

算法發揮について

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第 I 部

算法發揮の消去法について

井関知辰の『算法發揮』(元禄 3 年, 1690) は上巻に未知数の消去法を述べ, 中巻に問題 7 題そして下巻にその解法を載せている. 上巻に載せられている消去法は, 現代風に言えば終結式を求めている. 行列式によって計算したと言われているが, 井関がどのようにして終結式を求めたかを考えてみたい. 算法發揮上巻に載せられている消去の方法に関して, 3 次式について考えてみる.

$$ax^3 + bx^2 + cx + d = 0 \cdots (I)$$

$$kx^3 + lx^2 + mx + n = 0 \cdots (II)$$

より x を消去する方法について, 本文とは異なるが次数の高いほうより書く.

$$a \times (II) - k \times (I)$$

$$(al - bk)x^2 + (am - ck)x + (an - dk) = 0 \cdots (1)$$

$$(1) \times x + b \times (II) - l \times (I)$$

$$(am - ck)x^2 + \{(bm - cl) + (an - dk)\}x + (bn - dl) = 0 \cdots (2)$$

$$(2) \times x + c \times (II) - m \times (I)$$

$$(an - dk)x^2 + (bn - dl)x + (cn - dm) = 0 \cdots (3)$$

$$al - bk = p \quad am - ck = q \quad an - dk = r$$

$$bm - cl = u \quad bn - dl = s \quad cn - dm = t \text{ とする.}$$

(1), (2), (3) 式を書く

$$px^2 + qx + r = 0 \dots (1)$$

$$qx^2 + (u + r)x + s = 0 \dots (2)$$

$$rx^2 + sx + t = 0 \dots (3)$$

(1), (3)2 式より 2 次の場合と同様に計算すれば x を消去することができ、このことは『算法天元録』に 2 つの 3 次式より 2 つの 2 次式を導き、2 つの 2 次式より 2 つの 1 次式を導き x を消去することが載せられている。この方法では p, q, r, s, t, u の 4 次式になる。(a, b, c, d, k, l, m, n については 8 次式になる。)

ところが、算法發揮や解伏題では (1), (2), (3)3 式を用いて x を消去しており、この方法では p, q, r, s, t, u の 3 次式になる。(a, b, c, d, k, l, m, n については 6 次式になる。)

以上の説明では算法發揮や解伏題の消去法が有効である事はわかりますが、なぜ (1), (2), (3)3 式を用いて x を消去することに考えが及んだのは不明のままでした。

以下に「なぜ (1), (2), (3)3 式を用いたのか」、について考えてみる。

まずはじめに (1), (3)2 式より x を消去してみる。

$$(pt - r^2)^2 + (qr - ps)(qt - rs) = 0$$

上式は p, q, r, s, t, u の 4 次式になるが、

$q \times s - p \times t = u \times r$ の関係があるので

(これは $(am - ck)(bn - dl) - (al - bk)(cn - dm) = (bm - cl)(an - dk)$ より明らか)

r で約せ 3 次式、 $-ptu - 2prt + ps^2 + q^2t - qrs + r^3 = 0 \dots (4)$ となる。

同様に (1), (2) 式, (2), (3) 式から x を消去しても上記 (4) 式となる。次に (4) 式を p, q, r について整理してみる。

$$p\{s^2 - (u + r)t\} - q(rs - qt) + r\{(u + r)r - qs\} = 0 \text{ と整理できる。}$$

この式が、算法發揮に載せられている結果である。このように先の (1)(2)(3) 式のいずれの 2 つを組み合わせても結果は (4) 式になること、(4) 式を上記のように整理することを考えれば (1)(2)(3)3 式を用いて計算することに考えが及んだものと思われる。

第 II 部

算法發揮術文の注

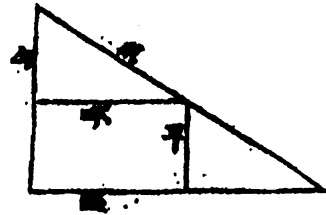
「氏(てい)」を見かけの近い文字「氏」としてしています。

1 第一問

1.1 問題

今図の如く勾股（直角三角形）内に長平（長方形）を内接させたものがある。

只 = 弦 + 外余積 又 = 勾 + 長 別 = 股 + 平
 が与えられたとき、勾、股、弦、長、平の長さを問う。



1.2 術文

勾を未知数とする。

$$\text{又} - \text{勾} = \text{長} \quad (\text{別} - \text{長}) \text{勾} = \text{角}$$

$$\text{勾} + \text{別} = \text{亢} \quad 2\text{只} - \text{勾} \times \text{別} = \text{氏}$$

$$\text{勾} + 2\text{長} = \text{房} \quad 4\text{勾}^2 + 4\text{別}^2 - \text{氏}^2 = \text{心}$$

$$8\text{別} + 2\text{氏} \times \text{房} = \text{尾} \quad \text{房}^2 - 4 = \text{箕}$$

$$\text{亢} \times \text{心} - \text{角} \times \text{尾} = \text{計} \quad \text{角} \times \text{箕} + \text{心} = \text{牛}$$

$$\text{亢} \times \text{箕} + \text{尾} = \text{女} \quad \text{計} \times \text{女} = \text{寄左}$$

$$\text{牛}^2 - (\text{寄左}) = 0$$

1.3 解説・演段

平を補助の未知数とする。

$$\text{勾} : \text{股} = \text{平} : (\text{股} - \text{長}) \quad \text{股} \times \text{平} = \text{勾}(\text{股} - \text{長})$$

$$(\text{別} - \text{平}) \text{平} = \text{勾}(\text{別} - \text{平} - \text{長})$$

$$-(\text{勾}^2 + \text{別} \times \text{勾} - \text{勾} \times \text{又}) + (\text{別} + \text{勾}) \text{平} - \text{平}^2 = 0$$

$$-\text{角} + \text{亢} \times \text{平} - \text{平}^2 = 0 \dots \text{前式}$$

$$\frac{1}{2} \text{勾} \times \text{股} = (\text{外余積}) + \text{長} \times \text{平} = (\text{只} - \text{弦}) + \text{長} \times \text{平}$$

$$2\text{弦} = 2\text{只} + 2\text{長} \times \text{平} - \text{勾} \times \text{股} \quad \text{弦}^2 = \text{勾}^2 + \text{股}^2$$

$$(2\text{只} + 2\text{長} \times \text{平} - \text{勾} \times \text{股})^2 = 4\text{勾}^2 + 4(\text{別} - \text{平})^2$$

$$\begin{aligned}
& 2\text{只} + 2(\text{又} - \text{勾})\text{平} - \text{勾}(\text{別} - \text{平}) = (2\text{只} - \text{勾} \times \text{別}) + (2\text{長} + \text{勾})\text{平} \\
& = 2\text{只} + 2\text{又} \times \text{平} - \text{勾} \times \text{平} - \text{勾} \times \text{別} = (2\text{只} - \text{勾} \times \text{別}) + (2\text{又} - \text{勾})\text{平} = \text{氏} + \text{房} \times \text{平} \\
& (2\text{只} - \text{勾} \times \text{別})^2 + 2(2\text{只} - \text{勾} \times \text{別})(2\text{又} - \text{勾})\text{平} + (2\text{又} - \text{勾})^2\text{平}^2 \\
& = 4\text{勾}^2 + 4(\text{別}^2 - 2\text{別} \times \text{平} + \text{平}^2) \\
& (4\text{勾}^2 + 4\text{別}^2 - \text{氏}^2) + (-8\text{別} - 2\text{氏} \times \text{房})\text{平} + (4 - \text{房}^2)\text{平}^2 = 0 \\
& (\text{氏}^2 - 4\text{勾}^2 - 4\text{別}^2) + (8\text{別} + 2\text{氏} \times \text{房})\text{平} + (\text{房}^2 - 4)\text{平}^2 = 0 \\
& -\text{心} + \text{尾} \times \text{平} + \text{箕} \times \text{平}^2 = 0 \dots \text{後式} \\
& \text{前式} \quad -\text{角} + \text{亢} \times \text{平} - \text{平}^2 = 0 \\
& \text{後式} \quad -\text{心} + \text{尾} \times \text{平} + \text{箕} \times \text{平}^2 = 0 \\
& \text{平陽図} \quad (\text{亢} \times \text{心} - \text{角} \times \text{尾}) + (-\text{心} - \text{角} \times \text{箕})\text{平} = 0 \dots \text{第一式} \\
& \quad \quad (-\text{心} - \text{角} \times \text{箕}) + (\text{尾} + \text{亢} \times \text{箕})\text{平} = 0 \dots \text{第二式} \\
& \text{陰図} \quad \text{計} \times \text{女} - \text{牛}^2 = 0
\end{aligned}$$

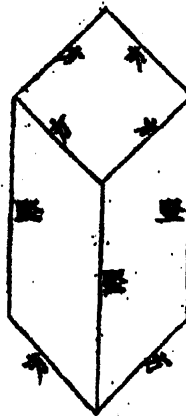
2 第二問

2.1 問題

今図のように方豎（底面が正方形の角柱）がある。

$$\text{只} = \text{積} \quad \text{又} = \sqrt{\text{方}} + \sqrt{\text{豎}} + \sqrt{\text{豎} - \text{方}}$$

が与えられたとき、方、豎の長さを問う。



2.2 術文

未知数を方とする。

$$4\text{方}^2 + \text{又}^4 = \text{角} \quad 4\text{又}^2 - 4\text{方} = \text{亢}$$

$$\text{角} \times \text{方}^2 - \text{亢} \times \text{積} = \text{氏} \quad \text{方}^2 \times \text{積} \times 16\text{又}^6 = \text{寄左}$$

$$\text{氏}^2 - (\text{寄左}) = 0$$

2.3 解説・演段

$\sqrt{\text{豎}}$ を補助の未知数とする.

$$-\text{只} + \text{方}^2 (\sqrt{\text{豎}})^2 = 0 \dots \text{前式}$$

$$\text{又} - \sqrt{\text{豎}} = \sqrt{\text{方}} + \sqrt{\text{豎} - \text{方}}$$

$$\text{又}^2 - 2 \text{又} \sqrt{\text{豎}} + \text{豎} = \text{方} + 2 \sqrt{\text{方}} \sqrt{\text{豎} - \text{方}} + \text{豎} - \text{方}$$

$$\text{又}^2 - 2 \text{又} \sqrt{\text{豎}} = 2 \sqrt{\text{方}} (\text{豎} - \text{方})$$

$$\text{又}^4 - 4 \text{又}^3 \sqrt{\text{豎}} + 4 \text{又}^2 \times \text{豎} = 4 \text{方} \times \text{豎} - 4 \text{方}^2$$

$$(\text{又}^4 + 4 \text{方}^2) - 4 \text{又}^3 \sqrt{\text{豎}} + (4 \text{又}^2 - 4 \text{方}) (\sqrt{\text{豎}})^2 = 0 \dots \text{後式}$$

$$\text{前式} \quad - \text{積} + \text{方}^2 (\sqrt{\text{豎}})^2 = 0$$

$$\text{後式} \quad \text{角} - 4 \text{又}^3 \sqrt{\text{豎}} + \text{亢} (\sqrt{\text{豎}})^2 = 0$$

$$\text{陽図} \quad 4 \text{積} \times \text{又}^3 + (-\text{角} \times \text{方}^2 - \text{亢} \times \text{積}) \sqrt{\text{豎}} = 0 \dots \text{第一式}$$

$$(-\text{角} \times \text{方}^2 - \text{亢} \times \text{積}) + 4 \text{方}^2 \times \text{又}^3 \sqrt{\text{豎}} = 0 \dots \text{第二式}$$

$$\text{陰図} \quad 16 \text{積} \times \text{又}^6 \times \text{方}^2 - \text{氏}^2 = 0$$

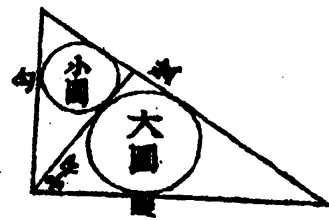
3 第三問

3.1 問題

今図のように勾股（直角三角形）内に中勾を隔てて大小円を内接させたものがある.

$$\text{只} = \text{外余積} + (\text{股} - \text{勾}) \quad \text{又} = (\text{大} - \text{小}) + (\text{弦} - \text{股})$$

が与えられたとき, 勾, 股, 弦, 大, 小の長さを問う.



3.2 術文

勾を未知数とする.

$$\text{勾} + 2 = \text{角} \quad 8 \text{勾} \times \text{円積法} - \text{角} = \text{亢}$$

$$\text{勾} - \text{又} = \text{氏} \quad 2 \text{勾} + 2 \text{勾}^2 \times \text{円積法} + 2 \text{只} = \text{房}$$

$$2 \text{角} \times \text{勾}^2 - 2 \text{氏} \times \text{房} = \text{心} \quad \text{亢} \times \text{氏} - \text{角} \times \text{氏} = \text{尾}$$

$$\text{角} + 2 \text{氏} \times \text{円積法} = \text{箕} \quad 2 \text{勾}^2 \times \text{亢} + 2 \text{勾}^2 \times \text{箕} - 2 \text{氏} \times \text{房} = \text{計}$$

$$4 \text{勾}^2 \times \text{円積法} + 2 \text{房} = \text{牛} \quad \text{亢} + 2 \text{氏} \times \text{円積法} = \text{女}$$

$$4 \text{勾}^2 \times \text{尾} \times \text{箕} \times \text{牛} + 2 \text{心} \times \text{牛}^2 = \text{寄左}$$

$$\text{勾}^2 \times \text{尾}^2 \times \text{女} + 2 \text{勾}^2 \times \text{箕}^2 \times \text{計} + \text{心} \times \text{計} \times \text{女} - (\text{寄左}) = 0$$

3.3 解説・演段

小: 本 = 勾 : 弦 小 × 弦 = 本 × 勾 本は直角三角形の内接円の直径

大: 本 = 股 : 弦 大 × 弦 = 本 × 股

$$(\text{大} - \text{小}) \text{弦} = \text{本} (\text{股} - \text{勾})$$

$$(\text{大}^2 + \text{小}^2) \text{弦}^2 = \text{本}^2 (\text{股}^2 + \text{勾}^2) = \text{本}^2 \times \text{弦}^2 \quad \text{大}^2 + \text{小}^2 = \text{本}^2$$

弦 - 股 を補助の未知数とする.

$$\text{本} = \text{勾} + \text{股} - \text{弦} = \text{勾} - (\text{弦} - \text{股}) = \text{甲}$$

股 - 勾 を弦 - 股 で表す. そのために股を求める.

$$\text{勾}^2 - (\text{弦} - \text{股})^2 = \text{勾}^2 - \text{弦}^2 - \text{股}^2 + 2 \text{弦} \times \text{股} = 2 \text{股} (\text{弦} - \text{股}) = \text{乙}$$

$$\text{乙} - 2 \text{勾} (\text{弦} - \text{股}) = 2 (\text{股} - \text{勾}) (\text{弦} - \text{股}) = \text{丙}$$

$$\text{只} = \text{外余積} + (\text{股} - \text{勾})$$

$$= \frac{1}{2} \text{勾} \times \text{股} - (\text{大}^2 + \text{小}^2) \text{円積法} + (\text{股} - \text{勾})$$

$$= \frac{\text{乙} \times \text{勾}}{4 (\text{弦} - \text{股})} - \text{本}^2 \times \text{円積法} + \frac{\text{丙}}{2 (\text{弦} - \text{股})}$$

$$4 \text{只} (\text{弦} - \text{股}) = \text{乙} \times \text{勾} - 4 \text{本}^2 \times \text{円積法} (\text{弦} - \text{股}) + 2 \text{丙}$$

$$= \{ \text{勾}^2 - (\text{弦} - \text{股})^2 \} \text{勾} - 4 \{ \text{勾} - (\text{弦} - \text{股}) \}^2 \text{円積法} (\text{弦} - \text{股})$$

$$+ 2 \{ \text{勾}^2 - (\text{弦} - \text{股})^2 - 2 \text{勾} (\text{弦} - \text{股}) \}$$

$$(\text{勾}^3 + 2 \text{勾}^2) + (-4 \text{勾}^2 \times \text{円積法} - 4 \text{勾} - 4 \text{只}) (\text{弦} - \text{股})$$

$$+ (-\text{勾} + 8 \text{勾} \times \text{円積法} - 2) (\text{弦} - \text{股})^2 - 4 \text{円積法} (\text{弦} - \text{股})^3 = 0 \dots \text{前式}$$

$$\text{又} = (\text{大} - \text{小}) + (\text{弦} - \text{股})$$

$$= \text{本} \frac{\text{股} - \text{勾}}{\text{弦}} + (\text{弦} - \text{股})$$

$$\{ \text{又} - (\text{弦} - \text{股}) \} \text{弦} = \text{本} (\text{股} - \text{勾})$$

$$\{ \text{又} - (\text{弦} - \text{股}) \} \text{弦} = \{ \text{勾} - (\text{弦} - \text{股}) \} \frac{\text{丙}}{2(\text{弦} - \text{股})}$$

$$\text{弦} = (\text{弦} - \text{股}) + \text{股} = (\text{弦} - \text{股}) + \frac{\text{乙}}{2(\text{弦} - \text{股})}$$

$$\{ \text{又} - (\text{弦} - \text{股}) \} \{ 2(\text{弦} - \text{股})^2 + \text{勾}^2 - (\text{弦} - \text{股})^2 \}$$

$$= \{ \text{勾} - (\text{弦} - \text{股}) \} \{ \text{勾}^2 - (\text{弦} - \text{股})^2 - 2 \text{勾} (\text{弦} - \text{股}) \}$$

$$(-\text{勾}^2 \times \text{又} + \text{勾}^3) + (-2 \text{勾}^2)(\text{弦} - \text{股}) + (-\text{又} + \text{勾})(\text{弦} - \text{股})^2 + 2(\text{弦} - \text{股})^3 = 0 \quad \dots \text{後式}$$

$$\text{前式} \quad \text{勾}^2 \times \text{角} - 2 \text{房} (\text{弦} - \text{股}) + \text{亢} (\text{弦} - \text{股})^2 - 4 \text{円積法} (\text{弦} - \text{股})^3 = 0$$

$$\text{後式} \quad \text{勾}^2 \times \text{氏} - 2 \text{勾}^2 (\text{弦} - \text{股}) + \text{氏} (\text{弦} - \text{股})^2 + 2(\text{弦} - \text{股})^3 = 0$$

陽図

$$2 \text{勾}^2 \times \text{房} - 2 \text{勾}^4 \times \text{角} + (-\text{勾}^2 \times \text{亢} \times \text{氏} + \text{勾}^2 \times \text{角} \times \text{氏})(\text{弦} - \text{股})$$

$$+ (4 \text{勾}^2 \times \text{氏} \times \text{円積法} + 2 \text{勾}^2 \times \text{角})(\text{弦} - \text{股})^2 = 0$$

$$(-\text{勾}^2 \times \text{亢} \times \text{氏} + \text{勾}^2 \times \text{角} \times \text{氏})$$

$$+ (4 \text{勾}^2 \times \text{氏} \times \text{円積法} + 2 \text{勾}^2 \times \text{角} + 2 \text{勾}^2 \times \text{亢} - 2 \text{氏} \times \text{房})(\text{弦} - \text{股})$$

$$+ (-8 \text{勾}^2 \times \text{円積法} - 4 \text{房})(\text{弦} - \text{股})^2 = 0$$

$$(4 \text{勾}^2 \times \text{氏} \times \text{円積法} + 2 \text{勾}^2 \times \text{角})$$

$$+ (-8 \text{勾}^2 \times \text{円積法} - 4 \text{房})(\text{弦} - \text{股}) + (4 \text{氏} \times \text{円積法} + 2 \text{亢})(\text{弦} - \text{股})^2 = 0$$

別図

$$-\text{心} - \text{尾} (\text{弦} - \text{股}) + 2 \text{箕} (\text{弦} - \text{股})^2 = 0$$

$$-\text{勾}^2 \times \text{尾} + \text{計} (\text{弦} - \text{股}) - 2 \text{牛} (\text{弦} - \text{股})^2 = 0$$

$$\text{勾}^2 \times \text{箕} - \text{牛} (\text{弦} - \text{股}) + \text{女} (\text{弦} - \text{股})^2 = 0$$

陰率

$$-\text{心} \times \text{計} \times \text{女} + 2 \text{勾}^2 \times \text{尾} \times \text{箕} \times \text{牛} + 2 \text{勾}^2 \times \text{尾} \times \text{箕} \times \text{牛}$$

$$2 \text{心} \times \text{牛}^2 - \text{勾}^2 \times \text{尾} \times \text{女} - 2 \text{勾}^2 \times \text{箕}^2 \times \text{計}$$

$$4 \text{勾}^2 \times \text{尾} \times \text{箕} \times \text{牛} + 2 \text{心} \times \text{牛} = \text{寄左}$$

$$\text{勾}^2 \times \text{尾}^2 \times \text{女} + 2 \text{勾}^2 \times \text{箕}^2 \times \text{計} + \text{心} \times \text{計} \times \text{女} - (\text{寄左}) = 0$$

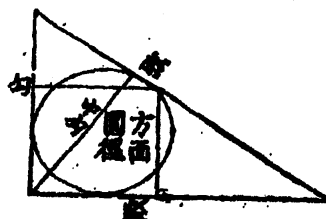
4 第四問

4.1 問題

今図のように勾股（直角三角形）内に方（正方形）と円を内接させたものがある。

只 = 方の外余積 + 圓 又 = 圓の外余積 + 中勾

勾, 股, 弦, 中勾, 圓, 方の長さを問う。



4.2 術文

方を未知数とする。

$$\text{方}^2 + \text{只} = \text{角} \quad 4\text{方} + 4\text{角} = \text{亢}$$

$$\text{方} + 4 = \text{氏} \quad (\text{方} + 2) \text{円積法} = \text{房}$$

$$2\text{角} \times \text{円積法} - (\text{方} + 2) = \text{心} \quad \text{方} + \text{角} - \text{又} = \text{尾}$$

$$\text{方} \times \text{角} + 2\text{方} \times 4\text{角} - (\text{方} \times \text{又} + 2\text{又}) = \text{箕}$$

$$\text{亢} \times \text{尾} - 2\text{方} \times \text{箕} = \text{計} \quad \text{氏} \times \text{尾} + 2\text{方} \times \text{心} = \text{牛}$$

$$4\text{方} \times \text{角} \times \text{房} + \text{氏} \times \text{箕} + \text{亢} \times \text{心} = \text{女}$$

$$8\text{方} \times \text{角} \times \text{亢} \times \text{房} \times \text{牛} + \text{氏} \times \text{計} \times \text{女} = \text{寄左}$$

$$8\text{方}^2 \times \text{角} \times \text{房} \times \text{女} + 2\text{角} \times \text{氏} \times \text{牛}^2 + \text{亢}^2 \times \text{房} \times \text{計} = \text{寄左相消}$$

4.3 解説・演段

$$\text{方} = \frac{\text{勾} \times \text{股}}{\text{勾} + \text{股}} \quad \text{円を補助の未知数とする。}$$

$$2\text{只} = \text{勾} \times \text{股} - 2\text{方}^2 + 2\text{円}$$

$$2\text{又} = \text{勾} \times \text{股} - 2\text{円}^2 \times \text{円積法} + 2\text{中勾}$$

$$\text{勾} \times \text{股} = \text{中勾} \times \text{弦} \quad \text{勾} + \text{股} = \text{円} + \text{弦}$$

$$2\text{勾} \times \text{股} = (\text{勾} + \text{股})^2 - \text{弦}^2 = (\text{円} + \text{弦})^2 - \text{弦}^2 = \text{円}^2 + 2\text{円} \times \text{弦}$$

$$2(\text{円} + \text{弦})\text{方} = \text{円}^2 + 2\text{円} \times \text{弦} \quad 2\text{弦}(\text{円} - \text{方}) = \text{円}(2\text{方} - \text{円})$$

$$\text{弦} = \frac{\text{円}(2\text{方} - \text{円})}{2(\text{円} - \text{方})}$$

$$2 \text{ 勾} \times \text{股} = \text{円}^2 + \frac{\text{円}^2(2 \text{ 方} - \text{円})}{\text{円} - \text{方}}$$

$$= \frac{\text{円}^2}{\text{円} - \text{方}} \times (\text{円} - \text{方} + 2 \text{ 方} - \text{円}) = \frac{\text{方} \times \text{円}^2}{\text{円} - \text{方}}$$

$$4 \text{ 只} = \frac{\text{方} \times \text{円}^2}{\text{円} - \text{方}} - 4 \text{ 方}^2 + 4 \text{ 円}$$

$$4 \text{ 只} (\text{円} - \text{方}) = \text{方} \times \text{円}^2 - 4 \text{ 方}^2 (\text{円} - \text{方}) + 4 \text{ 円} (\text{円} - \text{方})$$

$$4 \text{ 只} \times \text{円} - 4 \text{ 只} \times \text{方} = \text{方} \times \text{円}^2 - 4 \text{ 方}^2 \times \text{円} + 4 \text{ 方}^3 + 4 \text{ 円}^2 - 4 \text{ 円} \times \text{方}$$

$$(-4 \text{ 方}^3 + 4 \text{ 只} \times \text{方}) + (4 \text{ 只} + 4 \text{ 方} + 4 \text{ 方}^2) \text{ 円} + (-\text{方} - 4) \text{ 円}^2 = 0$$

$$-4 \text{ 方} \times \text{角} + \text{亢} \times \text{円} - \text{氏} \times \text{円}^2 = 0 \dots \text{前式}$$

$$(\text{只} - \text{円}) + \text{方}^2 = \text{甲} = \text{勾股積}$$

$$\text{中勾} = \frac{\text{勾} \times \text{股}}{\text{弦}} \quad \text{中勾} \times \text{弦} = 2 \text{ 甲}$$

$$\text{方} = \frac{\text{勾} \times \text{股}}{\text{勾} + \text{股}} \quad \text{方} (\text{勾} + \text{股}) = 2 \text{ 甲} \quad \text{方} (\text{円} + \text{弦}) = 2 \text{ 甲}$$

$$\text{中勾} \times \text{弦} = \text{方} \times \text{円} + \text{方} \times \text{弦} \quad (\text{中勾} - \text{方}) \text{ 弦} = \text{方} \times \text{円}$$

$$(\text{中勾} - \text{方}) \text{ 中勾} \times \text{弦} = \text{方} \times \text{円} \times \text{中勾}$$

$$(\text{中勾} - \text{方}) 2 \text{ 甲} = \text{方} \times \text{円} \times \text{中勾}$$

$$\text{又} + \text{円}^2 \times \text{円積法} - \text{甲} = \text{中勾} = \text{乙}$$

$$2\{\text{又} + \text{円}^2 \times \text{円積法} - (\text{只} - \text{円}) - \text{方}^2 - \text{方}\}(\text{只} - \text{円} + \text{方}^2)$$

$$= \text{方} \times \text{円} (\text{又} + \text{円}^2 \times \text{円積法} - \text{只} + \text{円} - \text{方}^2)$$

$$2(\text{又} + \text{円}^2 \times \text{円積法} + \text{円} - \text{角} - \text{方})(\text{角} - \text{円}) = \text{方} \times \text{円} (\text{又} + \text{円}^2 \times \text{円積法} + \text{円} - \text{角})$$

$$2 \text{ 又} \times \text{角} + 2 \text{ 円}^2 \times \text{円積法} \times \text{角} + 2 \text{ 円} \times \text{角} - 2 \text{ 角}^2 - 2 \text{ 方} \times \text{角} - 2 \text{ 円} \times \text{又}$$

$$- 2 \text{ 円}^3 \times \text{円積法} - 2 \text{ 円}^2 + 2 \text{ 円} \times \text{角} + 2 \text{ 方} \times \text{円}$$

$$= \text{又} \times \text{方} \times \text{円} + \text{方} \times \text{円}^3 \times \text{円積法} + \text{方} \times \text{円}^2 - \text{方} \times \text{円} \times \text{角}$$

$$(2 \text{ 又} \times \text{角} - 2 \text{ 方} \times \text{角} - 2 \text{ 角}^2) + (4 \text{ 角} - 2 \text{ 又} + 2 \text{ 方} - \text{又} \times \text{方} + \text{方} \times \text{角}) \text{ 円}$$

$$+ (2 \text{ 角} \times \text{円積法} - 2 - \text{方}) \text{ 円}^2 + (-2 \text{ 円積法} - \text{方} \times \text{円積法}) \text{ 円}^3 = 0$$

$$- 2 \text{ 角} \times \text{尾} + \text{箕} \times \text{円} + \text{心} \times \text{円}^2 - \text{房} \times \text{円}^3 = 0 \dots \text{後式}$$

前式 $-4 \text{方} \times \text{角} + \text{亢} \times \text{円} - \text{氏} \times \text{円}^2 + 0 \times \text{円}^3 = 0$

後式 $-2 \text{角} \times \text{尾} + \text{箕} \times \text{円} + \text{心} \times \text{円} - \text{房} \times \text{円}^3 = 0$

陽図

$(2 \text{角} \times \text{亢} \times \text{尾} - 4 \text{方} \times \text{角} \times \text{箕}) + (-2 \text{角} \times \text{氏} \times \text{尾} - 4 \text{方} \times \text{角} \times \text{心}) \text{円}$

$+ 4 \text{方} \times \text{角} \times \text{房} \times \text{円} = 0$

$(-2 \text{角} \times \text{氏} \times \text{尾} - 4 \text{方} \times \text{角} \times \text{心}) + (4 \text{方} \times \text{角} \times \text{房} + \text{氏} \times \text{箕} + \text{亢} \times \text{心}) \text{円}$

$- \text{亢} \times \text{房} \times \text{円}^2 = 0$

$4 \text{方} \times \text{角} \times \text{房} + (-\text{亢} \times \text{房}) \text{円} + \text{氏} \times \text{房} \times \text{円}^2 = 0$

別図

$\text{計} - \text{牛} \times \text{円} + 2 \text{方} \times \text{房} \times \text{円}^2 = 0$

$-2 \text{角} \times \text{牛} + \text{女} \times \text{円} - \text{亢} \times \text{房} \times \text{円}^2 = 0$

$4 \text{方} \times \text{角} - \text{亢} \times \text{円} + \text{氏} \times \text{円}^2 = 0$

陰図

$\text{氏} \times \text{計} \times \text{女} \quad 4 \text{方} \times \text{角} \times \text{亢} \times \text{房} \times \text{牛} \quad 4 \text{方} \times \text{角} \times \text{亢} \times \text{房} \times \text{牛}$

$- \text{亢}^2 \times \text{房} \times \text{計} \quad -2 \text{角} \times \text{氏} \times \text{牛}^2 \quad -8 \text{方} \times \text{角} \times \text{房} \times \text{女}$

$8 \text{方} \times \text{角} \times \text{亢} \times \text{房} \times \text{牛} + \text{氏} \times \text{計} \times \text{女} = \text{寄左}$

$8 \text{方}^2 \times \text{角} \times \text{房} \times \text{女} + 2 \text{角} \times \text{氏} \times \text{牛}^2 + \text{亢}^2 \times \text{房} \times \text{計} - (\text{寄左}) = 0$

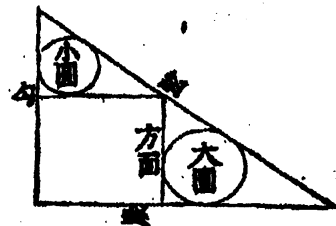
5. 第五問

5.1 問題

今図のように勾股（直角三角形）内に方（正方形）を内接させ正方形と直角三角形に内接するように大，小円を容れたものがある。

$\text{只} = \text{方} \text{の外余積} \quad \text{又} = \text{大} - \text{小}$

勾，股，弦，圓，方の長さを問う。



5.2 術文

勾を未知数とする.

$$4 \text{ 勾} + \text{又} = \text{角} \quad 2 \text{ 勾} \times \text{又} - \text{又}^2 = \text{亢}$$

$$(\text{勾}^2 + 4 \text{ 外余積}) - 2 \text{ 勾} \times \text{又} = \text{氏} \quad 3 \text{ 勾}^2 + \text{外余積} = \text{房}$$

$$2 \text{ 亢} - 2 \text{ 勾} \times \text{又} = \text{心} \quad \text{角} \times \text{又} + 2 \text{ 氏} = \text{尾}$$

$$\text{勾} \times \text{又} + 2 \text{ 房} = \text{箕} \quad 2 \text{ 勾} \times \text{氏} + \text{角} \times \text{亢} - \text{箕} \times \text{又} = \text{計}$$

$$(\text{亢} + 2 \text{ 房}) - 2 \text{ 勾} \times \text{又} = \text{牛} \quad (2 \text{ 勾}^3 + \text{角} \times \text{房}) - \text{勾} \times \text{氏} = \text{女}$$

$$\text{心} \times \text{計} \times \text{女} + 4 \text{ 勾}^3 \times \text{角} \times \text{心} \times \text{牛} + \text{勾} \times \text{角}^2 \times \text{尾}^2 \times \text{又} + 2 \text{ 勾} \times \text{尾} \times \text{箕} \times \text{牛} \times \text{又}$$

$$+ \text{勾}^3 \times \text{箕}^2 \times \text{又} + 8 \text{ 勾}^3 \times \text{尾} \times \text{女} \times \text{又} + 4 \text{ 勾} \times \text{角} \times \text{箕} \times \text{計} \times \text{又} + 4 \text{ 勾}^3 \times \text{牛}^2 \times \text{又} = \text{寄左}$$

$$\text{勾} \times \text{角}^2 \times \text{心} \times \text{計} + \text{勾}^2 \times \text{心} \times \text{牛}^2 + 4 \text{ 勾}^3 \times \text{心} \times \text{女} + \text{尾}^2 \times \text{女} \times \text{又}$$

$$+ 4 \text{ 勾}^2 \times \text{角} \times \text{尾} \times \text{箕} \times \text{又} + \text{箕}^2 \times \text{計} \times \text{又} + 4 \text{ 勾}^2 \times \text{角} \times \text{尾} \times \text{牛} \times \text{又}$$

$$+ 4 \text{ 勾} \times \text{計} \times \text{女} \times \text{又} + 8 \text{ 勾}^3 \times \text{箕} \times \text{牛} \times \text{又} - (\text{寄左}) = 0$$

5.3 解説・演段

本を補助の未知数とする.

$$\text{只} = \frac{1}{2} \text{ 勾} \times \text{股} - \text{方}^2 \quad \text{方} = \frac{\text{勾} \times \text{股}}{\text{勾} + \text{股}}$$

$$\text{股} - \text{方} : \text{股} = \text{大} : \text{本} \quad \text{大} = \frac{\text{本}}{\text{股}} (\text{股} - \text{方}) = \frac{\text{本} \times \text{股}}{\text{勾} + \text{股}} \quad \text{勾} \times \text{大} = \text{方} \times \text{本}$$

$$\text{勾} - \text{方} : \text{勾} = \text{小} : \text{本} \quad \text{小} = \frac{\text{本}}{\text{勾}} (\text{勾} - \text{方}) = \frac{\text{本} \times \text{勾}}{\text{勾} + \text{股}}$$

$$\text{大} + \text{小} = \frac{\text{本} (\text{股} + \text{勾})}{\text{勾} + \text{股}} = \text{本} \quad \text{大} - \text{小} = \text{又}$$

$$\text{弦} = (\text{勾} + \text{股}) - \text{本} \quad \text{弦}^2 = \text{勾}^2 + \text{股}^2$$

$$(\text{勾} + \text{股})^2 - 2(\text{勾} + \text{股}) \text{本} + \text{本}^2 = \text{勾}^2 + \text{股}^2$$

$$2 \text{ 股} (\text{勾} - \text{本}) - 2 \text{ 本} \times \text{勾} + \text{本}^2 = 0 \quad \text{股} = \frac{\text{本} (2 \text{ 勾} - \text{本})}{2(\text{勾} - \text{本})} \quad (\text{股を勾, 本で表した})$$

$$\text{勾} + \text{股} = \frac{2 \text{ 勾}^2 - 2 \text{ 勾} \times \text{本} + 2 \text{ 勾} \times \text{本} - \text{本}^2}{2(\text{勾} - \text{本})} = \frac{2 \text{ 勾}^2 - \text{本}^2}{2(\text{勾} - \text{本})}$$

$$\text{又} = \text{大} - \text{小} = \frac{\text{本}(\text{股} - \text{勾})}{\text{股} + \text{勾}}$$

$$\text{又}(\text{勾} + \text{股}) = \text{本}(\text{股} - \text{勾})$$

$$\text{又}(2\text{勾}^2 - \text{本}^2) = \text{本}\{\text{本}(2\text{勾} - \text{本}) - 2\text{勾}(\text{勾} - \text{本})\}$$

$$2\text{又} \times \text{勾}^2 - \text{又} \times \text{本}^2 = \text{本}(2\text{勾} \times \text{本} - \text{本}^2 - 2\text{勾}^2 + 2\text{勾} \times \text{本})$$

$$-2\text{又} \times \text{勾}^2 - 2\text{勾}^2 \times \text{本} + (\text{又} + 4\text{勾})\text{本}^2 - \text{本}^3 = 0 \dots \text{前式}$$

$$\text{本} + \text{又} = 2\text{大} \quad \text{勾}(\text{本} + \text{又}) = 2\text{大} \times \text{勾} = 2\text{本} \times \text{方}$$

$$4\text{本}^2 \times \text{方}^2 = \text{勾}^2 \times \text{本}^2 + 2\text{勾}^2 \times \text{本} \times \text{又} + \text{勾}^2 \times \text{又}^2$$

$$4\text{本}^2 \times \text{只} = 4\text{本}^2 \times \text{勾股積} - (\text{勾}^2 \times \text{本}^2 + 2\text{勾}^2 \times \text{本} \times \text{又} + \text{勾}^2 \times \text{又}^2)$$

$$= \text{本}^2 \times \text{勾} \frac{\text{本}(2\text{勾} - \text{本})}{\text{勾} - \text{本}} - (\text{勾}^2 \times \text{本}^2 + 2\text{勾}^2 \times \text{本} \times \text{又} + \text{勾}^2 \times \text{又}^2)$$

$$4\text{本}^2 \times \text{只}(\text{勾} - \text{本}) = \text{本}^3 \times \text{勾}(2\text{勾} - \text{本}) - (\text{勾}^2 \times \text{本}^2 + 2\text{勾}^2 \times \text{本} \times \text{又} + \text{勾}^2 \times \text{又}^2)(\text{勾} - \text{本})$$

$$4\text{本}^2 \times \text{只} \times \text{勾} - 4\text{本}^3 \times \text{只} = 2\text{勾}^2 \times \text{本}^3 - \text{勾} \times \text{本}^4 - \text{勾}^3 \times \text{本}^2 - 2\text{勾}^3 \times \text{本} \times \text{又} - \text{勾}^3 \times \text{又}^2$$

$$+ \text{勾}^2 \times \text{本}^3 + 2\text{勾}^2 \times \text{本}^2 \times \text{又} + \text{勾}^2 \times \text{又}^2 \times \text{本}$$

$$\text{勾}^3 \times \text{又}^2 + (2\text{勾}^3 \times \text{又} - \text{勾}^2 \times \text{又}^2)\text{本} + (\text{勾}^3 - 2\text{勾}^2 \times \text{又} + 4\text{只} \times \text{勾})\text{本}^2$$

$$+ (-3\text{勾}^2 - 4\text{只})\text{本}^3 + \text{勾} \times \text{本}^4 = 0 \dots \text{後式}$$

$$\text{前式} \quad -2\text{勾}^2 \times \text{又} - 2\text{勾}^2 \times \text{本} + \text{角} \times \text{本}^2 - \text{本}^3 = 0$$

$$\text{後式} \quad \text{勾}^3 \times \text{又}^2 + \text{勾}^2 \times \text{亢} \times \text{本} + \text{勾} \times \text{氏} \times \text{本}^2 - \text{房} \times \text{本}^3 + \text{勾} \times \text{本}^4 = 0$$

陽圖

$$(2\text{勾}^5 \times \text{又}^2 - 2\text{勾}^4 \times \text{亢} \times \text{又}) + (-\text{勾}^3 \times \text{角} \times \text{又}^2 - 2\text{勾}^3 \times \text{氏} \times \text{又})\text{本}$$

$$+ (\text{勾}^3 \times \text{又}^2 + 2\text{勾}^2 \times \text{房} \times \text{又})\text{本}^2 - 2\text{勾}^3 \times \text{又} \times \text{本}^3 = 0 \dots \text{第一式}$$

$$(-\text{勾}^3 \times \text{角} \times \text{又}^2 - 2\text{勾}^3 \times \text{氏} \times \text{又}) + (\text{勾}^3 \times \text{又}^2 + 2\text{勾}^2 \times \text{房} \times \text{又} - \text{勾}^2 \times \text{角} \times \text{亢} - 2\text{勾}^3 \times \text{氏})\text{本}$$

$$+ (-2\text{勾}^3 \times \text{又} + \text{勾}^2 \times \text{亢} + 2\text{勾}^2 \times \text{房})\text{本}^2 - 2\text{勾}^3 \times \text{本}^3 = 0 \dots \text{第二式}$$

$$(\text{勾}^3 \times \text{又}^2 + 2\text{勾}^2 \times \text{房} \times \text{又}) + (-2\text{勾}^3 \times \text{又} + \text{勾}^2 \times \text{亢} + 2\text{勾}^2 \times \text{房})\text{本}$$

$$+ (-2\text{勾}^3 + \text{勾} \times \text{氏} - \text{角} \times \text{房})\text{本}^2 + \text{勾} \times \text{角} \times \text{本}^3 = 0 \dots \text{第三式}$$

$$-2\text{勾}^3 \times \text{又} - 2\text{勾}^3 \times \text{本} + \text{勾} \times \text{角} \times \text{本}^2 - \text{勾} \times \text{本}^3 = 0 \dots \text{第四式}$$

別図

$$-\text{勾}^2 \times \text{心} - \text{勾} \times \text{尾} \times \text{本} + \text{箕} \times \text{本}^2 - 2 \text{勾} \times \text{本}^3 = 0 \quad \text{ル} + \text{ヲ} x + \text{ワ} x^2 + \text{カ} x^3 = 0$$

$$-\text{勾} \times \text{尾} \times \text{又} - \text{計} \times \text{本} + \text{牛} \times \text{本}^2 - 2 \text{勾} \times \text{本}^3 = 0 \quad \text{ヨ} + \text{タ} x + \text{レ} x^2 + \text{ソ} x^3 = 0$$

$$\text{勾}^2 \times \text{又} \times \text{箕} + \text{勾}^2 \times \text{牛} \times \text{本} - \text{女} \times \text{本}^2 - \text{勾} \times \text{角} \times \text{本}^3 = 0 \quad \text{ツ} + \text{子} x + \text{ナ} x^2 + \text{ラ} x^3 = 0$$

$$2 \text{勾}^2 \times \text{又} - 2 \text{勾}^2 \times \text{本} + \text{角} \times \text{本}^2 - \text{本}^3 = 0 \quad \text{ム} + \text{ウ} x + \text{井} x^2 + \text{ノ} x^3 = 0$$

陰率

正

$$\text{ルタナノ} \quad \text{勾}^2 \times \text{心} \times \text{計} \times \text{女}$$

$$\text{ルレラウ} \quad 2 \text{勾}^5 \times \text{角} \times \text{心} \times \text{牛}$$

$$\text{ルソ子井} \quad 2 \text{勾}^5 \times \text{角} \times \text{心} \times \text{牛}$$

$$\text{ヲヨラ井} \quad \text{勾}^3 \times \text{角}^2 \times \text{尾}^2 \times \text{又}$$

$$\text{ヲレツノ} \quad \text{勾}^3 \times \text{尾} \times \text{箕} \times \text{牛} \times \text{又}$$

$$\text{ヲソナム} \quad 4 \text{勾}^4 \times \text{尾} \times \text{女} \times \text{又}$$

$$\text{ワヨ子ノ} \quad \text{勾}^3 \times \text{尾} \times \text{箕} \times \text{牛} \times \text{又}$$

$$\text{ワタラム} \quad 2 \text{勾}^3 \times \text{角} \times \text{箕} \times \text{計} \times \text{又}$$

$$\text{ワソツウ} \quad 4 \text{勾}^5 \times \text{箕}^2 \times \text{又}$$

$$\text{カヨナウ} \quad 4 \text{勾}^4 \times \text{尾} \times \text{女} \times \text{又}$$

$$\text{カタツ井} \quad 2 \text{勾}^3 \times \text{角} \times \text{箕} \times \text{計} \times \text{又}$$

$$\text{カレ子ム} \quad 4 \text{勾}^5 \times \text{牛}^2 \times \text{又}$$

負

$$\text{ルタラ井} \quad \text{勾}^3 \times \text{角}^2 \times \text{心} \times \text{計}$$

$$\text{ルレ子ノ} \quad \text{勾}^4 \times \text{心} \times \text{牛}^2$$

$$\text{ルソナウ} \quad 4 \text{勾}^5 \times \text{心} \times \text{女}$$

$$\text{ヲヨナノ} \quad \text{勾}^2 \times \text{尾}^2 \times \text{女} \times \text{又}$$

$$\text{ヲレラム} \quad 2 \text{勾}^4 \times \text{角} \times \text{尾} \times \text{牛} \times \text{又}$$

ヲソツ井 $2 \text{ 勾}^4 \times \text{角} \times \text{尾} \times \text{箕} \times \text{又}$

ヲヨラウ $2 \text{ 勾}^4 \times \text{角} \times \text{尾} \times \text{箕} \times \text{又}$

ヲタツノ $\text{勾}^2 \times \text{箕}^2 \times \text{計} \times \text{又}$

ヲソ子ム $4 \text{ 勾}^5 \times \text{箕} \times \text{牛} \times \text{又}$

カヨ子井 $2 \text{ 勾}^4 \times \text{角} \times \text{尾} \times \text{牛} \times \text{又}$

カタナム $4 \text{ 勾}^3 \times \text{計} \times \text{女} \times \text{又}$

カレツク $4 \text{ 勾}^5 \times \text{箕} \times \text{牛} \times \text{又}$

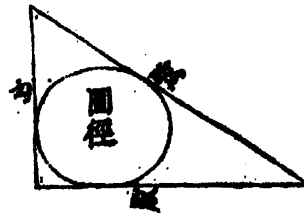
6 第六問

6.1 問題

今図のように勾股（直角三角形）内に円を内接させたものがある。

$$\text{只} = \text{外余積} + \text{勾}^2 \quad \text{又} = \text{弦}^2 + \text{股}^2$$

勾、股、弦、圓の長さを問う。



6.2 術文

弦を未知数とする。

$$\text{又} - \text{弦}^2 = \text{股}^2 \quad \text{弦}^2 - (\text{又} - \text{弦}^2) = \text{勾}^2$$

$$\text{勾}^2 \times \text{股}^2 = \text{角} \quad (\text{又} - \text{弦}^2)(2 \text{ 弦}^2 - \text{又}) = \text{角}$$

$$\text{只} - \text{勾}^2 = \text{外余積} = \text{亢} \quad 4 \text{ 円積法} - 1 = \text{氏}$$

$$\text{角} \times \text{氏} + 4 \text{ 亢} \times \text{弦}^2 = \text{房} \quad 4 \text{ 弦}^3 - 8 \text{ 亢} \times \text{弦} = \text{心}$$

$$4 \text{ 弦}^2 - 2 \text{ 亢} = \text{尾} \quad 4 \text{ 弦} \times \text{氏} - 2 \text{ 弦} = \text{箕}$$

$$\text{氏}^2 \times \text{房}^2 + 64 \text{ 亢}^2 \times \text{弦}^4 + 4 \text{ 亢}^2 \times \text{尾}^2 + 2 \text{ 角} \times \text{箕} \times \text{弦}^3 + 4 \text{ 亢} \times \text{氏} \times \text{房} \times \text{尾}$$

$$+ 16 \text{ 亢} \times \text{氏} \times \text{房} \times \text{弦}^2 + 2 \text{ 亢}^2 \times \text{心} \times \text{箕} = \text{寄左}$$

$$(4 \text{ 角} \times \text{氏} \times \text{尾} \times \text{弦} + 4 \text{ 亢} \times \text{房} \times \text{箕} + 16 \text{ 亢}^2 \times \text{氏} \times \text{心} + \text{角} \times \text{氏}^2 \times \text{心}$$

$$+ 32 \text{ 亢}^2 \times \text{尾} \times \text{弦}) \text{ 弦} = \text{寄左相消}$$

6.3 解説・演段

円を補助の未知数とする

$$\text{股}^2 = \text{又} - \text{弦}^2 \quad \text{勾}^2 = \text{弦}^2 - \text{股}^2 \quad \text{外余積} = \text{只} - \text{勾}^2 = \text{亢}$$

$$\text{勾股積} - \text{円}^2 \times \text{円積法} = \text{只} - \text{勾}^2$$

$$2 \text{勾} \times \text{股} = 4(\text{只} - \text{勾}^2) + 4 \text{円}^2 \times \text{円積法}$$

$$(\text{股} + \text{勾})^2 - \text{弦}^2 = 2 \text{勾} \times \text{股} = (\text{円} + \text{弦})^2 - \text{弦}^2 = \text{円}^2 + 2 \text{円} \times \text{弦}$$

$$\text{円}^2 + 2 \text{円} \times \text{弦} = 4(\text{只} - \text{勾}^2) + \text{円}^2 \times \text{円積法} \quad \text{外余積} = \text{只} - (2 \text{弦} - \text{又})$$

$$-4 \text{外余積} + 2 \text{弦} \times \text{円} + (-4 \text{円積法} + 1) \text{円}^2 = 0 \quad \dots \quad \text{前式}$$

$$(\text{円}^2 + 2 \text{円} \times \text{弦})^2 = 16(\text{勾股積})^2 = 4 \text{勾}^2 \times \text{股}^2$$

$$-4 \text{勾}^2 \times \text{股}^2 + 4 \text{弦}^2 \times \text{円}^2 + 4 \text{弦} \times \text{円}^3 + \text{円}^4 = 0 \quad \dots \quad \text{後式}$$

$$\text{前式} \quad -4 \text{亢} + 2 \text{弦} \times \text{円} - \text{氏} \times \text{円}^2 = 0$$

$$\text{後式} \quad -4 \text{角} + 4 \text{弦}^2 \times \text{円}^2 + 4 \text{弦} \times \text{円}^3 + \text{円}^4 = 0$$

陽図

$$8 \text{弦} \times \text{角} + (-4 \text{角} \times \text{氏} - 16 \text{亢} \times \text{弦}^2) \text{円} - 16 \text{亢} \times \text{弦} \times \text{円}^2 - 4 \text{亢} \times \text{円}^3 = 0 \quad \text{第一式}$$

$$(-4 \text{角} \times \text{氏} - 16 \text{亢} \times \text{弦}^2) + (-16 \text{亢} \times \text{弦} + 8 \text{弦}^3) \text{円} + (-4 \text{亢} + 8 \text{弦}^2) \text{円}^2 + 2 \text{弦} \times \text{円}^3 = 0 \quad \text{第二式}$$

$$-16 \text{亢} \times \text{弦} + (-4 \text{亢} + 8 \text{弦}^2) \text{円} + (2 \text{弦} - 4 \text{氏} \times \text{弦}) \text{円}^2 - \text{氏} \times \text{円}^3 = 0 \quad \text{第三式}$$

$$-4 \text{亢} + 2 \text{弦} \times \text{円} - \text{氏} \times \text{円}^2 = 0 \quad \text{第四式}$$

別図

$$2 \text{弦} \times \text{角} - \text{房} \times \text{円} - 4 \text{亢} \times \text{弦} \times \text{円}^2 - \text{亢} \times \text{円}^3 = 0 \quad \text{ル} + \text{ヲ} x + \text{ワ} x^2 + \text{カ} x^3 = 0$$

$$-2 \text{房} + \text{心} \times \text{円} + \text{尾} \times \text{円}^2 + \text{弦} \times \text{円}^3 = 0 \quad \text{ヨ} + \text{タ} x + \text{レ} x^2 + \text{ソ} x^3 = 0$$

$$-16 \text{亢} \times \text{弦} + 2 \text{尾} \times \text{円} - \text{箕} \times \text{円}^2 - \text{氏} \times \text{円}^3 = 0 \quad \text{ツ} + \text{子} x + \text{ナ} x^2 + \text{ラ} x^3 = 0$$

$$-4 \text{亢} + 2 \text{弦} \times \text{円} - \text{氏} \times \text{円}^2 = 0 \quad \text{ム} + \text{ウ} x + \text{井} x^2 = 0 \quad (\text{ノ} = 0)$$

陰率

正

$$\text{ルレラウ} \quad -4 \text{角} \times \text{氏} \times \text{尾} \times \text{弦}^2$$

ルソ子井 $-4 \text{ 角} \times \text{氏} \times \text{尾} \times \text{弦}^2$

ヲヨラ井 $2 \text{ 氏}^2 \times \text{房}^2$

ヲソナム $-4 \text{ 亢} \times \text{房} \times \text{箕} \times \text{弦}$

ワタラム $-16 \text{ 亢}^2 \times \text{氏} \times \text{心} \times \text{弦}$

クソツウ $128 \text{ 亢}^2 \times \text{弦}^4$

カヨナラ $-4 \text{ 亢} \times \text{房} \times \text{箕} \times \text{弦}$

カタツ井 $-16 \text{ 亢}^2 \times \text{氏} \times \text{心} \times \text{弦}$

カレ子ム $8 \text{ 亢}^2 \times \text{尾}^2$

負

ルタラ井 $-2 \text{ 角} \times \text{氏}^2 \times \text{心} \times \text{弦}$

ルソナウ $4 \text{ 角} \times \text{箕} \times \text{弦}^3$

ヲレラム $4 \text{ 亢} \times \text{氏} \times \text{房} \times \text{尾}$

ヲソツ井 $16 \text{ 亢} \times \text{氏} \times \text{房} \times \text{弦}^2$

ヲヨラウ $16 \text{ 亢} \times \text{氏} \times \text{房} \times \text{弦}^2$

ワソ子ム $-32 \text{ 亢}^2 \times \text{尾} \times \text{弦}^2$

カヨ子井 $4 \text{ 亢} \times \text{氏} \times \text{房} \times \text{尾}$

カタナム $4 \text{ 亢}^2 \times \text{心} \times \text{箕}$

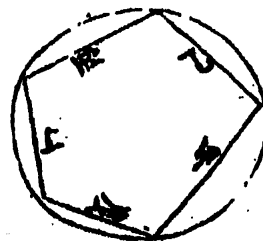
カレツウ $-32 \text{ 亢}^2 \times \text{尾} \times \text{弦}^2$

7 第七問

7.1 問題

今円内に五斜（五角形）を内接させたものがある。

甲、乙、丙、丁、戊、の長さが与えられたとき円の直径を問う。



7.2 術文

$$\text{円}^2 = \text{角} \quad (\text{乙}^4 + \text{丙}^4 - 2 \text{乙}^2 \times \text{丙}^2) \text{角} = \text{亢}$$

$$\text{角} \times \text{乙}^2 + \text{角} \times \text{丙}^2 - 2 \text{乙}^2 \times \text{丙}^2 = \text{氏}$$

$$\text{丁}^4 + \text{戊}^4 - 2 \text{丁}^2 \times \text{戊}^2 = \text{房}$$

$$\text{角} \times \text{丁}^2 + \text{角} \times \text{戊}^2 - 2 \text{丁}^2 \times \text{戊}^2 = \text{心}$$

$$\text{角} - 2 \text{甲}^2 = \text{尾}$$

$$2 \text{角}^2 \times \text{甲}^2 + 2 \text{心} \times \text{尾} - \text{角} \times \text{氏} = \text{箕}$$

$$(6 \text{角}^2 \times \text{甲}^4 + 8 \text{心} \times \text{尾} \times \text{甲}^2 + 4 \text{房} \times \text{尾}^2 + 4 \text{心}^2)$$

$$-(2 \text{角}^2 \times \text{房} + 4 \text{角} \times \text{心} \times \text{甲}^2 + \text{角} \times \text{亢}) = \text{計}$$

$$(\text{角}^2 \times \text{甲}^6 + \text{房} \times \text{心} \times \text{尾} + \text{心} \times \text{尾} \times \text{甲}^4 + 2 \text{心}^2 \times \text{甲}^2)$$

$$-(\text{角}^2 \times \text{房} \times \text{甲}^2 + 2 \text{角} \times \text{房} \times \text{尾} \times \text{甲}^2 + 2 \text{角} \times \text{心} \times \text{甲}^4) = \text{牛}$$

$$(\text{角}^2 \times \text{房}^2 - \text{角}^2 \times \text{甲}^8 + 4 \text{心}^2 \times \text{甲}^4 + 2 \text{角}^2 \times \text{房} \times \text{甲}^4)$$

$$-(4 \text{角} \times \text{房} \times \text{心} \times \text{甲}^2 + 4 \text{角} \times \text{心} \times \text{甲}^6) = \text{女}$$

$$4 \text{亢} \times \text{牛} - 2 \text{氏} \times \text{女} = \text{虚}$$

$$\text{亢} \times \text{計} - \text{角} \times \text{女} = \text{危}$$

$$\text{亢} \times \text{箕} + \text{氏} \times \text{計} - 2 \text{角} \times \text{牛} = \text{室}$$

$$\text{角} \times \text{危}^2 + 4 \text{亢}^2 \times \text{箕} \times \text{室} + 8 \text{氏}^2 \times \text{箕} \times \text{虚} = \text{寄左}$$

$$2 \text{角} \times \text{室} \times \text{虚} + 8 \text{亢} \times \text{氏} \times \text{箕} \times \text{危} = \text{寄左相消}$$

7.3 解説・演段

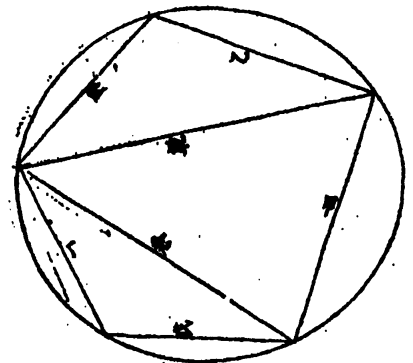
亢を未知数とする

$$\text{亢}^2 + \text{丁}^2 - \text{戊}^2 = 2 \text{亢} \times \text{離}$$

$$(\text{戊}^4 - 2 \text{戊}^2 \times \text{丁}^2 + \text{丁}^4) + 2(-\text{戊}^2 + \text{丁}^2) \text{亢}^2 + \text{亢}^4 = 4 \text{亢}^2 \times \text{離}^2 = \text{天}$$

$$4 \text{丁}^2 \times \text{亢}^2 - \text{天} = 4 \text{亢}^2 \times \text{震}^2$$

$$4 \text{亢}^2 \times \text{震}^2 \times \text{円}^2 = (-\text{角} \times \text{戊}^4 + 2 \text{角} \times \text{丁}^2 \times \text{戊}^2 - \text{角} \times \text{丁}^4)$$



$$+2(\text{角} \times \text{丁}^2 + \text{角} \times \text{戊}^2) \text{兌}^2 - \text{角} \times \text{兌}^4 = \text{寄左}$$

$$\text{丁}^2 \times \text{戊}^2 = \text{戊}^2 \times \text{震}^2 \quad 4 \text{丁}^2 \times \text{戊} \times \text{兌}^2 = \text{寄左相消}$$

$$(-\text{角} \times \text{戊}^4 + 2 \text{角} \times \text{戊}^2 \times \text{丁}^2 - \text{角} \times \text{丁}^4)$$

$$+(-4 \text{丁}^2 \times \text{戊}^2 + 2 \text{角} \times \text{丁}^2 + 2 \text{角} \times \text{戊}^2) \text{兌}^2 - \text{角} \times \text{兌}^4 = 0 \dots \text{進前式}$$

$$\text{兌}^2 + \text{乾}^2 - \text{甲}^2 = 2 \text{兌} \times \text{巽} \quad \{(-\text{甲}^2 + \text{乾}^2) + \text{兌}^2\}^2 = 4 \text{兌}^2 \times \text{巽}^2 = \text{地}$$

$$(\text{甲}^4 - 2 \text{甲}^2 \times \text{乾}^2 + \text{乾}^4) + (-2 \text{甲}^2 + 2 \text{乾}^2) \text{兌}^2 + \text{兌}^4 = \text{地}$$

$$4 \text{乾}^2 \times \text{兌}^2 - \text{地} = 4 \text{兌}^2 \times \text{坎}^2$$

$$= (-\text{甲}^4 + 2 \text{甲}^2 \times \text{乾}^2 - \text{乾}^4) + (2 \text{甲}^2 + 2 \text{乾}^2) \text{兌}^2 - \text{兌}^4$$

$$4 \text{兌}^2 \times \text{坎}^2 \times \text{戊}^2 = (-\text{角} \times \text{甲}^4 + 2 \text{角} \times \text{甲}^2 \times \text{乾}^2 - \text{角} \times \text{乾}^4)$$

$$+(2 \text{角} \times \text{甲}^2 + 2 \text{角} \times \text{乾}^2) \text{兌}^2 - \text{角} \times \text{兌}^4 = \text{寄再}$$

$$\text{乾}^2 \times \text{甲}^2 = \text{戊}^2 \times \text{坎}^2 \quad 4 \text{乾}^2 \times \text{甲}^2 \times \text{兌}^2 = \text{寄再相消}$$

$$(-\text{角} \times \text{甲}^4 + 2 \text{角} \times \text{甲}^2 \times \text{乾}^2 - \text{角} \times \text{乾}^4)$$

$$+(-4 \text{甲}^2 \times \text{乾}^2 + 2 \text{角} \times \text{甲}^2 + 2 \text{角} \times \text{乾}^2) \text{兌}^2 - \text{角} \times \text{兌}^4 = 0 \dots \text{進後式}$$

$$\text{進前式} \quad \text{角} \times \text{子} + \text{丑} \times \text{兌}^2 - \text{角} \times \text{兌}^4 = 0$$

$$\text{後前式} \quad \text{角} \times \text{寅} + \text{卯} \times \text{兌}^2 - \text{角} \times \text{兌}^4 = 0$$

$$(-\text{丑} \times \text{寅} + \text{子} \times \text{卯}) + (\text{角} \times \text{寅} - \text{角} \times \text{子}) \text{兌}^2 = 0$$

$$(\text{角} \times \text{寅} - \text{角} \times \text{子}) + (\text{卯} - \text{丑}) \text{兌}^2 = 0$$

$$(-\text{丑} \times \text{寅} + \text{子} \times \text{卯})(\text{卯} - \text{丑}) = \text{丑}^2 \times \text{寅} - \text{子} \times \text{卯} \times \text{丑} - \text{丑} \times \text{寅} \times \text{卯} + \text{子} \times \text{卯}^2$$

$$-(\text{角} \times \text{寅} - \text{角} \times \text{子})(\text{角} \times \text{寅} - \text{角} \times \text{子}) = -\text{角}^2 \times \text{寅}^2 + 2 \text{角}^2 \times \text{寅} \times \text{子} - \text{角}^2 \times \text{子}^2$$

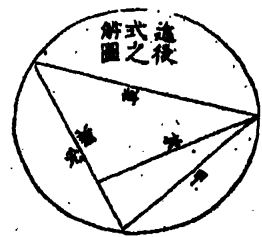
$$\text{乾}^2 + \text{乙}^2 - \text{丙}^2 = 2 \text{乾} \times \text{艮} \quad (\text{乾}^2 + \text{乙}^2 - \text{丙}^2)^2 = 4 \text{乾}^2 \times \text{艮}^2 = \text{上}$$

$$= (\text{丙}^4 - 2 \text{乙}^2 \times \text{丙}^2 + \text{乙}^4) + (-2 \text{丙}^2 + 2 \text{乙}^2) \text{乾}^2 + \text{乾}^4 = \text{上}$$

$$4 \text{乙}^2 \times \text{乾}^2 - \text{上} = 4 \text{乾}^2 \times \text{坤}^2 \quad 4 \text{乾}^2 \times \text{坤}^2 \times \text{戊}^2 = \text{下}$$

$$(-\text{角} \times \text{丙}^4 + 2 \text{角} \times \text{乙}^2 \times \text{丙}^2 - \text{角} \times \text{乙}^4) + (2 \text{角} \times \text{丙}^2 + 2 \text{角} \times \text{乙}^2) \text{乾}^2 - \text{角} \times \text{乾}^4 = \text{下}$$

$$\text{乙}^2 \times \text{丙}^2 = \text{戊}^2 \times \text{坤}^2 \quad 4 \text{乾}^2 \times \text{乙}^2 \times \text{丙}^2 = \text{下位相消}$$



$$(-\text{角} \times \text{丙}^4 + 2\text{角} \times \text{乙}^2 \times \text{丙}^2 - \text{角} \times \text{乙}^4)$$

$$+(-4\text{乙}^2 \times \text{丙}^2 + 2\text{角} \times \text{丙}^2 + 2\text{角} \times \text{乙}^2) \text{乾}^2 - \text{角} \times \text{乾}^4 = 0 \dots \text{退前式}$$

$$-\text{亢} + 2\text{氏} \times \text{乾}^2 - \text{角} \times \text{乾}^4 = 0$$

$$-\text{丁}^4 + 2\text{丁}^2 \times \text{戊}^2 - \text{戊}^4 = \text{子} \quad \text{子} = -\text{房}$$

$$2\text{円}^2 \times \text{丁}^2 + 2\text{円}^2 \times \text{戊}^2 - 4\text{丁}^2 \times \text{戊}^2 = \text{丑}$$

$$2\text{角} \times \text{丁}^2 + 2\text{角} \times \text{戊}^2 - 4\text{丁}^2 \times \text{戊}^2 = \text{丑} \quad \text{丑} = 2\text{心}$$

$$-\text{乾}^4 + 2\text{甲}^2 \times \text{乾}^2 - \text{甲}^4 = \text{寅}$$

$$2\text{円}^2 \times \text{乾}^2 + 2\text{円}^2 \times \text{甲}^2 - 4\text{甲}^2 \times \text{乾}^2 = 2\text{角} \times \text{甲}^2 + (-4\text{甲}^2 + 2\text{角}) \text{乾}^2 = \text{卯}$$

$$\text{丑}^2 \times \text{寅} + \text{子} \times \text{卯}^2 + 2\text{円}^4 \times \text{子} \times \text{寅} = \text{辰}$$

$$\text{丑}^2 \times \text{寅} = 4\text{心}^2(-\text{乾}^4 + 2\text{甲}^2 \times \text{乾}^2 - \text{甲}^4) = -4\text{心}^2 \times \text{甲}^4 + 8\text{心}^2 \times \text{甲}^2 \times \text{乾}^2 - 4\text{心}^2 \times \text{乾}^4$$

$$\text{子} \times \text{卯}^2 = -\text{房}(2\text{角} \times \text{甲}^2 + 2\text{尾} \times \text{乾}^2)^2$$

$$= -4\text{角}^2 \times \text{房} \times \text{甲}^4 - 8\text{角} \times \text{房} \times \text{尾} \times \text{甲}^2 \times \text{乾}^2 - 4\text{房} \times \text{尾}^2 \times \text{乾}^4$$

$$2\text{円}^4 \times \text{子} \times \text{寅} = 2\text{角}^2(-\text{房})(-\text{乾}^4 + 2\text{甲}^2 \times \text{乾}^2 - \text{甲}^4)$$

$$= 2\text{角}^2 \times \text{房} \times \text{甲}^4 - 4\text{角}^2 \times \text{房} \times \text{甲}^2 \times \text{乾}^2 + 2\text{角}^2 \times \text{房} \times \text{乾}^4$$

$$\text{辰} = (-4\text{心} \times \text{甲}^4 - 2\text{角}^2 \times \text{房} \times \text{甲}^4) + (8\text{心}^2 \times \text{甲}^2 - 8\text{角} \times \text{房} \times \text{尾} \times \text{甲}^2$$

$$-4\text{角}^2 \times \text{房} \times \text{甲}^2) \text{乾}^2 + (-4\text{心}^2 - 4\text{房} \times \text{尾}^2 + 2\text{角}^2 \times \text{房}) \text{乾}^4$$

$$\text{丑} \times \text{寅} \times \text{卯} + \text{子} \times \text{丑} \times \text{卯} + \text{円}^4 \times \text{寅}^2 + \text{円}^4 \times \text{子}^2 = \text{辰相消}$$

$$\text{丑} \times \text{寅} \times \text{卯} = 2\text{心}(-\text{乾}^4 + 2\text{甲}^2 \times \text{乾}^2 - \text{甲}^4)(2\text{角} \times \text{甲}^2 + 2\text{尾} \times \text{乾}^2)$$

$$= -4\text{角} \times \text{心} \times \text{甲}^6 + (8\text{角} \times \text{心} \times \text{甲}^4 - 4\text{心} \times \text{尾} \times \text{甲}^4) \text{乾}^2$$

$$+(-4\text{角} \times \text{心} \times \text{甲}^2 + 8\text{心} \times \text{尾} \times \text{甲}^2) \text{乾}^4 - 4\text{心} \times \text{尾} \times \text{乾}^6$$

$$\text{子} \times \text{丑} \times \text{卯} = -\text{房} \times 2\text{心}(2\text{角} \times \text{甲}^2 + 2\text{尾} \times \text{乾}^2)$$

$$= -4\text{角} \times \text{房} \times \text{心} \times \text{甲}^2 - 4\text{房} \times \text{心} \times \text{尾} \times \text{乾}^2$$

$$\text{円}^4 \times \text{寅}^2 = \text{角}^2(-\text{乾}^4 + 2\text{甲}^2 \times \text{乾}^2 - \text{甲}^4)^2$$

$$= \text{角}^2(\text{甲}^8 + \text{乾}^8 + 4\text{甲}^4 \times \text{乾}^4 - 4\text{甲}^2 \times \text{乾}^6 - 4\text{甲}^6 \times \text{乾}^2 + 2\text{甲}^4 \times \text{乾}^4)$$

$$= \text{角}^2 \times \text{甲}^8 - 4 \text{角}^2 \times \text{甲}^6 \times \text{乾}^2 + 6 \text{角}^2 \times \text{甲}^4 \times \text{乾}^4 - 4 \text{角}^2 \times \text{甲}^2 \times \text{乾}^6 + \text{角}^2 \times \text{乾}^8$$

$$\text{円}^4 \times \text{子}^2 = \text{角}^2 \times \text{房}^2$$

$$(-4 \text{角} \times \text{心} \times \text{甲}^6 - 4 \text{角} \times \text{房} \times \text{心} \times \text{甲}^2 + \text{角}^2 \times \text{甲}^8 + \text{角}^2 \times \text{房}^2)$$

$$+(8 \text{角} \times \text{心} \times \text{甲}^4 - 4 \text{心} \times \text{尾} \times \text{甲}^4 - 4 \text{房} \times \text{心} \times \text{尾} - 4 \text{角}^2 \times \text{甲}^6) \text{乾}^2$$

$$+(-4 \text{角} \times \text{心} \times \text{甲}^2 + 8 \text{心} \times \text{尾} \times \text{甲}^2 + 6 \text{角}^2 \times \text{甲}^4) \text{乾}^4$$

$$+(-4 \text{心} \times \text{尾} - 4 \text{角}^2 \times \text{甲}^2) \text{乾}^6 + \text{角}^2 \times \text{乾}^8 = \text{辰位相消}$$

$$(-4 \text{心}^2 \times \text{甲}^4 - 2 \text{角}^2 \times \text{房} \times \text{甲}^4 + 4 \text{角} \times \text{心} \times \text{甲}^6 + 4 \text{角} \times \text{房} \times \text{心} \times \text{甲}^2$$

$$- \text{角}^2 \times \text{甲}^8 - \text{角}^2 \times \text{房}^2) + (8 \text{心}^2 \times \text{甲}^2 - 8 \text{角} \times \text{房} \times \text{尾} \times \text{甲}^2 - 4 \text{角}^2 \times \text{房} \times \text{甲}^2$$

$$- 8 \text{角} \times \text{心} \times \text{甲}^4 + 4 \text{心} \times \text{尾} \times \text{甲}^4 + 4 \text{房} \times \text{心} \times \text{尾} + 4 \text{角}^2 \times \text{甲}^6) \text{乾}^2$$

$$(-4 \text{心}^2 - 4 \text{房} \times \text{尾}^2 + 2 \text{角}^2 \times \text{房} + 4 \text{角} \times \text{心} \times \text{甲}^2 - 8 \text{心} \times \text{尾} \times \text{甲}^2 - 6 \text{角}^2 \times \text{甲}^4) \text{乾}^4$$

$$+(4 \text{心} \times \text{尾} + 4 \text{角}^2 \times \text{甲}^2) \text{乾}^6 - \text{角}^2 \times \text{乾}^8 = 0 \dots \text{退後式}$$

$$(\text{進前式}) \times \text{角} - \text{角} \times \text{亢} + 2 \text{角} \times \text{氏} \times \text{乾}^2 - \text{角}^2 \times \text{乾}^4 = 0 \quad \text{退後式から } \text{乾}^8 \text{ を消去する}$$

$$- \text{女} + 4 \text{牛} \times \text{乾}^2 + (\text{角} \times \text{亢} - 4 \text{心}^2 - 4 \text{房} \times \text{尾}^2 + 2 \text{角}^2 \times \text{房} + 4 \text{角} \times \text{心} \times \text{甲}^2$$

$$- 8 \text{心} \times \text{尾} \times \text{甲}^2 - 6 \text{角}^2 \times \text{甲}^4) \text{乾}^4 + (-2 \text{角} \times \text{氏} + 4 \text{心} \times \text{尾} + 4 \text{角}^2 \times \text{甲}^2) \text{乾}^6 = 0$$

乾 = x とする

$$- \text{亢} + 2 \text{氏} \times x - \text{角} \times x^2 = 0$$

$$- \text{女} + 4 \text{牛} \times x - \text{計} \times x^2 + 2 \text{箕} \times x^3 = 0$$

$$(-2 \text{氏} \times \text{女} - 4 \text{亢} \times \text{牛}) + (-\text{角} \times \text{女} + \text{亢} \times \text{計})x - 2 \text{亢} \times \text{箕} \times x^2 = 0$$

$$(-\text{角} \times \text{女} + \text{亢} \times \text{計}) + (-2 \text{亢} \times \text{箕} + 4 \text{角} \times \text{牛} - 2 \text{氏} \times \text{計})x + 4 \text{氏} \times \text{箕} \times x^2 = 0$$

$$- 2 \text{亢} \times \text{箕} + 4 \text{氏} \times \text{箕} x - 2 \text{角} \times \text{箕} x^2 = 0$$

$$- \text{虚} + \text{危} x - 2 \text{亢} \times \text{箕} x^2 = 0$$

$$\text{危} - 2 \text{室} x + 4 \text{氏} \times \text{箕} x^2 = 0$$

$$- \text{亢} + 2 \text{氏} x - \text{角} x^2 = 0$$

$$- 2 \text{虚} \times \text{室} \times \text{角} \quad - 4 \text{危} \times \text{氏} \times \text{亢} \times \text{箕} \quad - 4 \text{危} \times \text{氏} \times \text{亢} \times \text{箕}$$

$$8 \text{虚} \times \text{氏}^2 \times \text{箕} \quad \text{虚}^2 \times \text{角} \quad 4 \text{亢}^2 \times \text{室} \times \text{箕}$$

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