

# Closed-Form Critical Response of Undamped Bilinear Hysteretic MDOF System under Pseudo-Double Impulse for Estimating Resonant Response under One-Cycle Sine Wave

Hiroki Akehashi<sup>1</sup>, Izuru Takewaki<sup>1\*</sup>

<sup>1</sup>Department of Architecture and Architectural Engineering, Graduate School of Engineering, Kyoto University, Kyotodaigaku-Katsura, Nishikyo, Kyoto 615-8540, Japan

\* **Correspondence:** Izuru Takewaki, E-mail [takewaki@archi.kyoto-u.ac.jp](mailto:takewaki@archi.kyoto-u.ac.jp)

## Abstract

Approximate closed-form maximum interstory drifts under the critical pseudo-double impulse (PDI) are derived for undamped multi-story shear building models with bilinear hysteresis by using the proposed updated mode-controlled energy-based approach (UMEA). PDI was proposed in the previous paper to simulate the critical responses of elastic-plastic MDOF models under near-fault fling-step motions. Although a double impulse (DI) is treated as a ground motion, PDI is treated as a set of impulsive lateral forces. PDI excites only the fundamental-mode responses of elastic proportionally-damped MDOF models because the fundamental participation vector is applied to the influence coefficient vector of PDI. UMEA is a kind of displacement control analyses, and the displacement increment, which is proportional to the fundamental mode evaluated by the tangent story stiffnesses, is given to the model. It is demonstrated that UMEA effectively captures the main characteristics of the maximum interstory drifts under the critical PDI and the critical one-cycle sine wave because most of the kinetic energy, which the fundamental mode has just before yielding, is given to the plastic fundamental mode just after yielding except for the case that the post-yielding stiffness is almost zero. The use of UMEA leads to much more efficient estimation of the maximum interstory drifts than the time-history response analysis because UMEA requires only  $2N$ -time evaluations of the elastic and plastic fundamental modes at most for  $N$ -story shear mass models with bilinear hysteresis. It is shown through numerical examples that the maximum interstory drifts under the critical PDI and the corresponding one-cycle-sine wave correspond well to those evaluated by the proposed method. It is also shown that the correspondence of the responses of elastic-plastic MDOF models under recorded near-fault motions and the estimated responses by the proposed method is fairly good.

**Keywords:** Critical excitation, Elastic-plastic earthquake response, Nonlinear resonance, Double impulse, Pseudo-double impulse, Near-fault ground motion, Updated mode-controlled energy-based approach.

## 1. Introduction

Earthquake ground motions can be typically classified into three types based on their characteristics; pulse-like ground motions, long-period long-duration ground motions and ground motions of random nature. Many structural design methods such as design spectrum-based methods have been developed so far. Although such methods are effective for ground motions of random nature, they are not always effective for pulse-like ground motions and long-period long-duration motions because of the difference of the characteristics (especially phase properties) [1, 2]. When these ground motions exceed the level specified in the code for building structural design, a specific class of building structures may experience severe damage. The damage to buildings under these ground motions should be minimized through sophisticated structural design for new buildings and seismic retrofit for older buildings. Many researchers investigated the elastic-plastic response characteristics and the effectiveness of the structural control methods under pulse-like ground motions [3-22].

A one-cycle sine wave has been widely used as a substitute for a fling-step motion [23-26]. On the other hand, Kojima and Takewaki [27] introduced a set of impulses for the expression of pulse-like motions and long-period long-duration ground motions. These methods focusing on the critical resonant case are based on the concept of the critical excitation method by Drenick [28] and the critical elastic-plastic responses have been investigated for various types of SDOF models under a double impulse (DI) as a substitute of the fling-step motion [29]. However, a limited number of investigations on the critical response under DI for elastic-plastic MDOF models has been accumulated. Although the closed-form solutions of the critical deformation responses and the critical timing for elastic-plastic SDOF models are easily derived based on the energy balance law, the application of the energy balance law is difficult for elastic-plastic MDOF models because the contributions of the elastic and plastic higher-mode responses are not negligible. Moreover, the excitation of the higher-mode responses under DI leads to the overestimation of the maximum floor accelerations under the critical one-cycle sine wave due to its impulsive nature.

To overcome these difficulties, Akehashi and Takewaki [30] proposed a pseudo-double impulse (PDI) in order to simulate the critical responses of elastic-plastic MDOF models under fling-step motions. Although DI is treated as a ground motion, PDI is treated as a set of impulsive lateral forces. PDI excites only the fundamental-mode responses of elastic proportionally-damped MDOF models because the fundamental participation vector is used as the influence coefficient vector of PDI. It was demonstrated that the maximum interstory drifts and the maximum floor accelerations under the critical PDI correspond well to those under the critical one-cycle sine wave. The critical condition of the impulse timing, which maximizes the total input energy, has also been derived. Moreover, the elastic responses have been formulated analytically for proportionally damped MDOF models and non-proportionally damped MDOF models. However, the critical maximum responses have never been derived for elastic-plastic MDOF models.

In this paper, approximate closed-form maximum interstory drifts under the critical PDI are derived for undamped multi-story shear building models with bilinear hysteresis. In Section 2, the relation between DI and PDI is explained briefly. Then, the modal coupling effect due to the elastic-plastic responses is explained when the fundamental participation vector is applied to the influence

coefficient vector. In Section 3, the updated mode-controlled energy-based approach (UMEA) is presented for deriving the approximate closed-form maximum interstory drifts under the critical PDI. The relation between UMEA and the modal coupling effect under the critical PDI and the critical one-cycle sine wave is also explained. In Section 4, the approximate closed-form maximum interstory drifts under the critical PDI are derived by using UMEA. In Section 5, the time-history response analysis is conducted to investigate the accuracy of the proposed closed-form expression for elastic-plastic MDOF models under the critical PDI and the corresponding one-cycle sine wave. The criticality of the input timing and the applicability of the proposed method to recorded near-fault motions are also investigated.

## 2. Pseudo-double impulse

A one-cycle sine wave has been widely used as a substitute for a fling-step earthquake ground motion [23-26]. Although the one-cycle sine wave effectively expresses the main characteristic of the fling-step motion, the iteration of the time-history response analysis is required to obtain the critical resonant maximum responses of elastic-plastic models. On the other hand, Kojima and Takewaki [27] introduced DI as another type of the substitute for the fling-step motion to effectively capture the critical maximum responses. The energy-based approach leads to the closed-form expression of the maximum deformations and the critical time interval for elastic-plastic SDOF models because DI consists of a set of two impulses and only the free vibration occurs after each impulse input. Moreover, the maximum responses under the critical one-cycle sine wave and those under the critical DI with adjusted  $V$  (: the velocity amplitude) correspond well for elastic-plastic SDOF models [27, 29]. This is because the Fourier amplitude of DI and that of the one-cycle sine wave correspond well in the range of  $0 \leq \omega \leq 2\pi / t_0$  ( $t_0$ : the time interval between two impulses) as shown in Figure 1a. However, in the case of elastic-plastic MDOF models, the maximum interstory drifts and the maximum floor accelerations under DI do not correspond well to those under the one-cycle sine wave since DI has multiple frequency components and excites the higher-mode responses.

To overcome these difficulties, PDI was introduced by Akehashi and Takewaki [30]. Although DI is treated as a ground motion, PDI is treated as a set of impulsive lateral forces (Figure 1b). PDI excites only the fundamental-mode responses of elastic proportionally-damped MDOF models because  $\beta_1 \phi_1$  (the fundamental participation vector) is used as the influence coefficient vector  $\mathbf{u}$  of PDI. It has been demonstrated that the maximum interstory drifts and the maximum floor accelerations of the elastic-plastic MDOF models under the critical PDI correspond well to those under the critical one-cycle sine wave. Moreover, the critical condition of the impulse timing, which maximizes the total input energy, has also been derived.

In Section 2.1, the relation between DI and PDI is explained briefly. In Section 2.2, the modal coupling effect due to the elastic-plastic responses under the external force with  $\mathbf{u} = \beta_1 \phi_1$  is explained.

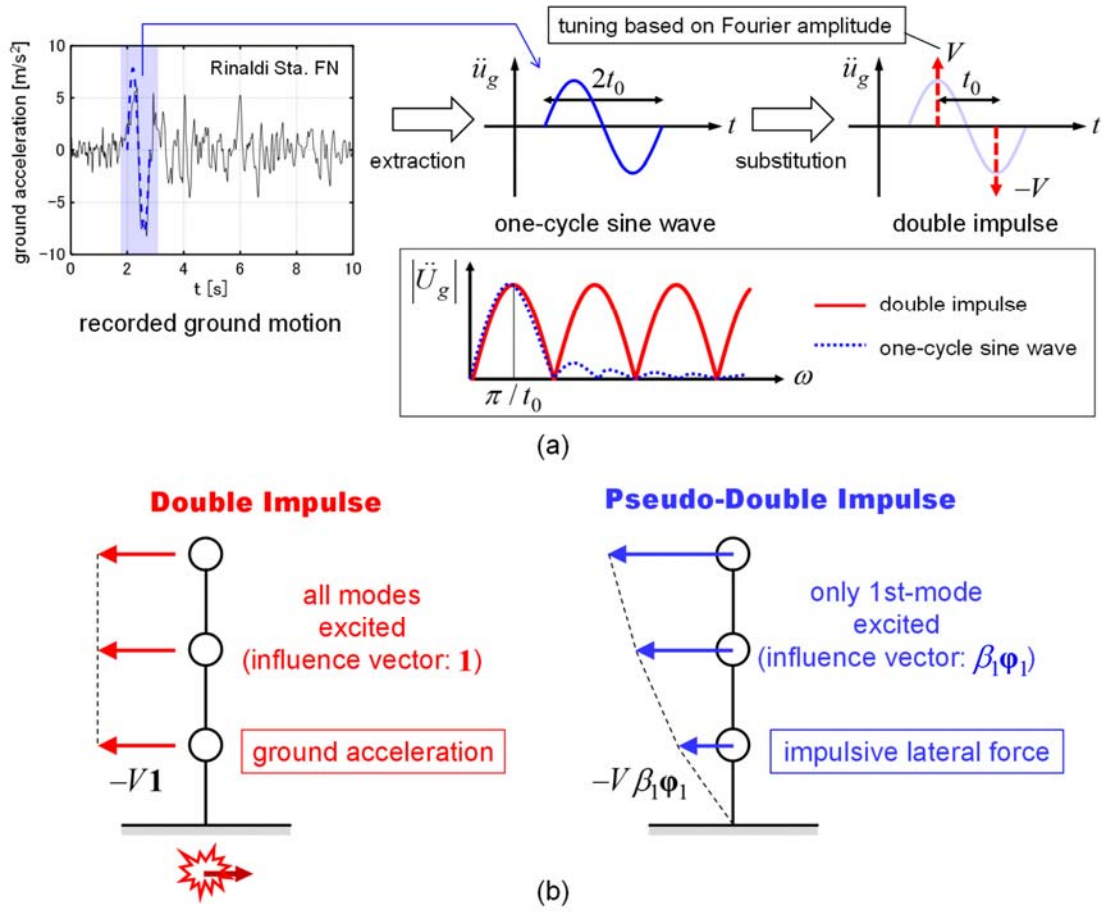


Figure 1 Double impulse (DI) and pseudo-double impulse (PDI).

## 2.1 Double impulse and pseudo-double impulse

Consider an undamped  $N$ -story shear mass model with bilinear hysteresis. Let  $m_i, k_i, d_{y,i}, \alpha_i$  denote the mass, the story stiffness, the yield interstory drift and the post-yield stiffness ratio in the  $i$ -th story, and  $\mathbf{M}, \mathbf{K}, \mathbf{u}$  denote the mass matrix, the stiffness matrix and the displacement vector. When the model responds elastically, the equations of motion under DI are expressed by

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}\mathbf{u} = -\mathbf{M}\mathbf{1}(V\delta(t) - V\delta(t - t_0)) \quad (1)$$

where  $\delta(t)$  is the Dirac delta function. The influence coefficient vector is  $\mathbf{1} = \mathbf{1}$  (one at every component). On the other hand, the equations of motion under PDI are expressed as follows.

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}\mathbf{u} = -\mathbf{M}(\beta_1\phi_1)(V\delta(t) - V\delta(t - t_0)) \quad (2)$$

where  $\phi_i = (\phi_{i,1}, \dots, \phi_{i,N})^T$  is the  $i$ -th undamped eigenmode vector and  $\beta_i\phi_i = \{(\phi_i^T \mathbf{M}\mathbf{1}) / (\phi_i^T \mathbf{M}\phi_i)\}\phi_i$  is the  $i$ -th participation vector. The influence coefficient vector is set to  $\mathbf{1} = \beta_1\phi_1$  in the case of PDI. It is noted that the critical conditions of the impulse timing, which

maximizes the total input energy by DI and PDI, have been derived [19, 30]. Especially in the case of PDI, the total input energy is maximized when the sum of the elastic fundamental-mode inertial forces attains zero.

The maximum interstory drifts and the maximum floor accelerations of elastic-plastic MDOF models under DI do not correspond well to those under the corresponding one-cycle sine wave since DI has multiple frequency components as shown in Figure 1a. On the other hand, the maximum interstory drifts and the maximum floor accelerations under the critical PDI correspond well to those under the critical one-cycle sine wave since PDI hardly excites the higher-mode responses [30].

## 2.2 Modal coupling effect under excitation with $\mathbf{u} = \beta_1 \boldsymbol{\phi}_1$

Consider the case where the model shows the free vibration under PDI and the interstory drift in the  $n$ -th story attains the yield deformation  $d_{y,n}$  at time  $\bar{t}$ . If all the stories respond elastically before  $\bar{t}$ , the displacement, the velocity and the acceleration at  $\bar{t}$  can be expressed as  $\mathbf{u} = \gamma_{\mathbf{u}} \boldsymbol{\phi}_1, \dot{\mathbf{u}} = \gamma_{\dot{\mathbf{u}}} \boldsymbol{\phi}_1, \ddot{\mathbf{u}} = -\omega_1^2 \gamma_{\mathbf{u}} \boldsymbol{\phi}_1$ , where  $\omega_1$  is the undamped fundamental natural circular frequency, and  $\gamma_{\mathbf{u}}, \gamma_{\dot{\mathbf{u}}}$  denote the coefficients depending on  $V$ .  $\gamma_{\mathbf{u}}, \gamma_{\dot{\mathbf{u}}}$  satisfy  $\gamma_{\mathbf{u}} \gamma_{\dot{\mathbf{u}}} \geq 0$  here because the model is in the loading process. The equations of motion just after  $\bar{t}$  are expressed by

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}^n \mathbf{u} + \mathbf{f}^n = \mathbf{0} \quad (3)$$

$\mathbf{K}^n$  is the stiffness matrix where all of  $k_n$  are replaced with the plastic stiffness  $\alpha_n k_n$ .  $\mathbf{f}^n$  is the vector which consists of zero except for the  $n$ -th and the  $(n-1)$ -th components. The  $n$ -th and the  $(n-1)$ -th components are  $\pm(1-\alpha_n)k_n d_{y,n} \cdot \text{sign}(\gamma_{\mathbf{u}})$  (see Figure 2a).

Let us define  $\mathbf{u}^n = (u_1, \dots, u_n + \{(1-\alpha_n)/\alpha_n\}d_{y,n} \cdot \text{sign}(\gamma_{\mathbf{u}}), \dots, u_N)^T$ . By using  $\mathbf{u}^n$ , Equation (3) can be expressed as

$$\mathbf{M}\ddot{\mathbf{u}}^n + \mathbf{K}^n \mathbf{u}^n = \mathbf{0} \quad (4)$$

Equation (4) means that the change of the reference coordinate system provides the equation of free vibration. The decomposition of  $\mathbf{u}, \dot{\mathbf{u}}, \ddot{\mathbf{u}}$  by the plastic modes (the modes evaluated by using the post-yield story stiffness) leads to

$$\mathbf{u}^n = \sum_{j=1}^N \beta_{\mathbf{u}^n, j}^n \boldsymbol{\phi}_j^n, \beta_{\mathbf{u}^n, j}^n = \{(\boldsymbol{\phi}_j^n)^T \mathbf{M} \mathbf{u}^n\} / \{(\boldsymbol{\phi}_j^n)^T \mathbf{M} \boldsymbol{\phi}_j^n\} \quad (5)$$

$$\dot{\mathbf{u}}^n = \dot{\mathbf{u}} = \sum_{j=1}^N \beta_{\dot{\mathbf{u}}, j}^n \boldsymbol{\phi}_j^n, \beta_{\dot{\mathbf{u}}, j}^n = \{(\boldsymbol{\phi}_j^n)^T \mathbf{M} \dot{\mathbf{u}}\} / \{(\boldsymbol{\phi}_j^n)^T \mathbf{M} \boldsymbol{\phi}_j^n\} \quad (6)$$

$$\ddot{\mathbf{u}}^n = \ddot{\mathbf{u}} = \sum_{j=1}^N \beta_{\ddot{\mathbf{u}}, j}^n \boldsymbol{\phi}_j^n, \beta_{\ddot{\mathbf{u}}, j}^n = \{(\boldsymbol{\phi}_j^n)^T \mathbf{M} \ddot{\mathbf{u}}\} / \{(\boldsymbol{\phi}_j^n)^T \mathbf{M} \boldsymbol{\phi}_j^n\} \quad (7)$$

where  $\boldsymbol{\varphi}_i^n = (\varphi_{i,1}^n, \dots, \varphi_{i,N}^n)^T$  is the  $i$ -th eigenmode evaluated by  $k_1, \dots, k_{n-1}, \alpha_n k_n, k_{n+1}, \dots, k_N$ . Since  $\mathbf{u}, \dot{\mathbf{u}}, \ddot{\mathbf{u}}$  are proportional to  $\boldsymbol{\varphi}_1$  and  $(\boldsymbol{\varphi}_j^n)^T \mathbf{M} \boldsymbol{\varphi}_1 \neq 0$  (for all  $j = 1, \dots, N$ ), the change of the reference coordinate system and the yielding give the kinetic energy and the (apparent) strain energy to all the plastic modes. This phenomenon is called modal coupling or nonlinear coupling.

The distribution of the strain energy to the plastic modes depends only on the change of the reference coordinate system. On the other hand, the distribution of the kinetic energy to the plastic modes depends only on the velocity response  $\dot{\mathbf{u}}$  at the moment of yielding. Except for the case of  $\alpha_i \approx 0$  (elastic perfectly-plastic model), most of the kinetic energy, which the fundamental mode has just before yielding, is given or transmitted to the plastic fundamental mode just after yielding (see Figure 2b). In the case of  $\alpha_i \approx 0$ , the interstory drift of the yielding story in the plastic fundamental mode is much larger than those of the unyielding stories, and the plastic fundamental mode differs largely from the fundamental mode just before yielding. Therefore, the kinetic energy given to the plastic higher modes becomes relatively large.

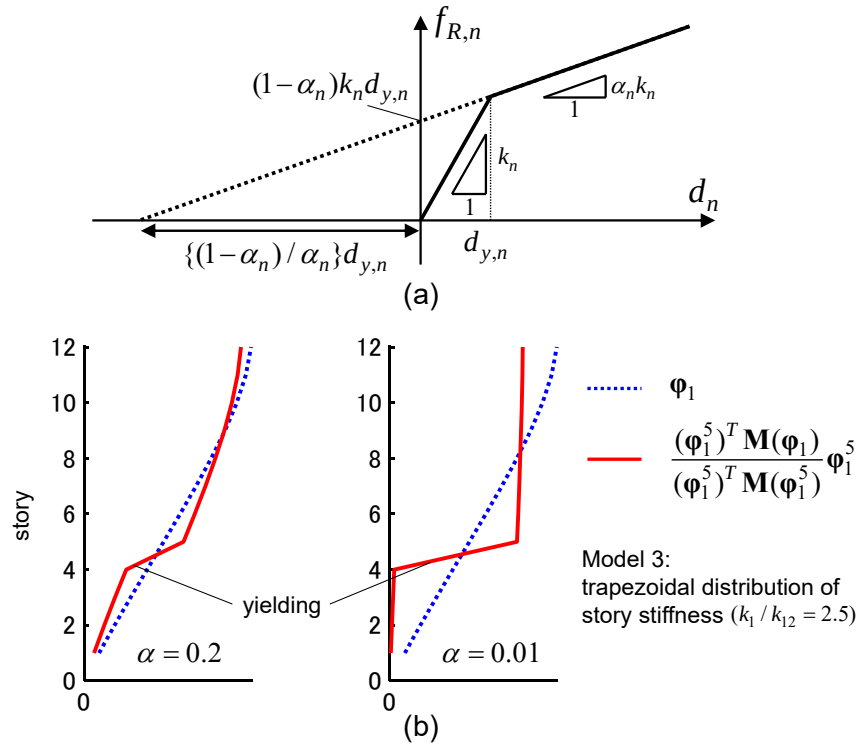


Figure 2 Restoring force-deformation relation in  $n$ -th story and example of decomposition of elastic fundamental mode by plastic fundamental mode, (a) restoring force-deformation relation in  $n$ -th story, (b) example of elastic fundamental mode and plastic fundamental mode.

### 3. Updated mode-controlled energy-based approach (UMEA)

In this section, UMEA is newly presented for deriving approximate closed-form maximum interstory drifts under the critical PDI. UMEA effectively expresses the plastic deformation concentration in specific stories under the critical PDI and the critical one-cycle sine wave. In Section 3.1, the concept of UMEA is explained. Then the relation between UMEA and the modal coupling effect under the critical PDI and the critical one-cycle sine wave is explained in Section 3.2.

#### 3.1 Concept of UMEA

UMEA is a kind of displacement-controlled analyses, and the displacement increment, which is proportional to the fundamental mode evaluated by the tangent story stiffnesses, is given to the model.  $k_n$  corresponds to the tangent story stiffness of the  $n$ -th story when the  $n$ -th story remains elastic, and  $\alpha_n k_n$  corresponds to the tangent story stiffness when the  $n$ -th story yields. In the beginning of the analysis, the displacement increment, which is proportional to the elastic fundamental mode  $\phi_1$ , is given until one of the stories yields. Then, the fundamental mode is re-evaluated by  $k_1, \dots, k_{n-1}, \alpha_n k_n, k_{n+1}, \dots, k_N$  ( $n$ : yielding story number) and the displacement increment is updated. The same procedure is repeated until the total hysteretic energy attains the targeted value (initial kinetic energy).

Consider the case where UMEA is applied to a 3DOF shear mass model (see Figures 3, 4).  $d_i, f_{R,i}$  in Figures 3a, b denote the interstory drift and the restoring force in the  $i$ -th story. In the beginning of UMEA, the displacement increment, which is proportional to  $\phi_1$ , is given. Then the first story yields first (point 1 in Figure 3a). The displacement increment is updated to be proportional to  $\phi_1^1$ , which is evaluated by  $\alpha_1 k_1, k_2, k_3$ . Next, the second story yields (point 2), and the displacement increment is updated to be proportional to  $\phi_1^{1,2}$ , which is evaluated by  $\alpha_1 k_1, \alpha_2 k_2, k_3$ . Before the third story yields, the total hysteretic energy attains the targeted value (initial kinetic energy) at point 3.

Hereafter, the analysis goes into the unloading and reloading process. The displacement increment is set to be proportional to  $\phi_1$  in the beginning of the unloading process. The first story yields again at point 4, then the displacement increment is updated to be proportional to  $\phi_1^1$ . The second story yields at point 5, and the displacement increment is updated to be proportional to  $\phi_1^{1,2}$ .

In this paper,  $N$ -story shear mass models with bilinear hysteresis is treated. The unidirectional application of UMEA to such models requires the  $N$ -time evaluations of the elastic and plastic fundamental modes at most. Thus, the estimation of the maximum interstory drifts under the critical PDI by UMEA requires  $2N$ -time evaluations of the fundamental modes at most. On the other hand, the time-history response analysis requires the repetition of the matrix multiplication as many times as the number of the time steps. Especially in the case of  $\alpha_i \simeq 0$ , it requires the convergence analysis to cope with the yielding. Therefore, the former is much more efficient in the evaluation of the maximum interstory drifts than the latter.

‘Modal pushover analysis’ is one of the well-known pushover analyses and has been widely used [31]. Modal pushover analysis treats the elastic eigenmodes and does not treat the plastic modes of the model in which the stiffnesses of some elements are changed to the post-yield stiffnesses. Moreover, the coupling between the elastic modes due to the elastic-plastic responses is neglected. On the other hand, UMEA uses the elastic and plastic fundamental modes, and focuses on the coupling effect between them, as explained in Section 3.2.

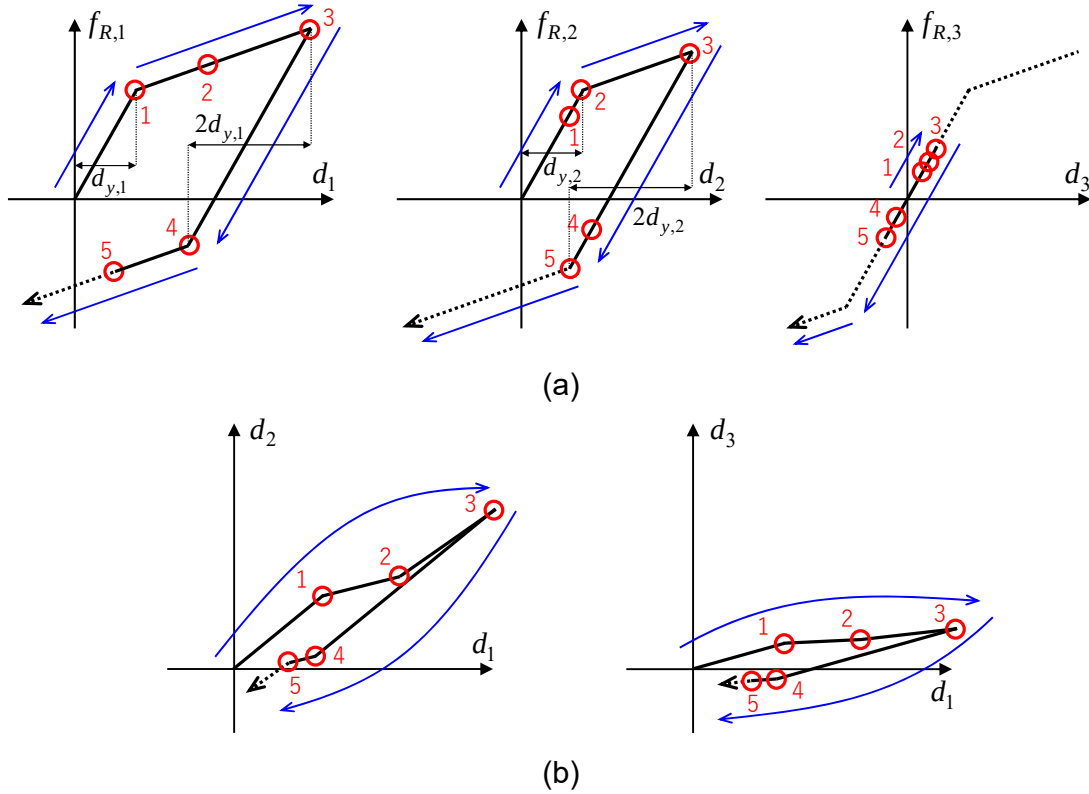


Figure 3 Overview of UMEA, (a) restoring force-deformation relation, (b) relation between  $d_1, d_2, d_3$ .

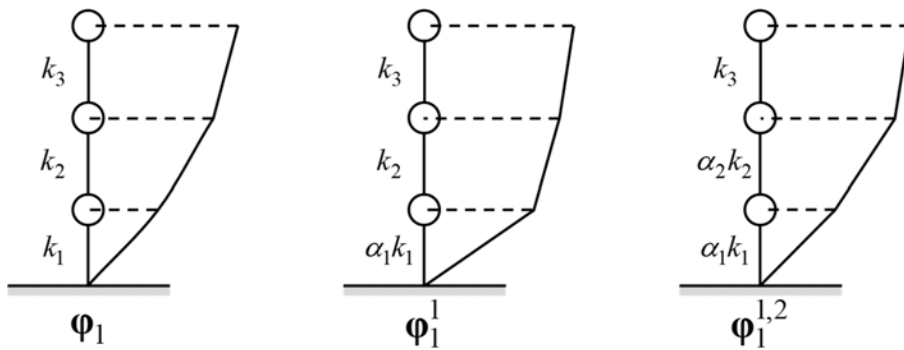


Figure 4 Elastic fundamental mode  $\phi_1$  and plastic fundamental modes  $\phi_1^1, \phi_1^{1,2}$ .

### 3.2 Relation between modal coupling effect and UMEA

The strain energy in the plastic fundamental mode is increased through UMEA. At the moment of yielding, the (apparent) strain energy is given to the plastic higher modes by the change of the reference coordinate system. However, except for that moment, the strain energy in the plastic higher modes is not increased through UMEA. On the other hand, the elastic mode responses and the plastic mode responses are coupled under PDI and the one-cycle sine wave as stated in Section 2.2. However, the plastic higher modes are hardly excited under the critical PDI and the critical one-cycle sine wave. This is because the influence coefficient vector is set to be proportional to  $\phi_1$  in the case of PDI and the input period of the critical one-cycle sine wave is relatively long. Moreover, most of the kinetic energy, which the fundamental mode has just before yielding, is given to the plastic fundamental mode just after yielding, and the kinetic energy given to the plastic higher modes are small enough as explained in Section 2.2. Therefore, UMEA effectively captures the main characteristics of the displacement responses under the critical PDI and the critical one-cycle sine wave.

## 4. Derivation of approximate closed-form maximum deformation under critical PDI

In this section, a set of approximate closed-form maximum interstory drifts under the critical PDI is derived by using UMEA. The word ‘closed-form’ means the solution of explicit mathematical expression, and it does not mean the ‘totally accurate solution’. It is noted that, due to the plastic higher-mode responses, the interstory velocity responses in specific stories under the critical PDI may temporarily reverse before attaining the maximum deformations. However, the excitation of the plastic higher modes is excluded in the evaluation of the distribution of the maximum interstory drifts because the kinetic energy given to the plastic higher modes are small enough as explained in Section 3.2.

The distribution  $\mathbf{d}_{\max}^{(1)}$  of the maximum interstory drifts after the first impulse is derived by UMEA in the negative direction. Let us define  $s_1^{(1)}$  as the story number which yields first (see Figure 5a), and  $s_i^{(1)}$  denotes the story number which yields after the story  $s_{i-1}^{(1)}$  ( $i = 2, \dots, N$ ). For example, Figures 5b, c illustrate an example of the restoring force-deformation relations of a 3DOF model for the derivation of  $\mathbf{d}_{\max}^{(1)}$  and  $\mathbf{d}_{\max}^{(2)}$  of the maximum interstory drifts after the second impulse. Assume  $s_1^{(1)} = 1, s_2^{(1)} = 2, s_3^{(1)} = 3$  here. In other words, the first story yields first (point 1 in Figure 5b), then the second story yields (point 2). Before the third story yields, the total hysteretic energy attains the targeted value at point 3. In the case of the story  $s_i^{(1)}$  attaining the yield deformation  $d_{y,s_i^{(1)}}^{(1)}$  (see

Figure 5a), the distribution  $\mathbf{d}^{(1),s_i^{(1)}} = (d_1^{(1),s_i^{(1)}}, \dots, d_N^{(1),s_i^{(1)}})^T$  of the interstory drifts is expressed as

$$\mathbf{d}^{(1),s_1^{(1)}} = -d_{y,s_1^{(1)}} \{\Delta_1 / \Delta_{1,s_1^{(1)}}\} \quad (8a)$$

$$\mathbf{d}^{(1),s_i^{(1)}} = \mathbf{d}^{(1),s_{i-1}^{(1)}} - (d_{y,s_i^{(1)}} + d_{s_i^{(1)}}^{(1),s_{i-1}^{(1)}}) \{ \Delta_1^{s_1^{(1)}, \dots, s_{i-1}^{(1)}} / \Delta_{1,s_i^{(1)}}^{s_1^{(1)}, \dots, s_{i-1}^{(1)}} \} \quad (i=2, \dots, N) \quad (8b)$$

where  $\Delta_1$  is the following interstory drift vector in the elastic fundamental mode  $\boldsymbol{\phi}_1$  and  $\Delta_1^{s_1^{(1)}, \dots, s_i^{(1)}}$  is that in the inelastic fundamental mode  $\boldsymbol{\phi}_1^{s_1^{(1)}, \dots, s_i^{(1)}}$  specified by the plastic state  $s_1^{(1)}, \dots, s_i^{(1)}$

$$\Delta_1 = \mathbf{T} \boldsymbol{\phi}_1 = (\Delta_{1,1}, \dots, \Delta_{1,N})^T \quad (8c)$$

$$\Delta_1^{s_1^{(1)}, \dots, s_i^{(1)}} = \mathbf{T} \boldsymbol{\phi}_1^{s_1^{(1)}, \dots, s_i^{(1)}} = (\Delta_{1,1}^{s_1^{(1)}, \dots, s_i^{(1)}}, \dots, \Delta_{1,N}^{s_1^{(1)}, \dots, s_i^{(1)}})^T \quad (i=1, \dots, N) \quad (8d)$$

$$\mathbf{T} = \begin{pmatrix} 1 & & & \mathbf{O} \\ -1 & \ddots & & \\ & \ddots & \ddots & \\ \mathbf{O} & & -1 & 1 \end{pmatrix} \quad (8e)$$

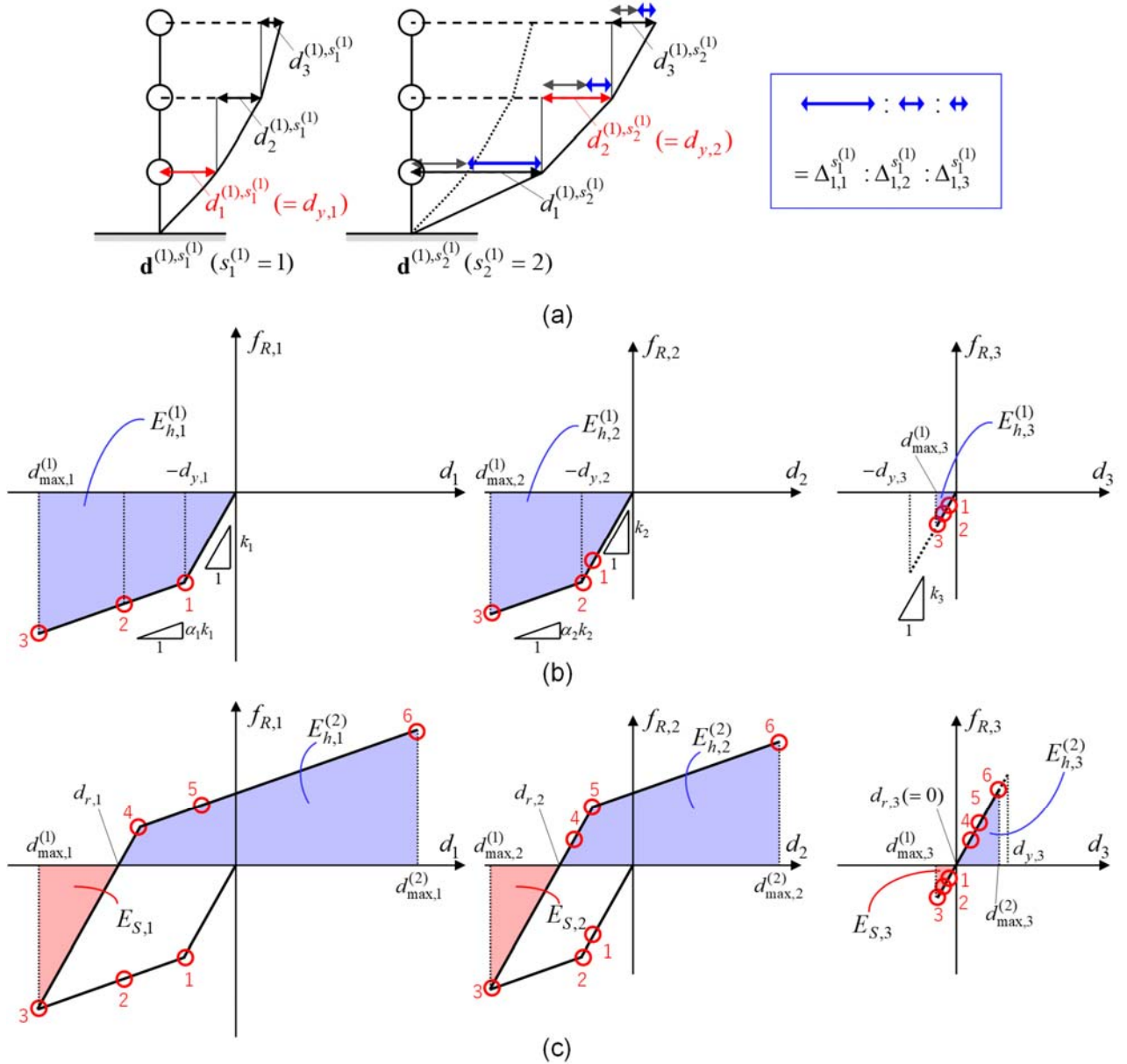


Figure 5 Displacement increment and examples of restoring force-deformation relations of 3DOF model, (a) displacement increment between  $\mathbf{d}^{(1),s_1^{(1)}}$  and  $\mathbf{d}^{(1),s_2^{(1)}}$ , (b) evaluation of  $\mathbf{d}_{\max}^{(1)}$  by UMEA, (c) evaluation of  $\mathbf{d}_{\max}^{(2)}$  by UMEA

Note that  $\boldsymbol{\phi}_1^{s_1^{(1)}, \dots, s_i^{(1)}}$  is the lowest eigenmode of the model in which the story stiffnesses corresponding to the plastic state  $s_1^{(1)}, \dots, s_i^{(1)}$  are replaced by the post-yield stiffnesses, as defined in Section 2.2. In the application of UMEA, the displacement increment, which is proportional to the fundamental mode evaluated by the tangent story stiffnesses, is given to the model. In other words, when one of the stories yields, the fundamental mode is re-evaluated by using the post-yield stiffness

of that story and the displacement increment is updated. Equation 8b indicates that the displacement increment, which is proportional to  $\phi_1^{s_1^{(1)}, \dots, s_{i-1}^{(1)}}$ , is given from the point at which the story  $s_{i-1}^{(1)}$  attains the yield deformation  $d_{y, s_{i-1}^{(1)}}$  to the point at which the story  $s_i^{(1)}$  attains the yield deformation  $d_{y, s_i^{(1)}}$  (Figure 5a).

In the case of Figure 5b, the displacement increment, which is proportional to  $\phi_1$ , is given at first. After the first story yields (point 1), the displacement increment is updated to be proportional to  $\phi_1^1$  (between points 1, 2). Next, the second story yields (point 2), and the displacement increment is updated to be proportional to  $\phi_1^{1,2}$  (between points 2, 3). Before the third story yields, the total hysteretic energy attains the targeted value (initial kinetic energy) at point 3.

It is noted that the distributions of the interstory drifts like  $\mathbf{d}^{(1), s_i^{(1)}}$  are not expressed as the absolute values, and they include the plus and minus signs.

The restoring forces  $\mathbf{f}_R^{(1), s_i^{(1)}} = (f_{R,1}^{(1), s_i^{(1)}}, \dots, f_{R,N}^{(1), s_i^{(1)}})^T$  corresponding to  $\mathbf{d}^{(1), s_i^{(1)}}$  are expressed by

$$\mathbf{f}_R^{(1), s_1^{(1)}} = -d_{y, s_1^{(1)}} \begin{pmatrix} k_1 & & \mathbf{O} \\ & \ddots & \\ \mathbf{O} & & k_N \end{pmatrix} \{\Delta_1 / \Delta_{1, s_1^{(1)}}\} \quad (9a)$$

$$\mathbf{f}_R^{(1), s_i^{(1)}} = \mathbf{f}_R^{(1), s_{i-1}^{(1)}} - (d_{y, s_i^{(1)}} + d_{s_i^{(1)}}^{(1), s_{i-1}^{(1)}}) \begin{pmatrix} p_1^{(1), s_{i-1}^{(1)}} k_1 & & \mathbf{O} \\ & \ddots & \\ \mathbf{O} & & p_N^{(1), s_{i-1}^{(1)}} k_N \end{pmatrix} \{\Delta_1^{s_1^{(1)}, \dots, s_{i-1}^{(1)}} / \Delta_{1, s_i^{(1)}}^{s_1^{(1)}, \dots, s_{i-1}^{(1)}}\} \quad (i = 2, \dots, N) \quad (9b)$$

where

$$p_j^{(1), s_i^{(1)}} = \begin{cases} \alpha_j & j \in (s_1^{(1)}, \dots, s_i^{(1)}) \\ 1 & \text{otherwise} \end{cases} \quad (10)$$

The total hysteretic energy  $E_h^{(1), s_i^{(1)}}$  can also be derived as

$$E_h^{(1), s_i^{(1)}} = \sum_{j=1}^N E_{h,j}^{(1), s_i^{(1)}} \quad (11a)$$

$$E_{h,j}^{(1), s_1^{(1)}} = \frac{1}{2k_j} (f_{R,j}^{(1), s_1^{(1)}})^2 \quad (11b)$$

$$E_{h,j}^{(1), s_i^{(1)}} = E_{h,j}^{(1), s_{i-1}^{(1)}} + \frac{(f_{R,j}^{(1), s_i^{(1)}} + f_{R,j}^{(1), s_{i-1}^{(1)}})(f_{R,j}^{(1), s_i^{(1)}} - f_{R,j}^{(1), s_{i-1}^{(1)}})}{2p_j^{(1), s_{i-1}^{(1)}} k_j} \quad (i = 2, \dots, N) \quad (11c)$$

On the other hand, the input energy  $E_I^{(1)}$  by the first impulse is expressed by

$$E_I^{(1)} = \frac{1}{2}(\beta_1 \Phi_1)^T \mathbf{M}(\beta_1 \Phi_1) V^2 \quad (12)$$

When  $E_h^{(1),s_i^{(1)}} \leq E_I^{(1)} \leq E_h^{(1),s_{i+1}^{(1)}}$  is satisfied, the distribution  $\mathbf{d}_{\max}^{(1)}$  of the maximum interstory drifts after the first impulse can be expressed as

$$\mathbf{d}_{\max}^{(1)} = (1-q)\mathbf{d}^{(1),s_i^{(1)}} + q\mathbf{d}^{(1),s_{i+1}^{(1)}} \quad (0 \leq q \leq 1) \quad (13)$$

The maximum restoring forces  $\mathbf{f}_R^{(1)} = (f_{R,1}^{(1)}, \dots, f_{R,N}^{(1)})^T$  corresponding to  $\mathbf{d}_{\max}^{(1)}$  are also expressed by

$$\mathbf{f}_R^{(1)} = (1-q)\mathbf{f}_R^{(1),s_i^{(1)}} + q\mathbf{f}_R^{(1),s_{i+1}^{(1)}} \quad (14)$$

The total hysteretic energy  $E_h^{(1)}$  can also be derived as

$$\begin{aligned} E_h^{(1)} &= \sum_{j=1}^N E_{h,j}^{(1)} \\ &= \sum_{j=1}^N \left\{ E_{h,j}^{(1),s_i^{(1)}} + \frac{1}{2p_j^{(1),s_i^{(1)}} k_j} (f_{R,j}^{(1)} + f_{R,j}^{(1),s_i^{(1)}})(f_{R,j}^{(1)} - f_{R,j}^{(1),s_i^{(1)}}) \right\} \end{aligned} \quad (15)$$

The application of the energy balance law leads to

$$E_I^{(1)} = E_h^{(1)} \quad (16)$$

Equation (16) can be solved easily since it is a quadratic equation of  $q$ .  $\mathbf{d}_{\max}^{(1)}, \mathbf{f}_R^{(1)}$  are derived by substituting the derived value of  $q$  into Equations (13), (14), and the distribution  $\mathbf{u}_{\max}^{(1)} (= \mathbf{T}^{-1}\mathbf{d}_{\max}^{(1)})$  of the maximum displacements is also derived.

It is noted that the case of  $E_I^{(1)} > E_h^{(1),s_N^{(1)}}$  (all stories yield) and the case of  $E_I^{(1)} < E_h^{(1),s_1^{(1)}}$  (all stories remain elastic) are not formulated here. However, the maximum responses in these cases can also be derived in a similar way.

Next, the distribution  $\mathbf{d}_{\max}^{(2)} (= \mathbf{T}\mathbf{u}_{\max}^{(2)})$  of the maximum interstory drifts after the second impulse is derived by UMEA in the positive direction. In other words, the unloading and the reloading are conducted. The updated central coordinates in the unloading process are expressed by

$$d_{r,j} = d_{\max,j}^{(1)} - (f_{R,j}^{(1)} / k_j) \quad (i = 1, \dots, N) \quad (17)$$

342 An example of  $d_r$  is shown in Figure 5c.  $|d_{r,1}|, |d_{r,2}| > 0$  because  $|d_{\max,1}^{(1)}|, |d_{\max,2}^{(1)}| > d_{y,1}, d_{y,2}$  and  
 343  $d_{r,3} = 0$  because  $|d_{\max,3}^{(1)}| < d_{y,3}$ . Hereafter, the case of  $d_{\max,j}^{(1)} / d_{y,j} \leq (1 + \alpha_j) / \alpha_j$  (for all  $j=1, \dots, N$ )  
 344 is treated. When  $d_{\max,j}^{(1)} / d_{y,j} > (1 + \alpha_j) / \alpha_j$ , the restoring force reaches zero at the post-yield  
 345 stiffness range. Such case is unrealistic since the interstory drifts are too large.

346 Let us define  $s_1^{(2)}$  as the story number which goes into the reloading process first, and  $s_i^{(2)}$  denotes  
 347 the story number which goes into the reloading process after the story  $s_{i-1}^{(2)}$  ( $i = 2, \dots, N$ ). For  
 348 example, Figure 5c illustrates an example of the restoring force-deformation relations of the 3DOF  
 349 model for the derivation of  $\mathbf{d}_{\max}^{(2)}$ , and  $s_1^{(2)} = 1, s_2^{(2)} = 2, s_3^{(2)} = 3$  here. In other words, the first story  
 350 yields first (point 4 in Figure 5c), then the second story yields (point 5). Before the third story yields,  
 351 the total hysteretic energy attains the targeted value at point 6. Just when the story  $s_i^{(2)}$  goes into the  
 352 reloading process, the distribution  $\mathbf{d}^{(2),s_i^{(2)}} = (d_1^{(2),s_i^{(2)}}, \dots, d_N^{(2),s_i^{(2)}})^T$  of the interstory drifts is  
 353 expressed as

$$354 \quad \mathbf{d}^{(2),s_1^{(2)}} = \begin{cases} \mathbf{d}_{\max}^{(1)} + (d_{y,s_1^{(2)}} - d_{\max,s_1^{(2)}}^{(1)}) \{ \Delta_1 / \Delta_{1,s_1^{(2)}} \} & (|d_{\max,s_1^{(2)}}^{(1)}| < d_{y,s_1^{(2)}}) \\ \mathbf{d}_{\max}^{(1)} + (2d_{y,s_1^{(2)}}) \{ \Delta_1 / \Delta_{1,s_1^{(2)}} \} & (|d_{\max,s_1^{(2)}}^{(1)}| \geq d_{y,s_1^{(2)}}) \end{cases} \quad (18a)$$

$$355 \quad \mathbf{d}^{(2),s_i^{(2)}} = \begin{cases} \mathbf{d}^{(2),s_{i-1}^{(2)}} + (d_{y,s_i^{(2)}} - d_{s_i^{(2)}}^{(2),s_{i-1}^{(2)}}) \{ \Delta_1^{s_1^{(2)}, \dots, s_{i-1}^{(2)}} / \Delta_{1,s_i^{(2)}}^{s_1^{(2)}, \dots, s_{i-1}^{(2)}} \} & (|d_{\max,s_i^{(2)}}^{(1)}| < d_{y,s_i^{(2)}}) \\ \mathbf{d}^{(2),s_{i-1}^{(2)}} + (d_{s_i^{(2)}}^{(1)} + 2d_{y,s_i^{(2)}} - d_{s_i^{(2)}}^{(2),s_{i-1}^{(2)}}) \{ \Delta_1^{s_1^{(2)}, \dots, s_{i-1}^{(2)}} / \Delta_{1,s_i^{(2)}}^{s_1^{(2)}, \dots, s_{i-1}^{(2)}} \} & (|d_{\max,s_i^{(2)}}^{(1)}| \geq d_{y,s_i^{(2)}}) \end{cases} \quad (i = 2, \dots, N) \quad (18b)$$

359 In the case of Figure 5c, the displacement increment, which is proportional to  $\phi_1$ , is given at first  
 360 (between points 3, 4). After the first story yields (point 4), the displacement increment is updated to  
 361 be proportional to  $\phi_1^1$  (between points 4, 5). Next, the second story yields (point 5), and the  
 362 displacement increment is updated to be proportional to  $\phi_1^{1,2}$  (between points 5, 6). Before the third  
 363 story yields, the total hysteretic energy attains the targeted value at point 6.

364 The restoring forces  $\mathbf{f}_R^{(2),s_i^{(2)}} = (f_{R,1}^{(2),s_i^{(2)}}, \dots, f_{R,N}^{(2),s_i^{(2)}})^T$  corresponding to  $\mathbf{d}^{(2),s_i^{(2)}}$  are expressed by

$$365 \quad \mathbf{f}_R^{(2),s_1^{(2)}} = \mathbf{f}_R^{(1)} + \begin{pmatrix} k_1 & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & k_N \end{pmatrix} (\mathbf{d}^{(2),s_1^{(2)}} - \mathbf{d}_{\max}^{(1)}) \quad (19a)$$

$$\mathbf{f}_R^{(2),s_i^{(2)}} = \mathbf{f}_R^{(2),s_{i-1}^{(2)}} + \begin{pmatrix} p_1^{(2),s_{i-1}^{(2)}} k_1 & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & p_N^{(2),s_{i-1}^{(2)}} k_N \end{pmatrix} (\mathbf{d}^{(2),s_i^{(2)}} - \mathbf{d}^{(2),s_{i-1}^{(2)}}) \quad (i = 2, \dots, N) \quad (19b)$$

where

$$p_j^{(2),s_i^{(2)}} = \begin{cases} \alpha_j & j \in (s_1^{(2)}, \dots, s_i^{(2)}) \\ 1 & \text{otherwise} \end{cases} \quad (20)$$

The total hysteretic energy  $E_h^{(2),s_i^{(2)}}$  can also be derived as

$$E_h^{(2),s_i^{(2)}} = \sum_{j=1}^N E_{h,j}^{(2),s_i^{(2)}} \quad (21a)$$

$$E_{h,j}^{(2),s_i^{(2)}} = \frac{1}{2k_j} (f_{R,j}^{(2),s_i^{(2)}}) |f_{R,j}^{(2),s_i^{(2)}}| \quad (21b)$$

$$E_{h,j}^{(2),s_i^{(2)}} = \begin{cases} E_{h,j}^{(2),s_{i-1}^{(2)}} + \frac{(f_{R,j}^{(2),s_i^{(2)}} + f_{R,j}^{(2),s_{i-1}^{(2)}})(f_{R,j}^{(2),s_i^{(2)}} - f_{R,j}^{(2),s_{i-1}^{(2)}})}{2p_j^{(2),s_{i-1}^{(2)}} k_j} & j \in (s_1^{(2)}, \dots, s_{i-1}^{(2)}) \\ \frac{1}{2k_j} (f_{R,j}^{(2),s_i^{(2)}}) |f_{R,j}^{(2),s_i^{(2)}}| & \text{otherwise} \end{cases} \quad (i = 2, \dots, N) \quad (21c)$$

It is noted that, when the restoring force of the  $j$ -th story is negative,  $E_{h,j}^{(2),s_i^{(2)}}$  is also negative in Equations (21a, b, c). On the other hand, the total strain energy  $E_S$  stored in the beginning of the unloading process can be expressed as

$$E_S = \sum_{j=1}^N E_{S,j} = \frac{1}{2} \sum_{j=1}^N (f_{R,j}^{(1)})^2 / k_j \quad (22)$$

Then the total strain energy  $E_{S\phi_1}$  stored in the elastic fundamental mode in the beginning of the unloading process can be expressed as

$$E_{S\phi_1} = \frac{1}{2} (\beta_1' \beta_1 \phi_1)^T \mathbf{K} (\beta_1' \beta_1 \phi_1), \beta_1' = \{(\beta_1 \phi_1)^T \mathbf{M} (\mathbf{u}_{\max}^{(1)} - \mathbf{T}^{-1} \mathbf{d}_r)\} / \{(\beta_1 \phi_1)^T \mathbf{M} (\beta_1 \phi_1)\} \quad (23)$$

The second impulse of the critical PDI acts at the timing when the sum of the elastic fundamental mode inertial forces attains zero as explained in Section 2.1. This means the maximization of the total input energy [30]. The distribution of the elastic fundamental-mode velocity responses just before the

second impulse is  $-\omega_1 \beta_1' \beta_1 \Phi_1$ . It is noted that each component of  $-\omega_1 \beta_1' \beta_1 \Phi_1$  is positive because  $\beta_1' < 0$ . Therefore, the input energy  $E_I^{(2)}$  by the second impulse is expressed by

$$E_I^{(2)} = \frac{1}{2}(-\omega_1 \beta_1' \beta_1 \Phi_1 + \beta_1 \Phi_1 V)^T \mathbf{M}(-\omega_1 \beta_1' \beta_1 \Phi_1 + \beta_1 \Phi_1 V) - \frac{1}{2}(-\omega_1 \beta_1' \beta_1 \Phi_1)^T \mathbf{M}(-\omega_1 \beta_1' \beta_1 \Phi_1) \\ = (-\omega_1 \beta_1' V + \frac{1}{2} V^2)(\beta_1 \Phi_1)^T \mathbf{M}(\beta_1 \Phi_1) \quad (24)$$

$E_I^{(2)} + E_S$  is the total mechanical energy just after the second impulse input. When  $E_h^{(2), s_i^{(2)}} \leq E_I^{(2)} + E_S \leq E_h^{(2), s_{i+1}^{(2)}}$  is satisfied, the distribution  $\mathbf{d}_{\max}^{(2)}$  of the maximum interstory drifts after the second impulse can be expressed as

$$\mathbf{d}_{\max}^{(2)} = (1-q)\mathbf{d}^{(2), s_i^{(2)}} + q\mathbf{d}^{(2), s_{i+1}^{(2)}} \quad (0 \leq q \leq 1) \quad (25)$$

The maximum restoring forces  $\mathbf{f}_R^{(2)}$  and the total hysteretic energy  $E_h^{(2)} (= \sum_{j=1}^N E_{h,j}^{(2)})$  when the model reaches  $\mathbf{d}_{\max}^{(2)}$  can also be derived in a similar way. The following quadratic equation in terms of  $q$  is formulated based on the energy balance law.

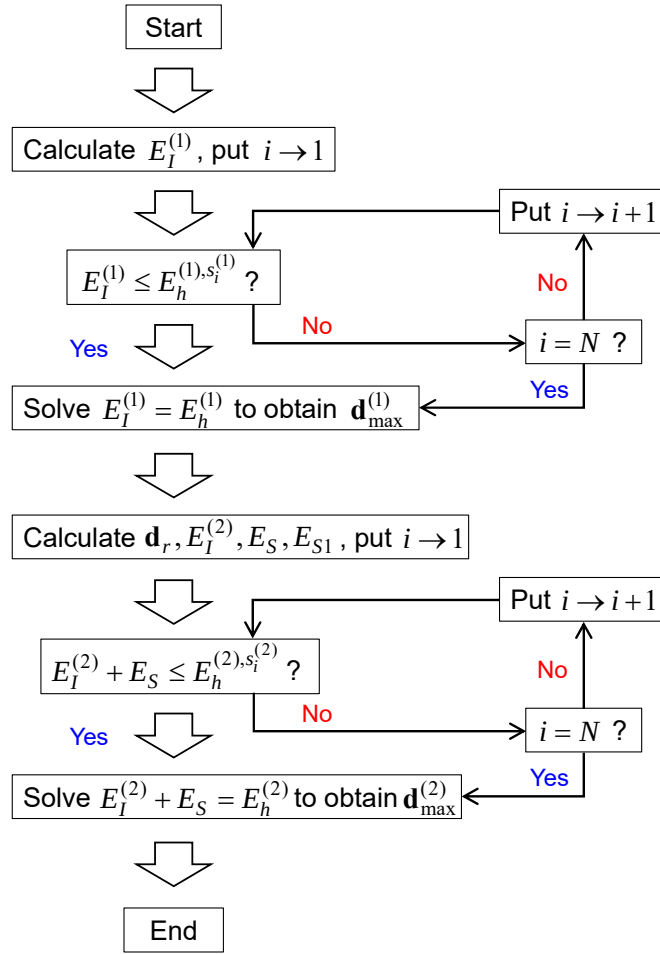
$$E_I^{(2)} + E_S = E_h^{(2)} \quad (26)$$

By solving Equation (26) and substituting the obtained  $q$  into Equation (25),  $\mathbf{d}_{\max}^{(2)}$  is derived. It is noted that the case of  $E_I^{(2)} + E_S > E_h^{(2), s_N^{(2)}}$  (all the stories yield) and the case of  $E_I^{(2)} + E_S < E_h^{(2), s_1^{(2)}}$  (all stories remain elastic) are not formulated here. However, the maximum responses in these cases can also be derived in a similar way. Figure 6 shows the flowchart for deriving  $\mathbf{d}_{\max}^{(1)}, \mathbf{d}_{\max}^{(2)}$ .

It should be pointed out that the accurate evaluation of the distributions of the maximum floor accelerations by UMEA is difficult. Although the distributions of the floor accelerations corresponding to  $\mathbf{d}_{\max}^{(1)}, \mathbf{d}_{\max}^{(2)}$  can be derived easily by equating the inertial forces with the restoring forces, the modal coupling effect and the free vibration responses after attaining  $\mathbf{d}_{\max}^{(1)}, \mathbf{d}_{\max}^{(2)}$  are neglected in the evaluation. The excitation of the elastic and plastic higher-mode responses affects largely the distributions of the maximum floor accelerations.

It is noted that  $\mathbf{d}_{\max}^{(1)}$  corresponds approximately to the maximum interstory drifts under a single impulse input or a half-cycle sine wave with the critical input period. Although the half-cycle sine wave with the critical input period excites higher-mode responses slightly, its effect may be negligible.

It is also noted that the effect of the structural damping on the maximum interstory drifts under PDI and one-cycle sine wave will be investigated in Appendix 2.

Figure 6 Flowchart for deriving  $\mathbf{d}_{\max}^{(1)}, \mathbf{d}_{\max}^{(2)}$ .

## 5. Numerical examples

In Sections 5.1 and 5.2, the time-history response analysis is conducted to check the accuracy of the proposed approximate closed-form expression for elastic-plastic MDOF models under the critical PDI and the corresponding one-cycle sine wave. As stated in Sections 2.1 and 4, the second impulse of the critical PDI acts at the timing where the sum of the elastic fundamental mode inertial forces attains zero to maximize the total input energy. The amplitude of the one-cycle sine wave is modulated so that the Fourier amplitude of the one-cycle sine wave corresponds to that of DI in the range of  $0 \leq \omega \leq 2\pi / t_0$  (see Figure 1) [27]. In Section 5.3, the criticality of the input timing  $t_0$  is investigated by the time-history response analysis for the elastic-plastic MDOF models under PDI and the one-cycle sine wave with various  $t_0$ .  $V = 0.3, 0.6, 0.9, 1.2, 1.5$  [m/s] are used in Sections 5.1-5.3. In Section 5.4, the maximum interstory drifts under recorded near-fault ground motions are compared with those under PDI and the corresponding one-cycle sine wave to investigate the applicability of the proposed method to recorded ground motions.

Three undamped shear building models of 12 stories with different story stiffness distributions are treated in Sections 5.1-5.4. Model 1 has a uniform distribution of story stiffnesses. Model 2 has the uniform story stiffness distribution at every four stories (1-4, 5-8, 9-12: stiffness ratio is 2:1.5:1 from the bottom). Model 3 has a trapezoidal distribution of story stiffnesses ( $k_1 / k_{12} = 2.5$ ). The undamped fundamental natural period of these models is 1.2[s]. All the floor masses have the same value ( $m_i = 400 \times 10^3$  [kg]). The common story height is 4[m] and the common yield interstory drift  $d_y$  is set to 4/150 [m]. The models have bilinear hysteresis and  $\alpha = 0.01, 0.1, 0.2$  are adopted. It is noted that  $\alpha = 0.01$  is unrealistic for the modeling of the multi-story frame structures as shear mass models. In other words, it is not appropriate to regard the maximum restoring force as the yield force for multi-story frame structures because the plastic hinges are not formed simultaneously in all the beams on the same floor. Moreover, the full range of the cross section seldom reaches the yield stress at once. Therefore,  $\alpha = 0.1 \sim 0.3$  may be appropriate although the case of  $\alpha = 0.01$  is treated just for reference.

It is noted that the maximum floor accelerations under the critical PDI and those under the corresponding one-cycle sine waves will be investigated in Appendix 1. Moreover, the applicability of the proposed method to the elastic-plastic moment-resisting frame will be investigated in Appendix 3.

## 5.1 PDI and proposed method

Figure 7 shows the distributions of the maximum interstory drifts by the time-history response analysis (THRA) under the critical PDI with  $V = 0.3, 0.6, 0.9, 1.2, 1.5$  [m/s] and those evaluated by the proposed method. In the case of  $V = 0.3$  [m/s], the distributions calculated by THRA coincide with the proposed closed-form solutions because the models remain elastic. In the case of  $\alpha = 0.01$ , the proposed method tends to overestimate the interstory drifts in the story which yields first, and the distributions of the maximum interstory drifts evaluated by the proposed method and those calculated by THRA do not correspond well. This is because the interstory drift of the yielding story in the plastic fundamental mode is much larger than those of the unyielding stories and the distributed kinetic energy to the plastic higher modes by yielding is relatively large. In the cases of  $\alpha = 0.1, 0.2$ , the distributions of the maximum interstory drifts evaluated by the proposed method correspond well to those calculated by THRA.

It is noted that the total computational time by the proposed method using UMEA is about 1/200 of that by THRA (time step increment: 0.001s,  $0 \leq t \leq 3$  s). In addition, the repetition of the convergence calculation due to the elastic-plastic responses is required for THRA. On the other hand, UMEA does not require such repetition, and the analysis is carried out easily and stably.

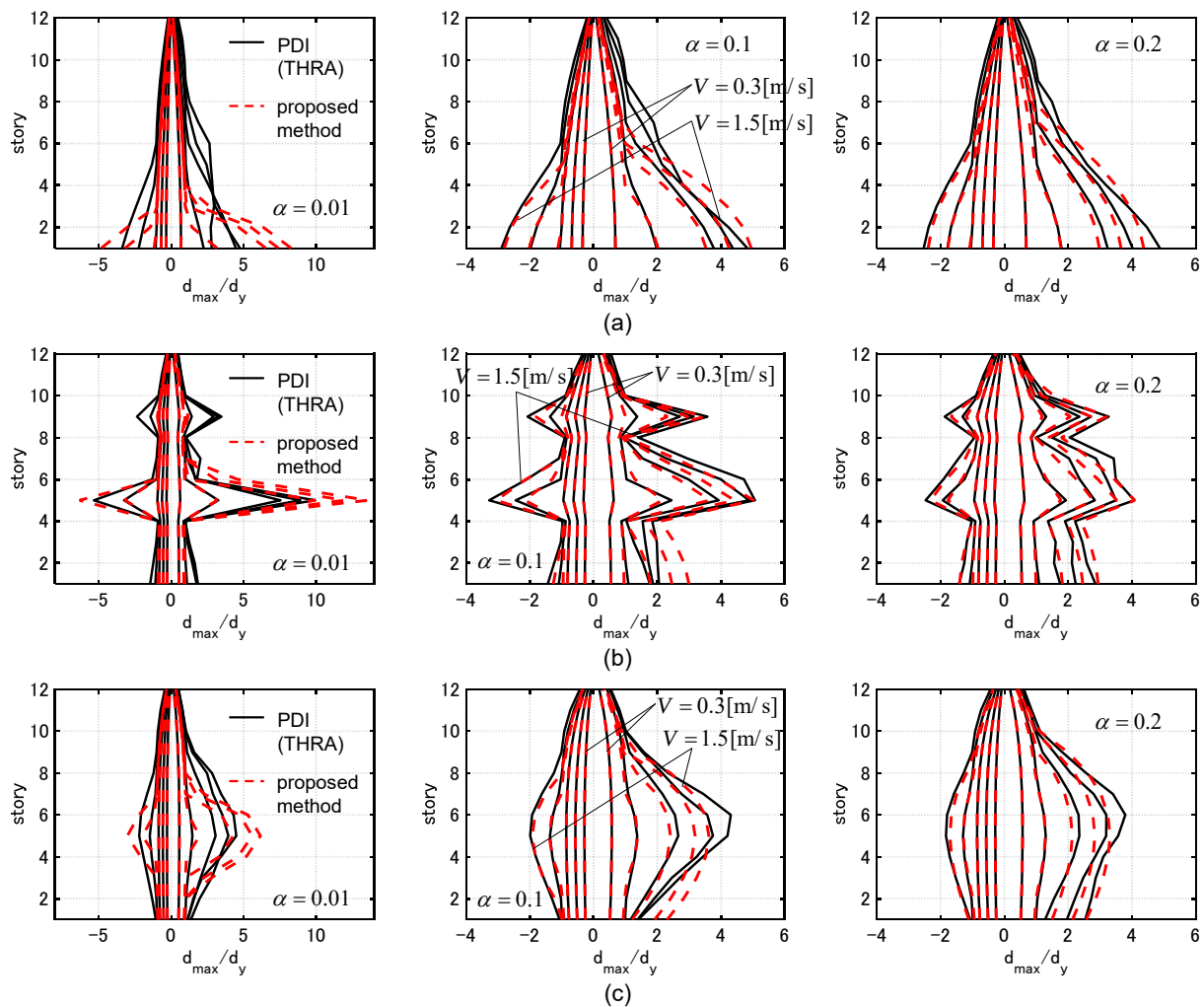


Figure 7 Distributions of maximum interstory drifts under critical PDI and those evaluated by proposed method, (a) Model 1, (b) Model 2, (c) Model 3.

## 5.2 Corresponding one-cycle sine wave and proposed method

Figure 8 shows the distributions of the maximum interstory drifts by THRA under the corresponding one-cycle sine wave with  $V = 0.3, 0.6, 0.9, 1.2, 1.5$  [m/s] and those evaluated by the proposed method. In the case of  $V = 0.3$  [m/s], the models remain elastic and the distributions calculated by THRA correspond well to the proposed closed-form solutions. In the case of  $\alpha = 0.01$ , the proposed method tends to overestimate the interstory drifts in the story which yields first for Models 2 and 3, as in the case of the critical PDI. It is pointed out that, the proposed method tends to underestimate the interstory drifts in the lower stories and those under the critical PDI (Figure 7) tend to be smaller than those under the corresponding one-cycle sine wave. This is because the responses in the lower stories are sensitive to the ground motions and the second stiffness is almost zero. In the cases of  $\alpha = 0.1, 0.2$ , the distributions of the maximum interstory drifts evaluated by the proposed method correspond well to those calculated by THRA. Moreover, the distributions of the maximum interstory

drifts under the critical PDI correspond well to those under the one-cycle sine wave. It is noted that the plastic deformation concentration in the lower stories is prevented under the one-cycle sine wave when the second stiffness is not too small.

Figure 9 shows the displacement response time-history in the first story and the top story for Model 3 ( $\alpha = 0.01, 0.2$ ) under the critical PDI and the corresponding one-cycle sine wave with  $V = 1.5[\text{m/s}]$ . The displacement response in the first story under the critical PDI does not correspond well to that under the one-cycle sine wave in the case of  $\alpha = 0.01$ . However, the top displacement responses under the critical PDI correspond well to those under the one-cycle sine wave in both cases of  $\alpha = 0.01, 0.2$  except for the phase lag between them.

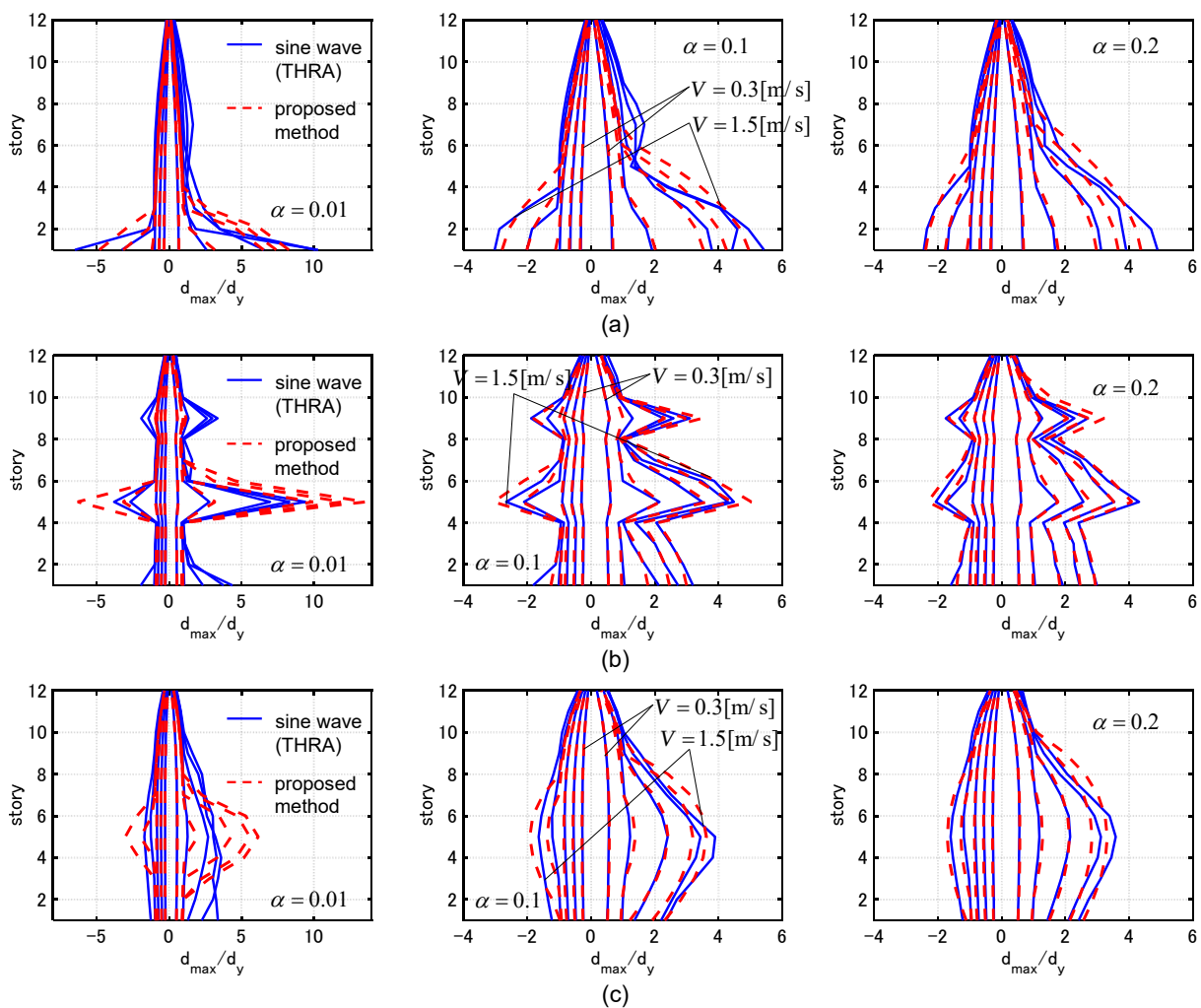


Figure 8 Distributions of maximum interstory drifts under corresponding one-cycle sine waves and those evaluated by proposed method, (a) Model 1, (b) Model 2, (c) Model 3.

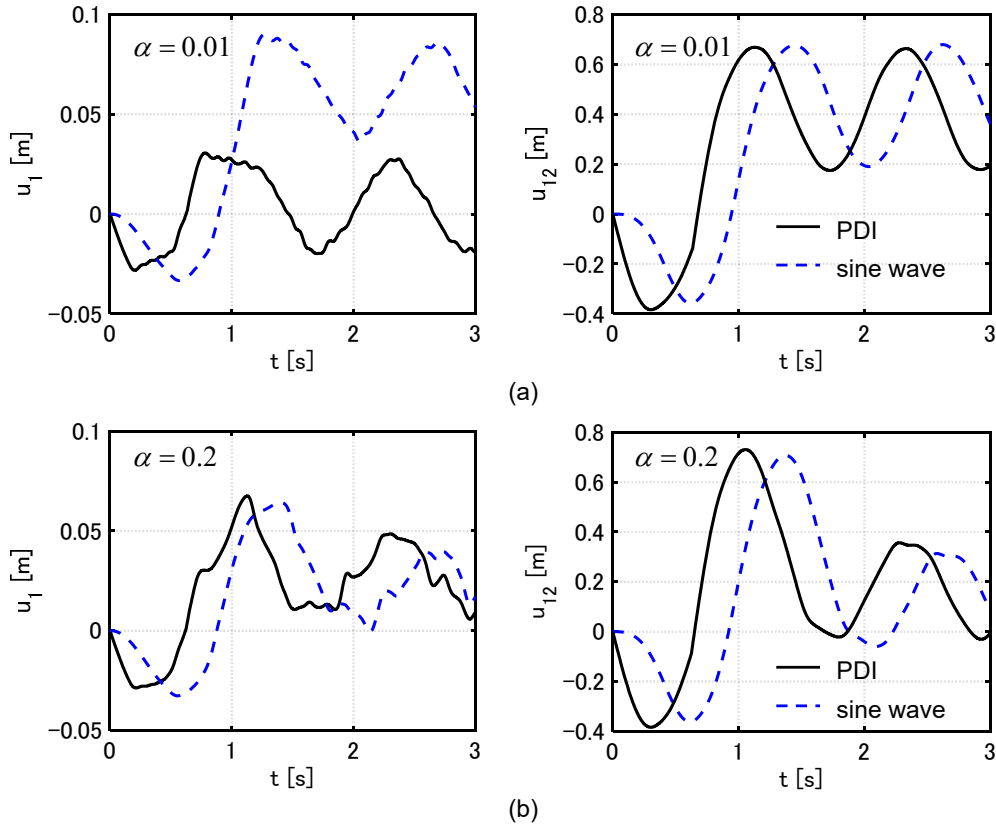


Figure 9 Displacement response time-history of first story and top story under critical PDI and corresponding one-cycle sine wave with  $V = 1.5[\text{m/s}]$  (Model 3), (a)  $\alpha = 0.01$ , (b)  $\alpha = 0.2$

### 5.3 Accuracy check of criticality in maximum response

Figures 10-12 show the maximum interstory drifts among all the stories with respect to  $t_0$  under PDI and the one-cycle sine wave. The estimated maximum interstory drifts by the proposed method, which are obtained in Sections 5.1 and 5.2, are also plotted with respect to  $t_0$ . In the case of  $\alpha = 0.01$ , the proposed method overestimates the maximum interstory drifts as explained in Section 5.1 and 5.2. In the case of Model 2 with  $\alpha = 0.1$  (Figure 11),  $d_{\max}$  has multiple peaks with respect to  $t_0$  under the one-cycle sine wave with  $V = 1.2, 1.5$ . This is because the interstory drift in the 9-th story, which has relatively small stiffness, tends to exhibit large plastic deformation under the one-cycle sine wave with small  $t_0$ . Except for the above-mentioned cases, the maximum interstory drifts calculated by THRA correspond well to those evaluated by the proposed method. Moreover, the proposed method effectively captures the peak values of  $d_{\max}$  under the condition of constant  $V$ . In these cases, the errors between  $d_{\max}$  by THRA and those estimated by the proposed method are within about 10 %.

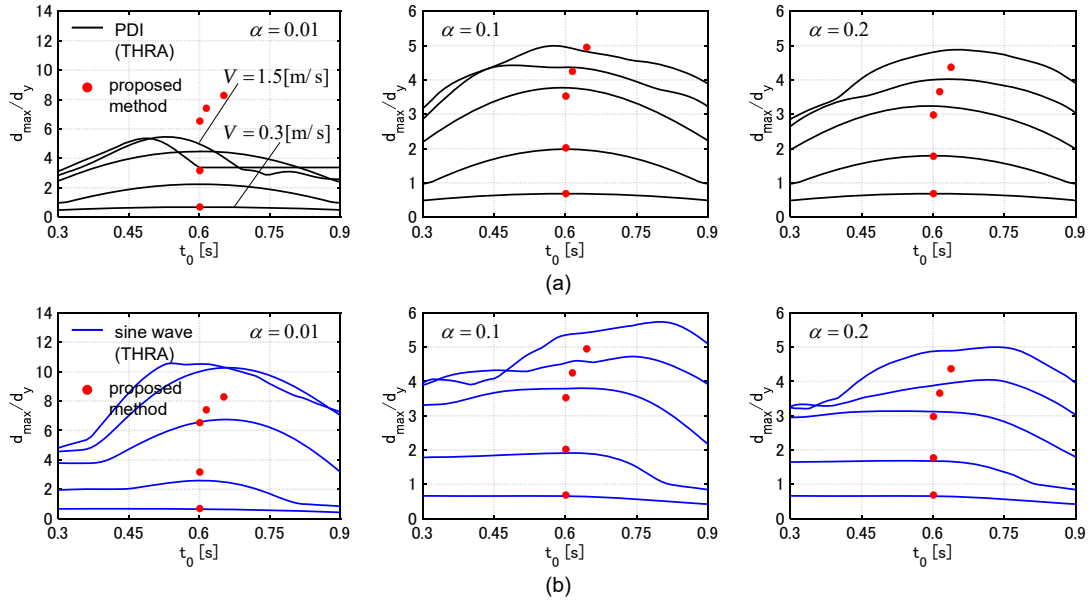


Figure 10 Maximum interstory drifts with respect to  $t_0$  (Model 1), (a) PDI and proposed method, (b) corresponding one-cycle sine wave and proposed method.

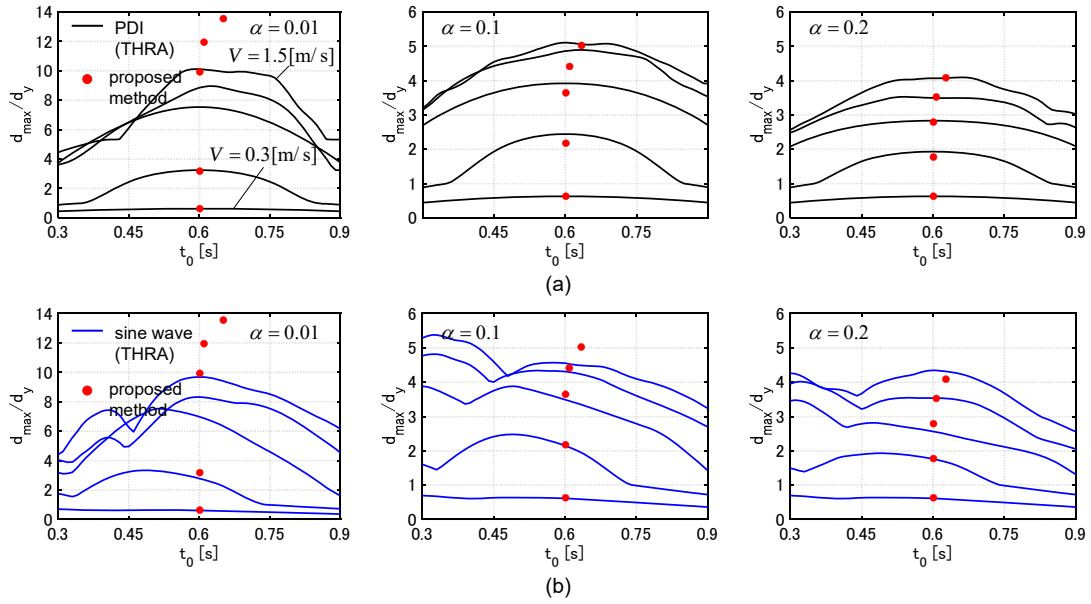


Figure 11 Maximum interstory drifts with respect to  $t_0$  (Model 2), (a) PDI and proposed method, (b) corresponding one-cycle sine wave and proposed method.

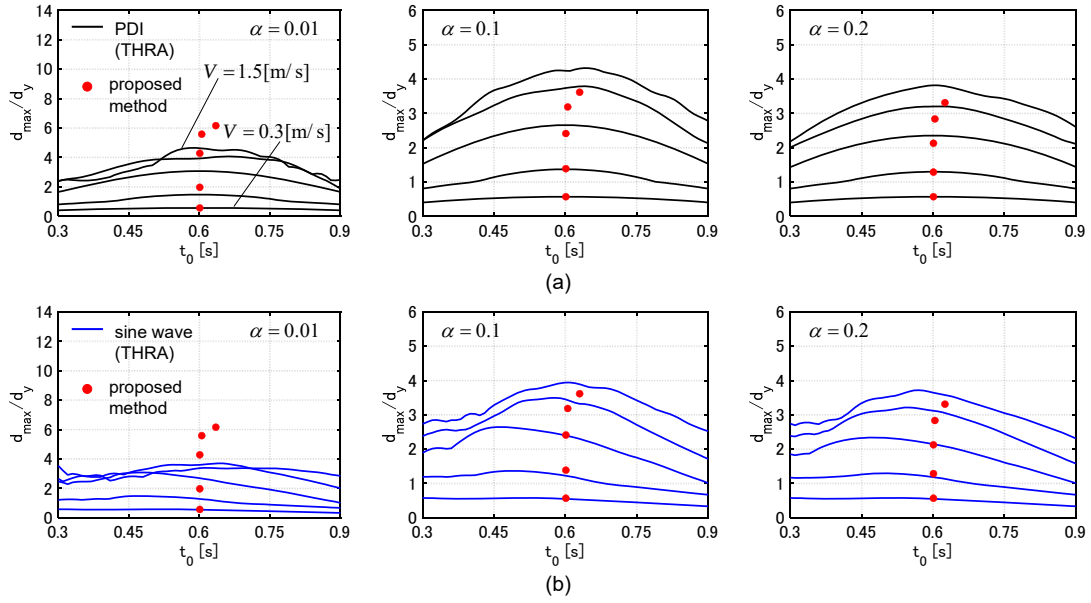


Figure 12 Maximum interstory drifts with respect to  $t_0$  (Model 3), (a) PDI and proposed method, (b) corresponding one-cycle sine wave and proposed method.

#### 5.4 Comparison of responses under recorded ground motions, PDI and one-cycle sine wave

In this section, the Rinaldi station fault-normal component during the Northridge earthquake in 1994 (Rinaldi Sta. FN) and the Kobe University NS component during the Hyogoken-Nanbu (Kobe) earthquake in 1995 (Kobe Univ. NS) and KMMH16 EW component during the Kumamoto earthquake (4/16/2016) are employed as the recorded near-fault motions. The main part of each motion is approximated by a one-cycle sine wave following the procedure shown in [27, 29]. The acceleration amplitude and the period of the approximated one-cycle sine wave for the Rinaldi station fault-normal component are  $A_p = 7.85[\text{m/s}^2]$ ,  $T_p = 0.8[\text{s}]$  and those for the Kobe University NS component are  $A_p = 2.6[\text{m/s}^2]$ ,  $T_p = 1.0[\text{s}]$ , and those for KMMH16 EW component are  $A_p = 7.5[\text{m/s}^2]$ ,  $T_p = 0.8[\text{s}]$ . The time interval  $t_0$  of impulses in PDI is given by  $T_p / 2$ . The velocity amplitudes  $V = 0.25, 1.0[\text{m/s}]$  are selected for DI and PDI. The amplitudes of the ground motions and the approximated one-cycle sine waves are adjusted to the levels of PDI. The models perform elastically under PDI and the one-cycle sine wave with  $V = 0.25[\text{m/s}]$ . Figure 13 shows the adjusted ground accelerations of the recorded motions and the approximated one-cycle sine waves.

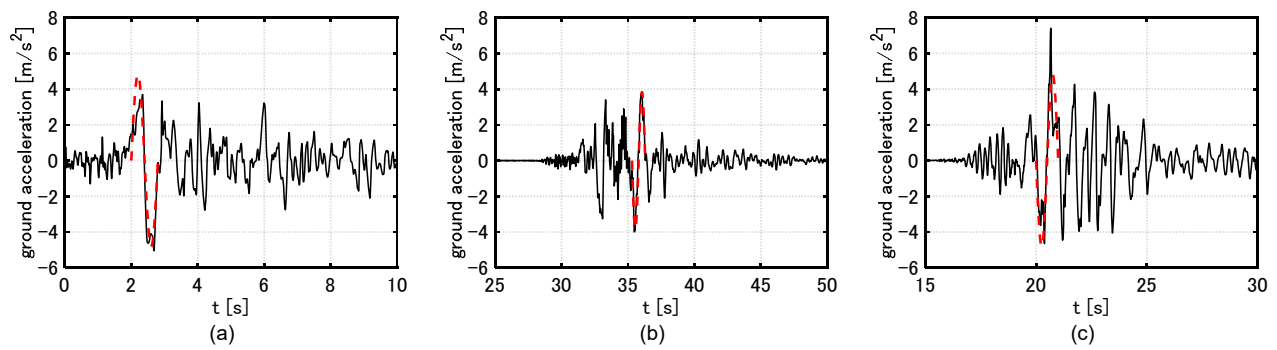


Figure 13 Ground accelerations of recorded ground motions and approximated one-cycle sine waves whose amplitudes are adjusted to the maximum velocity level  $V = 1.0[\text{m/s}]$  of DI and PDI, (a) Rinaldi Sta. FN, (b) Kobe Univ. NS, (c) KMMH16 EW.

Figure 14 shows the distributions of the maximum interstory drifts by THRA of Models 1, 2 and 3 ( $\alpha = 0.1$ ) under PDI, the approximated one-cycle sine waves and Rinaldi Sta. FN with  $V = 0.25, 1.0[\text{m/s}]$ . The estimated distributions under the critical PDI with  $V = 0.25, 1.0[\text{m/s}]$  are also plotted. The maximum interstory drifts under PDI are slightly smaller than those evaluated by the proposed method because the time intervals  $t_0$  of PDI are not critical ones. The maximum interstory drifts under the one-cycle sine wave with  $V = 0.25[\text{m/s}]$  are also slightly smaller than the estimated values. This is because  $T_p (= 2t_0)$  is about 1.8-2.0 times longer than the second natural periods of the models and the higher-mode displacement responses are hardly excited. On the other hand, the maximum interstory drifts in the upper stories under the one-cycle sine wave with  $V = 1.0[\text{m/s}]$  tend to be larger than those evaluated by the proposed method due to the excitation of the plastic higher modes. The maximum interstory drifts in the upper stories under Rinaldi Sta. FN with  $V = 0.25[\text{m/s}]$  also tend to be larger than those evaluated by the proposed method because the higher-mode responses are excited. As in the case of the one-cycle sine wave with  $V = 1.0[\text{m/s}]$ , the maximum interstory drifts in the upper stories under Rinaldi Sta. FN with  $V = 1.0[\text{m/s}]$  tend to be larger than those evaluated by the proposed method. However, regarding the maximum interstory drifts among all the stories, the estimated values by the proposed method correspond relatively well to those under the one-cycle sine wave and Rinaldi Sta. FN.

Figure 15 presents the distributions of the maximum interstory drifts by THRA of Models 1, 2 and 3 ( $\alpha = 0.1$ ) under PDI, the approximated one-cycle sine waves and Kobe Univ. NS with  $V = 0.25, 1.0[\text{m/s}]$ . The estimated distributions under the critical PDI with  $V = 0.25, 1.0[\text{m/s}]$  are also plotted. The distributions of the maximum interstory drifts under PDI and the one-cycle sine wave with  $V = 0.25, 1.0[\text{m/s}]$  correspond well to those evaluated by the proposed method because the time intervals  $t_0$  are relatively close to the critical ones. On the other hand, the maximum interstory drifts under Kobe Univ. NS with  $V = 0.25[\text{m/s}]$  are 2-3 times larger than those evaluated by the proposed method. This is because the ground acceleration of Kobe Univ. NS becomes large at multiple timings and the hysteretic energy dissipation does not occur in the elastic response range. In the case of  $V = 1.0[\text{m/s}]$ , the maximum interstory drifts in the upper stories under Kobe Univ. NS

tend to be larger than those evaluated by the proposed method because of the excitation of the plastic higher-mode responses. However, the maximum interstory drifts among all the stories under Kobe Univ. NS correspond well to the estimated values due to the hysteretic energy dissipation and the cumulative plastic deformations.

Figure 16 shows the distributions of the maximum interstory drifts by THRA of Models 1, 2 and 3 ( $\alpha = 0.1$ ) under PDI, the approximated one-cycle sine waves and KMMH16 EW with  $V = 0.25, 1.0$  [m/s]. When  $V = 0.25$  [m/s], the maximum interstory drifts under KMMH16 EW are about 1.5 times larger than those evaluated by the proposed method. This comes from the absence of the hysteretic energy dissipation, as in the cases of Kobe Univ. NS with  $V = 0.25$  [m/s]. On the other hand, in the cases of  $V = 1.0$  [m/s], the maximum interstory drifts evaluated by the proposed method are about 1.5 times larger than those under KMMH16 EW. This is because KMMH16 EW is not critical for the models.

It is noted that recorded ground motions have their own natures, and they are not always critical input for the models. To directly treat the uncertainty of the inputs, the critical PDI was adopted in this paper. The critical excitation method can rationally estimate the upper limit of the maximum responses. From this point of view, it can be said that the proposed method is helpful for improving the seismic evaluation because it can efficiently obtain the critical maximum responses.

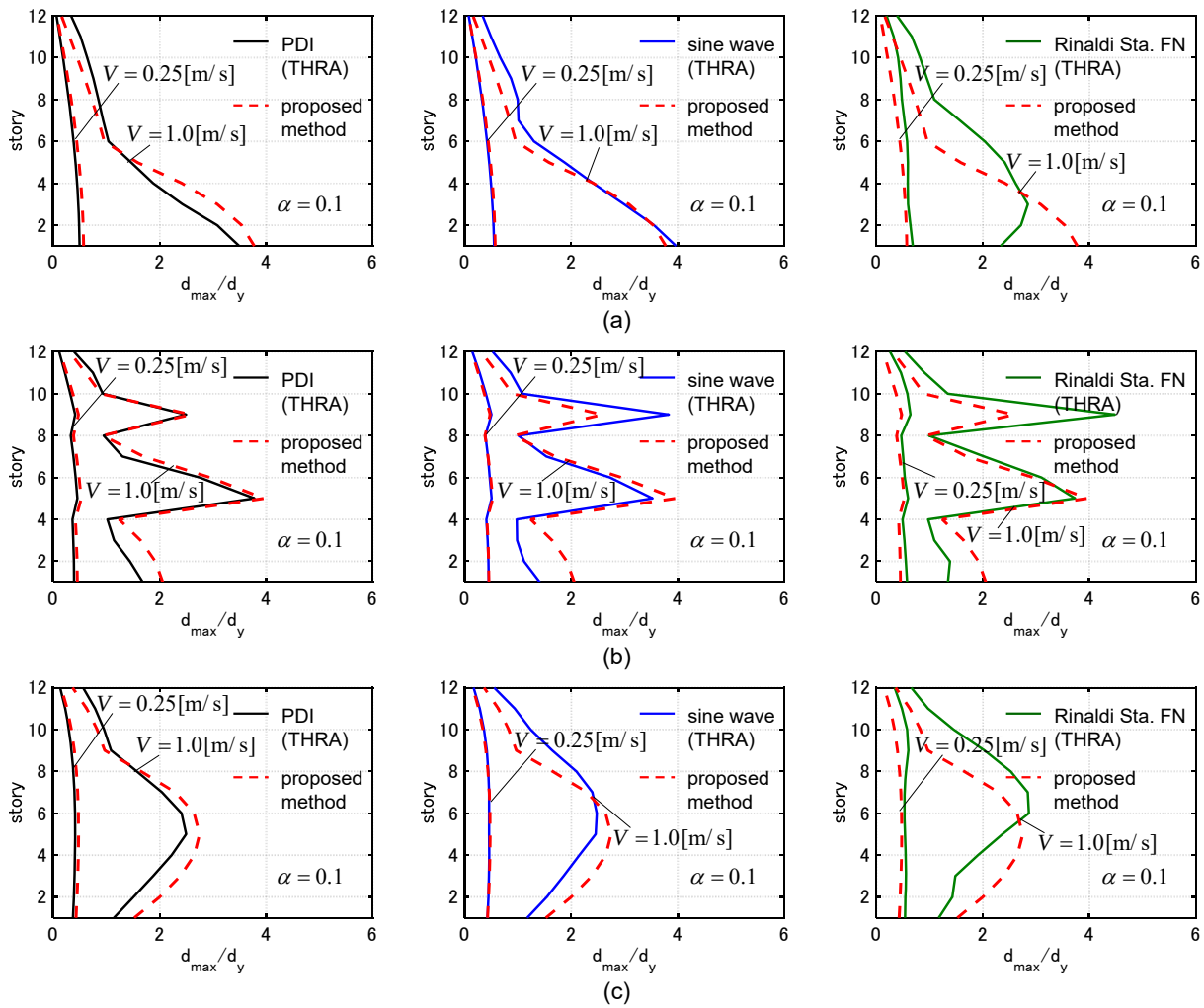


Figure 14 Distributions of maximum interstory drifts under Rinaldi Sta. FN, approximated one-cycle sine wave and PDI, (a) Model 1, (b) Model 2, (c) Model 3.

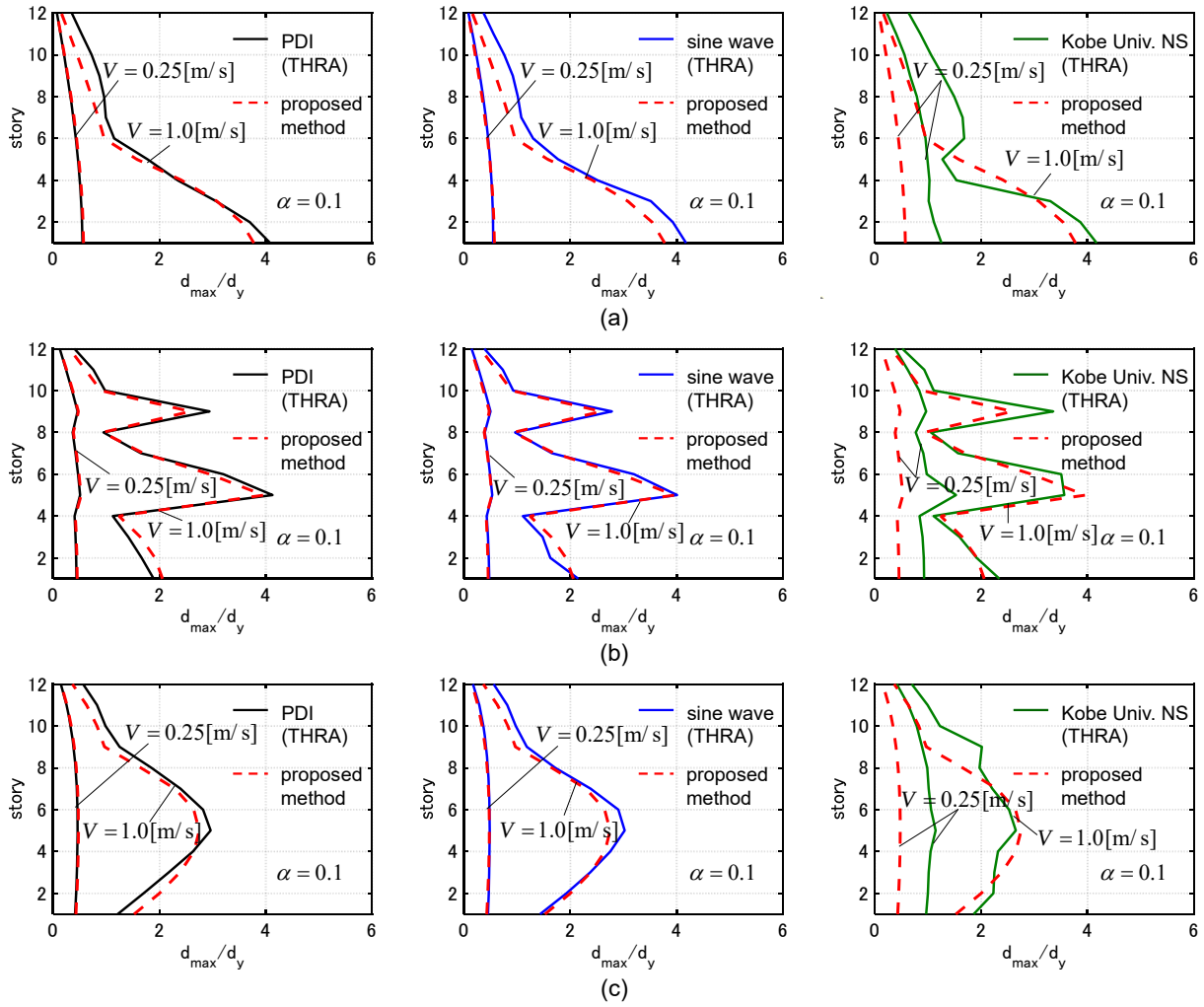


Figure 15 Distributions of maximum interstory drifts under Kobe Univ. NS, approximated one-cycle sine wave and PDI, (a) Model 1, (b) Model 2, (c) Model 3.

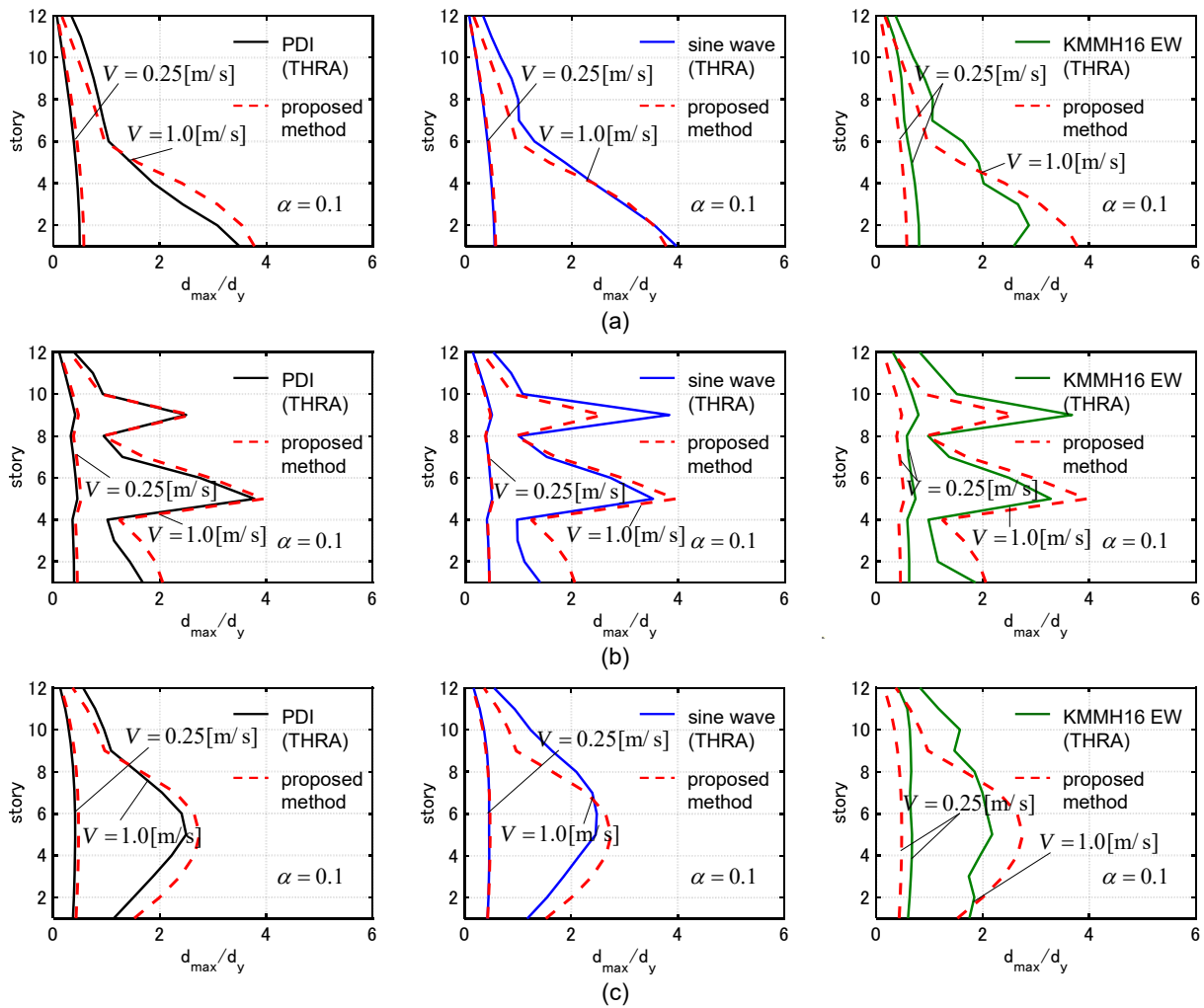


Figure 16 Distributions of maximum interstory drifts under KMMH16 EW, approximated one-cycle sine wave and PDI, (a) Model 1, (b) Model 2, (c) Model 3.

## 6. Conclusions

In this paper, approximate closed-form maximum interstory drifts under the critical pseudo-double impulse (PDI) were derived for undamped multi-story shear building models with bilinear hysteresis to estimate simply the response under the resonant one-cycle sine wave. The main conclusions can be summarized as follows.

- (1) PDI was proposed in order to treat the critical responses of elastic-plastic MDOF models under the one-cycle sine wave in the previous paper. In this paper, approximate closed-form maximum interstory drifts under the critical PDI were derived by using the updated mode-controlled energy-based approach (UMEA). It has demonstrated through numerical examples that the distributions of the maximum interstory drifts by the time-history analysis (THRA) under the critical PDI and the

corresponding one-cycle-sine wave correspond well to those evaluated by the proposed method except for the case of  $\alpha_i \approx 0$  (elastic perfectly-plastic model).

- (2) UMEA was newly proposed for deriving the approximate closed-form maximum interstory drifts under the critical PDI. UMEA is a kind of displacement-controlled analyses, and the displacement increment, which is proportional to the fundamental mode evaluated by the tangent story stiffnesses, is given to the model. In the beginning of the analysis, the displacement increment, which is proportional to the elastic fundamental mode, is given. When one of the stories yields, the fundamental mode is re-evaluated by using the post-yield stiffness of that story and the displacement increment is updated. UMEA effectively expresses the plastic deformation concentration in specific stories under the critical PDI and the critical one-cycle sine wave.
- (3) In the case of  $N$ -story shear mass models with bilinear hysteresis, the estimation of the maximum interstory drifts under the critical PDI by UMEA requires  $2N$ -time evaluations of the elastic and plastic fundamental modes at most. UMEA is much more efficient in the evaluation of the maximum interstory drifts than THRA because THRA requires repetition of the matrix multiplication as many times as the number of time steps. Moreover, UMEA does not require the convergence analysis due to the yielding although THRA requires.
- (4) The distribution of kinetic energy to the plastic modes depends only on the velocity response at the moment of yielding. Except for the case of  $\alpha_i \approx 0$  (elastic perfectly-plastic model), most of the kinetic energy, which the fundamental mode has just before yielding, is given to the plastic fundamental mode just after yielding. This provides the validity of using UMEA to estimate the distributions of the maximum interstory drifts.
- (5) The criticality of the input timing has been demonstrated through THRA for elastic-plastic MDOF models under PDI with various time intervals  $t_0$  and the corresponding one-cycle sine waves. The maximum interstory drifts  $d_{\max}$  among all the stories calculated by THRA correspond well to those evaluated by the proposed method except for models with relatively small story stiffness in the upper stories. Moreover, the proposed method effectively captures the peak values of  $d_{\max}$  under the condition of constant input velocity amplitude  $V$ .
- (6) It has been demonstrated through THRA for elastic-plastic MDOF models that the maximum interstory drifts under recorded near-fault ground motions do not always correspond well to those evaluated by the proposed method because the influence of the main part (pulse-like motion) of near-fault ground motions on the peak response of interstory drifts depends on the correspondence between such main part and the corresponding sine wave.

UMEA was developed to capture the ‘critical’ responses of elastic-plastic MDOF models under the one-cycle sine wave. It should be emphasized that the critical excitation method can rationally treat various types of uncertainties induced during the seismic design of buildings [28]. This aspect may be important. It should also be pointed out that the total computational time by the proposed method using UMEA is much smaller than that by THRA. In addition, UMEA does not require the repetition of the

convergence calculation due to the elastic-plastic responses, and the analysis is carried out easily and stably.

## 7. Acknowledgement

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## Appendix 1: Comparison of maximum floor accelerations under PDI and one-cycle sine wave

Figure A1 shows the distributions of the maximum floor accelerations by the time-history response analysis (THRA) under the critical PDI with  $V = 0.3, 0.6, 0.9, 1.2, 1.5$  [m/s] and the corresponding one-cycle sine waves.  $\alpha = 0.1$  has been adopted here. As stated in Section 4, the accurate evaluation of the distributions of the maximum floor accelerations by UMEA is difficult because the modal coupling effect and the free vibration responses after attaining  $\mathbf{d}_{\max}^{(1)}, \mathbf{d}_{\max}^{(2)}$  are not negligible. It can be observed from Figure A1 that the correspondence between the maximum floor accelerations under the critical PDI and those under the one-cycle sine waves is fairly well except a few cases for larger input levels. While the input velocity level smaller than  $V = 0.6$  [m/s] is usually used for the design earthquake ground motions in the structural design of most buildings in the earthquake-prone countries, the cases for the input velocities larger than  $V = 0.6$  [m/s] are shown for additional information. Furthermore, interstory drifts are mainly used for the safety check of main structures and accelerations are used for the check of functionality and damage of non-structural members and facilities. It may be rational to think that, even if only the safety check of main structures is made, its role is important.

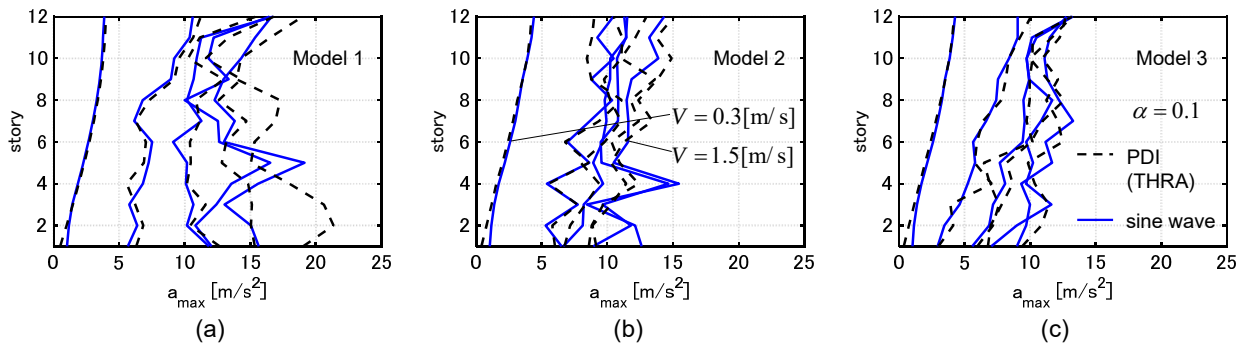


Figure A1 Distributions of maximum floor accelerations under critical PDI and corresponding one-cycle sine wave, (a) Model 1, (b) Model 2, (c) Model 3.

## Appendix 2: Effect of structural damping on maximum interstory drifts under PDI and one-cycle sine wave

The approximate closed-form expression of the maximum interstory drifts under the critical PDI cannot be applied to the models with structural damping. In this Appendix, a set of approximate closed-form expressions for the models with structural damping is derived.

The maximum displacement responses  $d_{\max, \text{SDOF}}^{(1)}(h), d_{\max, \text{SDOF}}^{(2)}(h)$  for the damped elastic SDOF model [29] are expressed by

$$d_{\max, \text{SDOF}}^{(1)}(h) = \left\{ -(4/3)h + \sqrt{(16/9)h^2 + 1} \right\} (V / \omega_1) \quad (\text{A1a}),$$

$$d_{\max, \text{SDOF}}^{(2)}(h) = \left\{ -(4/3)h + \sqrt{(16/9)h^2 + 1} \right\} \left( 1 + e^{-\pi h / \sqrt{1-h^2}} \right) (V / \omega_1) \quad (\text{A1b}).$$

$d_{\max, \text{SDOF}}^{(1)}(h) / d_{\max, \text{SDOF}}^{(1)}(0), d_{\max, \text{SDOF}}^{(2)}(h) / d_{\max, \text{SDOF}}^{(2)}(0)$  can then be obtained as follows.

$$d_{\max, \text{SDOF}}^{(1)}(h) / d_{\max, \text{SDOF}}^{(1)}(0) = -(4/3)h + \sqrt{(16/9)h^2 + 1} \quad (\text{A2a}),$$

$$d_{\max, \text{SDOF}}^{(2)}(h) / d_{\max, \text{SDOF}}^{(2)}(0) = \left\{ -(4/3)h + \sqrt{(16/9)h^2 + 1} \right\} \left( 1 + e^{-\pi h / \sqrt{1-h^2}} \right) / 2 \quad (\text{A2b}).$$

In the cases of the damped elastic-plastic MDOF models, the maximum interstory drifts  $\mathbf{d}_{\max}^{(1)}(h), \mathbf{d}_{\max}^{(2)}(h)$  in terms of the fundamental mode damping ratio  $h$  are approximated by

$$\mathbf{d}_{\max}^{(1)}(h) = \left\{ -(4/3)h + \sqrt{(16/9)h^2 + 1} \right\} \mathbf{d}_{\max}^{(1)}(0) \quad (\text{A3a}),$$

$$\mathbf{d}_{\max}^{(2)}(h) = \left\{ -(4/3)h + \sqrt{(16/9)h^2 + 1} \right\} \left\{ \left( 1 + e^{-\pi h / \sqrt{1-h^2}} \right) / 2 \right\} \mathbf{d}_{\max}^{(2)}(0) \quad (\text{A3b}).$$

where  $\mathbf{d}_{\max}^{(1)}(0), \mathbf{d}_{\max}^{(2)}(0)$  are the maximum interstory drifts of the undamped elastic-plastic MDOF models, which have been derived in Section 4. The stiffness-proportional type damping is assumed. Equations (A3a, b) means that the damping effect on the reduction of the maximum displacement responses for the SDOF models was applied to the elastic-plastic MDOF models. It is pointed out that such damping effect is not large for ground motions with short durations. In such cases, most of the input energies are dissipated by the hysteretic energies. Note that the values of Equations (A2a, b) are 0.987 and 0.972 when  $h = 0.01$ , and are 0.974 and 0.944 when  $h = 0.02$ .

Figure A2 shows the maximum interstory drifts of Model 3 under the critical PDI and the corresponding one-cycle sine waves with  $V = 0.3, 0.6, 0.9, 1.2, 1.5$  [m/s] compared to those by the proposed method.  $h = 0.01, 0.02$  and  $\alpha = 0.1$  are adopted. It can be observed that the maximum interstory drifts evaluated by the proposed method correspond fairly well to those under the critical PDI and those under the one-cycle sine waves. While the direct inclusion of viscous damping in the proposed formulation leads to much complicated expression on the maximum interstory drifts, the

approximation shown in this appendix can provide simple, but relatively accurate results as shown in Figure A2.

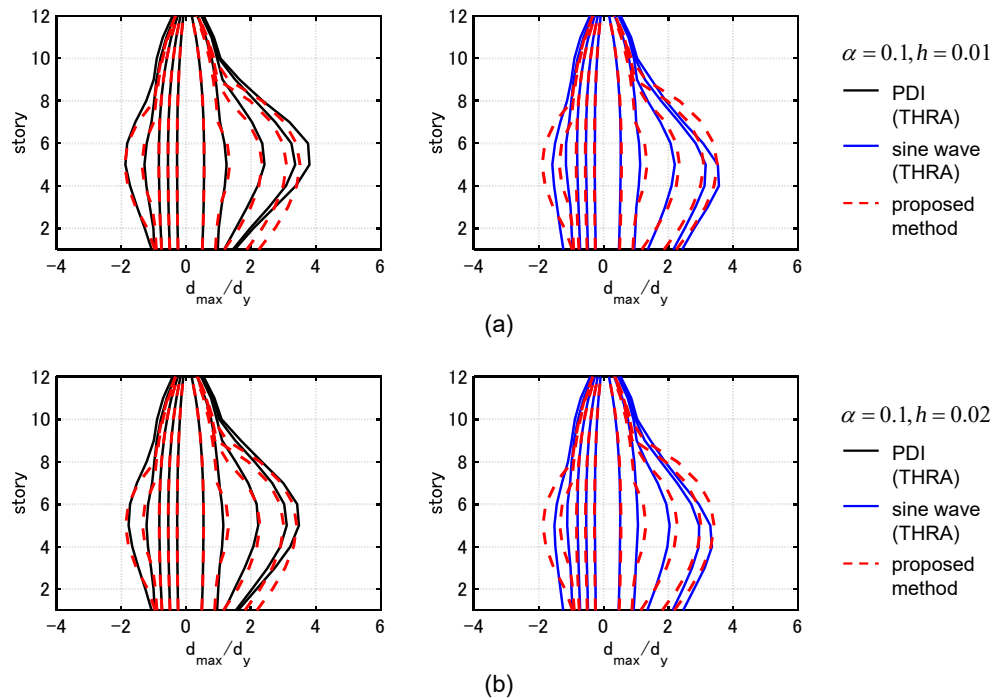


Figure A2 Distributions of maximum interstory drifts of Model 3 under critical PDI and corresponding one-cycle sine wave compared to those by proposed method, (a)  $h = 0.01$ , (b)  $h = 0.02$ .

### Appendix 3: Practical application of proposed method to elastic-plastic moment-resisting frame

In this section, the applicability of the proposed method to elastic-plastic moment-resisting frames is investigated.

In Japan, a shear mass system equivalently modeled from the moment-resisting frame is usually used for the preliminary seismic resistant design. Simple estimation of the maximum responses by using the equivalent shear mass system enables a prediction of the maximum responses with smaller computational time and leads to the better understanding of the dynamic response characteristics of the moment-resisting frame. The equivalent shear mass systems can be obtained through the following procedure; (1) the static pushover analysis to the moment-resisting frame, (2) the approximation of the story shear-story drift relation of each story by bilinear or trilinear hysteresis. It is noted that the distribution of the horizontal forces in the static pushover analysis is specified in the code for building seismic resistant design. It is also noted the values of the yield interstory deformation and the post-yielding stiffness ratio are determined based on the equivalence of the hysteretic energy of the original hysteresis of the frame model and that of the shear mass system.

An example of the transformation of a moment-resisting frame into an equivalent shear mass system is shown in Figure A3a. A 10-story 3-span planar moment-resisting frame is adopted here, and the details of the frame are shown in [32]. The parameters of the equivalent shear mass system are shown in Figure A3b. The values of  $d_y$  in the middle stories are relatively large, and those in the first and top stories are relatively small. In addition, the values of the post-yielding stiffness ratio  $\alpha$  are relatively small in the first and 9th, 10th stories. It is pointed out that, in the cases of 3D building frames, the values of  $\alpha$  become larger than those of planar frames because each vertical frame in the 3D building frames has different stiffness and strength properties and some of frames remain elastic after the yield deformation.

Figure A3c presents the maximum interstory drifts of the moment-resisting frame and the equivalent shear mass system under the one-cycle sine wave, which corresponds to the critical PDI with  $V = 0.9[\text{m/s}]$ . The structural damping ratio is set to 0.02 (stiffness-proportional type). It can be observed that the proposed method developed for the shear mass system can accurately estimate the maximum interstory drifts of the moment-resisting frame under the one-cycle sine wave.

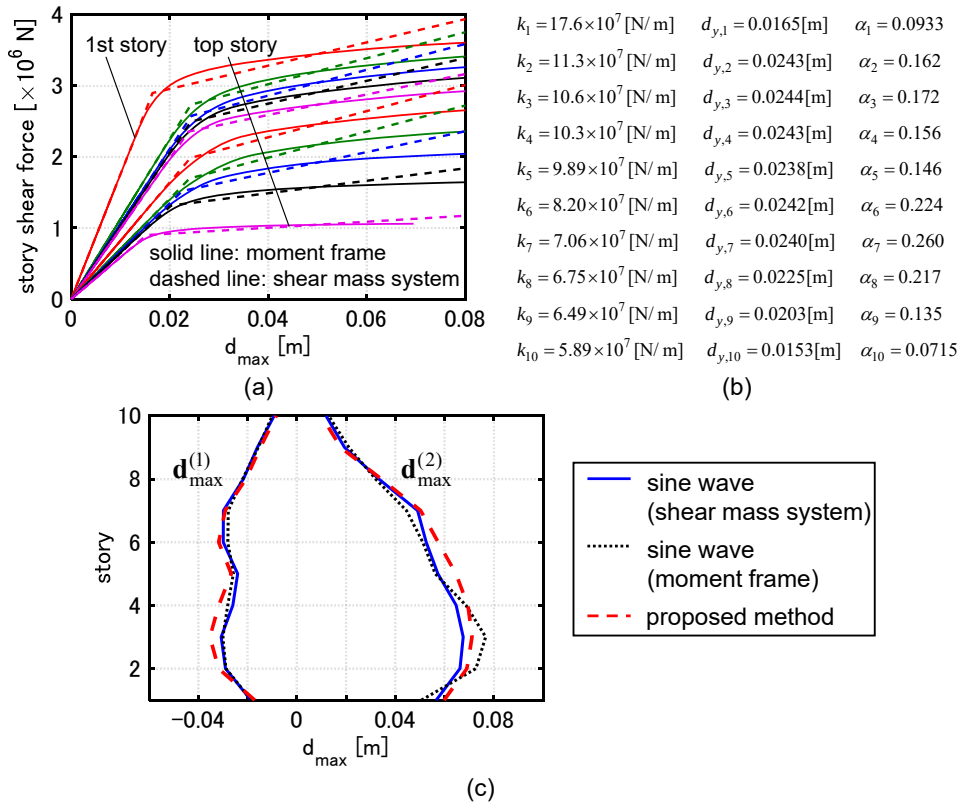


Figure A3 Comparison of results for moment-resisting frame and those for equivalent shear mass system, (a) story shear-story drift relation, (b) parameters of equivalent shear mass system, (c) distributions of maximum interstory drifts  $d_{\max}^{(1)}$ ,  $d_{\max}^{(2)}$  of moment-resisting frame and equivalent shear mass system compared to those by proposed method.