Control Parameter Dependence of the Work in a Process under Stochastic Control

Graduate School of Fundamental Sci & Tech. Keio Univ.
Hiroyuki Suzuki[1] and Youhei Fujitani

コロイド粒子を, 外場を制御しながら運ぶことを考えよう. 制御に要するエネルギーまで考えれば第二法則に縛られるが, 粒子がされる力学的仕事は自由エネルギー差より小さくなるだろう. この様子を見るため, 我々は粒子を有限時間で一次元的に一定距離運ぶことを考える. 粒子の位置と速度を観測しながら, それを基に外場を制御して、要する仕事の統計平均が最小になるように制御法を作る. 数値計算によると, このときの最小仕事は自由エネルギー差より小さい. また, 粒子速度を観測しないで制御すると最小仕事は上昇した.

The second law of thermodynamics tells that the quasistatic process minimizes the macroscopic work done to a system among all the isothermal process starting from one of its equillibrium state to another. This can be reproduced by the stochastic energetics in terms of the classical Langevin dynamics. Mesoscopically, we can control the external force by observing one or some of the fluctuating physical quantities. This control can reduce the minimum of mechanical work, which does not include the work needed for the control. We are interested in how the minimum depends on the kinds of physical quantities observed for the control.

We study a one-dimensional shift of a point-charge, with the charge q and the mass m, for a given distance a and during a given time-lapse from $t = 0$ to t_f. We control an external electric field E to minimize the mechanical work done to the point-charge. Its position x follows

$$m\frac{d^2x(t)}{dt^2} = -m\gamma\frac{dx(t)}{dt} + qE(t) + \xi(t) \tag{1}$$

where the noise ξ is assumed to be Gaussian white. The control theory requires definition of the evaluation functional $J[E]$. Although this should be the mechanical work done to the point-charge in our formulation, we push other two terms into the functional. One represents the constraint on the strength of the external field, which is required to converte the functional into a positive quadratic form. The other represents the constraint at the end-point, which is introduced to avoid two-point boundary-value problem. Thus, we define

[1]E-mail: hiroyuki@beer.appi.keio.ac.jp

$$J[E] = \langle \int_0^{t_f} qE(t)\frac{dx(t)}{dt} + R_1E(t)^2dt + R_2(x(t_f) - x_f)^2 \rangle \tag{2}$$

where R_1 and R_2 are control parameters without physical meanings, and $x(t)$ is stochastically determined by $E(\tau)$ with $\tau < t$. Because of (1).

Let us first observe both position x and velocity $\dot{x}$ of the point-charge, and the solution of the variational problem is given by a set of simultaneous equations of (1) and

$$E_{opt}(t) = -\frac{q}{R_1}\left[p_1(t)x(t) + \left(p_2(t) + \frac{1}{2}\right)\frac{dx(t)}{dt}\right] \tag{3}$$

where $p_1(t)$ and $p_2(t)$ satisfy Ricatti equations, appearing in the standard control theory. Equation (3) represents how we control $E(t)$ by observing $x(t)$ and $\dot{x}(t)$ to minimize $J[E]$.

The limit of $R_1 \to 0$ and $R_2 \to \infty$, where the strength of E is not constrained and the boundary condition at $t = t_f$ is imposed, is physically meaningful. Our numerical results are shown below. As $R_1 \to 0$ with a large enough R_2 value, the time average of E^2 increases as expected (Fig.1a), while the mechaniclal work $W = \int \mathrm{d}t\ qE(t)\dot{x}(t)$ approaches a finite value (Fig.1b). This finite value is found to remain unchanged as R_2 increases. We thus find physically meaningful minimum of the mechanical work to be well-defined, and to be smaller than the increase of the free energy during this process.

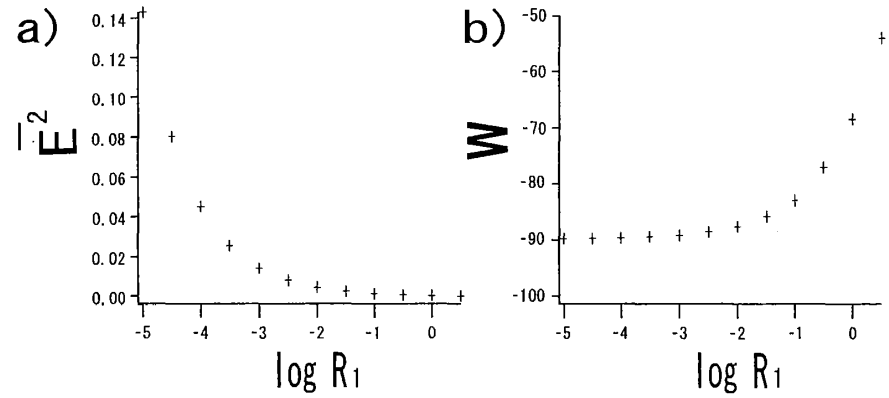

Figure 1: Cheap-Control Limit

The control above utilizes two observed quantities, $x(t)$ and $\dot{x}(t)$; Other control systems can be formulated by changing observed quantities. We find that the minimum increases when only $x(t)$ is utilized. The difference may reflect that of some information quantity between the two control systems.

We thank K. Sekimoto and S. Adachi for helpful discussions.

References

[1] K. Sekimoto, J.Pys.Soc.Jpn. **66** (1997), 1234.

[2] Y. Fujitani, T. Kojou and K. Inoue, J.Pys.Soc.Jpn. **72** (2003), 1300.