

Okishio's Theorem Generalized

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1. Linear case with joint production

1.1. Okishio(1961) showed in a linear model without joint production that when firms adopt cost-reducing new processes the rate of profit will rise provided the real wage rate remain fixed. Let this result be expressed by simple equations. The symbol A denotes the m by n augmented input coefficient matrix. The price equation before technical progress is:

$$p^0 = (1+r^0)p^0 A^0.$$

Cost-reducing implies

$$p^0 \geq (1+r^0)p^0 A^*.$$

The superscripts 0 and * indicate those symbols before and after technical progress respectively. Given the two equations above, Okishio theorem states that

$$p^* = (1+r^*)p^* A^* \text{ with } r^* > r^0$$

if A^* is indecomposable. (The first equation is meaningful only in an economic story.)

1.2. Now let us shift to a linear model with joint production. In such a model we can allow for fixed capital as explained by von Neumann and P.Sraffa. Notation is:

B : output coefficient matrix(m by n) : a process columnwise.

A : input coefficient matrix(m by n). includes workers' feeding stuff.

x : output column n -vector, p : price row m -vector.

r : uniform rate of profit.

Again the superscripts 0 and * for old and new processes respectively.

The price equation or the equilibrium condition(price side only) is:

$$pB = (1+r)pA.$$

To obtain a generalization of Okishio's theorem we introduce

Quantity Augmenting Property(QAP): A technology (B^*, A^*) is said to have the Quantity Augmenting Property if there exists a column n^* -vector $x^+ \geq 0$ such that

$$B^* x^+ \gg (1+r^0)A^* x^+,$$

where r^0 is among the possible equilibrium rates of profit with the old technology (B^0, A^0) .

Theorem. If the new technology satisfies the QAP, $r^* > r^0$.

Proof. Almost tautological.

1.3. It may be desirable to obtain a condition on the price side because managers are tempted to introduce new processes depending on cost calculation.:

Generalized Profitability Condition(GPC): The technology (B^*, A^*) is said to have the Generalized Profitability Condition over the technology (B^0, A^0) , if for no price vector(semi-positive) is it true that $pB^* \leq (1+r^0)pA^*$.

Theorem. QAP and GPC are equivalent.

Proof. (A theorem due to A.Tucker).

A natural economic interpretation of GPC is that there is no price vector under which every process makes losses or breaks even with at least one process making losses. In a sense, to that extent new processes are productive.

2. Nonlinear case

2.1. To allow for economic externalities and variable returns to scale, one may wish to consider nonlinear input-output model. Thus, B and A are now dependent upon the activity level vector x , and written $B(x)$ and $A(x)$. Let us define

$$H(x; r) \equiv B(x) - (1+r)A(x).$$

$H(x; r)$ may be written simply $H(x)$ when r is not relevant. We make the following Assumptions:

- (1) $H_i(x)$ is differentiable for every i .
 - (2) For each i , if $H_i(x) < 0$ at some $x \in D \equiv \mathbb{R}_+^n - \{0\}$, then $\nabla H_i(x) \cdot x < 0$ at the same x .
- ((2) is satisfied ,e.g., by functions homogeneous of positive degrees, or pseudoconcave functions such that $H_i(x) \geq 0$. See Mangasarian(1969).)

Theorem. If the system of inequalities $H(x) \geq 0$ has no solution on $S \equiv \{x \in D | \sum x_i = 1\}$, then there exists a semi-positive vector $p \in \mathbb{R}_+^m$ and $x^* \in S$ such that $p \nabla H(x^*) < 0$.

Proof. Please refer to Fujimoto(1980).

Now it is not difficult to have a theorem similar to that in section 1.2 above.=

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