

A metrical result on transcendence measures in certain fields

(Abstract)

by

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Let F_q be a finite field of characteristic p with q elements. Then we denote by $A = F_q[T]$ the ring of polynomials in a variable T over F_q , $K = F_q(T)$ the quotient field of A , and $K_\infty = F_q((1/T))$ the field of formal power series in $1/T$ over F_q .

The purpose of the present talk is to give transcendence measures for almost all elements of K_∞ with respect to a Haar measure on K_∞ .

As in Sprindžuk [4], we have a non-archimedean absolute value $|\cdot|$ on K_∞ . That is, for any element $\omega \in K_\infty$ of the form

$$\omega = \sum_{v=\ell}^{\infty} a_v T^{-v} \quad (a_v \in F_q, v \in \ell, \ell + 1, \dots),$$

we define $|\omega| = q^{-\ell}$ if $a_\ell \neq 0$, $|\omega| = 0$ if $a_v = 0$ for all $v \geq \ell$. We know that K_∞ is complete and locally compact with respect to the metric defined by this absolute value. Let μ be the Haar measure normalized by

$$\mu(D) = 1, \text{ where } D = \{\omega \in K_\infty; |\omega| \leq q^{-1}\}.$$

For any nonzero polynomial $P(x) = \sum_{i=1}^d a_i x^i \in A[x]$, we define logarithmic height $\hat{h}(P)$ of P by

$$\hat{h}(P) = \log_q \max_{0 \leq i \leq d} |a_i|,$$

where $\log_q x$ means the logarithmic function with base q . Then our main result is stated as follows.

Theorem. *Let ε be an arbitrary positive number. Then, for almost all $\omega \in K_\omega$ (w.r.t. μ), we have*

$$|P(\omega)| \geq q^{-(3+\varepsilon)dh} \min(1, |\tilde{\omega}|)^d$$

for all nonzero polynomials $P \in A[x]$ with $\deg P \leq d$, $\hat{h}(P) \leq h$, and $\max(d, h) \geq c(\omega, \varepsilon)$, where $\tilde{\omega} = \omega -$ (the constant term of ω) and $c(\omega, \varepsilon)$ is a positive constant depending only on ω and ε .

We see that this lower bound is fairly good. In fact, by Hilfssatz 3 of Bundschuh [2], for any $\omega \in K_\omega$ and any positive integers d, h , there exists a nonzero polynomial $P \in A[x]$ with $\deg P \leq d$, $\hat{h}(P) \leq h$ satisfying

$$|P(\omega)| \leq q^{-d(h-1)-1} \max(1, |\omega|)^d.$$

For the proof of the theorem, We refer to the paper [1].

References

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