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Abstract	This study aims to investigate feasibility of a drive-by bridge inspection method using dynamic vehicle force. The dynamic vehicle force is identified using accelerations of a traveling vehicle. Dynamic vehicle forces when the vehicle travels on healthy and damage states of bridges are first identified. Differences of the identified dynamic vehicle forces from healthy and damage states of bridges are then considered as a damage sensitive feature for the drive-by bridge inspection. A least square minimization with Tikhonov regularization was utilized to estimate the dynamic vehicle force from acceleration data of the vehicle. A band pass filter is applied to the dynamic vehicle force before estimating the difference of the dynamic vehicle force. The range of the band pass filter was set around 1st natural frequency of the bridge. The feasibility of the proposed method was verified through simulation and a laboratory experiment. It was observed that the severer the bridge damage is, the larger the difference of dynamic vehicle force is, implying a possibility to detect bridge damages successfully by the proposed method.
Keywords (separated by '-')	Drive-by bridge inspection - Bridge damage detection - Dynamic vehicle force - Band pass filter



Structural Health Monitoring of Bridges Using Dynamic Vehicle Force

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Abstract. This study aims to investigate feasibility of a drive-by bridge inspection method using dynamic vehicle force. The dynamic vehicle force is identified using accelerations of a traveling vehicle. Dynamic vehicle forces when the vehicle travels on healthy and damage states of bridges are first identified. Differences of the identified dynamic vehicle forces from healthy and damage states of bridges are then considered as a damage sensitive feature for the drive-by bridge inspection. A least square minimization with Tikhonov regularization was utilized to estimate the dynamic vehicle force from acceleration data of the vehicle. A band pass filter is applied to the dynamic vehicle force before estimating the difference of the dynamic vehicle force. The range of the band pass filter was set around 1st natural frequency of the bridge. The feasibility of the proposed method was verified through simulation and a laboratory experiment. It was observed that the severer the bridge damage is, the larger the difference of dynamic vehicle force is, implying a possibility to detect bridge damages successfully by the proposed method.

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1 Introduction

Many bridges have been loaded by the increase in traffic volume and vehicle weights since commencement of the bridge, and as a result a number of bridges need to be monitored and repaired. One of the conventional bridge monitoring methods is the visual inspection of target bridges, but it is time-consuming and inefficient way of bridge inspection. In order to cope with the weakness of the visual inspection, sensors are installed on a bridge to monitor changes in physical properties of the bridge. However, it is still costly, and also is time- and labor-consuming to install sensors on the bridge, even though quantitative information is available.

For higher efficiency, Yang et al. [1] proposed an indirect method using vehicle vibrations during the vehicle traveling on the bridge, which especially focuses on identification of the bridge natural frequency. Since then, many researches on such a bridge inspection method using vehicle vibrations [2–13], and those are also called as “drive-by

bridge inspection” [5], which aims to obtain a variety of information of a bridge, such as natural frequency, stiffness, and damping of the target bridge, as well as vehicle wheel loads.

One of the challenges of the drive-by bridge inspection is how to obtain higher accuracy on changes in structural performance of bridges from the measured vehicle vibrations that is a result of the vehicle–bridge interaction. Vehicle and bridge vibrations are greatly affected by road roughness on the bridge, and therefore, how to reduce influences of the road roughness from the measured data is a crucially important issue to realize drive-by bridge inspection. In this study, a novel approach to detect existence of bridge damage utilizing dynamic vehicle forces is proposed. First, when the vehicle travels on the target bridge in intact state or reference state, the reference dynamic vehicle force is estimated. When the bridge is to be inspected, the dynamic vehicle force is identified again for test. Then the difference of identified forces of reference and test states is calculated in time domain.

If the flexural rigidity of the bridge is decreased due to damage, the vibration characteristics and the dynamic vehicle force will be changed from the reference condition. Therefore the difference of the dynamic vehicle force is proposed as a damage sensitive feature in the drive-by bridge inspection. A band pass filter was applied to identified dynamic vehicle force so as to decrease influences of road roughness before estimating difference of dynamic vehicle force. To investigate possibility of damage quantitatively, the difference of identified dynamic vehicle forces is converted to power spectral density (PSD), and the power of the PSD is investigated. Simulation and laboratory experiments have been carried out to investigate feasibility of the proposed damage sensitive feature for the drive-by bridge inspection. Changes in the power of the PSD of the difference of dynamic vehicle forces were observed due to damage of the bridge both in the simulation and the laboratory experiment.

2 Methodology

2.1 State-Space Equation

In order to estimate the vehicle dynamic force, firstly a discrete state-space equation of a vehicle motion is derived. Dynamic equations of motion of a vehicle are written as Eq. (1).

$$\mathbf{M}_v \ddot{\mathbf{y}}_v + \mathbf{C}_v \dot{\mathbf{y}}_v + \mathbf{K}_v \mathbf{y}_v = \mathbf{f}_v \quad (1)$$

where, $\mathbf{M}_v$, $\mathbf{C}_v$ and $\mathbf{K}_v$ indicate mass, damping and stiffness matrices of the vehicle, respectively. $\mathbf{y}_v$ denotes the displacement vector of the vehicle. The dot above variables indicates derivative with respect to time. $\mathbf{f}_v$ represents the dynamic vehicle force vector. Equation (1) can be converted to the continuous state-space equation as Eq. (2).

$$\dot{\mathbf{X}} = \mathbf{A}\mathbf{X} + \mathbf{g} \quad (2)$$

where, $\mathbf{X}$, $\mathbf{A}$ and $\mathbf{g}$ are defined as Eq. (3), Eq. (4) and Eq. (5), respectively.

$$\mathbf{X} = \{\mathbf{y}_v \dot{\mathbf{y}}_v\}^T \quad (3)$$

$$A = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}_v^{-1}\mathbf{K}_v & -\mathbf{M}_v^{-1}\mathbf{C}_v \end{bmatrix} \quad (4)$$

$$\mathbf{g} = \begin{bmatrix} \mathbf{0} \\ -\mathbf{M}_v^{-1}\mathbf{f}_v \end{bmatrix} \quad (5)$$

Solution of Eq. (2) as a 1st-order discrete-time state-space model can be obtained with using exponential matrix as Eq. (6).

$$\mathbf{X}_{j+1} = \mathbf{M}\mathbf{X}_j + \mathbf{G}\mathbf{f}_{v,j} \quad (6)$$

where, $\mathbf{M} = e^{A_h}$ and h is a time step. $\mathbf{f}_{v,j}$ is the dynamic vehicle force vector at a time j . $\mathbf{G}$ is calculated as Eq. (7).

$$\mathbf{G} = \mathbf{A}^{-1}(\mathbf{M} - \mathbf{I}) \begin{bmatrix} \mathbf{0} \\ -\mathbf{M}_v^{-1} \end{bmatrix} \quad (7)$$

Equation (6) is rewritable as Eq. (8), defining a new state vector shown in Eq. (9).

$$\mathbf{Z}_{j+1} = \begin{bmatrix} \mathbf{M} & \mathbf{G} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \mathbf{Z}_j + \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix} \Delta \mathbf{f}_{v,j} \quad (8)$$

$$\mathbf{Z}_j = \left\{ \mathbf{X}_j \mathbf{f}_{v,j} \right\}^T \quad (9)$$

where, $\Delta \mathbf{f}_{v,j}$ is defined as Eq. (10), which is a time-step difference of the dynamic vehicle force vector.

$$\Delta \mathbf{f}_{v,j} = (\mathbf{f}_{v,j+1} - \mathbf{f}_{v,j})/h \quad (10)$$

Equation (8) is used in the least square minimization in order to estimate the dynamic vehicle force.

2.2 Regularized Least Square Minimization

To estimate the dynamic vehicle force, regularized least square minimization is applied. A cost function in the regularized least square minimization is defined as Eq. (11).

$$CF = \sum_{j=1}^N [(d_j - \mathbf{Q}\mathbf{Z}_j, d_j - \mathbf{Q}\mathbf{Z}_j) + (\Delta \mathbf{f}_{v,j}, \lambda \Delta \mathbf{f}_{v,j})] \quad (11)$$

where, d_j is a measured vehicle acceleration vector. The notation $(\cdot, \cdot)$ means an inner product. $\mathbf{Q}$ is a selection matrix of d_j and $\mathbf{Z}_j$, which is defined as Eq. (12).

$$\mathbf{Q} = [-\mathbf{M}_v^{-1}\mathbf{K}_v \quad -\mathbf{M}_v^{-1}\mathbf{C}_v \quad \mathbf{M}_v^{-1}] \quad (12)$$

λ in Eq. (11) is the regularization parameter, which has a role of balancing volumes of 1st and 2nd terms in Eq. (11). The 2nd term in Eq. (11) is named a regularization

term, which decreases the ill-posedness in the inverse problem [14]. The optimal value of λ is obtained by Hansen's L-curve method [15]. The L-curve is plotted on a log-log scale using the residual least squares norm (E_{norm}) and solution norm (F_{norm}), given in Eq. (13) and Eq. (14), respectively.

$$E_{norm} = \sqrt{\sum_{j=1}^N (d_j - QZ_j, d_j - QZ_j)} \quad (13)$$

$$F_{norm} = \sqrt{\sum_{j=1}^N (\Delta f_{v,j}, \lambda \Delta f_{v,j})} \quad (14)$$

The optimal value for λ corresponds to the point of maximum positive curvature on the L-curve plot. This λ value is selected to provide the optimal force vector estimate [7]. The regularized least square minimization provides the optimal Z_j and $\Delta f_{v,j}$, and finally, $f_{v,j}$ can be obtained from Eq. (10). Dynamic programming technique [16] is applied to calculate the solution for each candidate λ s. Details of the dynamic force estimation algorithm can be referred in [7, 9].

2.3 Damage Detection

In the proposed method, bridge damage, especially change in modal parameters is detected by the difference of identified dynamic vehicle forces of intact and damaged condition. The estimated dynamic vehicle force when the vehicle travels on the healthy bridge is called as RF (Reference Force). The estimated dynamic vehicle force when the vehicle travels on the bridge to be inspected is called as TF (Test Force). Once the RF is estimated in advance, the difference of RF and TF, which is named as FD (Force Different), can be utilized for the bridge damage detection.

The FD is estimated under assumptions of constant vehicle speed and knowing the entry and exit times of the vehicle to the bridge. The band pass filter is applied to TF and RF in order to eliminate undesirable frequency components, and then the FD is calculated. The range of the band pass filter is set around the 1st natural frequency of the target bridge.

We expect that vibration components affected by road roughness will be canceled out by calculating the difference, and only vibration component relevant to the change in bridge modal parameters will remain. Therefore, the FD comprising large amplitude indicates possibility of bridge damage to some degrees. The power of PSD of the FD is used to clarify the level of possible damage. Measurement of vehicle vibrations can be repeated several times for statistical approaches. The flow of the proposed method is shown in Fig. 1.

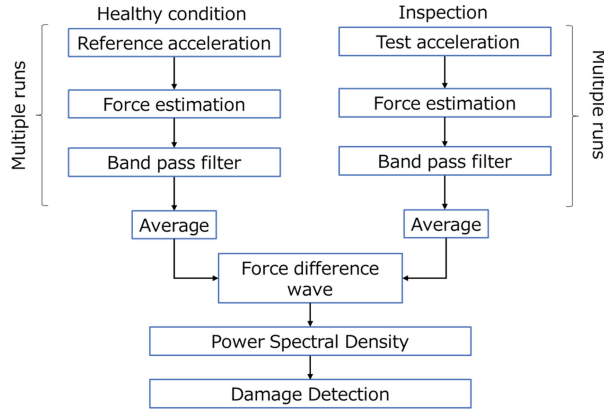


Fig. 1. Flow chart of the proposed damage detection method.

3 Feasibility Investigation by Simulation

3.1 Analytical Models

Numerical simulation of a vehicle-bridge interaction is carried out as a feasibility investigation. Accelerations of vehicle motions are estimated by solving equations of motion of the vehicle-bridge interaction. The Wilson-theta method was adopted as a numerical integration method, where 1.420815 is used as the value of theta following reference [7]. Properties of the vehicle and bridge are summarized in Table 1 and Table 2, respectively. The bridge is modeled as a simply supported beam, and the FE model of the beam consists of eight elements. The road roughness profile generated by means of the Monte Carlo simulation is shown in Fig. 2. Two-degree-of-freedom half-car (2DOF HC) model shown in Fig. 3 is used for simulation and the dynamic force identification.

To imitate the bridge damage with different damage severity, the flexural stiffness of the 4th element from the entry (see Fig. 4) was reduced by 10%, 20% and 30%. Those damage scenarios are named as “Damage1”, “Damage2” and “Damage3”, respectively. Intact scenario for reference and test states are named “Healthy” and “No damage”, respectively. The simulation was repeated 10 times per each damage scenario including the healthy state. Radom noise whose Signal-to-Noise Ratio (SNR) is 10 was added to each acceleration. SNR is a ratio of variances of the noise and the original data. The vehicle speed was assumed as 20 km/h. The range of the band pass filter is between 2 Hz and 3 Hz considering the bridge 1st natural frequency of 2.55 Hz.

Table 1. Vehicle properties in simulation

Property		Value
Body mass		18000 kg
Suspension stiffness	Front	2.0×10^6 N/m
	Rear	5.0×10^6 N/m
Suspension damping	Front	1.0×10^4 N s/m
	Rear	2.0×10^4 N s/m
Pitch moment inertia		103841 kg·m ²
Axle distance		4.75 m

Table 2. Bridge properties in simulation

Property	Value
Span length	41 m
Mass per unit length	24152 kg/m
Second moment of area	5.1305 m ⁴
Young's modulus	3.5×10^{10} N/m ²
Number of elements	8
1 st natural frequency	2.55 Hz

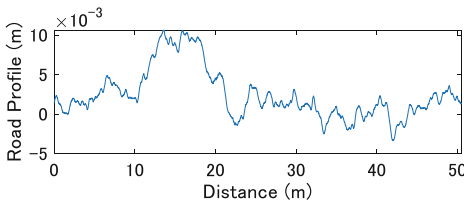


Fig. 2. Road roughness profile in simulation

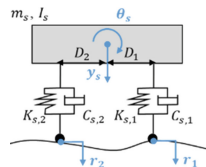


Fig. 3. 2DOF vehicle model

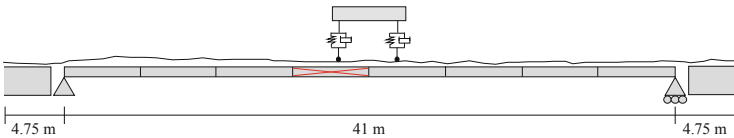


Fig. 4. Bridge scheme in simulation

3.2 Damage Detection

RF in bounce direction for 10 times running are shown in Fig. 5, applying low pass filter whose cutoff frequency is 10 Hz to input acceleration. Figure 5 shows the 10 times results have large deviation especially in low frequency range. This deviation is caused by the fact that the accuracy of dynamic vehicle force identification is lower in low frequency range than high frequency range. This accuracy deterioration is because the running vehicle acceleration is more sensitive to high frequency components of the road profile and bridge vibration.

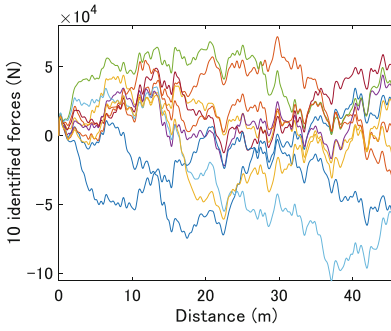


Fig. 5. RF before applying band pass filter

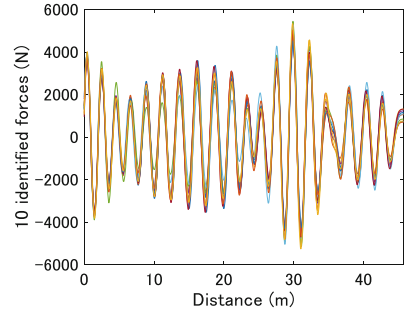


Fig. 6. RF after applying band pass filter

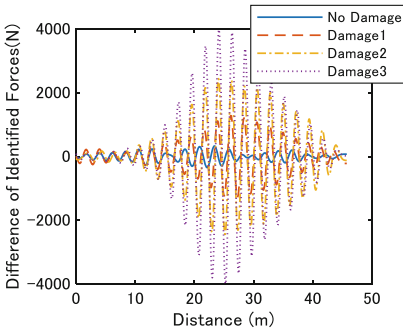


Fig. 7. FDs from simulation

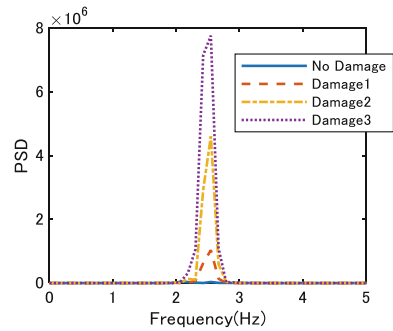


Fig. 8. PSDs of FDs from simulation

By applying band pass filter to these identified forces, this deviation is much reduced as shown in Fig. 6. After calculating the average of TF in each test scenarios, the FD is estimated and those time histories of the FD are shown in Fig. 7. It should be noted in Fig. 7 that when damage becomes more severe, the wave amplitude of FD also increases. It encourages the use of the proposed FD-based drive-by bridge inspection method. This tendency is clearly visualized from PSD of the FD as shown in Fig. 8.

4 Laboratory Experiment

4.1 Experiment Summary

The feasibility of the proposed method is also investigated through a laboratory experiment. Properties of the bridge and vehicle used in the laboratory experiment are summarized in Table 3 and Table 4, respectively. Table 4 shows the vehicle properties if the model vehicle is idealized as 2DOF-HC model. Those properties when the model vehicle is idealized as single-degree-of-freedom quarter-car (1DOF-QC) model shown in Fig. 9 are summarized in Table 5. The 1DOF-QC model is also used in the dynamic force

Table 3. Bridge properties in experiment

Property		Value
Span length		5.4 m
Bridge length		5.6 m
Mass per unit length (kg/m)		46.9
Second moment of area (m ⁴)		5.48×10^{-7}
Young's modulus (N/m ²)		2.1×10^{11}
Damping ratio		0.012
1 st natural frequency	Intact	2.81 Hz
	Damage1	2.71 Hz
	Damage2	2.59 Hz
	Damage3	2.59 Hz

Table 4. Properties of 2DOF-HC model

Property		Value
Body mass		22.9 kg
Suspension stiffness	Front	3480 N/m
	Rear	3531 N/m
Suspension damping	Front	3.51 N s/m
	Rear	13.2 N s/m
Axle distance		0.4 m
Distance of axle to center of gravity	Front	0.202 m
	Rear	0.198 m
Distance between sensor and center of gravity	Front	0.182 m
	Rear	0.188 m
Bounce frequency		2.83 Hz

identification in order to investigate a suitable model for the proposed drive-by bridge inspection method. The vehicle is composed by four axles, a body plate, steel weights, and two accelerometers as shown in Fig. 10. Two accelerometers measure vertical vibrations of the model vehicle, and they are converted to the accelerations for bounce and pitch motions of the model vehicle using geometric relationship when 2DOF-HC model is considered.

The model bridge used in the experiment is a simply-supported steel bridge whose span length is 5.4 m. Figure 11 shows the model bridge under the moving model vehicle. The model vehicle travels on tracks set up on model bridge. In order to model the road roughness profile on the track, the road roughness profile is engraved on the track by a laser. The road roughness profile on the tracks is shown in Fig. 12. Three slits were introduced to the model bridge as damage on the bridge, and those slits can be reinforced with plates and bolts as shown in Fig. 13. The locations of the slits are 1.68 m, 2.01 m, and 2.36 m from entry support of the bridge. The condition that all slits are reinforced is assumed as “No damage” scenario. The state that two of three slits are reinforced is “Damage 1” scenario. The state that one of three slits are reinforced is “Damage 2” scenario. Finally, the state that all slits are open is “Damage 3” scenario. All damage scenarios are summarized in Fig. 14. The vehicle speed was set to 1.26 m/s. The range of the band pass filter is 2.3 Hz to 3.3 Hz, considering bridge 1st natural frequency of 2.81 Hz in healthy case.

Table 5. Properties of 1DOF-QC model

Property	Value
Body mass	5.78 kg
Suspension stiffness	1766 N/m
Suspension damping	6.60 N s/m
Natural frequency	2.83 Hz

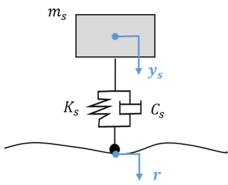


Fig. 9. 1DOF-QC model

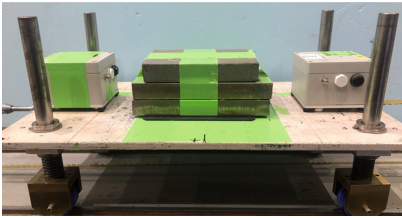


Fig. 10. Vehicle used in experiment



Fig. 11. Bridge used in experiment

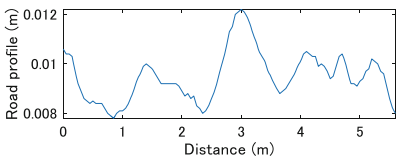


Fig. 12. Road roughness profile used in experiment

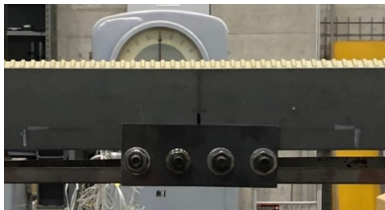


Fig. 13. Reinforced slit of bridge in experiment

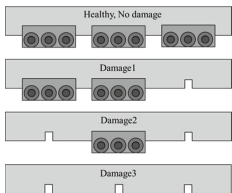


Fig. 14. Damage scenarios in experiment

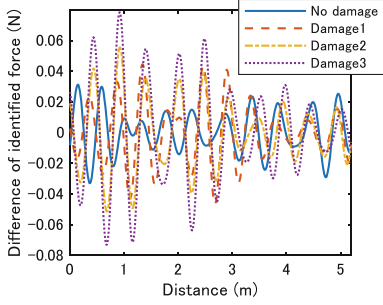


Fig. 15. FD by 1DOF-QC model

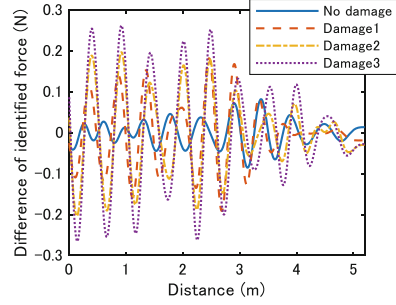


Fig. 16. FD by 2DOF-HC model

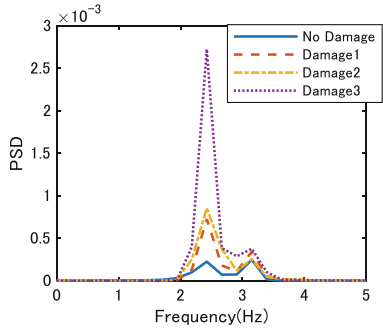


Fig. 17. PSD of FD by 1DOF-QC model in experiment

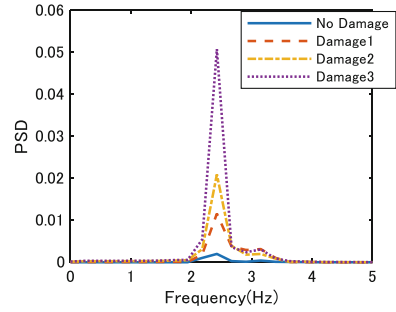


Fig. 18. PSD of FD by 2DOF-HC model in experiment

4.2 Damage Detection from Experiment

FDs identified utilizing 1DOF-QC and 2DOF-HC models are shown in Fig. 15 and Fig. 16, respectively. Those figures show that the amplitude of FD increases with the damage severity in both vehicle models. PSDs of the FD are shown in Fig. 17 and Fig. 18, respectively for 1DOF-QC and 2DOF-HC models, and ratios of PSD peaks in damage scenarios to the “No damage” scenario are summarized in Table 6. Change in PSD peaks along the change in damage levels was clearer in 2DOF-HC than 1DOF-QC model. Especially, the ratio of PSD peaks in damage scenarios to the “No damage” scenario of 1DOF-QC showed close values in Damage1 and Damage2, while the ratio of 2DOF-HC showed more clear difference.

Table 6. Ratio of PSD peaks of damage cases against no damage case in experiment

Model	Damage1	Damage2	Damage3
1DOF-QC	3.2	3.8	12.1
2DOF-HC	5.8	10.5	25.5

5 Conclusions

A drive-by bridge inspection method focusing on difference of dynamic vehicle forces of healthy and damaged bridge conditions is proposed. Dynamic vehicle forces are identified using measured vehicle acceleration by means of regularized least square minimization. The band pass filter is applied to identified vehicle forces before estimating the force difference in order to reduce undesirable vibration components. Observations from the numerical simulation and laboratory experiment showed the feasibility of damage detection using the proposed drive-by bridge inspection method. 2DOF-HC model detected the damage more clearly than 1DOF-QC model in the experiment.

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