

CONSTITUTIVE MODEL FOR UNSATURATED CRUSHABLE SOILS

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Abstract

Particle crushing, occurring in crushable materials under high-stress conditions exceeding their crushing strength, leads to particle breakdown and reduction in peak shear strength. The presence of water further diminishes crushing strength. Additionally, particle crushing significantly alters the soil-water characteristic curve (SWCC). The combined effects of particle crushing and the degree of saturation changes induce excessive deformation and weaken the soil. While existing models can predict the behavior of unsaturated soil and particle crushing effects individually, a comprehensive model for unsaturated crushable soils is necessary. This study proposes a constitutive model for unsaturated crushable soils, integrating the effect of the degree of saturation on crushing strength by developing the crushing surface. It incorporates variations in the grading state index and the degree of saturation, affecting soil strength via state boundary surface movement. Validation is achieved through past experimental evidence. The model effectively captures key features of unsaturated crushable soils, including the reduction in crushing strength with increased degree of saturation, the evolution of SWCC due to particle crushing, and additional particle crushing during wetting. Furthermore, a parametric study offers insights into unsaturated crushable soil behavior, highlighting the combined effects of particle crushing and variations in the degree of saturation. When significant particle crushing occurs, increased volumetric compression due to particle crushing leads to a higher degree of saturation and further strength reduction, amplifying soil deformation. Understanding these interactions is crucial for predicting the behavior of unsaturated crushable soils, emphasizing the significance of this study.

Keywords

Constitutive model; Crushing strength; Degree of saturation; Particle crushing; Unsaturated soils

List of notations

$\mathbf{D}^e$	elastic stiffness tensor;
e, e_{ref}	void ratio and reference void ratio;
f	yield function;
f_c	crushing surface function;
G	shear modulus;
H_c	crushing history;
$\mathbf{I}$	The fourth-order identity tensor;

I_G	grading state index;
I_h	ratio controlling the effect of hysteresis;
K	nonlinear elastic bulk modulus;
M	critical state stress ratio;
M_c	slope of crushing surface;
p', p''	Terzaghi's and Bishop's mean effective stress;
p_a	atmospheric pressure;
p_c	current state of stress;
p_0	stress history;
p_r	parameter for particle crushing resistance;
p_s	particle crushing strength (defined as size of the initial crushing surface);
p_s^{sat}	particle crushing strength under saturated condition;
p_s^{dry}	particle crushing strength under dry condition;
q	deviatoric stress;
s, s^*	suction and modified suction;
S_r	degree of saturation;
u_a, u_w	pore air pressure and pore water pressure;
v	specific volume;
v_e	Poisson's ratio;
v_{sbs}	specific volume on the state boundary surface of unsaturated crushable soil;
v_{sbs}^{sat}	specific volume on the state boundary surface of saturated uncrushable soil;
ξ_c	the difference between the particle crushing strength of saturated soil and dry state;
ξ_e	material parameter for modified suction;
ξ_h	material parameter for the effect of hysteresis;
ξ_{I_G}	the volumetric distance between the SBS of non-crushed soil and completely crushed soil;
ξ_{S_r}	the volumetric distance between the SBS of saturated and dry soil;
α, m, n	material parameters for van Genuchten's SWCC model;
ϵ	strain tensor;
ϵ_v	volumetric strain;
ζ	monotonic increasing function of stress ratio;

η', η''	Terzaghi's and Bishop's stress ratio;
κ	swelling index;
$d\Lambda$	plastic multiplier;
λ	compression index;
N, Γ	reference specific volume on NCL and CSL of saturated non-crushed soil under the reference pressure;
σ	Cauchy's total stress tensor;
σ', σ''	Terzaghi's and Bishop's effective stress tensor;
Ψ	state parameter represents the upward movement of SBS due to the degree of saturation and grading state index;
Ψ_1	state parameter that satisfies 0 at saturated state and $\xi_{s,r}$ at dry state;
Ψ_2	state parameter that satisfies 0 at non-crushed state and $-\xi_{IG}$ at completely crushed state;
Ψ_c	the difference between the particle crushing strength of saturated soil and the current state;
Ω	the difference between the specific volume at the current state and the specific volume on the state boundary surface;
ω	sub-loading surface parameter controlling the evolution rate of Ω ;
$\mathbf{1}$	The second-order identity tensor.

1. Introduction

Particle crushing is a phenomenon observed in crushable soils (such as carbonate sands, silica sands, volcanic soils, and decomposed granite) under high stress levels exceeding crushing strength. This phenomenon leads to the fragmentation of soil particles into smaller pieces and subsequent alterations in grading. Particle crushing can cause various geotechnical problems, including reductions in bearing capacity of piles [18], and deformations in earth dam structures [19]. High-pressure oedometer test on Ottawa quartz sand [29] reveal initially exhibits low compressibility. However, beyond a certain stress level, there is a dramatic increase in compressibility accompanied by significant changes in grading due to crushing. Ring shear tests on Dog's Bay Sand [7] demonstrate the evolution of grading with increased shear strain, indicating that both compression and shearing induce particle crushing. This phenomenon significantly impacts the mechanical behavior of soils, causing a decrease in void ratio and a downward shift of the critical state line (CSL) in $e-\ln p'$ plane [15, 24] as smaller particles filling void spaces, and reduction of peak shear strength as observed in triaxial compression tests on decomposed granite soil [22].

Previous studies have suggested that water significantly influences crushing strength. Nobari and Duncan [27] investigated the factors influencing the collapse of rockfill upon flooding. Sieve analyses conducted before and after flooding the sample showed that some particle crushing occurs during the collapse. Single particle crushing tests have been conducted under different water conditions to investigate the effect of water on crushing strength [10, 20]. The main conclusion was that the presence of water significantly reduces the crushing strength of particles. Nara et al. [25] conducted an experimental study of the humidity effects on the crack propagation of igneous rock. The results showed that an increase in humidity significantly accelerates crack propagation. Atkinson [3] supported the idea that cracks in rocks can propagate even under crushing strength due to environmental conditions, known as subcritical crack growth due to stress corrosion [21]. Additionally, suction occurs between crack planes in unsaturated particles, providing additional compressive stress at the crack tip [25, 37]. This suction tends to close the crack, thereby increasing the particle's ability to resist crushing forces. In the other hand, when the particles are saturated, these suctions forces are reduced or eliminated, leading to a decrease in the particle's ability to resist crushing forces. Therefore, the presence of water is a critical factor influencing the particle crushing behavior of crushable granular materials. An increase in the degree of saturation contributes to a decrease in crushing strength, and vice versa.

Compression and soaking tests on air-dried silt [13] indicated that samples soaked at a constant applied pressure undergo significant volumetric deformation, known as hydraulic collapse. This occurs because

unsaturated soil typically exhibits higher stiffness, and a looser state compared to saturated soil at the same stress level. Isotropic compression tests at different constant suctions [32] showed that the normally consolidated line (NCL) of saturated samples forms a linear relationship in the $e-\ln p'$ plane. In contrast, the NCL of unsaturated samples exhibits curved behavior, with slopes decreasing until approaching the NCL of saturated soils. Based on this result, the CSL of unsaturated soils is unique regardless of suctions. This suggests that even under constant suction, volumetric compression can increase the degree of saturation, ultimately causing soil saturation [35]. Previous research has linked suction to the degree of saturation through the soil water characteristic curve (SWCC) [36]. Numerous experiments have explored the evolution of the SWCC of crushable granular materials. These studies reveal that particle crushing significantly alters SWCC, leading to a notable increase in the degree of saturation during particle crushing under constant suction [9, 38]. As particle crushing induces a change in grading, these findings align with various grain-size-dependent SWCC models capable of estimating the curve under different gradings [2, 8].

When considering the combined effects of particle crushing and unsaturated soils, void ratio and peak shear strength can decrease due to particle crushing. Additionally, this reduction in void ratio may lead to an increase in the degree of saturation, consequently resulting in further reduction in shear strength and crushing strength. The 2018 Hokkaido Eastern Iwate earthquake caused frequent slope failures, even on slopes with angles of 30 degrees or less. Sediment flows traveled longer distances compared to those caused by rainfall [12]. One of the causes of long-distance flow was particle breakage during the earthquake, followed by an increase in water content [1]. This highlights the importance of these combined effects on significant soil deformation and failure. Therefore, it is crucial to consider the interaction between these behaviors in dealing with unsaturated crushable soils.

Previous studies have proposed various constitutive models for predicting particle crushing behavior [15, 26, 30, 34]. However, these models tend to neglect the impact of degree of saturation on crushing strength, where crushing strength decreases as the degree of saturation increases, and vice versa. Oldecop and Alonso [28] proposed a one-dimensional compression model for rockfill materials accounting the effects of water on compressibility and collapse phenomena. The model highlights particle breakage as a key mechanism influencing rockfill behavior. They suggest that the yield locus of the rockfill material depends on the water content, when stress level approaches the crushing strength. Buscarnera and Einav [6] developed a constitutive model based on thermodynamic framework to explain the dependence of yielding on the degree of saturation in brittle granular materials. The model effectively describes the onset of particle crushing and its dependence on

saturation index. However, it assumes that particle breakage is the only source of energy dissipation at the onset of plastic behavior.

The objective of this study is to develop a constitutive model for unsaturated crushable soils that captures the combined effects of particle crushing and variations in the degree of saturation, as well as key features of unsaturated crushable soils, including the reduction in crushing strength with increased degree of saturation, the evolution of SWCC due to particle crushing, and additional particle crushing during wetting. The model was calibrated and validated using the existing experimental data. Additionally, this study aims to provide insights into the behavior of unsaturated crushable soil through a parametric study. When significant particle crushing occurs, an increase in volumetric compression due to particle crushing contributes to a rise in the degree of saturation, consequently resulting in further strength reduction and soil deformation. This parametric study underscores the importance of understanding these complex interactions for predicting the behavior of unsaturated crushable soils.

2. A model for unsaturated crushable soils

In this study, an elastoplastic model based on the critical state theory, which couples the behavior of particle crushing and unsaturated soils, is presented. The model incorporates the effect of particle crushing and the degree of saturation into the movement of the state boundary surface (SBS), which contains the NCL and the CSL. The crushing surface is developed to consider the effect of change in both state of stress and degree of saturation. Additionally, the alteration of SWCC due to particle crushing is incorporated by the change in the modified suction. By incorporating these aspects, the proposed constitutive model can predict the behavior of unsaturated crushable soils, which cannot be predicted by existing models due to the coupling effects of particle crushing and the variation of degree of saturation, which are interlinked.

2.1 Grading state index

To quantify the amount of particle crushing, the grading state index plays a crucial role. Muir Wood [23] introduced the grading state index (I_G), which is represented by the ratio of areas ABC and ABD as shown in Fig. 1. The concept assumes an evolution of the grain size distribution curve (GSD) from a single size grading (line AB) to a limiting grading (curve AD), with the grading state index (I_G) ranging from 0 to 1.

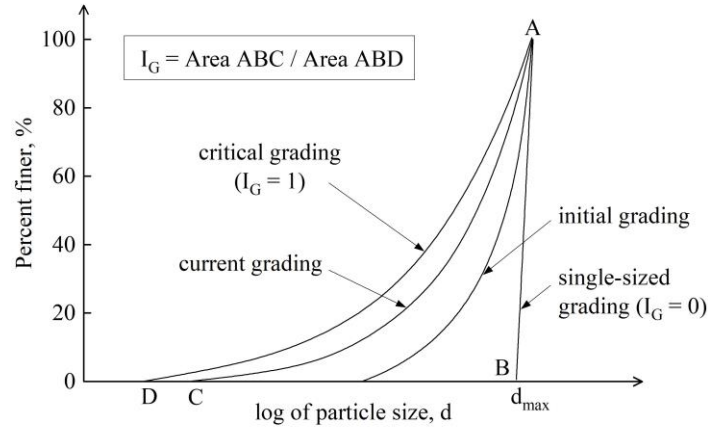


Fig. 1 The definition of grading state index (Muir Wood, 2007)

2.2 Crushing surface of unsaturated crushable soils

The crushing surface is essential for determining whether crushing occurs under the current state of stress. When the state of stress reaches this surface, particles experience crushing, which leads to the expansion of crushing surface. Previous studies [15, 26] proposed a specific form of the crushing surface, given by Eq. 1. The expression accounts for the effects of both mean effective stress and deviatoric stress on crushing behavior.

$$f_c(\sigma', p_0) = p' \left[1 + \frac{1}{2} \left(\frac{\eta'}{M_c} \right)^3 \right] - p_0 \quad (1)$$

In the equation, the shape of crushing surface is determined by the first term, while the size of the crushing surface depends on the stress history (p_0). It is important to note that the degree of saturation can also impact particle crushing. This effect is evident in changes in grading during flooding in earth dam structures [27], single particle crushing under different water conditions [10, 20], and investigations of subcritical crack growth due to environmental conditions [3, 25]. Consequently, the development of a crushing surface for unsaturated crushable soils should consider both the state of stress and degree of saturation. Oldecop and Alonso [28] provided a general expression for crack propagation velocity with water as corrosive agent by applying the thermodynamic formulation of the reaction rate theory to stress corrosion cracking. Their expression indicates that crack propagation depends on both stress and relative humidity. Buscarnera and Einav [6] extended the thermodynamic theory of breakage mechanics to unsaturated media to investigate the onset of particle breakage. Their formulation demonstrated that the crushing strength of unsaturated soils increases compared to saturated soils as a function of degree of saturation. Specifically, as the degree of saturation increases, the stress required to break a particle decreases monotonically. Therefore, it is reasonable to assume the particle crushing strength

(p_s) decreases monotonically with increasing degree of saturation where p_s represents the mean effective stress at which crushing initiates under isotropic stress condition, assuming a single-sized grading ($I_G = 0$). The non-negative state parameter Ψ_c is added to the particle crushing strength under saturated condition (p_s^{sat}) as follows:

$$p_s = p_s^{sat} + \Psi_c \quad (2)$$

The state parameter Ψ_c denotes the difference between the particle crushing strength of the saturated soil and the current state of the degree of saturation. For simplicity, a simple linear expression of Ψ_c , as a function of the degree of saturation, is assumed as:

$$\Psi_c = \xi_c (1 - S_r) \quad (3)$$

where ξ_c represents the difference between the particle crushing strength of the fully saturated and dry states. Fig. 2 illustrates how particle crushing strength varies with the degree of saturation. As the degree of saturation increases, the state parameter Ψ_c decreases monotonically until it reaches zero. At this point, Eq. 2 returns to the particle crushing strength under saturated condition.

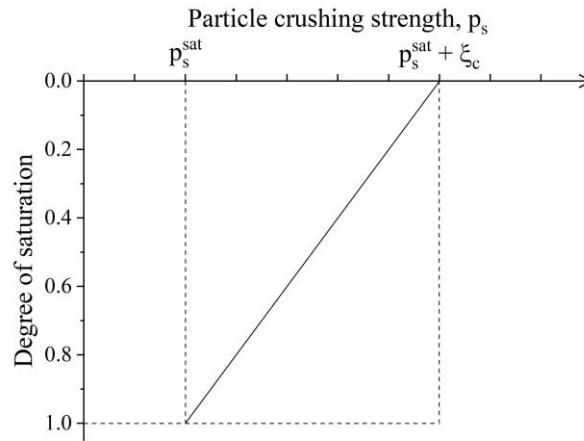


Fig. 2 The variation of particle crushing strength (p_s) as a function of degree of saturation

This particle crushing strength (p_s) corresponds to the size of initial crushing surface ($I_G = 0$) as shown in Fig. 3. The initial crushing surface can expand or shrink due to the state of degree of saturation through the state parameter Ψ_c . When the degree of saturation increases, the initial crushing surface shrinks (p_s decreases) until the soil reaches the fully saturated state ($p_s = p_s^{sat}$). Conversely, when the degree of saturation decreases, the initial crushing surface expands (p_s increases) until the soil reaches the fully dry state ($p_s = p_s^{dry}$). Establishing the relationship with Eq. 3 allows to qualitatively capture the reduction in crushing strength with increasing degree of saturation. It maintains model simplicity while satisfying the assumptions of monotonic variation and the limits of crushing strength at fully saturated and dry states.

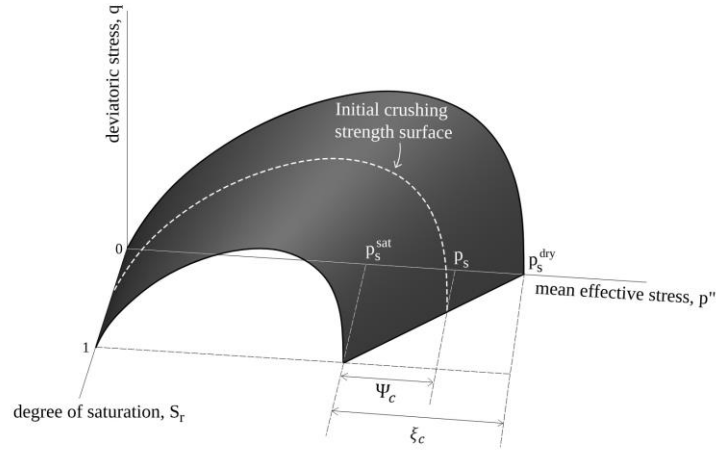


Fig. 3 Initial crushing strength surface as a function of degree of saturation

Any current state of stress can be simplified as a scalar value (p_c), defined by Eq. 4 and represented in Fig. 4, employing the same shape of the crushing surface as described in Eq. 1. However, to extend the concept of effective stress to unsaturated soils, we employ Bishop's effective stress (e.g., p'' and η'') which will be explained in a later section.

$$p_c(p'', \eta'') = p'' \left[1 + \frac{1}{2} \left(\frac{\eta''}{M_c} \right)^3 \right] \quad (4)$$

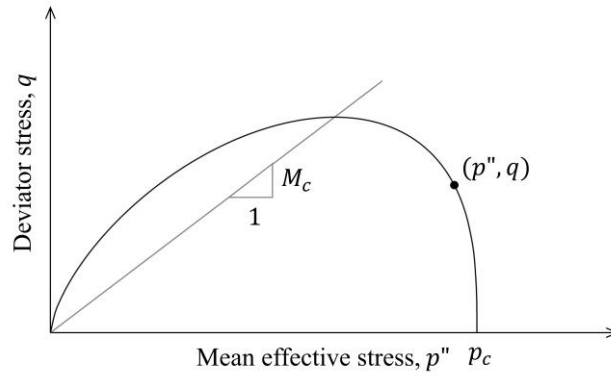


Fig. 4 Definition of the current state of stress (p_c)

After the initiation of crushing (when state of stress reaches the initial crushing surface), the concept of crushing history (H_c) is introduced to record the amount of particle crushing that takes place and establish the evolution law of the grading state index (I_G). Since particle crushing is an irreversible process, once a particle breaks, it cannot return to its original state. Therefore, crushing history (H_c) is defined as the highest recorded value (all-time high) of the difference between the current state of stress (p_c) and the particle crushing strength (p_s) from the beginning to the current state.

$$H_c = \max(p_c - p_s) \geq 0 \quad (5)$$

The particle crushing strength (p_s) undergoes changes due to variations in the degree of saturation, as described in Eq. 2, while changes in the state of stress result in alterations in p_c as given by Eq. 4. Consequently, crushing history (H_c) is influenced by both factors: degree of saturation and state of stress. Eq. 5 ensures that crushing history (H_c) monotonically increases from the initial value of 0 (when state of stress is under the particle crushing strength; $p_c < p_s$). The size of crushing surface of unsaturated crushable soils depends on both the particle crushing strength (p_s) and the crushing history (H_c). Therefore, the crushing surface of unsaturated crushable soils can be defined as shown in Eq. 6, with the same shape as the crushing surface described in Eq. 1.

$$f_c(\sigma'', p_s, H_c) = p'' \left[1 + \frac{1}{2} \left(\frac{\eta''}{M_c} \right)^3 \right] - (p_s + H_c) \quad (6)$$

when $f_c < 0$, no particle crushing takes place. When $f_c = 0$, particle crushing occurs. The condition $f_c > 0$ represents an impossible region.

2.3 Evolution law of grading state index

The grading state index (I_G) is proposed to exhibit a monotonic increase with respect to the crushing history (H_c). As the crushing history (H_c) increases, the grading state index (I_G) finally approaches a limit of 1. This relationship between the grading state index (I_G) and the crushing history (H_c) can be expressed as Eq. 7. The variation of grading index (I_G) with the changes in crushing history (H_c) and the effects of p_r are illustrated in Fig. 5.

$$I_G = 1 - \exp\left(-\frac{H_c}{p_r}\right) \quad (7)$$

where p_r represents a constant parameter which controls the rate of particle crushing after the initiation of crushing. By the definition of H_c given in Eq. 5, when the degree of saturation remains constant, Eq. 7 yields the same equation as proposed by Kikumoto et al. [15].

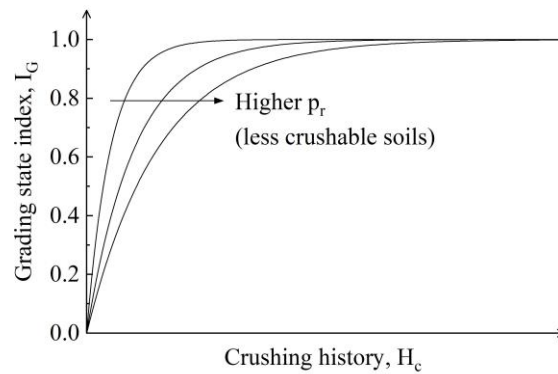


Fig. 5 The effect of p_r on the evolution of grading state index (I_G) with crushing history (H_c)

Eq. 7 ensures that I_G monotonically increases from 0 to 1 with an increase in H_c . Then, the evolution law of I_G can be derived by taking derivative of Eq. 7 as follows.

$$dI_G = \frac{1}{p_r} \exp\left(-\frac{H_c}{p_r}\right) dH_c = \left(\frac{1 - I_G}{p_r}\right) dH_c \quad (8)$$

When particle crushing occurs, $f_c(\sigma'', p_s, H_c) = 0$ and $dH_c > 0$. The increment of H_c can be determined using the consistency condition ($df_c = 0$) of crushing surface.

$$dH_c = \begin{cases} \frac{\partial f_c}{\partial \sigma''} d\sigma'' + \xi_c dS_r & \text{when } (p_c - p_s) > H_c \\ 0 & \text{when } (p_c - p_s) \leq H_c \end{cases} \quad (9)$$

2.4 Unsaturated soils

Bishop's effective stress (σ'') is selected to determine the effective stress of unsaturated soils [5] since the Bishop's critical state stress ratio is consistent regardless of the degree of saturation [32]. Bishop's effective stress (σ'') can be expressed by Eq. 10.

$$\sigma'' = (\sigma - u_a \mathbf{1}) + S_r(u_a - u_w) \mathbf{1} \quad (10)$$

The SWCC model is needed to describe the hydraulic behavior of unsaturated soils under various loading conditions. In this study, the classical SWCC model [36], which establishes a relationship between suction (s) and the degree of saturation (S_r), is used as follows:

$$S_r^A = S_{min} + (S_{max} - S_{min})[1 + (\alpha^A s)^n]^{-m} \quad (11)$$

where α , n , and m are material parameters, S_{max} and S_{min} are the maximum and minimum degree of saturation, superscripts A can be either d or w denote the main drying and wetting curves.

The classical SWCC has been extended to account the effect of void ratio and hysteresis [14, 16] by utilizing the modified suction (s^*) and ratio (I_h) as expressed in Eq. 12 and Eq. 13, respectively.

$$s^* = s \left(\frac{e}{e_{ref}} \right)^{\xi_e} \quad (12)$$

$$I_h = \frac{S_r - S_r^w}{S_r^d - S_r^w} \quad (13)$$

However, experimental findings have indicated that particle breakage significantly alters the SWCC, leading to a notable increase in the degree of saturation during particle crushing under constant suction [9, 38]. As particle crushing induces changes in the particle size distribution (PSD), numerous grain-size-dependent SWCC models have been proposed [2, 8]. However, these models are mostly empirical, involving many parameters to describe the SWCC, which adds to the complexity of the model. Since a decrease in void ratio due

to particle crushing directly impacts the modified suction (s^*). For the sake of simplicity, the evolution of the SWCC due to grain crushing will be automatically incorporated through the change of the modified suction (s^*).

2.5 Decomposition of strain and elasticity

The additive strain decomposition is assumed, the total strain rate tensor ($d\boldsymbol{\varepsilon}$) can be calculated by sum of elastic ($d\boldsymbol{\varepsilon}^e$) and plastic ($d\boldsymbol{\varepsilon}^p$) strain rate tensor.

$$d\boldsymbol{\varepsilon} = d\boldsymbol{\varepsilon}^e + d\boldsymbol{\varepsilon}^p \quad (14)$$

A conventional elastic stiffness tensor ($\mathbf{D}^e$) is assumed given as Eq. 15 where K is nonlinear elastic bulk modulus, and G represents shear modulus.

$$\mathbf{D}^e = K\mathbf{1} \otimes \mathbf{1} + 2G\left(\mathbf{I} - \frac{1}{3}\mathbf{1} \otimes \mathbf{1}\right) \quad (15)$$

The rate form of elastic stress-strain relationship is defined as Eq. 16.

$$d\boldsymbol{\sigma}'' = \mathbf{D}^e : d\boldsymbol{\varepsilon}^e \quad (16)$$

The plastic strain rate tensor is obtained by associated flow rule as:

$$d\boldsymbol{\varepsilon}^p = \langle d\Lambda \rangle \frac{\partial f}{\partial \boldsymbol{\sigma}''} \quad (17)$$

where $\langle \cdot \rangle$ is the macauley bracket define as $\langle x \rangle = \begin{cases} 0, & x < 0 \\ x, & x \geq 0 \end{cases}$.

2.6 State boundary surface and yield function

The state boundary surface (SBS), which includes the CSL and the NCL, is commonly used in the formulation of critical state models as illustrated in Fig. 6. The specific volume on the state boundary surface (v_{sbs}^{sat}) which defines the loosest state of saturated uncrushable soil at the current state of stress (p', η') is represented by Eq. 18.

$$v_{sbs}^{sat} = N - \lambda \ln \frac{p'}{p_a} + (\Gamma - N) \zeta(\eta') \quad (18)$$

which includes the stress ratio (η'), reference atmospheric pressure (p_a), λ is a compression index, N and Γ represent the specific volume on the NCL and the CSL at reference pressure, and $\zeta(\eta')$ is a monotonically increasing function of the stress ratio, which is equal to 0 at the NCL and 1 at the CSL. Srinil et al. [33] proposed a unified state boundary surface model adapting the basic structure of the sand model, which relies on strength dependency on the state parameter [4], to a critical state model for both saturated and unsaturated

conditions. It was proved that the models are virtually identical for both sand and clay. In this study, the nonlinear function based on the modified Cam clay model is used as Eq. 19.

$$\zeta(\eta') = \frac{\ln \left\{ 1 + \left(\frac{\eta'}{M} \right)^2 \right\}}{\ln 2} \quad (19)$$

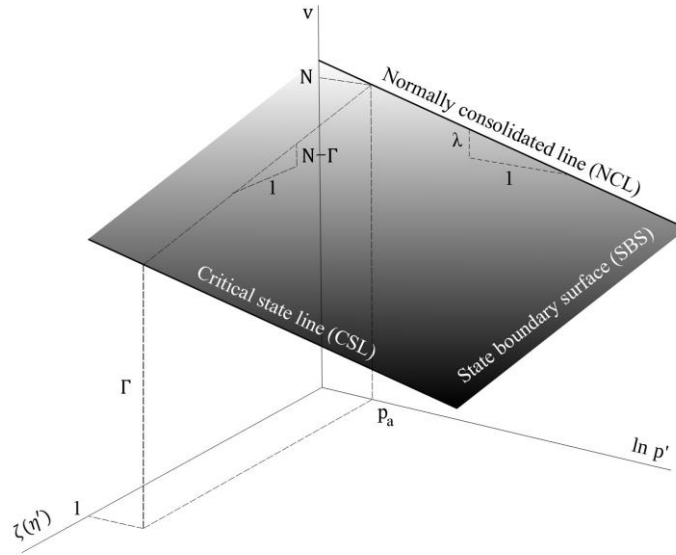


Fig. 6 State boundary surface in $\ln p'$ - $\zeta(\eta')$ - v space of saturated non-crushed soils

For unsaturated soils, SBS was defined using Bishop's effective stress. Compression and soaking tests on air-dried silt [13] indicate that when the unsaturated samples were soaked at the constant applied pressure, it is noticeable that the compression line of the unsaturated samples suddenly fall to some point on the compression line of saturated samples even under the constant applied pressure. Therefore, we assume that the SBS of unsaturated soils shifts upward due to a decrease in degree of saturation by the parameter $\Psi_1(S_r)$, which is a function of degree of saturation [14, 16].

$$\Psi_1(S_r) = \xi_{S_r}(1 - S_r) \quad (20)$$

where ξ_{S_r} represents the volumetric distance between the SBS of saturated and dry soil.

In contrast, particle crushing causes a downward parallel shift of the CSL in the specific volume (v) and logarithm of mean effective stress ($\ln p'$) plane [15, 24]. As a result, the SBS of crushable soils is assumed to shift downward due to an increase in grading state index by the parameter $\Psi_2(I_G)$, which is a function of grading state index [15, 26].

$$\Psi_2(I_G) = -\xi_{I_G} I_G \quad (21)$$

where ξ_{I_G} represents the volumetric distance between the SBS of non-crushed soil ($I_G = 0$) and crushed soil ($I_G = 1$).

To incorporate these effects into the movement of SBS, the new state parameter $\Psi(S_r, I_G)$ is assumed by linearly composing Eq. 20 and Eq. 21, as the function of degree of saturation and grading state index. This parameter represents the upward movement of SBS in the volumetric plane, as shown in Fig. 7 where the value of $\Psi(S_r, I_G)$ can be either positive or negative, depending on the state of the soil.

$$\Psi(S_r, I_G) = \Psi_1(S_r) + \Psi_2(I_G) \quad (22)$$

Since the parameter N is defined as the reference specific volume of saturated non-crushed soil, when $S_r = 1$ and $I_G = 0$, $\Psi = 0$, indicating that the SBS becomes the SBS of saturated uncrushable soil, as shown in Fig. 7. Then, the specific volume on the SBS of unsaturated crushable soil is defined as follows.

$$v_{sbs} = N - \lambda \ln \frac{p''}{p_a} + (\Gamma - N) \zeta(\eta'') + \Psi(S_r, I_G) \quad (23)$$

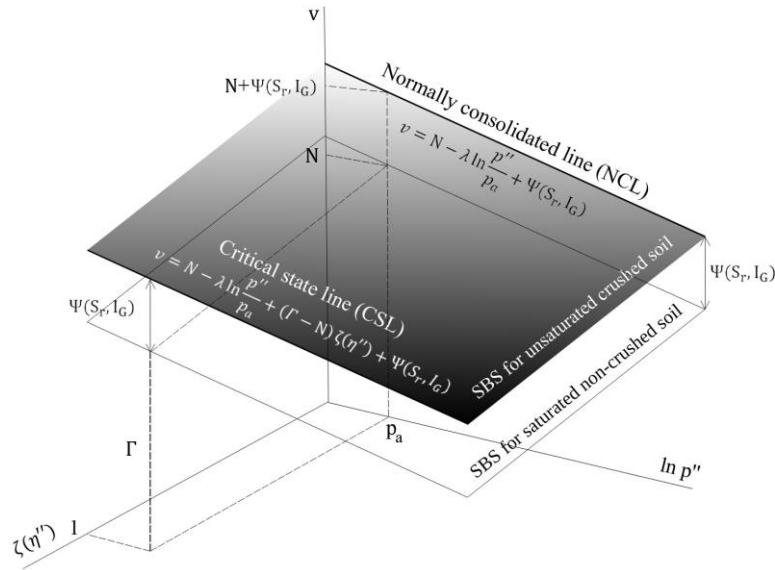


Fig. 7 State boundary surface in $\ln p'' - \zeta(\eta'')$ - v space of unsaturated crushed soils

Since soil exhibits elastoplastic behavior even under the yield surface, the model considers the effect of density to predict the behavior of over-consolidated soils by using sub-loading surface concept [11] and a state variable Ω is used to represent the difference in specific volume between the current state and specific volume on the SBS as shown in Fig. 8. The specific volume (v) of the current state of unsaturated crushable soil incorporating the sub-loading surface concept is defined as follows.

$$v = N - \lambda \ln \frac{p''}{p_a} + (\Gamma - N) \zeta(\eta'') + \Psi(S_r, I_G) - \Omega \quad (24)$$

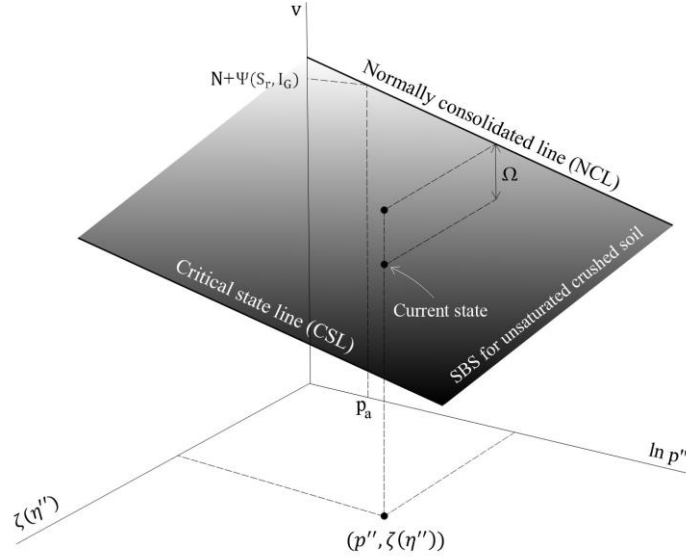


Fig. 8 State boundary surface considering sub-loading parameter

The initial specific volume (v_0) can be calculated by substituting the initial states ($p'' = p_0''$, $\eta'' = 0$, $\Psi = \Psi_0$, and $\Omega = \Omega_0$) into Eq. 24, we get Eq. 25.

$$v_0 = N - \lambda \ln \frac{p_0''}{p_a} + \Psi_0 - \Omega_0 \quad (25)$$

The volumetric strain (ε_v) can be defined by Eq. 26.

$$\varepsilon_v = -\frac{dv}{v_0} = \frac{v_0 - v}{v_0} \quad (26)$$

Substituting Eq. 24 and Eq. 25 into Eq. 26, we obtain the volumetric strain as Eq. 27.

$$\varepsilon_v = \frac{1}{v_0} \left\{ \lambda \ln \frac{p''}{p_0''} + (N - \Gamma) \zeta(\eta'') - (\Psi - \Psi_0) + (\Omega - \Omega_0) \right\} \quad (27)$$

The plastic volumetric strain is the difference between the total volumetric strain (ε_v) and the elastic volumetric strain (ε_v^e) following the additive strain decomposition. Finally, the yield function (f) for unsaturated crushable materials is derived as follows.

$$f = \left\{ (\lambda - \kappa) \ln \frac{p''}{p_0''} + (N - \Gamma) \zeta(\eta'') \right\} - (\Psi - \Psi_0) + (\Omega - \Omega_0) - v_0 \varepsilon_v^p \equiv 0 \quad (28)$$

2.8 Loading conditions for elastoplastic behavior and particle crushing

The Kuhn-Tucker conditions [17] must be satisfied in any loading path.

$$f \leq 0; \dot{\lambda} \geq 0; \dot{\lambda} f = 0 \quad (29)$$

Since the yield function in the proposed model (Eq. 28) is identically equal to zero, the loading condition simplifies to:

$$\dot{\Lambda} \geq 0 \quad (30)$$

where $\dot{\Lambda} = 0$ indicates elastic behavior (unloading), $\dot{\Lambda} > 0$ indicates plastic deformation (loading).

In case of no particle crushing, the following conditions must be satisfied:

$$f_c < 0 \text{ or } dH_c = 0 \quad (31)$$

where $f_c < 0$ indicates that there is no particle crushing which H_c remains constant. In case of particle crushing, the following conditions must be satisfied:

$$f_c = 0 \text{ and } dH_c > 0 \quad (32)$$

where $f_c = 0$ indicates that particle crushing occurs with an increase in H_c .

2.9 Consistency condition and hardening rules

When soil exhibits elastoplastic, the consistency condition of the yield function $df(\sigma'', \varepsilon_v^p, \Psi, \Omega) = 0$ needs to be satisfied, as shown in Eq. 33.

$$df = \frac{\partial f}{\partial \sigma''} d\sigma'' + \frac{\partial f}{\partial \varepsilon_v^p} d\varepsilon_v^p + \frac{\partial f}{\partial \Psi} d\Psi + \frac{\partial f}{\partial \Omega} d\Omega = 0 \quad (33)$$

Hardening rule of volumetric plastic strain rate ($d\varepsilon_v^p$) can be derived from the associated flow rule as Eq. 34.

$$\frac{\partial f}{\partial \varepsilon_v^p} d\varepsilon_v^p = -v_0(d\Lambda) \frac{\partial f}{\partial p} \quad (34)$$

As plastic deformation occurs, Ω decreases and eventually reaches 0 (on the SBS), resulting in the smooth transition between the elastic and plastic region. The hardening rule of Ω can be defined as:

$$\frac{\partial f}{\partial \Omega} d\Omega = -\omega v_0 \Omega |\Omega| \|d\varepsilon^p\| \quad (35)$$

where ω is the parameter that governs the effect of density.

While the state parameter Ψ is a function of degree of saturation (S_r) and grading state index (I_G) as expressed in Eq. 22, the hardening rule of Ψ is derived as Eq. 36.

$$\frac{\partial f}{\partial \Psi} d\Psi = \xi_{S_r} dS_r + \xi_{I_G} dI_G \quad (36)$$

By substituting Eq. 8 and Eq. 9, when particle crushing occurs, the hardening rule of Ψ can be rewritten as follows.

$$\frac{\partial f}{\partial \Psi} d\Psi = \xi_{S_r} dS_r + \xi_{I_G} \left(\frac{1 - I_G}{p_r} \right) \left[\frac{\partial f_c}{\partial \sigma''} d\sigma'' + \xi_c dS_r \right] \quad (37)$$

309

310 2.10 Elastoplastic stiffness tensor

311 From additive strain decomposition, Eq. 14, elastic stress-strain relationship, Eq. 16, and associated flow rule,
 312 Eq. 17, the effective stress rate tensor can be given as Eq. 38.

$$d\sigma'' = \mathbf{D}^e : d\epsilon^e = \mathbf{D}^e : (d\epsilon - d\epsilon^p) = \mathbf{D}^e : \left(d\epsilon - \langle d\Lambda \rangle \frac{\partial f}{\partial \sigma''} \right) \quad (38)$$

314 By substituting Eq. 34 – Eq. 38 in the consistency condition of yield function, Eq. 33, the plastic multiplier
 315 ($d\Lambda$) can be derived as follows.

$$d\Lambda = \frac{\left(\frac{\partial f^*}{\partial \sigma''} \right) : \mathbf{D}^e : d\epsilon + \left(\frac{\partial f^*}{\partial S_r} \right) dS_r}{(M_p) + \left(\frac{\partial f^*}{\partial \sigma''} \right) : \mathbf{D}^e : \frac{\partial f}{\partial \sigma''}} \quad (39)$$

317 where

$$\frac{\partial f^*}{\partial \sigma''} = \frac{\partial f}{\partial \sigma''} + \xi_{I_G} \left(\frac{1 - I_G}{p_r} \right) \frac{\partial f_c}{\partial \sigma''} \quad (40)$$

319

$$\frac{\partial f^*}{\partial S_r} = \xi_{S_r} + \xi_{I_G} \left\{ \left(\frac{1 - I_G}{p_r} \right) \xi_c \right\} \quad (41)$$

$$M_p = \omega v_0 \Omega |\Omega| \left| \frac{\partial f}{\partial \sigma''} \right| + v_0 \frac{\partial f}{\partial p} \quad (42)$$

322 By substituting Eq. 39 into Eq. 38, the rate form of elastoplastic stress–strain relationship when particle
 323 crushing occurs can be derived as follows.

$$d\sigma'' = \left[\mathbf{D}^e - \frac{\mathbf{D}^e : \frac{\partial f^*}{\partial \sigma''} : \frac{\partial f}{\partial \sigma''} : \mathbf{D}^e}{M_p + \frac{\partial f^*}{\partial \sigma''} : \mathbf{D}^e : \frac{\partial f}{\partial \sigma''}} \right] d\epsilon - \left[\frac{\mathbf{D}^e : \frac{\partial f}{\partial \sigma''} \left(\frac{\partial f^*}{\partial S_r} \right)}{M_p + \frac{\partial f^*}{\partial \sigma''} : \mathbf{D}^e : \frac{\partial f}{\partial \sigma''}} \right] dS_r \quad (43)$$

325

326 3. Model validation

327 3.1 Calibration of model parameters

328 In this study, Q-ROK sand [9] and Gushui rockfill materials [31] are used to validate the model. The model
 329 parameters are categorized into three sets: stress-strain characteristics, soil water characteristic curves, and
 330 particle crushing. Table 1 presents the parameters for stress- strain characteristics. The parameters λ , κ , and N
 331 are calibrated from the isotropic compression or one-dimensional compression curve in the $\ln p''$ - v plane. M

denotes the critical state ratio in the q - p'' plane, and v_e is the Poisson's ratio. ξ_{I_G} reflects the effect of grading state index (I_G) on the SBS position, determined through a high-pressure oedometer test; it represents the volumetric distance between the NCL of non-crushed soil (when $I_G = 0$) and fully crushed soil (when $I_G = 1$) at the same pressure. ξ_{S_r} represents the effect of degree of saturation (S_r) on the SBS position, determined through a wetting or constant suction compression test (for details, refer to [14, 16]). ω is the parameter controlling the transition zone between elastic and plastic behavior calibrated through trial and error to fit the simulation results with the compression curve.

Table 1 Parameters for stress-strain characteristics

Symbol	Q-ROK sand	Gushui rockfill materials	Description
λ	0.09	0.08	Compression index
κ	0.01	0.004	Swelling index
N	2.17	2.135	Reference specific volume on NCL under $p'' = p_a$, $q = 0$, $I_G = 0$, and $S_r = 1$
M	1.30	1.65	Critical state stress ratio
v_e	0.10	0.25	Poisson's ratio
ξ_{I_G}	0.35	0.25	Effect of I_G on SBS
ξ_{S_r}	0.15	0.03	Effect of S_r on SBS
ω	150	230	Sub-loading parameter

Table 2 presents the model parameters for the SWCC. The van Genuchten SWCC model parameters (S_{min} , S_{max} , α_d , α_w , n , m) are calibrated to align with the SWCC. ξ_e , parameter controlling the effect of void ratio and particle crushing on the SWCC, is calibrated to fit the simulated SWCC curve with the experimental data. Hysteresis of SWCC is represented by ξ_h , determined by fitting the simulated SWCC curve with experimental data from wetting and drying cyclic tests.

Table 2 Parameters for soil water characteristics curves (SWCC)

Symbol	Q-ROK sand	Gushui rockfill materials	Description
S_{max}	1.00	1.00	Parameters for van Genuchten SWCC model
S_{min}	0.10	0.10	
α_d	1.60	1.20	
α_w	3.20	2.40	
n	4.00	3.80	
m	0.80	1.00	
ξ_e	2.0	2.0	Effect of void ratio on SWCC
ξ_h	100.0	100.0	Effect of hysteresis on SWCC

Table 3 presents the model parameters for particle crushing. p_s^{sat} can be determined from the isotropic compression test; it represents the mean effective stress (p'') at which the saturated soil begin to crush, defining

particle crushing strength under saturated conditions. ξ_c can be measured as the difference in the particle crushing strength under saturated and dry conditions (for p_s^{sat} and ξ_c , refer to Fig. 3). M_c represents the parameter controlling the shape of the crushing surface (see Fig. 4). p_r controls the rate of crushing (see Fig. 5). M_c and p_r can be determined by fitting the simulation results with the experimental data.

Table 3 Parameters for particle crushing

Symbol	Q-ROK sand	Gushui rockfill materials	Description
p_s^{sat}	20	1.8	Particle crushing strength (MPa)
ξ_c	10	1.0	Effect of S_r on the particle crushing strength (MPa)
M_c	1.60	1.80	Slope of the crushing surface
p_r	50	70	Crushing resistance (MPa)

3.2 Q-ROK sand

The one-dimensional compression tests on the Q-ROK sand were conducted by Gao et al. [9], reaching vertical net stresses of 40, 70, and 110 MPa to study the evolution of SWCC due to grain crushing. The calibrated parameters for Q-ROK sand (Table 1-3) were utilized to simulate the one-dimensional compression tests with the initial void ratio of 0.78. Fig. 9 compares the simulated one-dimensional compression curve of saturated Q-ROK sand, up to a vertical net stress of 110 MPa, with the corresponding experimental data. During the one-dimensional compression tests, particle crushing occurs. Particle size distribution curves (PSD) were collected at the vertical net stress of 40, 70, and 110 MPa to determine the grading state index (I_G) at each stress level (see Fig. 10). SWCC measurements were also conducted at these stresses by controlling air pressure. The drying path of SWCC for Q-ROK sand at each vertical net stress level, along with the simulation results, is shown in Fig. 11. Even the PSD evolution is not considered in the evolution of SWCC due to particle crushing, however, the matching of the experimental results and simulation results (as shown in Fig. 10 and Fig. 11) confirm the validity of the evolution law of I_G and the change in SWCC due to particle crushing through the change of modified suction, as expressed by Eq. 7 and Eq. 12 respectively.

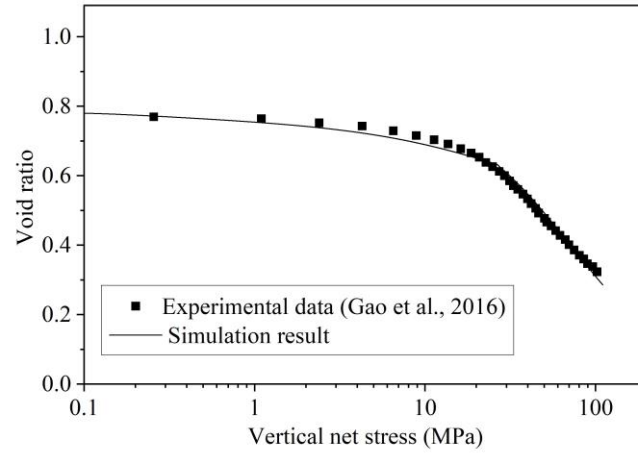


Fig. 9 One-dimensional compression curve of Q-ROK sand

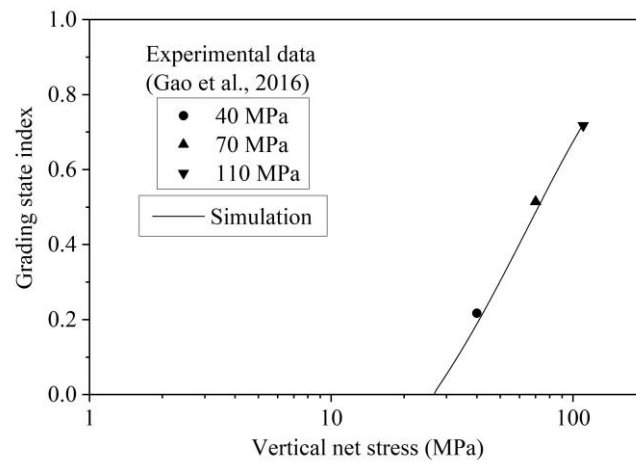


Fig. 10 Evolution of grading state index (I_G) of Q-ROK sand

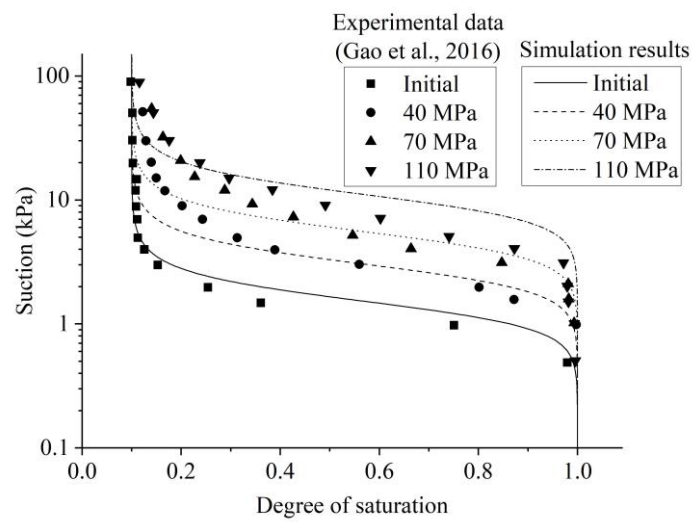


Fig. 11 SWCC (drying path) of Q-ROK sand at different vertical net stresses

3.3 Gushui rockfill materials

Shao et al. [31] conducted wetting tests on Gushui rockfill materials using triaxial CD tests at different stress levels, simulated through a well-calibrated discrete element method (DEM) model. Initially, the unsaturated samples were sheared up to specific stress conditions. Afterward, the samples were flooded to achieve a fully saturated state and then sheared until an axial strain of 0.08 was reached. The tests were performed under three different confining pressures: 0.5 MPa, 1.0 MPa, and 2.0 MPa. The calibrated parameters for Gushui rockfill materials were used to simulate the results with the initial void ratio of 0.80. The simulated results highlighted significant particle crushing during the wetting process. The definition of the crushing surface, as proposed in Eq. 6, allows the model to capture further particle crushing during the wetting process, as shown in Fig. 12, even when the deviatoric stress remains constant.

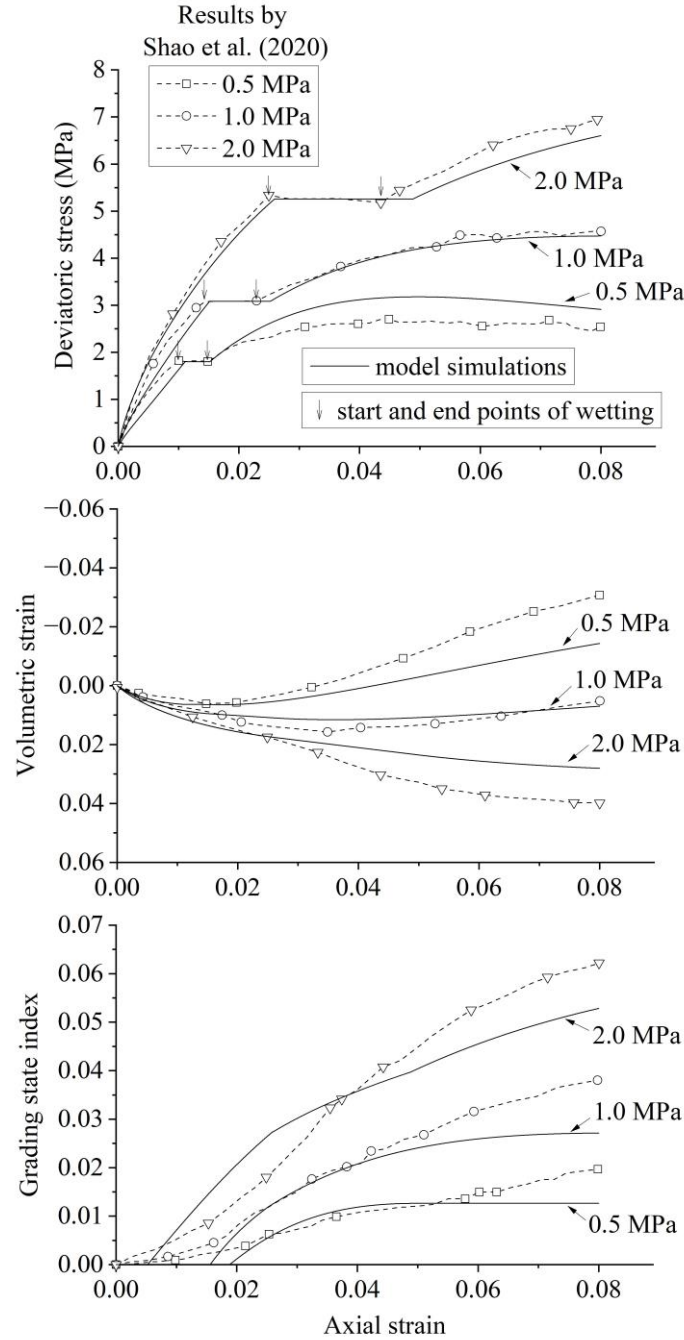


Fig. 12 Model simulations of wetting tests of Gushui rockfill materials under different confining pressures

4. Parametric study

The proposed constitutive model was employed to simulate a series of tests, including compression tests, triaxial shearing tests, and wetting tests. In conventional representations, stress in saturated soils is typically expressed as effective stress, while unsaturated soils are characterized using net stress. However, a comparison was made between unsaturated and saturated cases in this study. Given that the suction in the saturated case is zero, the net stress in the saturated cases is consequently equivalent to the effective stress.

4.1 Compression tests

Compression tests were simulated under fully drained conditions in 4 cases: unsaturated ($S_{r0} = 0.50$) and saturated soils ($S_{r0} = 1$), both with and without particle crushing. In the case without particle crushing, the value of p_s^{sat} was set to an extremely high value (e.g., 1000 MPa) to prevent particle crushing. The calibrated parameters for Q-ROK sand (Table 1-3) were utilized to simulate the isotropic compression tests with the initial void ratio of 0.78. The compression curves for these four cases are depicted in Fig. 13.

When comparing unsaturated and saturated cases, initially, unsaturated soil tends to exhibit higher stiffness compared to saturated soil. However, as the soil is compressed, the degree of saturation increases, and the compression line eventually converges to the saturated case as the degree of saturation approaches 1. On the other hand, when comparing the cases with and without crushing, the compression lines in the low-stress region are identical. Once crushing initiates, the crushable soil is significantly compressed compared to the non-crushable soil as indicated by the increase in the slope of the compression line. However, the slope of the compression line experiences a notable decline at a vertical net stress of around 200 MPa due to the limit of crushing at which no further significant crushing occurs.

The initial crushing point (the point at which grading state index begins to rise in Fig. 13) is influenced by variations in the initial degree of saturation. In case with a lower initial degree of saturation (e.g., unsaturated case), the initial crushing strength is higher, causing the initial crushing point to shift rightward. This observation confirms the validity of the crushing surface described in Eq. 6 and the evolution law of I_G in Eq. 7, consistent with previous studies that highlight the dominant role of water presence on crushing strength. Specifically, as particles approach saturation, their crushing strength decreases [10, 20, 25]. Once crushing initiates, the slope of the compression line changes instantaneously, accompanied by a gradual increase in the grading state index. However, as the vertical net stress increases, the degree of saturation increases due to volumetric compression and particle crushing. Eventually, the evolution of the grading state index for unsaturated case converges to that of the saturated case.

The impact of particle crushing on the degree of saturation is shown in Fig. 13 through the unsaturated case ($S_{r0} = 0.50$). After crushing initiates, the degree of saturation increases substantially compared to the case without crushing. In other words, as the particle crushing takes place, the soil saturates at a noticeably accelerated rate. By the end of the tests, the unsaturated crushable sample reaches a fully saturated state, while the unsaturated non-crushable sample remains unsaturated. This observation aligns with previous studies [9, 38] that noted a significant increase in the degree of saturation during particle crushing under constant suction.

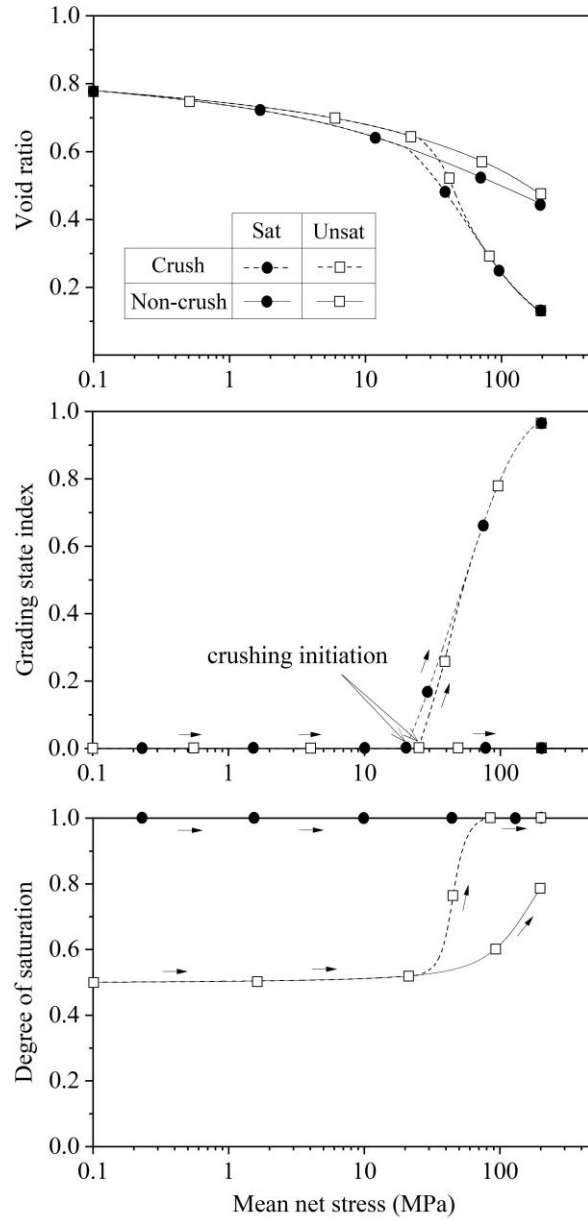


Fig. 13 Compression curves of Q-ROK sand under 4 cases

Additionally, compression tests were simulated under 5 different initial degrees of saturation (S_{r0}): 1 (case a), 0.8 (case b), 0.6 (case c), 0.4 (case d), and 0.2 (case e). The compression curves of each case are shown in Fig. 14. The results indicate that in the saturated case, the soil strength tends to be initially lower than that of unsaturated soils, and the volumetric strain gradually decreases throughout the tests. In contrast, as the soil becomes drier, the soil strength initially tends to be higher, however, once crushing initiates, the soil strength decreases, and an abrupt drop in volumetric strain occurs. This suggests that unsaturated soil exhibits more brittle behavior compared to saturated soil. The relationship between the degree of saturation, mean net stress, and grading state index is shown in Fig. 15. As shown in the contour of Fig. 15b, the initial crushing point depends on the degree of saturation, as described by the relationship in Eq. 2–3. Initially, as the mean net stress

increases, the degree of saturation gradually increases. However, after reaching the point where crushing initiates, the degree of saturation increases rapidly, eventually approaching 1. Therefore, it is important to note that when dealing with unsaturated crushable materials, the combined effects between particle crushing and variations in the degree of saturation play an important role. Engineers should be cautious about the stress point around the crushing initiation stress, as significant deformation can occur once the stress reaches the crushing point.

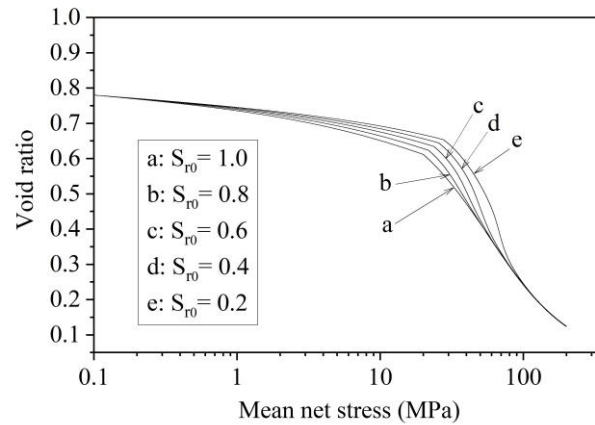


Fig. 14 Compression curves of Q-ROK sand under different initial degree of saturation

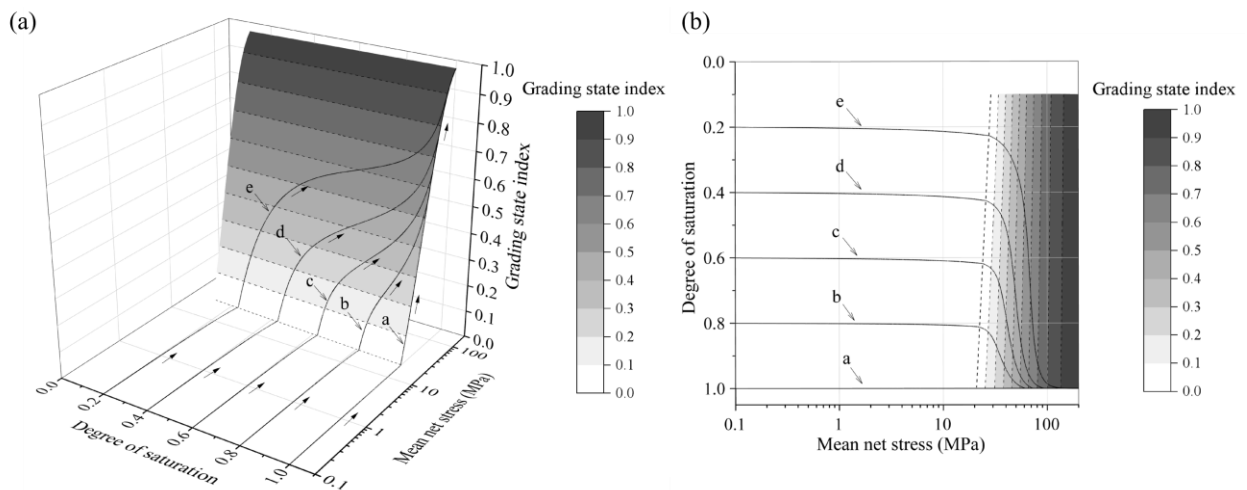
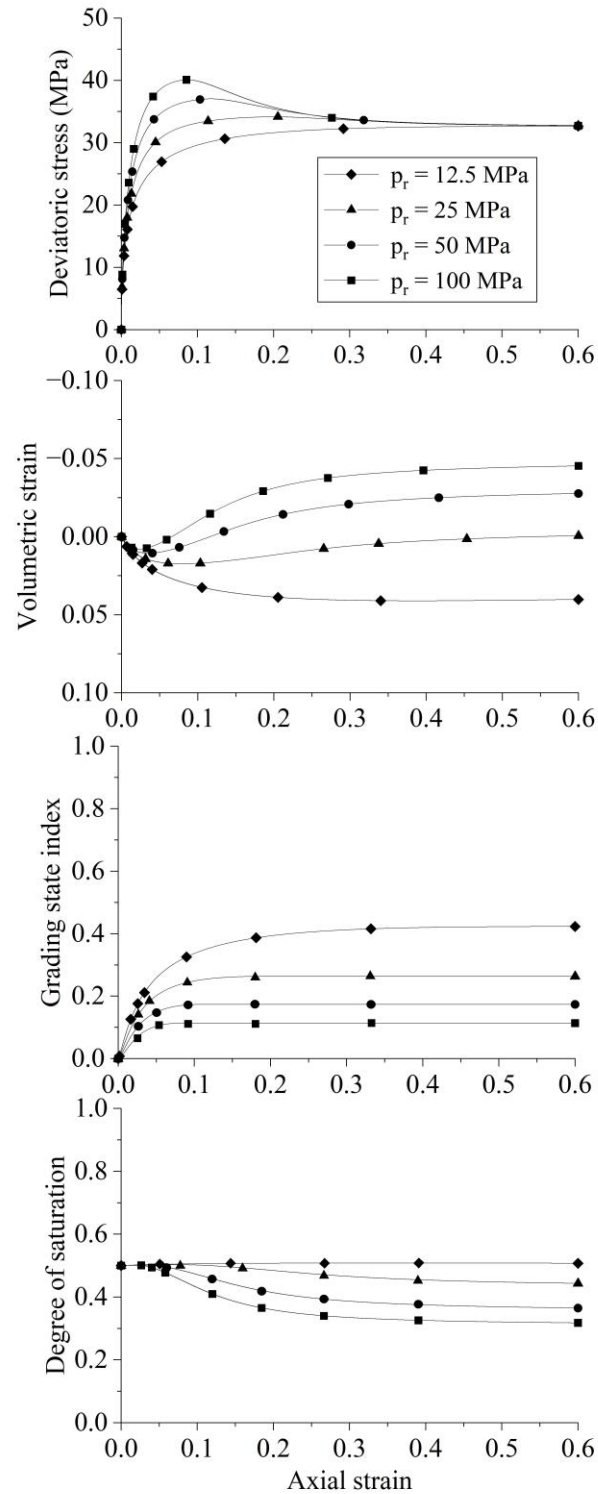


Fig. 15 The 3D results of compression tests of Q-ROK sand under different initial degree of saturation

4.2 Triaxial shearing tests

The importance of considering both particle crushing and variations in the degree of saturation as coupled behaviors when evaluating shear strength in unsaturated crushable soils is highlighted in this section. Triaxial shearing tests under constant mean stress were simulated using the calibrated parameters for Q-ROK sand

452 (Table 1-3). To illustrate these combined effects, the crushing resistance (p_r) of Q-ROK sand was varied to
 453 12.5, 25, 50, and 100 MPa to represent different levels of particle crushing. For all cases, the initial degree of
 454 saturation (S_{r0}) was set to 0.5, the initial void ratio was 0.60, and the confining pressure was 25 MPa.



455
 456 **Fig. 16** Effect of p_r on the results of triaxial shearing tests of Q-ROK sand

Fig. 16 presents the influence of p_r on the triaxial shearing results. As higher p_r indicates greater crushing resistance, less particle crushing and more dilative behavior are expected, leading to a significant decrease in the degree of saturation due to dilation. This decrease in the degree of saturation provides additional strength, resulting in higher shear strength and crushing strength, as unsaturated soils are typically stronger than saturated soils. A clear peak shear strength followed by strain softening can be observed in higher p_r case. In contrast, a lower p_r indicates lower crushing resistance, leading to a higher amount of particle crushing. This results in more compressive behavior, with the degree of saturation either slightly decreasing or even increasing due to compression. In this case, additional strength provided by the decrease in the degree of saturation is eliminated. Combined with the effects of particle crushing, a significant drop in peak shear strength is evident compared to the higher p_r case. In other words, as the material transitions from less crushable (higher p_r) to more crushable (lower p_r), the peak shear strength significantly decreases, the volumetric behavior becomes more compressive, and the stress-strain response tends to shift from strain softening to strain hardening, which is a crucial factor in evaluating the shear strength of unsaturated crushable materials.

4.3 Wetting tests

Wetting tests under anisotropic conditions were simulated using the calibrated parameters for Gushui rockfill materials (Table 1-3), with the initial void ratio of 0.80. The samples were first subjected to a confining pressure of 1.8 MPa. Dry samples were triaxially sheared up to an axial strain of 0.025, followed by soaking while controlling suction until reaching fully saturation. Finally, further shearing was conducted up to an axial strain of 0.10. The effect of the parameter ξ_c on anisotropic wetting was investigated by varying ξ_c to 0 and 2.5 MPa. Fig. 17 illustrates the results of wetting under different values of ξ_c .

For $\xi_c = 2.5$ MPa, the shear strength is initially slightly higher than the case of $\xi_c = 0$ MPa, due to less particle crushing. Since ξ_c provides additional crushing resistance in the unsaturated state, the onset of particle crushing occurs more slowly in the case of $\xi_c = 2.5$ MPa. Hydraulic collapse was observed in both cases, as indicated by the increased axial strain during wetting. For $\xi_c = 0$ MPa, no particle crushing was observed during soaking, as evidenced by a constant I_G . In contrast, for $\xi_c = 2.5$ MPa, significant particle crushing occurs since the particle crushing strength decreases as a function of saturation, resulting in a markedly larger collapse in axial strain during wetting. The case of $\xi_c = 0$ represents a condition where particle crushing strength (p_s) remains constant, independent of the degree of saturation, implies that Eq. 3 is inactive. Without the activation of Eq. 3, no further particle crushing occurs during wetting, resulting in less collapsed axial strain. This

underscores the importance of the parameter ξ_c in controlling the variation of particle crushing strength with the degree of saturation.

Interestingly, after soaking, the shear strength for $\xi_c = 2.5$ MPa appears lower than the case of $\xi_c = 0$ MPa. Although $\xi_c = 2.5$ MPa initially exhibits higher strength, the increased degree of saturation during wetting reduces the crushing strength and causes further particle crushing. This results in significantly larger collapsed axial strain during wetting and a substantial drop in shear strength. This parametric study highlights the combined effects of particle crushing and changes in saturation, which are particularly important for geotechnical structures subjected to tidal or heavy rainfall conditions.

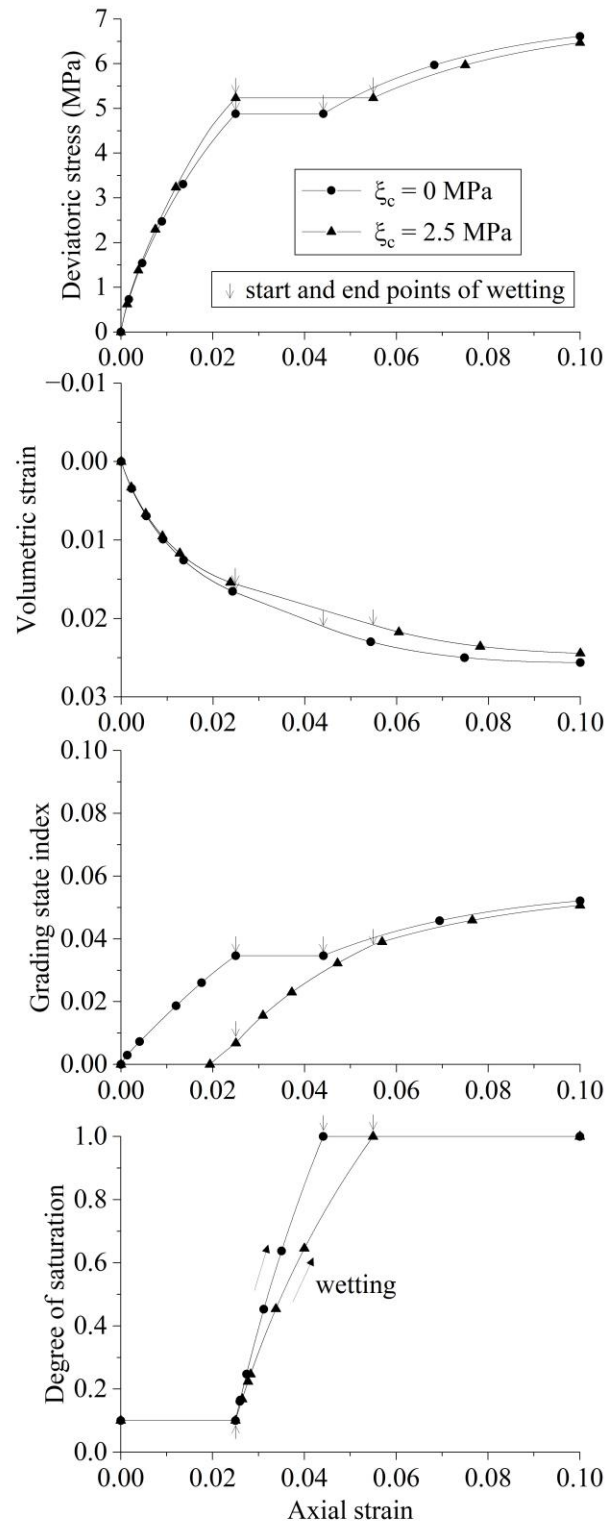


Fig. 17 The effect of ξ_c on the anisotropic wetting tests of Gushui rockfill materials

5. Conclusions

This study presents a constitutive model for unsaturated crushable soils, integrating the effect of degree of saturation on soil crushing strength by developing a crushing surface to consider the effect of change in both state of stress and degree of saturation. The model also incorporates changes in the soil-water characteristic curve (SWCC) due to particle crushing by the reflection in the volumetric compression via modified suction. It accounts for variations in grading state index and degree of saturation, influencing soil strength through state boundary surface movement. The model is validated against past experimental evidence.

The model effectively captures key features of unsaturated crushable soils, including the reduction in crushing strength with increased degree of saturation, the evolution of SWCC due to particle crushing, and particle crushing behavior during wetting. Simulation outcomes underscore the significant impact of particle crushing on soil behavior, leading to decreased peak shear strength and increased degree of saturation. Consequently, this increased degree of saturation further reduces strength and amplifies soil deformation. Moreover, findings from wetting tests suggest that soaking unsaturated crushable soils can further intensify particle crushing, resulting in a significant drop in shear strength and a larger deformation. By providing a comprehensive framework that accounts for the coupled nature of changes in degree of saturation and particle crushing, this model offers valuable insights into the behavior of unsaturated crushable soils. Understanding these interactions is crucial for accurately predicting unsaturated crushable soils behavior, particularly in highly crushable soils or geotechnical structures exposed to tidal or heavy rainfall conditions. As these soils may initially exhibit strong behavior, however, once particle crushing initiates or the degree of saturation changes suddenly, the combined effects of particle crushing and variations in the degree of saturation can lead to abrupt deformation, posing a significant risk to the structure.

Acknowledgements

This research is supported by the Civil Engineering Centennial Scholarship of Chulalongkorn University to the first author. Additionally, the research is supported by the NSRF (FRB660065/0258-RE-KRIS/FF66/35) granted to the third author. JSPS KAKENHI under grants 19H00780 and 23K26194 to the last author are also acknowledged.

Data availability

The datasets generated during and analyzed during the study are available from the corresponding author upon reasonable request.

Conflict of interest

The authors have no competing interests to declare relevant to this article's content.

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