

CFD Simulation of Grout Flow in Rough-Walled Rock Fractures: Influence of Water–Cement Ratio

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ABSTRACT

Rock grouting involves injecting grout material into a rock mass to reduce its permeability and increase its strength. A critical aspect of grout design is determining the water/cement ratio, which primarily governs the flowability and viscosity of the cement grout.

This paper examines the influence of the water/cement ratio on grout flow in a single rough-walled rock fracture using computational fluid dynamics (CFD) simulations. To better understand the grout flow within a rough-walled fracture, the fracture's roughness is modeled as a simplified sinusoidal geometry.

The study identifies four distinct flow regimes of cement slurry in a rough-walled fracture. As the Reynolds number increases, the flow transitions through the following regimes: (1) yield stress-dominated flow, (2) laminar flow, (3) laminar flow with recirculation zones, and (4) non-linear flow. Under the same pressure conditions, cement slurry with a higher water/cement ratio flows more rapidly due to its lower cohesion, making it more prone to transitioning into the non-linear flow regime. Furthermore, the study reveals that, as the fracture roughness increases, the transition to the next flow regime occurs at a lower Reynolds number.

The findings suggest that, in the presence of recirculation zones, there is a risk of insufficient sealing between the asperities of rough-walled fractures. This highlights the need to optimize the water/cement ratio and injection pressure, based on the fracture morphology, in order to achieve the effective sealing of these rough-walled fractures.

Keywords

Grout, rough-walled fracture, CFD, roughness, Bingham fluid, cement slurry, water/cement ratio

1 INTRODUCTION

For the treatment of dam foundations, grouting is performed to reduce permeability and enhance the strength of rock masses. Grouting involves injecting grout material, commonly cement slurry, into a rock mass to seal the fractures or voids within it. A critical aspect of grout design is determining the grout thickness. Grout thickness describes the flowability and viscosity of the grout — that is, how thick or thin it is when mixed and pumped. In the case of cement slurry, thickness is primarily controlled by the water/cement (w/c) ratio (Ewert and Hungsbers 2018). Accordingly, the selection of an appropriate w/c ratio is essential in grouting design. Cement slurries with a low w/c ratio (thick grout) tend to exhibit lower flowability and higher viscosity due to the greater presence of cement particles. Cement slurries with a high w/c ratio (thin grout) contain fewer cement particles and thus exhibit higher flowability and lower viscosity. Thin grout has the advantage of better penetration into rock fractures, while thick grout provides superior sealing capability.

The treatment of dam foundations in Japan involves a thin-to-thick grout procedure. The procedure starts with thin grout and the grout thickness is increased step by step after the specified injection time (Figure 1). For dam foundations in Japan, the grouting typically starts with a w/c ratio of 10 and ends with a w/c ratio of 2. This method is favored with the expectation that thin grout better penetrates through fractures with smaller apertures due to lower cohesion, while thick grout better seals fractures with larger apertures due to higher cohesion (Ewert and Hungsbers 2018; Nobuto et al. 2009). However, this procedure tends to be time-consuming and to require large amounts of cement as multiple w/c ratios are used in a single stage of grouting. Therefore, a new criterion to determine the grout thickness is needed to improve

dam foundation treatment operations in Japan.

Currently, there are no standard criteria for designing grout thickness based on the hydrogeological conditions at each site. This is because little is understood about the effects of grout thickness on the flow mechanisms in a fracture considering roughness. The influence of fracture roughness on fluid flow has been studied extensively for Newtonian fluids, such as groundwater. The Newtonian fluid flow in a fracture is known to follow linear Darcy's law at a low Reynolds number and, with an increasing Reynolds number, non-linearity is introduced due to the inertial effect (Zimmerman et al. 2004). It is known that fracture morphology has a significant effect on the transition of the flow regimes of the Newtonian fluid flow (Brush and Thomson 2003; Chen et al. 2017, Lee et al. 2014; Briggs et al. 2017). In particular, Zou et al. (2015) utilized the wavelet decomposition method for fracture roughness to show that secondary roughness plays a major role in the formation of recirculation zones and a transformation to the non-linear flow regime.

For non-Newtonian fluids flowing through a single fracture, such as cement slurry, closed-form analytical solutions exist for a limited number of rheological models, including those of the Power-law fluid, Bingham fluid, and Hershel-Bulkley fluid (Lavrov 2023). However, these solutions are formulated under the assumptions of smooth flat parallel plates, while actual fractures exhibit roughness. Although some semi-analytical expressions have been proposed for non-Newtonian fluid flows, considering fracture roughness, the application is limited to the estimation of the hydraulic aperture; the flow behaviors have not been closely investigated (Gustafson and Stille 1996; Di Federico 1997; Lenci and Di Federico 2020).

On the other hand, attempts have been made to study non-Newtonian fluid flows in a realistic fracture with roughness through three-dimensional numerical simulations and laboratory experiments

(Rodríguez de Castro and Radilla 2016; Rodriguez de Castro and Radilla 2017; Zhang et al. 2019; Zou et al. 2024; Duan et al. 2025). Rodriguez de Castro and Radilla (2016, 2017) conducted laboratory experiments on a non-Newtonian fluid flow in a rough-walled fracture and proposed a method to extend Darcy's law to the flow of yield stress and a Carreau fluid flow in a rough-walled fracture from the rheological properties of the fluid, the hydraulic aperture of the fracture, and the inertial coefficients. Zou et al. (2024) identified flow regimes for Bingham fluid and Hershel-Bulkley fluid in a rough-walled fracture through a numerical simulation using synthetic fractures. The boundaries among these three flow regimes (yield stress dominated regime, viscous stress dominated regime, and transition regime) are quantified by the Reynolds number and Bingham number or Hershel-Bulkley number.

The above studies employed experimental and numerical approaches for identifying non-Newtonian fluid flow regimes and their boundary conditions in realistic fractures with roughness. However, the models were based on a limited number of fracture samples. Thus, there is room for further discussion regarding the conditions under which these flow regimes occur. In particular, the authors believe that the relationship among the flow regimes, fracture roughness, and fluid properties needs greater examination with a simplified fracture geometry.

The aim of this paper is to understand the physical nature of the flow behavior of cement slurry at varying grout thicknesses in a rough-walled fracture using a simplified fracture geometry with a one-dimensional aperture variation. This is achieved by a CFD (computational fluid dynamics) simulation of the grout flow in a sinusoidal fracture shape at varying w/c ratios and degrees of fracture roughness under a wide range of pressure gradients. A detailed analysis is conducted on the interpretation of the simulation results to understand the flow characteristics of the grout flow in a fracture with roughness. The flow regimes and the conditions under which each flow regime occurs are discussed as well as their implications

in terms of grout thickness design for the treatment of dam foundations in Japan.

2 METHODOLOGY

2.1 Model geometry

The cement slurry flow in a single fracture with roughness is simulated through the CFD approach. Previous research has demonstrated that the geometry of fracture surfaces can be represented as a superposition of multiple sinusoidal curves with varying wavelengths and amplitudes (Zou et al. 2015). Accordingly, in studies of groundwater flow through rock fractures, fracture surfaces have often been modeled as sinusoidal curves as a means to simplify complex fracture geometry (Zimmerman and Paluszny 2023).

Following this approach, the authors modeled the fracture surface roughness using sinusoidal curves in an attempt to clarify the influence of fracture roughness on the flow behavior of cement slurry under varying w/c ratios and pressure gradients. This modeling approach allows for systematic investigation of the effects of surface roughness by adjusting the amplitude and wavelength of the sinusoidal curves. Moreover, it enables comparison with previous studies on groundwater flow. The following functions are used to represent the upper and lower shapes of the sinusoidal fracture.

$$Y_s = \delta \cos\left(\frac{2x\pi}{\lambda}\right) \quad (1)$$

$$Y_b = \delta \cos\left(\frac{2x\pi}{\lambda} + \pi\right) \quad (2)$$

where Y_s represents the y-coordinates of the upper boundary, Y_b represents the y-coordinates of the bottom boundary, x represents the x-coordinates, δ represents amplitude, and λ represents the wavelength.

The out of phase configuration of the upper and lower surfaces is chosen to represent the change in aperture due to fracture roughness. To investigate the influence of roughness over a relatively continuous

section, the sinusoidal sections are designed to consist of five waves. The roughness of the fracture is expressed by the ratio of amplitude (δ) to wavelength (λ). The δ/λ is varied at 0.02, 0.06, and 0.1 to represent different degrees of fracture roughness. The range in δ/λ is chosen to represent the range in the joint roughness coefficient (JRC). In this study, the corresponding JRC is calculated from the equation below (Tatone and Grasselli 2010).

$$JRC = 51.85 (Z_2)^{0.60} - 10.37 \quad (3)$$

where Z_2 is the root mean square first deviation, that represents the inclination of profiling line of rock surface, which can be determined from the equation below.

$$Z_2 = \left[\frac{1}{M-1} \sum_{i=1}^{M-1} \left(\frac{\Delta y}{\Delta x} \right)_i^2 \right]^{\frac{1}{2}} \quad (4)$$

where M is the number of sampling points along the profile, Δy is the height difference between two adjacent sampling points, and Δx is the interval of sampling points.

For the calculation of Z_2 , the sampling interval of 0.5 mm is used. From the above equations, the δ/λ of 0.02 corresponds to the JRC of 1.02, the δ/λ of 0.06 corresponds to the JRC of 11.7, and the δ/λ of 0.1 corresponds to the JRC of 19.6. The model geometry for δ/λ of 0.1 is shown in Figure 2. Entrance and exit lengths of 85 mm are assumed at both sides of the sinusoidal section to examine the influence of fracture roughness on the fully developed flow. The mean aperture is kept constant at 1.5 mm. The mesh in the domain consists of 205 elements in the x-direction and 45 elements in the y-direction. The mesh is set to be finer near the top and bottom walls to capture the flow behavior along the crests.

2.2 Simulation parameters

The cement slurry in a single fracture with roughness is simulated with multiple combinations

of key parameters, namely, injection pressure, w/c ratio of the cement slurry, and fracture roughness. A list of the key parameters used in the simulations is given in Table 1. A total of 108 simulations are conducted using the combinations of three w/c ratios, 12 injection pressures, and three fracture geometries. For the fluid properties of the cement slurry, the w/c ratios of 0.8, 1, and 2 are considered. Cement slurries with the w/c ratio of 2 or higher are not considered, as previous studies have shown that their rheological behavior does not differ significantly (Kishi et al. 2018).

For differential pressure, a range in pressure of 0.2 kPa to 140 kPa is considered. During the grouting of a fracture, the cement slurry undergoes a varying pressure gradient. Near the borehole, the cement slurry experiences a high pressure gradient upon injection. As the cement slurry travels within the fracture, the pressure gradient becomes lower and lower until the flow of cement slurry eventually stops. Therefore, in order to understand the cement slurry flow in a rough-walled fracture at varying stages of grouting, the authors decided to consider a total of 12 injection pressures in a low pressure range (0.2 kPa, 0.6 kPa, 1 kPa, and 1.4 kPa), medium pressure range (2 kPa, 6 kPa, 10 kPa, and 14 kPa), and high pressure range (20 kPa, 60 kPa, 100 kPa, and 140 kPa).

2.3 Fluid Properties

The fluid properties of cement slurry with w/c ratios of 0.8, 1, and 2 are determined from the viscosity measurements previously taken by Kishi et al. (2018) on Portland blast-furnace slag cement, which is a type of cement commonly used for the treatment of dam foundations in Japan. In the viscosity measurements, a viscometer was used for the low shear rate range, while a custom test apparatus consisting of parallel acrylic plates was used for the high shear rate range. In the parallel plates flow test, the apparent viscosity for each w/c ratio and injection pressure was calculated by flowing the slurry between parallel plates of varying gap widths.

From these experiments, the relationship between shear rate and shear displacement was established for each w/c ratio. The results indicated that the yield stress increased as the w/c ratio decreased. The yield stress was found to be 3.5 Pa for the w/c ratio of 0.8 and 1.1 Pa for the w/c ratio of 1. For slurries with the w/c ratio of 2 or greater, the yield stress was found to be negligible. Furthermore, the results indicated that the viscosity decreased with increasing w/c ratio, approaching the viscosity of water.

Previous studies have reported that cement slurry can be approximated as either a Newtonian fluid or a Bingham fluid. Bingham fluid is the fluid that is mobilized once the shear stress exceeds the yield stress, and is one of the commonly used models to represent cement slurry in the studies of rock grouting (Gustafson and Stille 1996; Rahman et al. 2015; Funchag and Thörn 2018; Shamu and Håkansson 2019; Zou et al. 2020). Based on the relationship between shear rate and shear stress established from experiments, the slurry with the w/c ratio of 2 is modeled as a Newtonian fluid due to its negligible yield stress, while slurries with the w/c ratios of 0.8 and 1 are modeled as Bingham fluids due to the presence of yield stress. The viscosity, density, and yield stress values used in the models for w/c ratios of 0.8, 1, and 2 are summarized in Table 2.

Since the numerical simulation becomes unstable when directly incorporating the Bingham model, the cement slurry with w/c ratios of 0.8 and 1 is modelled by following the bi-viscous Bingham model.

$$v = \min\left(v_o, \frac{\tau_0}{\dot{\gamma}}\right) \quad (5)$$

where v is the kinematic viscosity, v_o is the yielding kinematic viscosity, and $\dot{\gamma}$ is the shear rate.

The yielding kinematic viscosity represents the kinematic viscosity before yielding. In order to

approximate Bingham fluid, it is considered desirable to make the yielding kinematic viscosity of bi-viscous Bingham model as large as possible. For this simulation, the yielding kinematic viscosity is set to be 1000 times the kinematic viscosity to express the immobile state of Bingham fluid, after confirming that this setting yields results consistent with the analytical solution discussed in section 2.5.

2.4 Governing equations

The opensource CFD software OpenFOAM is used to run the simulations (Weller et al. 1998). Each simulation is performed in two dimensions with a finite volume method. The flow is introduced by differential pressure by setting a specified pressure at the inlet and 0 kPa at the outlet. The no-slip boundary condition is used for the upper and lower walls. The governing equations are 2D Navier stokes equations for incompressible flow. The following continuity and momentum equations are solved:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \quad (6)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \quad (7)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (8)$$

where u and v are velocities in the x and y directions, respectively, p is pressure, ρ is density, μ is the kinematics viscosity, and t is time. For the pressure-velocity solver, the PISO (pressure implicit with splitting operators) algorithm is used (Issa 1986).

2.5 Validation of proposed method

To verify the validity of the proposed method, a simulation of Bingham fluid flow between parallel plates is conducted. The objectives of this validation are to confirm the accuracy of the CFD simulation and to evaluate whether the bi-viscosity Bingham model appropriately represents Bingham fluid

behavior.

The analysis domain consists of two-dimensional parallel plates with an opening of 1.5 mm and a length of 90 mm. The mesh in the domain consists of 1215 elements in the x-direction and 45 elements in the y-direction. Cement slurry with the w/c ratio of 1 is introduced from one end of the domain under a pressure difference of 1 kPa. No-slip boundary conditions are applied at the top and bottom boundaries. The fluid properties of the slurry used in the simulation are listed in Table 2. The simulation is conducted under steady-state conditions.

Figure 3 shows a comparison between the simulated velocity profile and the analytically derived velocity profile assuming Bingham fluid flow between the parallel plates at a constant aperture of 1.5 mm under no-slip conditions. As shown in the figure, the simulation results closely match the analytical solution, confirming the validity of the CFD method. Additionally, the presence of a plug flow region—a characteristic feature of Bingham fluids—is accurately captured, indicating that the bi-viscosity model is appropriate for representing Bingham fluid behavior.

3 RESULTS

3.1 Reynolds number

Table 3 summarizes the Reynolds numbers for all cases. The injection pressures of 0.2 kPa to 140 kPa are labelled as P1 ~ P12 in ascending order. These labels are used to describe the injection pressure for the subsequent analysis. Some of the simulation results for the w/c ratios of 2 are not presented because the solutions did not converge within the maximum number of iterations.

The Reynolds number varies by the w/c ratio and δ/λ . As shown in Table 3, the Reynolds number

is in the range of $10^2 \sim 10^4$ for the w/c ratio of 2, $10^2 \sim 10^3$ for the w/c ratio of 1, and $10^3 \sim 10^3$ for the w/c ratio of 0.8 under the pressure gradient considered in the simulations. Cement slurry with lower w/c ratios flows more slowly due to greater cohesion. Among the cases with the same w/c ratio and injection pressure, the Reynolds number is smaller for a higher δ/λ . This decrease in the Reynolds number may be due to the loss of velocity due to the disturbance of the flow caused by the abrupt change in the shape of the fracture.

3.2 Friction factor

Figure 4 presents plots of the relationship between the friction factor and the Reynolds number for each δ/λ . The friction factor across the sinusoidal fracture is calculated by the Darcy-Weisbach equation.

$$f = - \frac{\frac{dp}{dx}}{\frac{1}{d} \left(\frac{1}{2} \rho U_0^2 \right)} \quad (9)$$

where $\frac{dp}{dx}$ is the pressure gradient across the sinusoidal fracture, d is the hydraulic diameter that is two times the fracture aperture (White 2003), ρ is the fluid density, and U_0 is the average velocity of the cross section. The relative error is calculated from the following equation:

$$E = \frac{f_{HP} - f}{f_{HP}} \quad (10)$$

where f_{HP} is the friction factor assuming the Hagen-Poiseuille flow.

The deviation from the Hagen-Poiseuille flow with increasing Reynolds number indicates the transition from laminar flow to non-linear flow. The critical Reynolds number is determined as the point where the friction factor deviates 25% from the Hagen-Poiseuille flow or when the friction factor increases with an increasing Reynolds number. The critical Reynolds number is 2×10^3 for the δ/λ of 0.02. This is approximately the same as the critical Reynolds number between the parallel plates, which is around $2 \sim 3 \times 10^3$. The critical Reynolds number is 1×10^3 for the δ/λ of 0.06 and 2×10^2 for the δ/λ of 0.1. The critical

Reynolds number decreases with increasing roughness. These observations indicate that roughness is the controlling factor of the critical Reynolds number. Additionally, it shows that the δ/λ of 0.02 has little effect on reducing the critical Reynolds number in comparison to the parallel plate flow. This observation is consistent with previous research that considers groundwater flow in a sinusoidal fracture, which showed that non-linearity is increasingly introduced with an increasing δ/λ (Brush and Thomson 2003; Mgaya et al. 2006).

It is observed that the friction factor for the w/c ratios of 0.8 and 1 deviates from Hagen-Poiseuille flow at a Reynolds number of around 10 in all plots. This deviation of the friction factor at low Reynolds numbers for the w/c ratios of 0.8 and 1 indicates that the flow regime is yield-stress-dominated. This flow behavior is characteristic of Bingham fluids; therefore, this trend is not observed for the w/c ratio of 2.

The results also revealed a clear dependency of the flow regimes on the w/c ratio because cement slurry with lower w/c ratios flows more slowly due to greater cohesion. For the δ/λ of 0.02, the cement slurry with the w/c ratio of 2 is mostly in the laminar flow regime to non-linear flow regime, the cement slurry with the w/c ratio of 0.8 is mostly in the yield-stress dominated regime to the laminar flow regime, and the cement slurry with the w/c ratio of 1 overlaps all flow regimes depending on the Reynolds number.

At the same Reynolds number, the w/c ratio has little effect on the friction factor. At the same Reynolds number, it is known that the friction factor of the Bingham plastic flow is greater at a higher Hedstrom number, which is the ratio of inertial force to viscous force (Tada et al. 1992).

$$He = \frac{D^2 \rho \tau_0}{\mu^2} \quad (11)$$

where D is the fracture aperture, ρ is the fluid density, τ_0 is the yield stress, and μ is the viscosity.

For the fluid properties considered in the simulation, the Hedstrom number is 149 for the w/c ratio of 1 and

197 for the w/c ratio of 0.8. Since the difference in the Hedstrom numbers is small, the w/c ratio has a negligible effect on the friction factor at the same Reynolds number in this simulation.

3.3 Velocity profiles and wall shear stress

Figures 5 and 6 show the streamlines of the middle section of the sinusoidal fracture ($\delta/\lambda=0.1$) and the velocity distributions at the center of the section for P5, P7, and P9 when the w/c ratios are 0.8 and 1. The velocity profiles are compared with the analytically derived velocity profiles assuming Bingham fluid flow between the parallel plates at a constant aperture of 1.5 mm under no-slip conditions.

For both w/c ratios, the velocity profiles show a deviation from the parabolic profile with an increasing Reynolds number. For the w/c ratio of 0.8, the velocity profile shows a negative value near the walls for the δ/λ of 0.1 in P7, and the δ/λ of 0.06 and 0.1 in P9. The cement slurry with the w/c ratio of 1 presents the same trend, except that the velocity distribution presents a larger deviation from the idealized parabolic flow in P9. At this pressure, the flow transforms to non-linear flow. Therefore, the velocity becomes smaller than the parabolic assumptions due to the loss of energy from the non-laminar flow disturbance.

The two important observations from the velocity profiles are 1) the formation of recirculation zones, and 2) the transformation to the non-linear flow regime. These observations are consistent with the previous studies on the Newtonian fluid flow in a rough-walled fracture. For example, the formation of recirculation zones in a rough-walled fracture is reported to be caused by the abrupt change in fracture shape (Lee et al. 2014). This can be explained by the theory of flow separation, which occurs when the pressure gradient becomes positive in the flow direction. When the fluid flows through a diverging channel, the fluid tries to compensate for the increase in energy due to pressure by decreasing the velocity of the fluid until flow separation occurs at the walls. This causes the formation of recirculation zones at the crests.

The flow separation at the walls is further examined with the wall shear stress. The wall shear stress at the top wall of the fracture ($\delta/\lambda=0.01$) at P7 for the w/c ratio of 1 and the corresponding velocity profile are shown in Figure 7. In the velocity profile plots, the analytically derived velocity profile assuming Bingham fluid flow between parallel plates is shown as a dotted line. The wall shear stress decreases towards the crests of the wave and increases towards the troughs. The point where the wall stress becomes negative is the separation point, and the point where the wall stress recovers from a negative value is the reattachment point. The region between these points is defined as the region of flow separation, with the formation of recirculation zones. As can be seen in the velocity profiles in Figure 7, negative x-velocity is observed at the walls in this flow separation region.

The x-components of the wall shear stress are plotted against the x-coordinates for the top wall of the middle section for the w/c ratio of 1 at P1 ~ P7 (Figure 8). When δ/λ is 0.02, the wall shear stress maintains a positive value, indicating no flow separation. When δ/λ is 0.06 or 0.1, the wall shear stress maintains a positive value at lower pressure, and begins to show a negative value as the pressure increases, suggesting the occurrence of flow separation and the formation of recirculation zones. This indicates that the formation of recirculation zones depends on the Reynolds number and the roughness of the fracture, especially the asperity angle of the fracture. With an increasing Reynolds number and increasing asperity angle, the length of these recirculation zones will increase.

Figure 9 shows the percentage of the area with the formation of recirculation zones in a sinusoidal fracture within the range of Reynolds numbers of 0~ 500. Similar to the wall shear stress, it shows that the region of recirculation zones increases with an increasing Reynolds number and increased roughness. When δ/λ is 0.02, recirculation zones do not develop at any Reynolds number. When δ/λ is 0.06, recirculation

zones start to develop at the Reynolds number of 250, and when δ/λ is 0.1, recirculation zones start to develop at the Reynolds number of 100. The plots show no significant difference between the w/c ratios, indicating that the formation of recirculation zones depends on the fracture roughness and Reynolds number. For a rough-walled fracture with a larger asperity angle, recirculation zones form at lower Reynolds numbers with increasing roughness, while for a smooth fracture with a lower asperity angle, recirculation zones would not form at any Reynolds number.

Figure 10 provides an enlarged image of the flow behavior at the crests for the w/c ratio of 1 at three pressures of P5, P7, and P9. The velocity in the negative x-direction is shown in black. When δ/λ is 0.02, the x-velocity is mostly in the positive x-direction, except for P9, where the flow regime transforms to the non-linear flow regime. For the δ/λ of 0.06 and 0.1, the region of negative x-velocity increases with the Reynolds number, which indicates the formation of recirculation zones. At P7, there is a clear line of separation between the main flow zone and the recirculation zone for the δ/λ of 0.06 and 0.1, and the interaction between these two zones is inactive. This indicates that the cement slurry preferentially flows within the main flow zone rather than the crests of the fracture in this flow regime. The line of separation is less clear at a higher pressure, and there is more active interaction between the main flow zone and the recirculation zone.

3.4 Flow rate

Figure 11 presents plots for the pressure gradient and discharge by the w/c ratio. The pressure gradient is the differential pressure across the whole domain per unit length. The black dotted line indicates the analytical solution assuming the Newtonian fluid flow between the parallel plates. These plots demonstrate the flow regimes by pressure gradient for each w/c ratio.

For the w/c ratio of 0.8, the discharge follows the analytical solution, except for the pressure gradients below 5×10^4 Pa/m and above 3×10^5 Pa/m. The deviation for the pressure gradient below 5×10^4 Pa/m is due to the Bingham characteristics of the fluid; the deviation occurs at a lower pressure gradient when the fluid regime is yield-stress dominated. The deviation for the pressure gradient above 3×10^5 Pa/m is due to the transformation to the non-linear laminar flow regime.

For the w/c ratio of 1, the discharge follows the analytical solution, except for the pressure gradients below 1×10^4 Pa/m and above 3×10^5 Pa/m. Similar to the case of the w/c ratio of 0.8, the deviation at lower pressure gradients is due to yield stress, and the deviation at higher pressure gradients is due to the transition from the laminar flow regime to the non-linear flow regime.

Lastly, for the w/c ratio of 2, the deviation from the analytical solution is observed at 2×10^4 Pa/m due to the transformation to the non-linear flow regime. Overall, the plots demonstrate the dependency of the flow regime on the w/c ratio and pressure gradient. Also, the results reveal that the discharge would be significantly lower than the analytical solution when the flow regime is yield-stress-dominated or in the non-linear flow regime.

4 DISCUSSIONS

4.1 Flow regimes

The authors identify four regimes for the cement slurry flow in a sinusoidal fracture. The flow transforms from Step 1 yield stress-dominated flow, to Step 2 laminar flow regime, to Step 3 laminar flow regime with recirculation zones, and to Step 4 non-linear flow regime. Although not considered in this paper, the flow would transform to turbulent flow with the further increase in pressure. The schematics of the flow regimes of the cement slurry at the transition from laminar to non-linear flow is shown in Figure 12.

Step 1 is the yield stress-dominated regime. This flow regime is observed for the low pressure gradient range with the w/c ratios of 0.8 and 1. In this flow regime, the lower flow rate is observed as compared to the analytical solution due to the influence of the yield stress. Step 2 is the laminar flow regime. In this flow regime, the cement slurry flows along the crests and troughs of the sinusoidal fracture without the formation of recirculation zones. With an increasing Reynolds number, the flow transforms to Step 3, which is the laminar flow regime with recirculation zones. This flow regime only occurs in a rough-walled fracture with an abrupt change in the fracture shape. Among the δ/λ considered in these simulations, the recirculation zones develop with the δ/λ of 0.06 and 0.1. With increasing roughness, recirculation zones would form at lower Reynolds numbers. In this regime, the cement slurry would preferentially flow at the center of the fracture, and would be less likely to penetrate the crests due to the formation of these recirculation zones.

With the further increase in the Reynolds number, a disturbance is introduced into the flow, until eventually the flow transforms to Step 4, the non-linear flow. In this flow regime, the flow rate is significantly reduced from the parallel plate flow rate due to the loss of energy brought about by the disturbance. The transition from linear to non-linear flow would also occur at lower Reynolds numbers with increasing roughness.

The condition under which the flow regimes transform is mainly a function of fracture geometry and the Reynolds number. In these simulations, the w/c ratio has a negligible effect on the flow behavior other than the Reynolds number, especially once the flow is in the laminar flow regime. The flow regime transforms to the next flow regime at a lower Reynolds number with an increasing δ/λ . When δ/λ is 0.02, no recirculation zones form and the flow regime transforms directly from Step 2 laminar flow regime to Step 4 non-linear flow regime at Reynolds numbers of 2000 ~ 3000. When δ/λ is 0.06, the flow transforms

to Step 3 laminar flow regime with recirculation zones forming at the Reynolds number of 250, and to Step 4 non-linear flow regime at the Reynolds number of 1000. When δ/λ is 0.1, the flow transforms to Step 2 at the Reynolds number of 50 and to Step 3 at the Reynolds number of 200.

4.2 Implications of rock grouting

Understanding the flow regimes of cement slurry flow is important for rock grouting design. The two important design parameters of rock grouting are penetration length and sealing effectiveness.

The flow regime and fracture roughness have significant effects on the penetration length of cement slurry. In the yield stress-dominated regime, the penetration length would be shorter than that assumed for the parallel plates. In the laminar flow regime, it was observed that the flow rate would be reduced to 0.8 of the parallel plate flow with increasing roughness. In the non-linear flow regime, the penetration length would be 0.3~0.5 of that assumed for the parallel plates. In a rough-walled fracture, cement slurry flow is likely to transform to the non-linear flow regime at lower Reynolds numbers. It is important to be aware of this difference when predicting the penetration length of rock grouting.

The sealing effectiveness of cement slurry is also influenced by fracture roughness and the flow regime. It is more difficult to seal a rough-walled fracture than a smooth fracture due to its irregular shape. Particularly in the Step 2 flow regime with recirculation zones, the flow would form recirculation zones at crests that are separated from the main flow zone. Although the simulation is based on the assumption of a one-dimensional single-phase flow, the results indicate that cement slurry is less likely to flow through recirculation zones and potentially leaves gaps in the fracture.

Based on the simulations, recirculation zones would form at Reynolds numbers of 50 ~ 250 when δ/λ is greater than 0.06. Thus, one way of avoiding the formation of these recirculation zones is by controlling the Reynolds number. However, the Reynolds number changes as the grout travels within the rock mass. Upon injection, the Reynolds number of cement slurry is at its maximum near the borehole and decreases with increasing distance from it. Therefore, it is not practical to control the injection pressure based on the Reynolds number for the purpose of preventing the formation of recirculation zones.

Alternatively, a combination of multiple thin-to-thick mixtures may be effective for preventing the formation of recirculation zones. The results show that cement slurry with different thicknesses would flow at different Reynolds numbers under the same pressure gradient. Therefore, with a combination of multiple mixtures, it is possible to compensate for the formation of recirculation zones resulting from single mix usage. For example, with high injection pressure, thinner grout would be in the non-linear flow regime and seal the crests, while thicker grout would be in the laminar flow regime and seal the main flow zone in the vicinity of the borehole. The results also show that cement slurry would flow in the same way when the w/c ratio is above 2; thus, the use of multiple w/c ratios above 2 would be ineffective in terms of fracture sealing.

4.3 Limitations

The proposed flow regimes and their boundaries are based on simulations that assume a simplified sinusoidal fracture geometry with a one-dimensional aperture variation. Although the simplification of the fracture geometry was effective for revealing the flow characteristics of the cement slurry flow in a rough-walled fracture, further research will need to be conducted to extend the understanding to three-dimensional grout flow behavior in a rough-walled fracture, including flow channeling and tortuosity, as reported in previous studies on groundwater flow (Wang et al. 2020; Tan et al. 2020; Rong et al. 2020; Sun et al. 2024).

Moreover, the simulations assumed that the rheological properties of the cement slurry depend on the w/c ratio, whereas in practice, the rheology may also be influenced by other factors such as additives or ambient temperature. (Stille et al. 2012; Zhang et al. 2022). Although the authors believe that these simulations provide sufficient insight into the influence of grout thickness on the flow behaviors in a rough-walled fracture, the model should be tested with a wider range of fluid properties in future research.

5 CONCLUSIONS

In this paper, CFD simulations of grout flow in a sinusoidal fracture were conducted to understand the physical nature of grout flow behaviors in a rough-walled fracture. The w/c ratio and fracture roughness, as well as the injection pressure, were varied in the simulations. From the simulations, the following findings were obtained.

- 1) At the same differential pressure, cement slurry with lower w/c ratios flowed at lower Reynolds numbers due to greater cohesion. In the range of differential pressure between 0.2 kPa and 140 kPa, the Reynolds number was found to be in the range of $10^2 \sim 10^4$ for the w/c ratio of 2, $10^{-2} \sim 10^3$ for the w/c ratio of 1, and $10^{-3} \sim 10^3$ for the w/c ratio of 0.8.
- 2) Four flow regimes were identified for the cement slurry flow in a sinusoidal fracture. With increasing pressure, the flow regime was seen to transform from Step 1 yield stress-dominated flow, to Step 2 laminar flow, to Step 3 laminar flow with recirculation zones, and to Step 4 non-linear flow. The transformation between each step was seen to occur at lower Reynold numbers with increasing roughness. Due to the difference in the Reynolds numbers depending on the w/c ratio, the grout with lower w/c ratios was more likely to stay in the laminar flow regime, while the grout with higher w/c ratios was likely to stay in the non-linear flow regime.
- 3) Our findings for the flow regimes suggest the possibility of insufficient sealing between the asperities

of rough-walled fractures upon the formation of recirculation zones under a certain range in Reynolds numbers. This may occur when δ/λ is greater than 0.06, and when the Reynolds number is in the order of $10^1 \sim 10^2$. This implies that the grout thickness and injection pressure should be optimized, based on the fracture roughness, to achieve the complete sealing of a rough-walled fracture. The Reynolds number might be controlled by adjusting the grout thickness and injection pressure, or by using multiple thin-to-thick mixtures of grout to compensate for the formation of recirculation zones that may occur from single mix usage.

Rock grouting has typically been designed based on the penetration length; therefore, less attention has been paid to the quality of the sealing of the rock fractures. The simulation results in this study were able to show the possibility that the quality of grouting in rough-walled rock fractures may be controlled by the flow regimes. In future research, with the extension of this study to three-dimensional fracture morphology, the authors will endeavor to develop a method for choosing the optimal grout thickness based on fracture morphology in order to achieve the high quality sealing of rock fractures.

FUNDING

The authors would like to express their appreciation for the funding received by JSPS 24K07657 “Development of a coupled simulation to reveal the filling and hardening behavior of cement slurry in rock grouting” and the funding by JFE Engineering.

DECLARATION OF COMPETING INTEREST

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Table 1 Simulation parameters

Parameters	Values
w/c ratio	0.8, 1, 2
δ/λ	0.02, 0.06, 0.1
Differential pressure (kPa)	Low pressure range: 0.2, 0.6, 1, 1.4 Medium pressure range: 2, 6, 10, 14 High pressure range: 20, 60, 100, 140

Table 2 Rheological properties of cement slurry

	w/c ratio = 2	w/c ratio = 1	w/c ratio = 0.8
Model	Newtonian	Bingham	Bingham
Viscosity (Pa · s)	0.0021	0.005	0.008
Yield stress (Pa)	0	1.1	3.5
Density (kg/m ³)	1250	1500	1600

Table 3 Reynolds numbers for all cases

		Differential pressure (kPa)											
		(P1)	(P2)	(P3)	(P4)	(P5)	(P6)	(P7)	(P8)	(P9)	(P10)	(P11)	(P12)
		0.2	0.6	1	1.4	2	6	10	14	20	60	100	140
w/c ratio = 2	$\delta/\lambda = 0.02$	86.3	259	432	604	866	1722	1967	2122	2183	3426	4551	7539
	$\delta/\lambda = 0.06$	85.4	255	424	593	851	1621	1841	1986	2210	-	-	-
	$\delta/\lambda = 0.1$	83.3	247	408	569	772	1500	1750	1932	2075	-	-	-
w/c ratio = 1	$\delta/\lambda = 0.02$	0.02	17.0	51.4	87.6	148	535	957	1594	1638	2142	2322	2680
	$\delta/\lambda = 0.06$	0.02	16.8	46.0	77.8	142	525	938	1441	1478	2063	2265	2502
	$\delta/\lambda = 0.1$	0.02	16.3	45.7	79.1	133	503	867	1154	1407	1947	2235	2449
w/c ratio = 0.8	$\delta/\lambda = 0.02$	0.008	0.02	0.95	7.19	18.9	126	340	496	730	1746	1896	1997
	$\delta/\lambda = 0.06$	0.008	0.02	0.84	6.93	18.9	122	322	487	715	1590	1874	1978
	$\delta/\lambda = 0.1$	0.008	0.02	0.73	6.59	18.5	119	314	467	681	1480	1787	1833

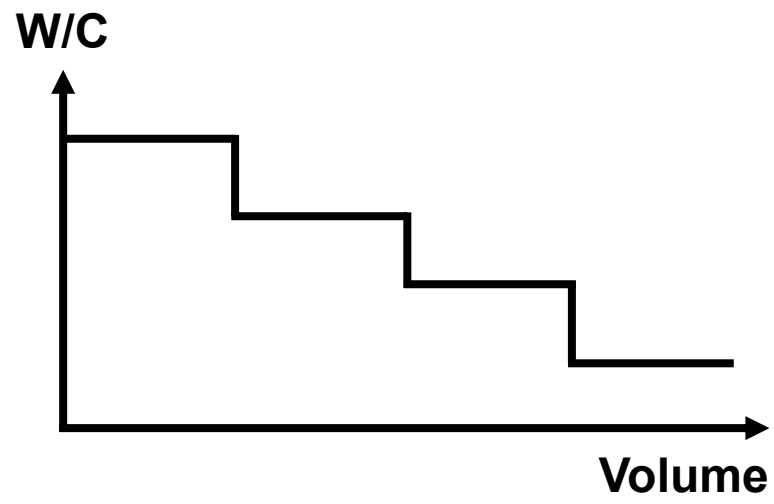


Figure 1. Schematics of thin-to-thick grout procedure

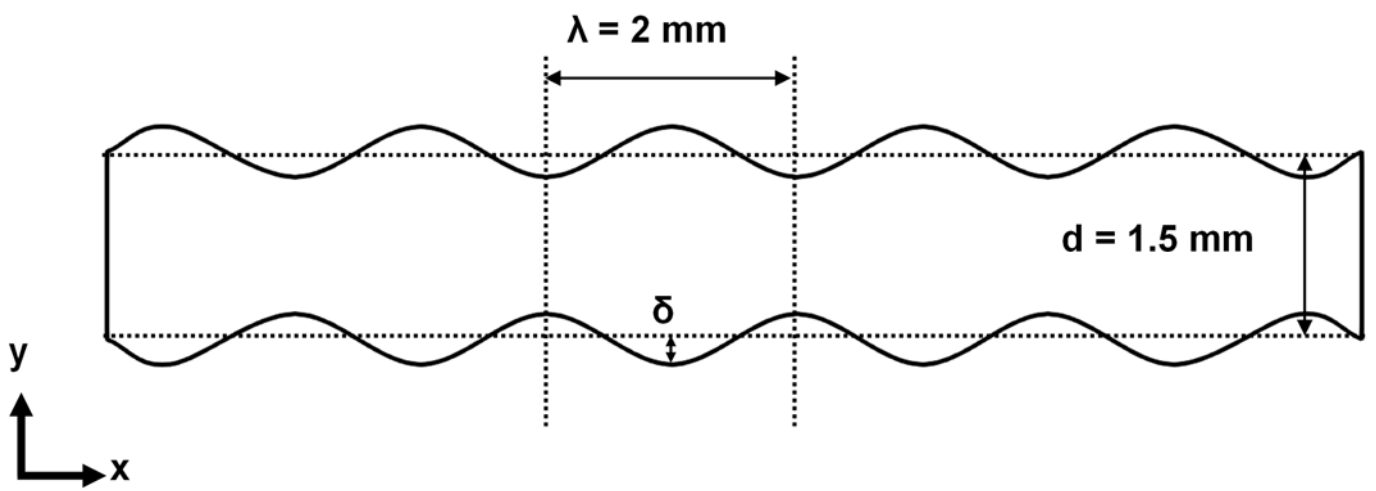


Figure 2. Model geometry of sinusoidal fracture ($\delta/\lambda=0.1$)

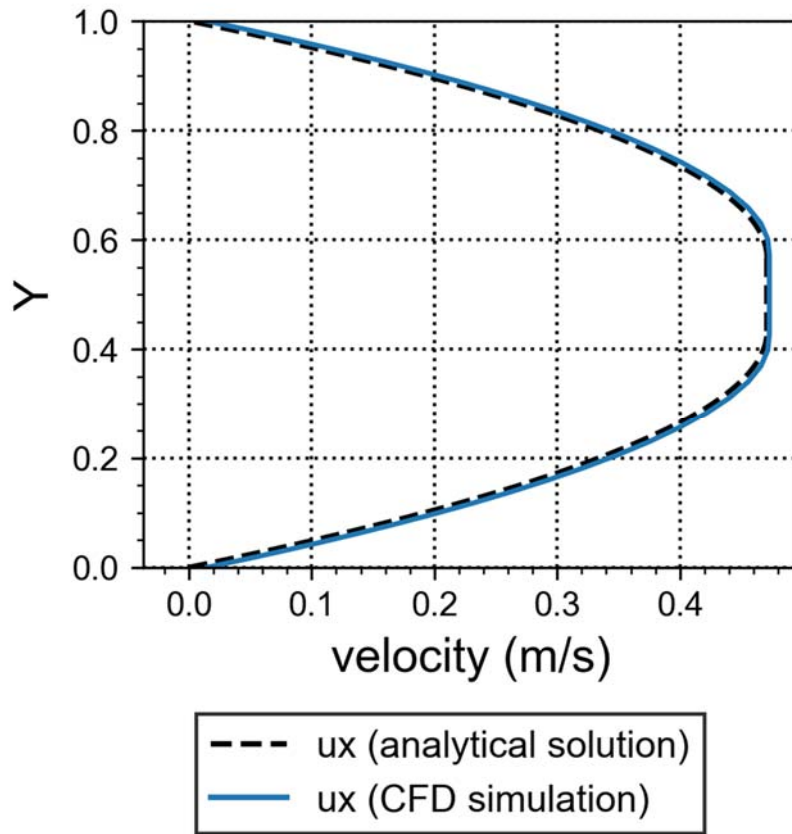


Figure 3. Comparison of velocity profiles from the analytical solution and CFD analysis

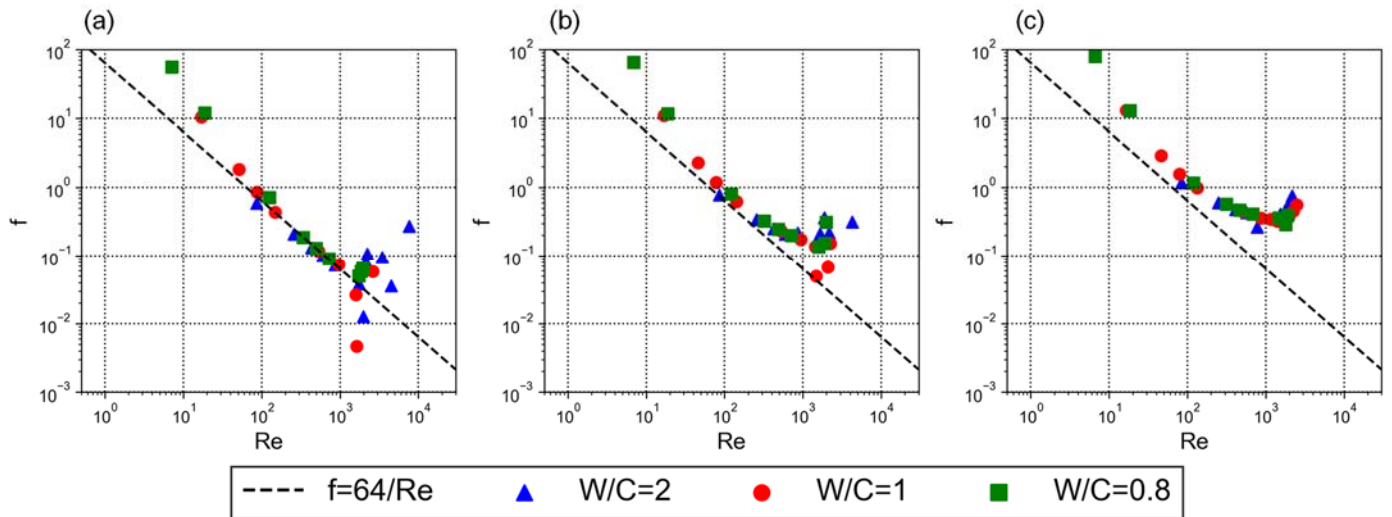


Figure 4. Relationship between friction factor and Reynolds number by δ/λ

(a) $\delta/\lambda=0.02$, (b) $\delta/\lambda=0.06$, and (c) $\delta/\lambda=0.1$

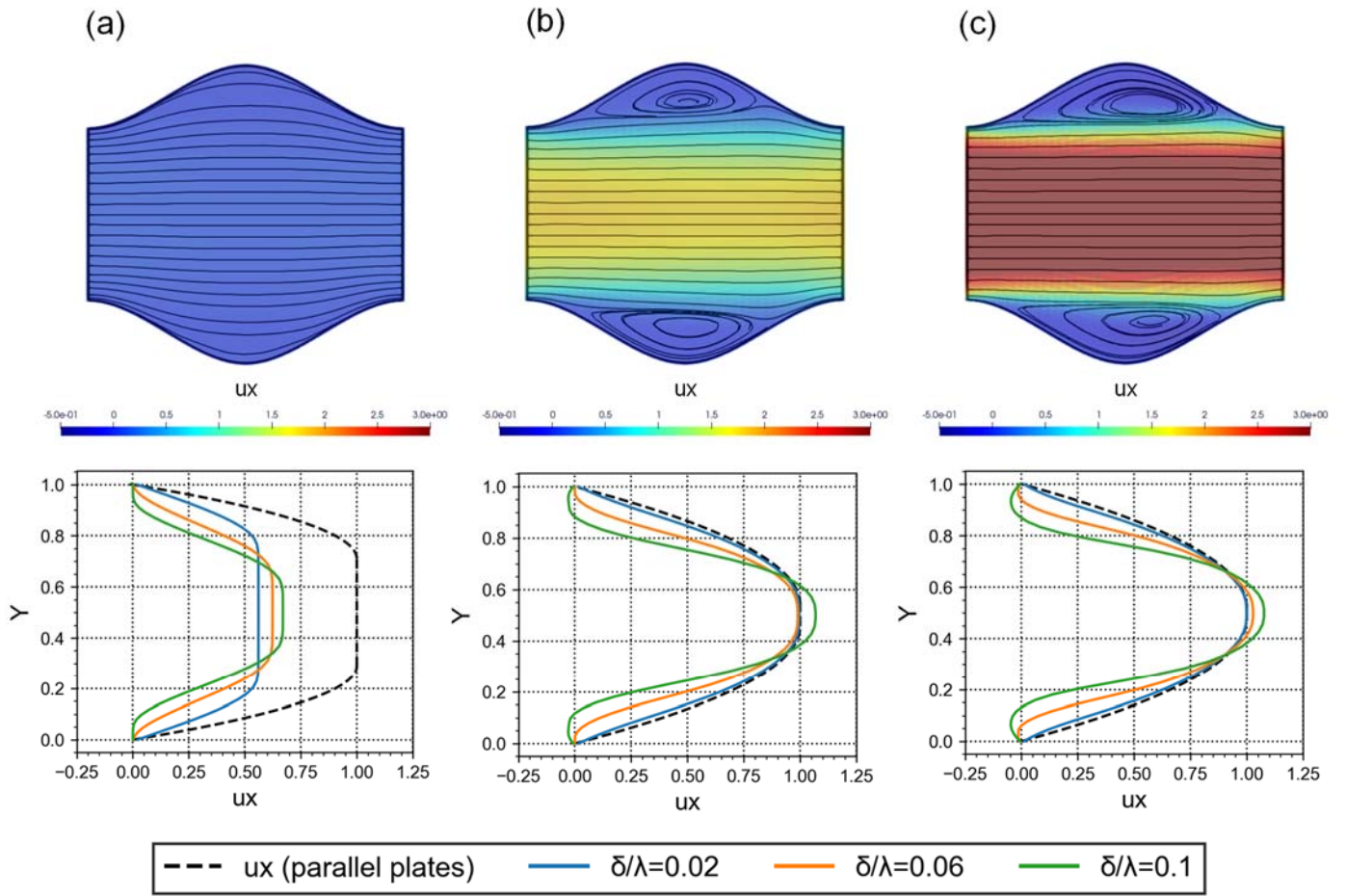


Figure 5. Velocity profiles and streamlines of middle section for w/c ratio of 0.8 and δ/λ of 0.1 at three different pressures: (a) P5, (b) P7, and (c) P9

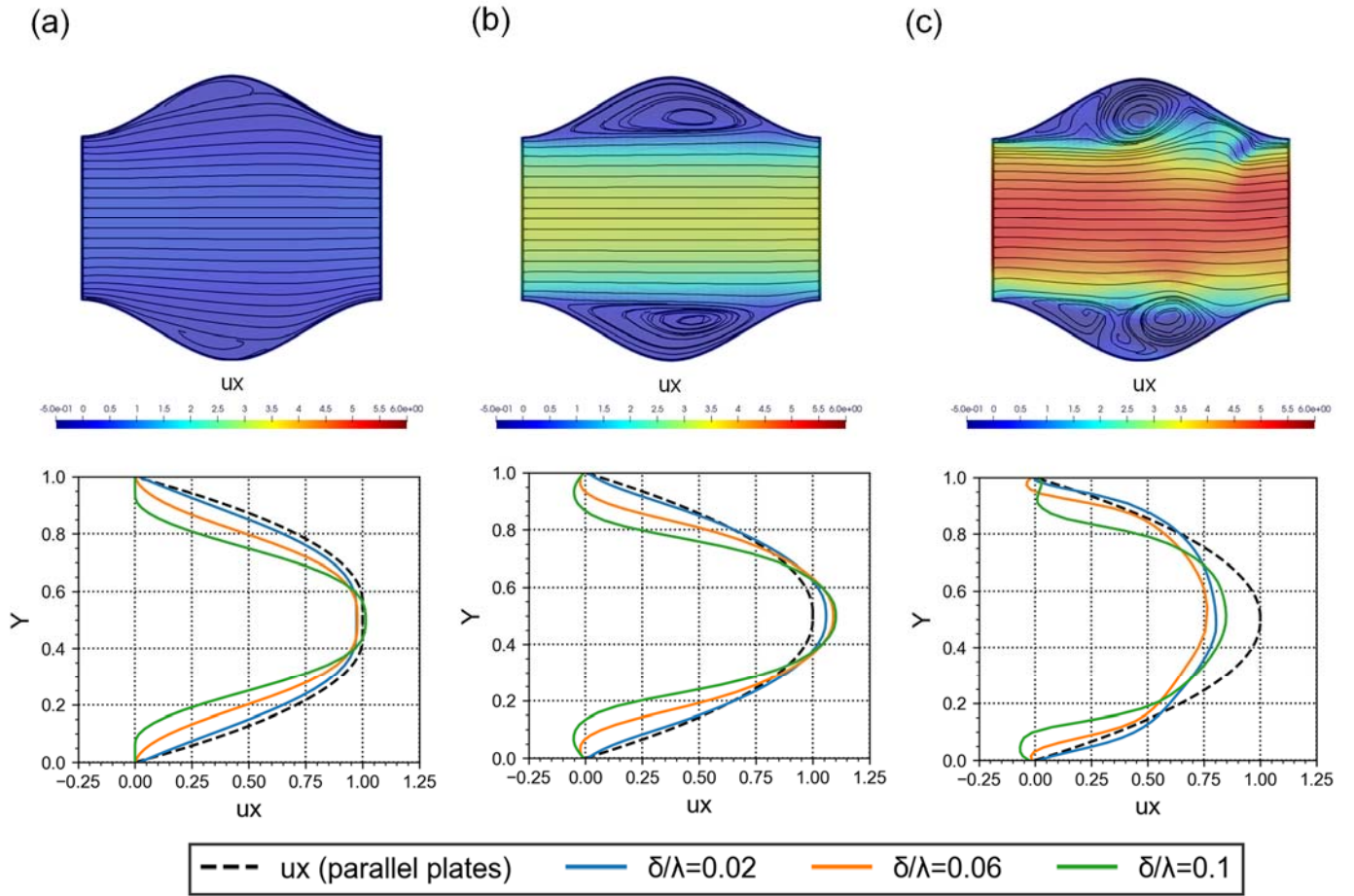


Figure 6. Velocity profiles and streamlines of middle section for w/c ratio of 1 and δ/λ of 0.1 at three different pressures: (a) P5, (b) P7, and (c) P9

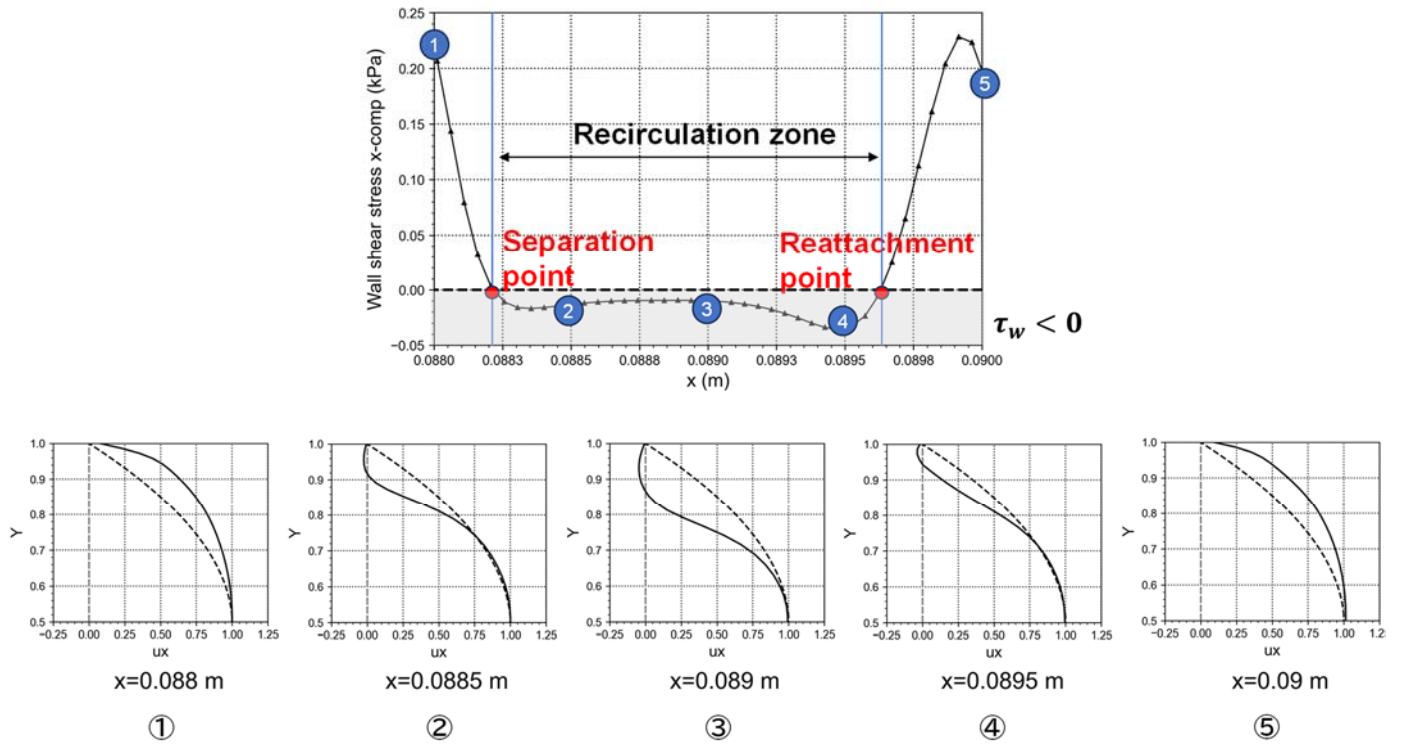


Figure 7. Relationship between wall shear stress and velocity profiles

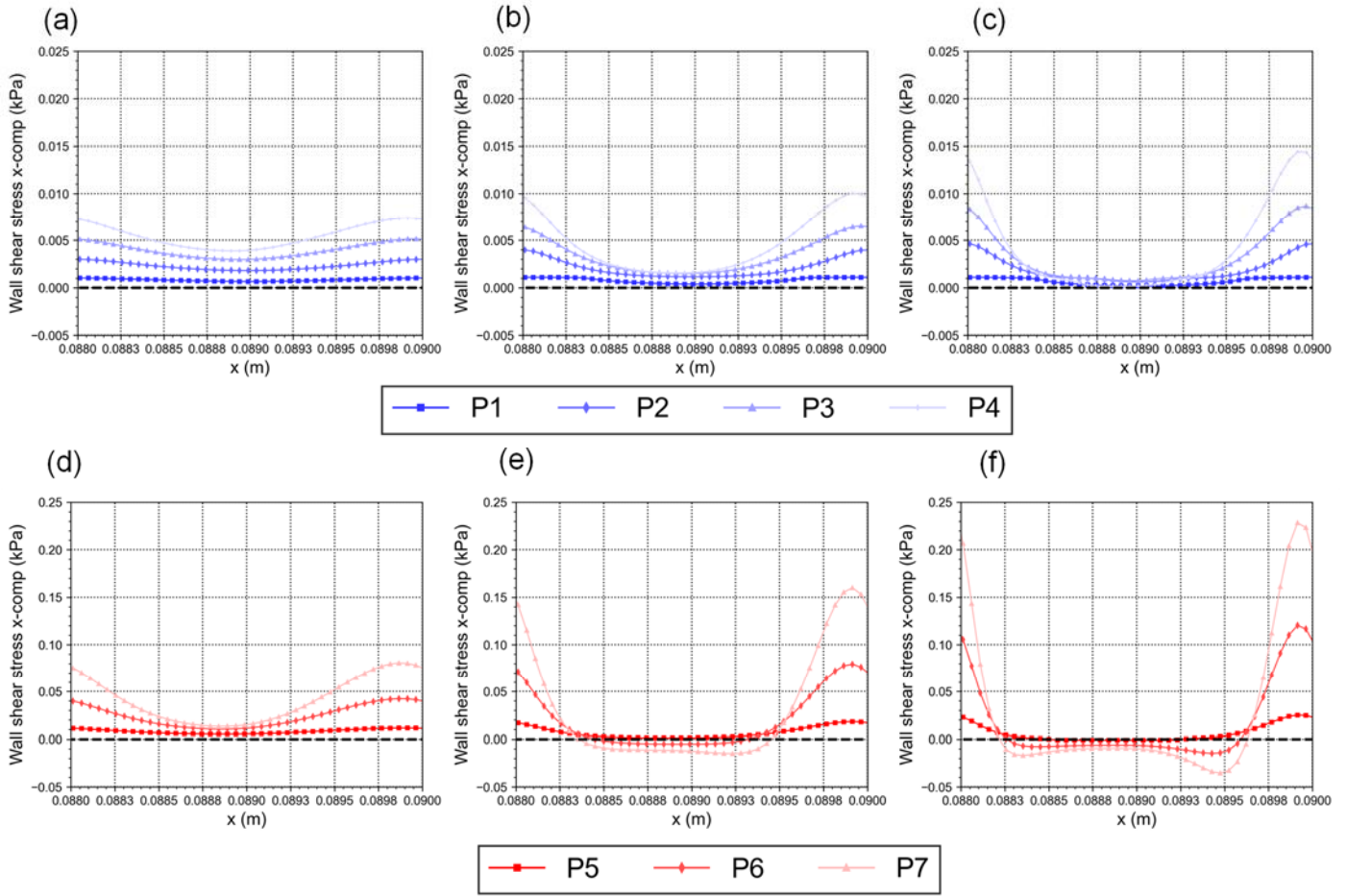


Figure 8. Wall shear stress of top wall for w/c ratio of 1 at different geometries and pressure ranges:

- (a) $\delta / \lambda = 0.02$, low pressure range; (b) $\delta / \lambda = 0.06$, low pressure range; (c) $\delta / \lambda = 0.1$, low pressure range; (d) $\delta / \lambda = 0.02$, medium pressure range; (e) $\delta / \lambda = 0.06$, medium pressure range; and (f) $\delta / \lambda = 0.1$, medium pressure range

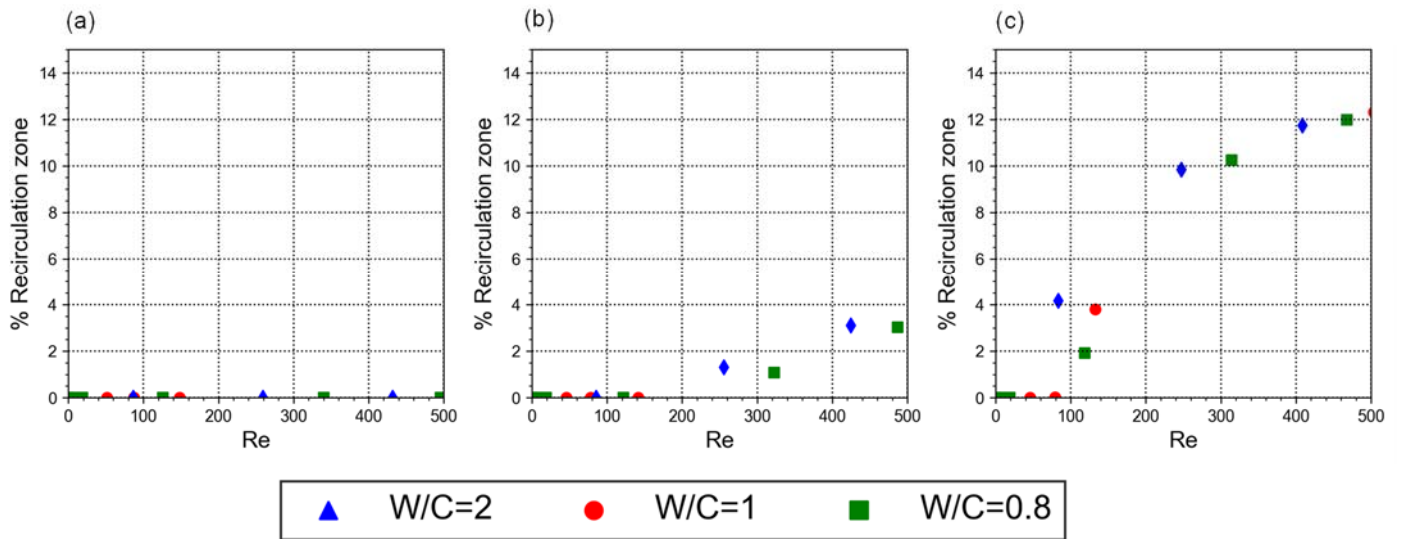


Figure 9. Percentage of area with formation of recirculation zones by δ/λ : (a) $\delta / \lambda = 0.02$, (b) $\delta / \lambda = 0.06$, and (c) $\delta / \lambda = 0.1$

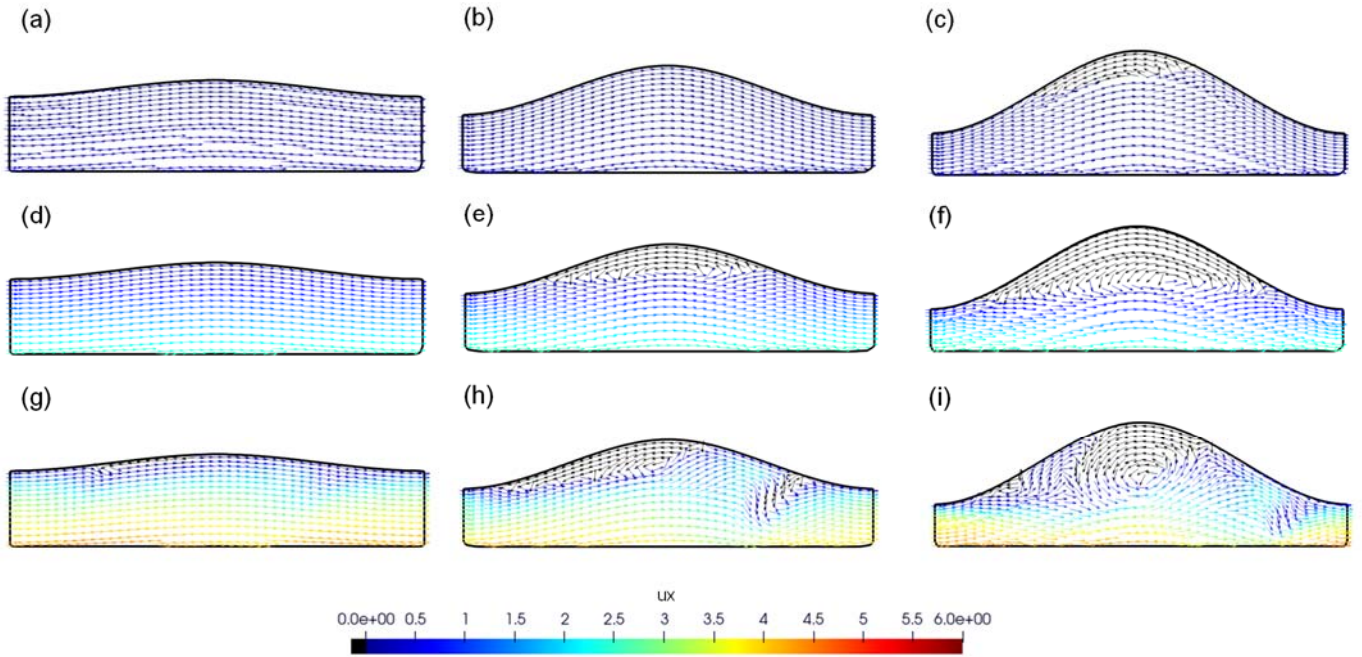


Figure 10. Velocity profiles at crests for w/c ratio of 1: (a) $\delta/\lambda=0.02$, P5; (b) $\delta/\lambda=0.06$, P5; (c) $\delta/\lambda=0.1$, P5; (d) $\delta/\lambda=0.02$, P7; (e) $\delta/\lambda=0.06$, P7; (f) $\delta/\lambda=0.1$, P7; (g) $\delta/\lambda=0.02$, P9; (h) $\delta/\lambda=0.06$, P9; and (i) $\delta/\lambda=0.1$, P9

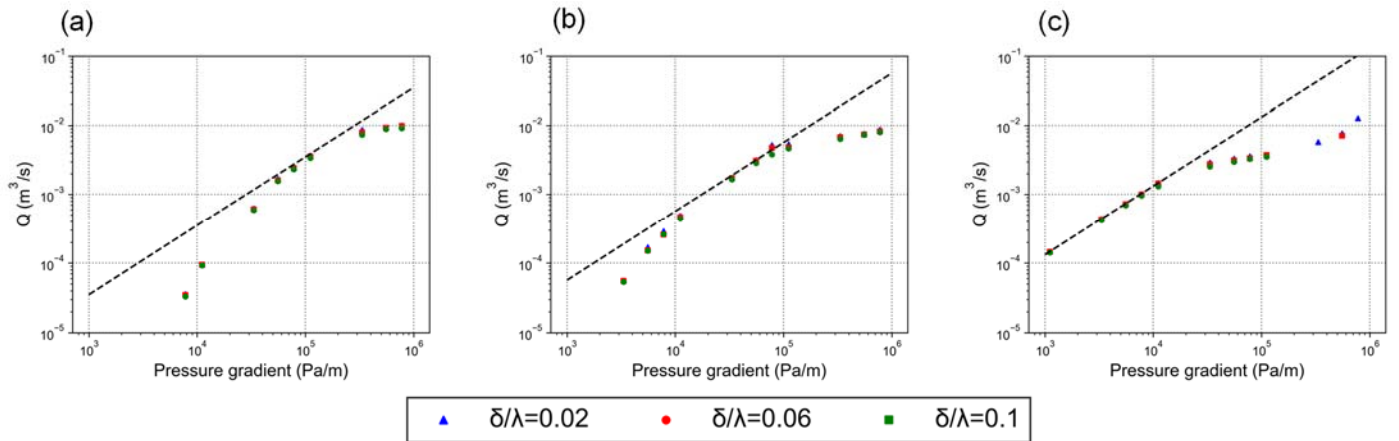
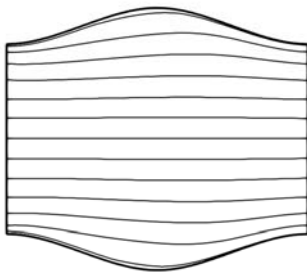
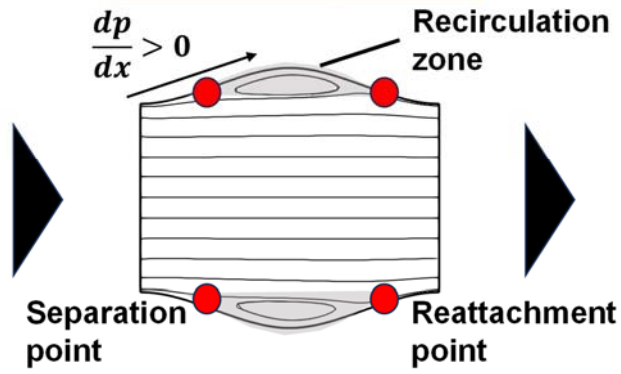


Figure 11. Relationship between pressure gradient and discharge by w/c ratio:
(a) w/c ratio =0.8, (b) w/c ratio =1, and (c) w/c ratio =2

Laminar flow regime



Laminar flow regime with flow separation



Non-linear flow regime

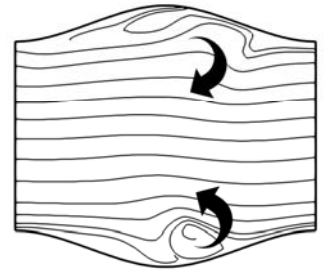


Figure 12. Schematic diagrams of flow regimes for cement slurry at transition from laminar to non-linear flow