

SEMIGROUPS PRESENTED BY FINITE CONGRUENCE CLASSES *

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In this article, we study semigroups presented by finite congruence classes.

Definition 1 (1) *Let X be finite alphabets and R a subset of $X^+ \times X^+$. Then R is called a rewriting system.*

(2) *For $u, v \in X^+$, $(w_1, w_2) \in R$, $uw_1v \Rightarrow_R uw_2v$.*

The congruence μ_R on X^+ generated by $\Rightarrow_R$ is called the Thue congruence defined by R .

(3) *A semigroup S is (finitely) presented if there exists a (finite) set of X , there exists a surjective homomorphism ϕ of X^+ to S and there exists a (finite) rewriting system R consisting of pairs of words over X such that the Thue congruence μ_R is the congruence $\{(w_1, w_2) \in X^+ \times X^+ \mid \phi(w_1) = \phi(w_2)\}$.*

In this case, we say that S has a presentation by X and R denoted by $S = \langle X : R \rangle$.

Definition 2 *A semigroup S has a presentation with regular [resp. finite] congruence classes if there exists a finite set X and there exists a surjective homomorphism ϕ of X^+ to M such that for each word $w \in X^+$, $\phi^{-1}(\phi(w))$ is a regular [resp. finite] language.*

In [3], we investigated the properties of semigroups presented by regular congruence classes. The class of such semigroups contains many important classes of semigroups, for example, Burnside semigroups ([1]), Baumslag-Solitar semigroups ([2]) and so on. Even the class of semigroups presented by finite congruence classes looks so interesting. So we give a characterization of one-relator rewriting systems all Thue congruence classes of that are finite.

*This is an abstract and the paper will appear elsewhere.

Definition 3 A word u over a finite alphabet X is unbordered if there exists no non-empty word v with $u \in vX^+ \cap X^+v$.

We have

Theorem. Let u, w be word over a finite alphabet X and $R = \{(u, w)\}$ a one-relator rewriting system. Assume that u is an unbordered and the length of u is shorter than one of w . Further, assume that u is not a subword of w . Then the rewriting system $R = \{(u, w)\}$ gives only finite congruence classes if and only if there are no non-empty words $l_{i,j}, r_{i,j}$ over X such that $u = l_{s,t}r_{s,t}$ ($1 \leq s \leq 2k, 1 \leq t \leq i_s$), $w \in r_{1,1} \cdots r_{1,i_1}X^+$, $w \in X^+l_{2,i_2} \cdots l_{2,1}l_{1,i_1}, \dots$, $w \in X^+l_{2k,i_k} \cdots l_{2k,1}l_{2k-1,i_{2k-1}}$ and $l_{2k,i_k} = l_{1,1}$.

References

- [1] V. Guba, *The word problem for the relatively free semigroup satisfying $T^m = T^{m+n}$ with $m \geq 4$ or $m = 3, n = 1$* Internat. J. Algebra Comput. **3**(1993), no. 2, 125-140.
- [2] D. Jackson, *Decision and separability problems for Baumslag-Solitar semigroups* Internat. J. Algebra Comput. **12**(2002), no. 1-2, 33-49.
- [3] K. Shoji, *Finitely generated semigroups which have such a presentation that all the congruence classes are regular languages*, Math. Japonicae **69**(2008), 73-78.