of missing YNN three-body forces. The present estimate is compatible with the actual contribution of an effective three-body force evaluated in Ref. [59], established within the assumption of decuplet saturation and based on LECs constrained from dimensional scaling arguments. It is also comparable to the value suggested in Ref. [142] within a phenomenological analysis. Comparing the uncertainty estimate with the NLO results for different cutoffs in Table 7.1, one can see that the cutoff variations are noticeably smaller. This confirms again the conclusion that the cutoff variation does not provide a reliable measure for estimating uncertainties, stressed already in earlier studies of nuclear matter [254, 256]. Indeed, as emphasized in Ref. [215], in general the residual cutoff dependence underestimates the theoretical uncertainty. Since the calculation at $N^2\text{LO}$ is incomplete, i.e. three-body forces are missing, one has to be somewhat cautious with the interpretation of the corresponding result. Strictly speaking, the actual uncertainty is still the one obtained for the result at NLO. This concerns not only the estimate at ρ_0 but also the one at the highest density considered by us. As one can see from Fig. 7.11, the uncertainty of U_Λ increases rapidly with density and amounts to roughly ± 20 MeV at $2\rho_0$.

In Fig. 7.14, we show the uncertainty band for different cutoffs of 500 and 600 MeV. For the Λ and Σ in SNM, which correspond to the upper and middle panels, the cutoff variation does not change the qualitative behaviors: The NLO bands of Λ in SNM touch the quasi-empirical value at ρ_0 , while the $N^2\text{LO}$ bands underestimate that value. The U_Σ in SNM by both NLO and $N^2\text{LO}$ underestimate the phenomenological value of $U_\Sigma(\rho_0) = 30 \pm 20$ MeV. Contrary, whereas the NLO bands of Σ^- are similar under the cutoff variation, the $N^2\text{LO}$ Σ^- in PNM varies its qualitative behavior. For the cutoff 500 MeV case, the $N^2\text{LO}$ band increases as density rises, while the 600 MeV case is almost stable.

Additional uncertainties arise due to the issues already discussed in Subsection 7.3.2. Since the P -wave interactions in the ΛN system are not directly constrained, only indirectly via $\Sigma^- p$ and $\Sigma^+ p$ differential cross sections and the assumed $\text{SU}(3)$ symmetry, their contribution to U_Λ at ρ_0 varies in the order of 4 MeV for the considered chiral YN potentials, cf. Table 7.2, and by almost 10 MeV at $2\rho_0$. In case of the $N^2\text{LO}$ calcula-

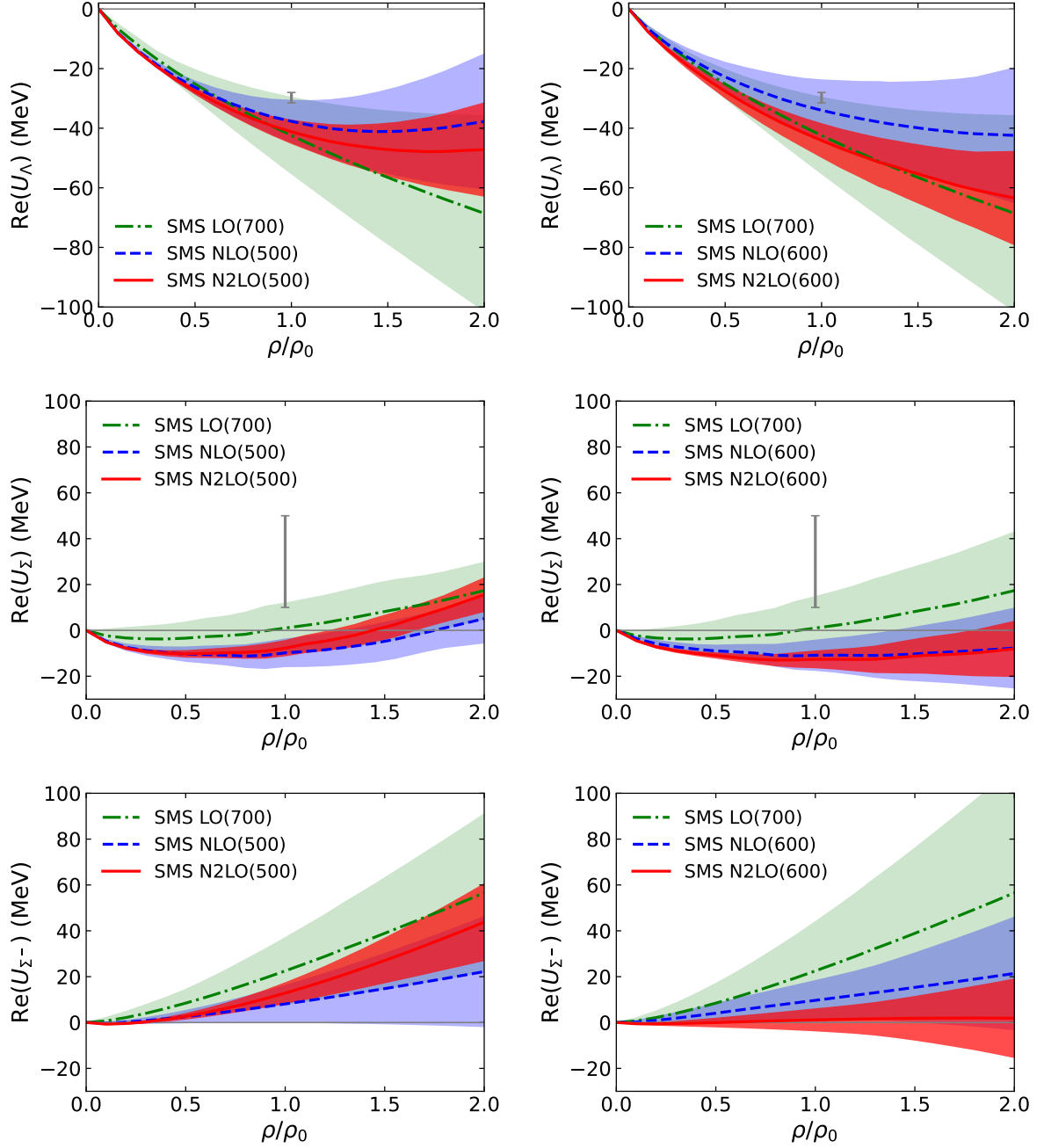


Figure 7.14: Uncertainty estimate of the hyperon single-particle potentials with different cutoffs of 500 MeV (left column) and 600 MeV (right column). From upper to lower, the figures correspond to Λ in SNM, Σ in SNM, and Σ^- in PNM.

tion even uncertainties in the D -waves might become relevant. Actually, in the context of the $N^2\text{LO}$ potential it is instructive to take a look at the nuclear matter case, where calculations have been performed up to $N^4\text{LO}$. There the result at $N^2\text{LO}$ sticks out from the general trend as being particularly attractive, see Fig. 1 in Ref. [256]. Thus, one can speculate that also for the Λ the $N^2\text{LO}$ result is somewhat exceptional and a fully converged calculation would rather yield values for U_Λ that are close to those obtained for the SMS NLO YN potentials.

Note that other uncertainties, like those associated with the use of the BHF approximation [232, 234, 241, 275], are not considered here.

The uncertainty estimation for Σ in SNM is presented in Fig. 7.12. The corresponding uncertainty at ρ_0 is

$$\begin{aligned} U_\Sigma^{\text{NLO}}(\rho_0) &= -11.3 \pm 6.7 \text{ MeV}, \\ U_\Sigma^{\text{N}^2\text{LO}}(\rho_0) &= -10.6 \pm 3.7 \text{ MeV}. \end{aligned} \quad (7.7)$$

The uncertainty bands for the NLO and $N^2\text{LO}$ potentials are similar because there is not much difference between the results at NLO and $N^2\text{LO}$, as can be seen from Fig. 7.6. Obviously, U_Σ is predicted to be attractive up to $1.5\rho_0$, even when considering the theoretical uncertainty.

In Fig. 7.13 we show the Σ^- single-particle potential in PNM. The values at ρ_0 in PNM are

$$\begin{aligned} U_{\Sigma^-}^{\text{NLO}}(\rho_0) &= 9.4 \pm 7.2 \text{ MeV}, \\ U_{\Sigma^-}^{\text{N}^2\text{LO}}(\rho_0) &= 4.1 \pm 4.0 \text{ MeV}. \end{aligned} \quad (7.8)$$

Compared to the case of U_Σ in SNM, the $N^2\text{LO}$ uncertainty band is noticeably larger due to the fact that for a given ρ_0 the Fermi momentum in PNM is larger. Obviously U_{Σ^-} is predominantly repulsive, for the SMS NLO as well as for the $N^2\text{LO}$ potentials.

We do not show results for Λ in PNM since the uncertainties are very similar to those in SNM, reflecting the isoscalar nature of the Λ hyperon.

We have made some exploratory calculations utilizing a Bayesian approach [276]. Those yield results very similar to the EKM method. Therefore, we postpone a full-fledged Bayesian analysis for the future.

7.5 Short summary

We have investigated the properties of hyperons in nuclear matter within the self-consistent Brueckner-Hartree-Fock approach using the continuous choice for intermediate states. We employed the recently established semilocal momentum-space regularized (SMS) hyperon-nucleon interaction up to next-to-next-to-leading order, enabling a systematic investigation of the order-by-order convergence. We also used the NN interaction by Reinert *et al.* [216], regularized in the same scheme as the hyperon-nucleon potential, to evaluate the nucleon single-particle potential.

The Λ single-particle potentials from the SMS YN interactions turned out to be qualitatively similar to those predicted by the NLO13 and NLO19 interactions, at least up

to nuclear matter saturation density. However, overall, the resulting in-medium potentials from the new YN interactions are slightly more attractive, in particular those from the N^2LO interaction. This concerns first of all the value of U_Λ at the momentum $k = 0$ MeV, associated with the binding energy of the Λ in infinite nuclear matter, but is reflected also in the momentum dependence. For the Σ case, the results for the SMS YN potentials differ noticeably from those for our earlier chiral YN potentials. Specifically, now the single-particle potential in symmetric nuclear matter at the saturation density is even attractive, $U_\Sigma \approx -10$ MeV, while results in the order of $U_\Sigma \approx +10$ MeV are predicted by the NLO13/NLO19 potentials. The reason for the difference are new constraints from Σ^+p and Σ^-p differential cross section data from the the J-PARC E40 experiment [104–106], which have been taken into account when the SMS YN potentials have been established.

In the present work we reported also on the first effort to estimate the error due to the truncation in the chiral expansion for the single-particle potentials. Thereby we followed a procedure proposed by Epelbaum, Krebs, and Meißner [215], a simple but efficient method which combines information about the expected size and the actual size of higher-order corrections. The estimated uncertainty relevant for the Λ is in the order of 7 MeV at nuclear matter saturation density. It is comparable with the difference of the theoretical results for U_Λ at NLO to the usually quoted quasi-empirical value of $U_\Lambda \approx -30$ MeV.

Chapter 8

Summary and outlook

We have investigated the hyperon single-particle potentials in nuclear matter, which is a key ingredient to solve the hyperon puzzle of neutron stars, based on the hypernuclear data and the baryon forces within chiral effective field theory (EFT). We aim to extend the constrained density region, which is currently only on the saturation density, for better extrapolation to denser environment.

The hypernuclear spectroscopy serves as a ‘boundary condition’ around the saturation density. For this purpose, we employ the spherical Skyrme-Hartree-Fock method. We constructed a model that fit to the results by chiral two- and three-body forces [59, 86]. This model of the Λ single-particle potential is found reproducing the experimental Λ binding energy data, at the same level of accuracy as a conventional attractive model. Thus, the chiral EFT Λ single-particle potential that suppresses Λ in dense neutron star matter is consistent with the Λ spectroscopy. Next, we search for the parameter space of the Λ single-particle potentials by varying the Taylor coefficients and the effective mass at the saturation density. The root-mean-square deviation ΔB_Λ is used to evaluate the consistency between the calculated Λ binding energies and experimental data. The well-known fact $U_\Lambda(\rho_0, k = 0 \text{ fm}^{-1}) = -30 \text{ MeV}$ [75] is confirmed. There are lower and upper limits for the normalized effective mass: $0.65 < m_\Lambda^*/m_\Lambda < 0.95$, reflecting the fact that the energy splitting between orbitals is sensitive to the effective mass. This provides an important constraints on the momentum dependence of the Λ single-particle potentials, which is important for the Λ collective flow created in heavy-ion collisions [88].

By employing the baryon interactions within chiral EFT, we have employed a conventional two-body force at next-to-leading order with the three-body forces whose low-energy constants are estimated via the decuplet saturation approximation. The Brueckner theory with the continuous choice of the single-particle potential is employed to evaluate the hyperon single-particle potentials in nuclear medium. It is shown that the low-energy constants of the three-body force are chosen in such a way that the Λ single-particle potential is enough repulsive to suppress Λ ’s in higher density region, and that the Σ single-particle potential at ρ_0 fulfills the phenomenological value, $U_\Sigma(\rho_0) = 30 \pm 20 \text{ MeV}$ [75]. They also fulfill the hypernuclear constraint on the effective mass elaborated in our hypernuclear study. Combining these facts with our hypernuclear study, the chiral $YN + YNN$ force can be constructed to be consistent with the Λ hypernuclear constraint, the Σ phenomenological value, and the hyperon puzzle of neutron stars.

We have also worked on the modern YN force at next-to-next-to-leading order developed in 2023, enabling a systematic investigation of the order-by-order convergence. The YN force is a modern semilocal momentum-space regularized (SMS) force. Furthermore, their low-energy constants are tuned to reproduce the recent ΣN differential cross section data measured by the J-PARC E40 experiment. We also used the NN interaction by the Reinert *et al.* [216], regularized in the same scheme as the hyperon-nucleon potential, to evaluate the nucleon single-particle potential. The Λ single-particle potentials from the SMS YN interactions turned out to be qualitatively similar to those predicted by the NLO13 and NLO19 interactions, at least up to nuclear matter saturation density. For the Σ case the results for the SMS YN forces differ noticeably from those for the earlier chiral YN forces. Specifically, now the single-particle potential in symmetric nuclear matter at the saturation density is even attractive, $U_\Sigma(\rho_0, k=0 \text{ fm}^{-1}) \approx -10 \text{ MeV}$, while results in the order of $U_\Sigma \approx +10 \text{ MeV}$ are predicted by the NLO13/NLO19 forces. The reason for the difference are new constraints from $\Sigma^+ p$ and $\Sigma^- p$ differential cross section data from the the J-PARC E40 experiment [104–106]. The Σ case may become different from the conventional thoughts, although plausible consequence can be obtained only after including the three-body force.

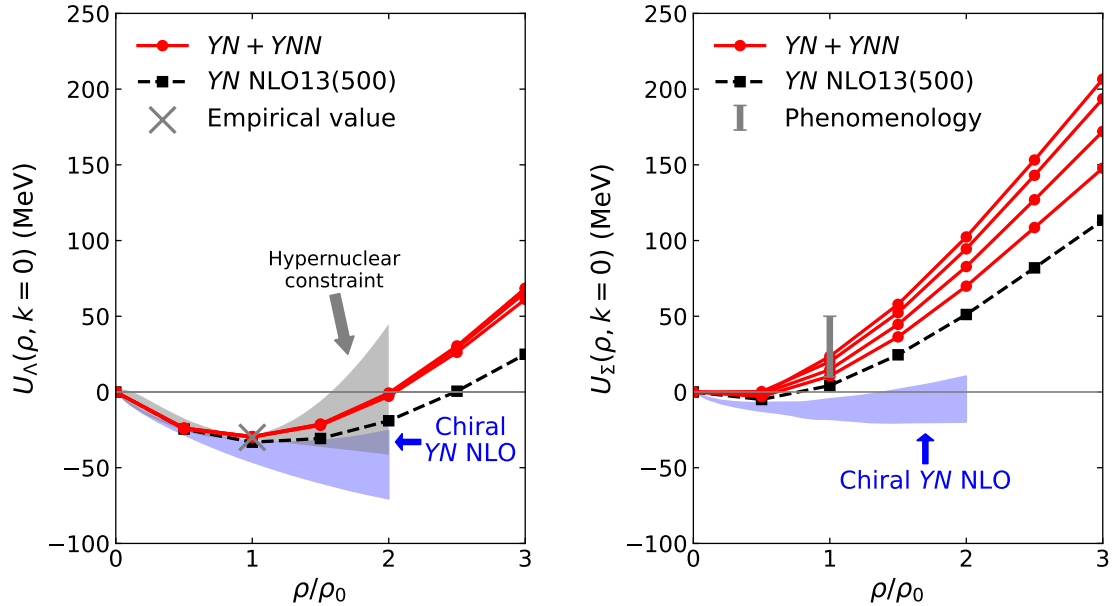


Figure 8.1: Summarized density dependence of Λ (left) and Σ (right) single-particle potentials in symmetric nuclear matter.

We summarize our important findings in Figs. 8.1 illustrating the constrained density dependencies for the Λ and Σ single-particle potentials. The two- and three-body force calculation by chiral EFT is consistent with the hypernuclear constraint. The upper band of the uncertainty of the SMS YN up to NLO is around the lower bound of the hypernuclear constraint. This uncertainty includes roughly the three-body force contribution at $N^2\text{LO}$. The summarized momentum dependence of the Λ single-particle potentials is shown in Fig. 8.2. The chiral force calculations are within the hypernuclear band calculated by the constrained effective mass $0.65 < m_\Lambda^*/m_\Lambda < 0.95$.

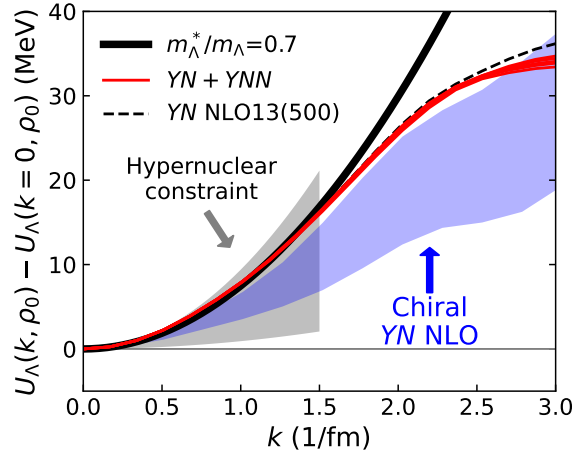


Figure 8.2: Summarized momentum dependence of Λ single-particle potentials in symmetric nuclear matter.

An straight future work is to implement the three-body force for the $N^2\text{LO}$ calculation. An explicit calculation of their effect is, however, extremely challenging: One needs to solve the Bethe-Faddeev equation [277, 278] with chiral three-body forces. So far pertinent investigations in the context of chiral NN [254, 279] and YN [59, 64] forces have been limited to an approximation, namely to the application of density-dependent two-body forces derived from the $N^2\text{LO}$ three-body force. Such an effective interaction can be obtained by summing one particle over the occupied states in the Fermi sea [131, 236–238]. In the on-shell approximation the resulting YN (NN) interaction can be expressed in analytical form with operator structures identical to those of free-space YN (NN) interactions, see Ref. [131] for details. This idea has been already exploited by Gerstung *et al.* [59]. One possible way for avoiding the on-shell approximation is to numerically integrate out one particle in medium. In this case, there would be a much more direct connection between the three-body forces as used in *ab initio* few-body calculations [217] and the ones that enter the G -matrix calculation. Work along this line is left for the future.

Other future work is to verify simultaneously both the Λ and Σ single-particle potentials from the hyperon collective flows in heavy-ion collisions. In our previous work [88] on the Λ directed flow, all hyperons and their resonances were assumed to feel the same single-particle potential as the Λ . Meanwhile, the Σ^0 contribution should have an impact because the Λ directed flow is in essence the $\Lambda + \Sigma^0$ directed flow due to the process $\Sigma^0 \rightarrow \gamma + \Lambda$. Also, the Σ single-particle potential is expected to be more repulsive compared to that of Λ as reflected in the non-existence of Σ hypernuclei except for ${}^4_\Sigma\text{He}$. Such repulsive nature of the Σ single-particle potential can be found in Fig. 8.1. Our first calculation shown in our proceeding [280] shows some sort of sensitivity of the $\Lambda + \Sigma^0$ flow on the Σ single-particle potentials. A more detailed analysis is now ongoing.

Appendix A

Conventions

- The Minkowski metric is given by $g^{\mu\nu} = \text{diag}(1, -1, -1, -1)$.
- The Pauli matrices are given as

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (\text{A.1})$$

which fulfill the relations

$$\sigma_i \sigma_j = \delta_{ij} + i\epsilon_{ijk} \sigma_k, \quad [\sigma_i, \sigma_j] = 2i\epsilon_{ijk} \sigma_k, \quad \{\sigma_i, \sigma_j\} = 2\delta_{ij}. \quad (\text{A.2})$$

- The gamma matrices are defined as a set of matrices fulfilling the anti-commutation relation $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$. They are represented in Dirac representation as

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}. \quad (\text{A.3})$$

It is convenient to additionally define the so-called chirality matrix

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

- The Gell-Mann matrices are given as

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (\text{A.4})$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad (\text{A.5})$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}. \quad (\text{A.6})$$

- The octet-baryon and pseudo-scalar meson fields are represented by a 3×3 matrix

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}, \quad (\text{A.7})$$

$$\phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}. \quad (\text{A.8})$$

- The decuplet-baryon field is represented by a totally-symmetric tensor (e.g. Ref. [249])

$$\begin{aligned} T^{111} &= \Delta^{++}, & T^{112} &= \frac{\Delta^+}{\sqrt{3}}, & T^{122} &= \frac{\Delta^0}{\sqrt{3}}, & T^{222} &= \Delta^-, \\ T^{113} &= \frac{\Sigma^{*+}}{\sqrt{3}}, & T^{123} &= \frac{\Sigma^{*0}}{\sqrt{6}}, & T^{223} &= \frac{\Sigma^{*-}}{\sqrt{3}}, \\ T^{133} &= \frac{\Xi^{*0}}{\sqrt{3}}, & T^{123} &= \frac{\Xi^{*-}}{\sqrt{3}}, \\ T^{333} &= \Omega^-, \end{aligned}$$

since the decuplet state corresponds to the Young tableau $\boxed{a}\boxed{b}\boxed{c}$ with the tensor basis as u -, d -, and s -quarks for $a = 1, 2$, and 3 , respectively. The dividing factors are motivated by the number of components:

$$\begin{aligned} \Delta^{++} &: \boxed{u}\boxed{u}\boxed{u} = uuu, \\ \Delta^+ &: \boxed{u}\boxed{u}\boxed{d} = uud + udu + duu, \\ \Delta^0 &: \boxed{u}\boxed{d}\boxed{d} = udd + dud + ddu, \\ \Delta^- &: \boxed{d}\boxed{d}\boxed{d} = ddd, \\ \Sigma^{*+} &: \boxed{u}\boxed{u}\boxed{s} = uus + usu + suu, \\ \Sigma^{*0} &: \boxed{u}\boxed{d}\boxed{s} = uds + usd + dus + dsu + sud + sdu, \\ \Sigma^{*-} &: \boxed{d}\boxed{d}\boxed{s} = dds + dsd + sdd, \\ \Xi^{*0} &: \boxed{u}\boxed{s}\boxed{s} = uss + sus + ssu, \\ \Xi^{*-} &: \boxed{d}\boxed{s}\boxed{s} = dss + sds + ssd, \\ \Omega &: \boxed{s}\boxed{s}\boxed{s} = sss. \end{aligned}$$

- The spin $\frac{1}{2}$ to $\frac{3}{2}$ transition operator $\vec{S}$ is given as 2×4 matrices

$$\begin{aligned} S_1 &= \begin{pmatrix} -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{6}} & 0 \\ 0 & -\frac{1}{\sqrt{6}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}, & S_2 &= \begin{pmatrix} -\frac{i}{\sqrt{2}} & 0 & -\frac{i}{\sqrt{6}} & 0 \\ 0 & -\frac{i}{\sqrt{6}} & 0 & -\frac{i}{\sqrt{2}} \end{pmatrix}, \\ S_3 &= \begin{pmatrix} 0 & \sqrt{\frac{2}{3}} & 0 & 0 \\ 0 & 0 & \sqrt{\frac{2}{3}} & 0 \end{pmatrix}. \end{aligned} \quad (\text{A.9})$$

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