

The Phase Shift Analysis on Alpha-Alpha Scattering in the Energy Range from 22.9 to 28.9 Mev

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The scattering of alpha particles from Helium has been experimentally studied with increased accuracy at five energies between 22.9 and 28.9 Mev. The phase shift analysis up to $L=6$ has been performed for each energy. The result shows that the S-, D- and G-wave phase shifts are necessary to fit the data and the I-wave phase shift is small in this energy range.

The level parameters for the G-state have been extracted using the dispersion theory and they are compatible with the alpha-alpha model of Be⁸.

INTRODUCTION

The scattering of two alpha particles has been studied for many years. Although accuracies of the early experiments¹⁾ were limited by the necessity of using natural alpha particles, these results are able to show the verification of the scattering theory of identical particles of Mott and the existence of nuclear forces other than the Coulomb force. Wheeler²⁾, in 1941, analyzed the early scattering data in terms of excited states in the compound nucleus Be⁸,

Recently, the precise alpha-alpha scattering experiments have been performed by Heydenburg and Temmer³⁾ at the Carnegie Institution in the region from 0.15 to 3 Mev; by Russell, Phillips, and Reich⁴⁾ at the Rice Institute in the region from 2.5 to 5.5 Mev; by Jones, Phillips, and Miller⁵⁾ at the same Institute in the region from 5 to 9 Mev; by Berk, Steigert, and Salinger⁶⁾ at Yale University at the energy of 7.56 Mev; by Nilson, Jentschke, Briggs, Kerman, and Snyder^{7,8)}, at the University of Illinois in the region from 12.3 to 22.9 Mev; by Bredin, Burcham, Evans, Gibson, McKee, Prowse, Rotblat, and Snyder⁹⁾ at the University of Birmingham in the region from 23.1 to 38.4 Mev; and by Conzett, Igo, Shaw, and Slobodrian¹⁰⁾ at Berkeley in the region from 36.8 to 47.3 Mev. All of these results have been analyzed in terms of partial wave phase shifts, and it is found that only nonzero phase shifts needed to fit the data to a laboratory energy of 47 Mev are those of $L=0, 2, 4, 6$, and 8 . The behavior of these phase shifts has given evidence for several virtual excited states in Be⁸ in this energy range; a narrow 0^+ ground state at 190 Kev bombarding energy, a broad 2^+ state at 3.1 Mev in Be⁸, and an even broader 4^+ state at approximately 12 Mev in Be⁸. The differential cross sections of alpha-alpha scattering are strongly energy-dependent, and consequently data at several neighboring energies are needed to explore resonance effects in the phase shifts. Therefore, sufficient justification was felt in repeating the

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measurement with increased accuracy at more closely spaced energy intervals above 23 Mev.

An attempt has been made to measure the differential cross section of alpha-alpha scattering at five energies between 23 and 29 Mev¹¹⁾. This paper reports the phase shifts analysis of these results.

DIFFERENTIAL CROSS SECTION MEASUREMENT

Alpha particles accelerated to an energy of 30 Mev by the Kyoto University 105cm cycrotron were used. The alpha particle beam was led into a scattering chamber in which a cylindrical Helium gas target was placed. After passing through the chamber, the beam was collected in a Faraday cup and integrated. A thin CsI crystal was used to detect elastically scattered alpha particles. A collimation system consisting of two rectangular slits was placed in front of the CsI crystal to determine the scattering volume and the solid angle subtended by the detector. The angular position of the detector was reproducible to $\pm 0.05^\circ$ in the laboratory system.

The incident alpha particle beam was degraded to the desirable energies by placing Aluminum absorbers in front of the scattering chamber, then the mean energy of the beam at the scattering chamber was determined from its range in Aluminum with the accuracy of about 1%.

Absolute differential cross sections were evaluated from the number of detector counts, the integrated charge and the Helium pressure as described in Reference 11. The uncertainty in the absolute values of differential cross sections were separated from those in the relative values. The absolute uncertainty amounts to

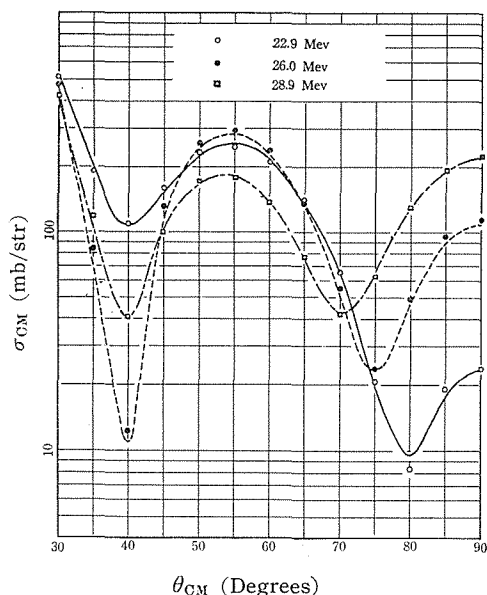


Fig. 1. Computed and observed differential cross sections of alpha-alpha scattering at 22.9, 26.0 and 28.9 Mev, curves computed with the best sets of the phase shifts obtained from the present work.

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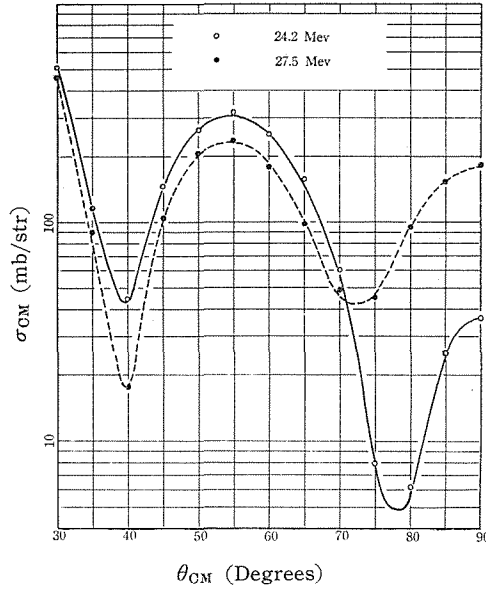


Fig. 2. Computed and observed differential cross sections of alpha-alpha scattering at 24.2 and 27.5 Mev, curves computed with the best sets of the phase shifts obtained from the present work.

3%, while the relative uncertainty varies from 1 to 10%. The measured cross sections at 22.9 ± 0.3 , 24.2 ± 0.3 , 26.0 ± 0.3 , 27.5 ± 0.3 , and 28.9 ± 0.3 Mev are plotted in Figures 1 and 2.

PHASE SHIFT ANALYSIS

The center of mass differential cross section σ_{OM} of alpha-alpha scattering can be expressed in terms of the nuclear phase shifts α_L as follows¹²⁾,

$$\begin{aligned} \sigma_{OM}(\theta) = & \frac{4e^4}{M^2 V^2} \left| \operatorname{cosec}^2(\theta/2) \exp[i\eta \ln \operatorname{cosec}^2(\theta/2)] \right. \\ & + \sec^2(\theta/2) \exp[i\eta \ln \sec^2(\theta/2)] \\ & \left. + \frac{2i}{\eta} \sum_{L=0,2,4}^{\infty} (2L+1) P_L(\cos \theta) \exp(2i\xi_L) [\exp(2i\alpha_L) - 1] \right|^2 \quad (1) \end{aligned}$$

with

$$\eta = \frac{4e^2}{\hbar V}, \quad \xi_L = \sum_{s=1}^L \tan^{-1}(\eta/s), \quad \xi_0 = 0,$$

where M denotes the mass of alpha particles, V is the relative velocity between alpha particles, θ is the center of mass scattering angle and $P_L(\cos \theta)$ is the Legendre polynomial of order L .

The best set of phase shifts was extracted from the measured cross sections in the following manner which has been successfully applied to the phase shift analysis of pion-proton scattering¹³⁾. Cramer¹⁴⁾ gives the method of the maximum likelihood to determine magnitudes from an indirect measurement. Suppose that we have a system,

to a certain minimum value as the calculation carried on. The theory of the least squares implies the minimum value of χ^2 to be $n-k$, if we know the true standard deviation of the measurement. Therefore, a more sensitive measure of the goodness of fit is obtained from the root-mean-squares error ε , (*rms* error), defined as follows,

$$\varepsilon = (\chi^2/n-k)^{1/2}.$$

Our results show that the *rms* error varies from 1 to 1.6.

The best sets of phase shifts at all energies thus found are listed in Table 1 with the corresponding *rms* errors. The analysis was limited up to the G-wave phase shift, since the results at Birmingham⁹⁾ and Berkeley¹⁵⁾ show that the K-wave phase shift ($L=8$) is essentially zero in the energy range from 30 to 44 Mev.

The errors in these phase shifts are assigned in the following manner. The values of σ_i^m and the corresponding relative errors were fed into the program as input data, then the computer found the best set of phase shifts and their errors defined by Eq. (3). This solution and error were accepted as the standard phase shift solution and the standard error. However, this error is only due to the relative error. Therefore, in order to evaluate the error arising from the absolute uncertainty, the values of σ_i^m were shifted by 3%, and the phase shift solution which corresponds to these σ_i^m was found. The difference between this phase shift solution and the standard solution is considered as the error arising from the absolute uncertainty. These errors, then, are combined with the standard errors in the usual manner, and these combined errors are listed in Table 1.

Table 1. The results of the phase shift analysis.

$E_{lab}(\text{Mev})$	$\alpha_0(\text{deg})$	$\alpha_2(\text{deg})$	$\alpha_4(\text{deg})$	$\alpha_6(\text{deg})$	ε
22.9 ± 0.3	-11.6 ± 1.7	94.2 ± 1.4	51.7 ± 2.0	0.7 ± 0.6	1.41
24.2 ± 0.3	-12.6 ± 1.9	93.0 ± 2.3	72.5 ± 2.5	2.1 ± 1.0	1.61
26.0 ± 0.3	-16.5 ± 4.2	88.0 ± 6.0	89.1 ± 6.0	2.4 ± 2.0	1.42
27.5 ± 0.3	-28.8 ± 4.2	81.1 ± 3.3	97.7 ± 4.5	0.4 ± 1.0	1.15
28.9 ± 0.3	-28.2 ± 1.4	81.1 ± 1.4	111.5 ± 3.0	0.4 ± 0.6	1.49

DISCUSSION

The phase shifts listed in Table 1 are plotted in conjunction with those from the results at Illinois and Birmingham in Figures 3 and 4.

The values of the S- and D-wave phase shifts seem to be consistent with each other and show no resonance behavior in this energy range. The I-wave phase shift is not accurately determined and shows no resonance behavior.

The values of the G-wave phase shift of the present analysis are systematically smaller than those derived at Birmingham. This discrepancy could be explained by the rather large uncertainty in the measured cross sections at Birmingham, since the absolute uncertainty is a serious source of errors in the phase shifts, as is pointed out in Reference 9.

The variation of the G-wave phase shift can be fitted by the single-level dispersion theory of Wigner-Eisenbud¹⁶⁾ with a hard-sphere radius of $3.5 \times 10^{-13} \text{ cm}$,

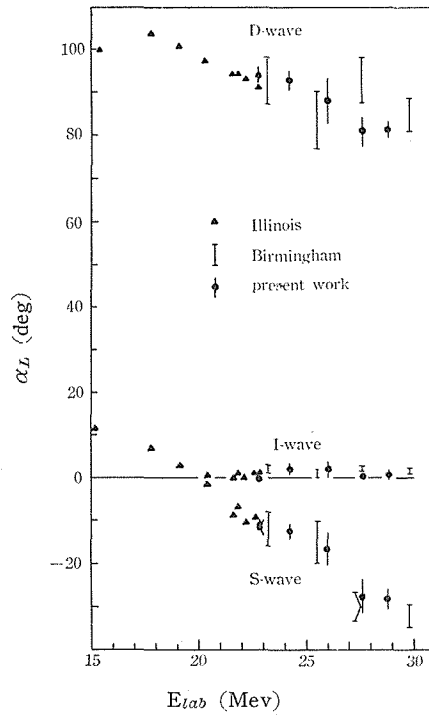


Fig. 3 Experimental alpha-alpha S-, D- and I-wave phase shifts.

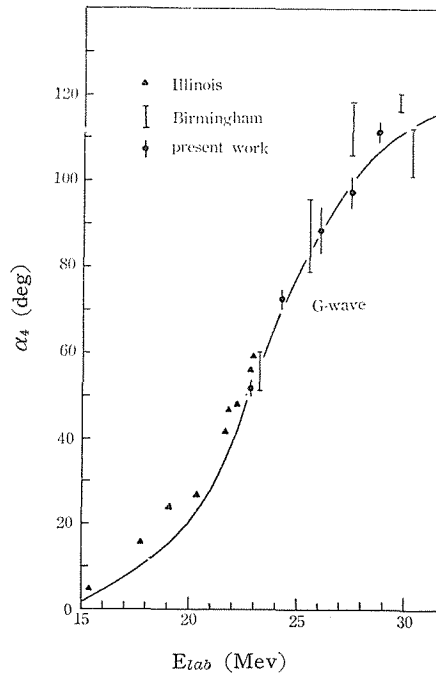


Fig. 4. Comparison of experimental G-wave phase shift to that obtained from the single-level dispersion theory with a hard-sphere radius of $3.5 \times 10^{-13} \text{cm}$, a reduced width of 3.5 Mev and a resonance energy of 12.5 Mev.

a reduced width of 3.5 Mev (1.3 Wigner limit) and a resonance energy of 12.5 Mev. Considering the reduced width of this state in comparison with the Wigner limit, $3\hbar^2/2\mu a^2$ (≈ 2.65 Mev), it is indicated that the G-state in Be^8 is almost wholly an alpha particle state. The lower states, the 0^+ ground state and the 2^+ excited state at 3.1 Mev in Be^8 , are also confirmed as alpha particle states from the previous studies by Heydenburg *et al.*¹³⁾, Russell *et al.*¹⁴⁾, Jones *et al.*¹⁵⁾, and Nilson *et al.*¹⁶⁾. These three states ($0^+ - 2^+ - 4^+$), are predicted by both the shell model¹⁷⁾ and the alpha particle model¹⁸⁾, however, these experimental results lend credence to the latter model.

An alpha particle potential which is able to explain the behavior of these three wave phase shifts is most desirable. Among various types of alpha particle potentials, the Haefner potential¹⁸⁾ has been studied in detail by many authors^{3,4,8)}. Quite good fits were obtained by this potential, however in order to fit to all the phase shifts, it is necessary to use different values of the well depth for different values of L . None of potential of this form which would fit all the phase shifts without changing parameters has found.

As an attempt to solve this problem, Russell *et al.*¹⁴⁾ and Jones *et al.*¹⁵⁾ discussed the qualitative feature of the potential of alpha-alpha interaction which could explain the behavior of the S- and D-wave phase shifts. Russell *et al.*, from their analysis of the low energy data, have presented a qualitative picture of the alpha-

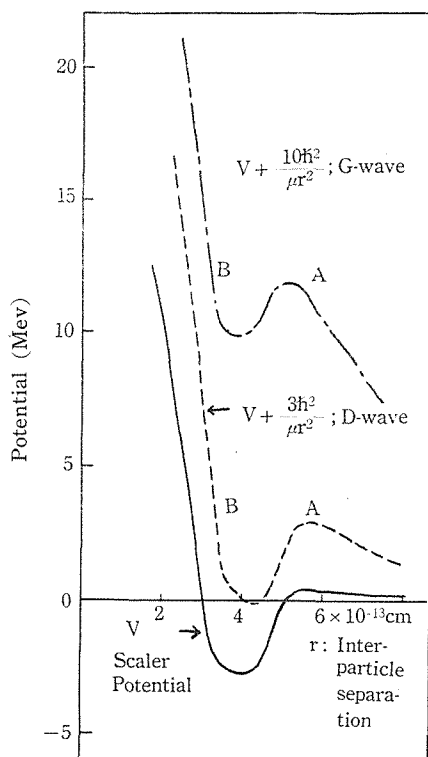


Fig. 5. Qualitative features of the alpha-alpha potential as presented by Russell *et al.* and Jones *et al.*

alpha potential as an attractive potential trough, of the order of $2 \times 10^{-13} \text{cm}$ wide and a few Mev deep, located at a radius of about $5 \times 10^{-13} \text{cm}$ and a potential core which must be more repulsive than the Coulomb potential. Jones et al. added a centrifugal potential, $L(L+1)\hbar^2/2\mu r^2$, to this scalar potential to explain the behavior of the D-wave phase shift. The qualitative shape of this potential is shown in Figure 5.

According to their analysis, the D-wave phase shift is fitted by the dispersion theory with a hard-sphere radius of $3.5 \times 10^{-13} \text{cm}$ up to 20 Mev in the laboratory system, while the lower energy D-wave phase shift (with $E_{lab} < 6 \text{ Mev}$) requires a larger hard-sphere radius. A possible explanation for this discrepancy is that at higher energies some of scattering does not occur at point A but rather at point B.

For the G-wave phase shift, this argument is also supported by the fact that the low energy G-wave phase shift (with $E_{lab} < 23 \text{ Mev}$) is fitted with a radius of $4.4 \times 10^{-13} \text{cm}$ as is described in Reference 8, while the present results are consistent with a radius of $3.5 \times 10^{-13} \text{cm}$. Therefore, it is hoped that these qualitative informations about the potential of alpha-alpha interaction would shed some light on finding a more sophisticated potential which can fit all these phase shifts.

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