

Network-based distributed direct load control guaranteeing fair welfare maximization

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Abstract: This paper examines a distributed direct load control (DLC) problem for maximizing customer welfare in a power system for the network communication of energy management controllers (EMCs). A model is first built to describe the dynamics and communication intervals of the EMCs with a distributed and uniform controller. The controller conditions are then derived to stabilize the system and to converge the power imbalance to zero at an assigned rate. The control condition that maximizes customer welfare is then found. Furthermore, an optimal controller that maximizes customer welfare over a given network communication is proposed, and the performance degradation caused by distributed management is evaluated. This paper reveals that even though moving from a centralized to a distributed DLC can degrade customer welfare, this degradation can be reduced by considering consumer properties and network topologies of the EMCs. Numerical examples with real consumption data are also presented to demonstrate the effectiveness of the proposed method.

1 Introduction

1.1 Motivation and incitement

Due to growing demand, increasing fuel prices, and heightened opposition to fossil and nuclear energy, electricity shortages are likely to occur in the future. Introducing various types of renewable energy sources, including solar, wind, and biomass energy, is a matter of great urgency [1], especially after the Fukushima earthquake in Japan on March 11, 2011 [2]. However, not all electricity shortage problems can be solved by renewable energy sources alone due to their capacity limitations. Furthermore, natural fluctuations in renewable energy production can destabilize power systems. To address these issues, focus has been given to demand response (DR), as summarized in [3–5], in which customers are incentivized (e.g., with monetary rewards) when they restrain their electricity demands. The effectiveness of DR has been demonstrated experimentally in the USA [6], United Kingdom [7], Ireland [8], and other countries [9]. Thus, DR is expected to be implemented in future smart grids that are equipped with bidirectional communication between suppliers and consumers over network technology [10] and advanced metering infrastructure [11].

Direct load control (DLC) is one of the most effective incentive-based DR programs. Under a DLC program, each customer is equipped with an energy management controller (EMC) that is controlled remotely by a system operator to directly manage end-use devices such as air conditioners, hot water systems, and electric vehicles (for charging) [3–5, 12]. Although a considerable incentive is provided to the customers for their participation, recruitment is often difficult. Based on site surveys, incentive and trust are both important factors that influence customers' willingness [13]. To establish trust, energy companies have established control policies to minimize outage time [14] or the amount of reduced consumption [15] to avoid causing inconvenience to customers with DLC as much as possible.

Conversely, DR programs including DLC are usually implemented by means of centralized management of system operators, including energy suppliers [16]. However, centralized management needs investment in communication systems [11] and raises security

issues concerning the accumulation of private information [17]. The implementation of distributed management is expected to resolve such issues. Distributed management style typically determines control signals via communication between customers' network devices, assuming that customers are provided with such devices (e.g., smart meters). This creates advantages of distributed computation and lower data transfer due to local data processing [18].

However, distributed management may lead to instabilities and performance degradation due to network communication and coordination time [19]. Haring *et al.* recently investigated the difference between centralized and distributed DLC programs on the basis of benchmark simulations and observed some performance degradation in distributed algorithms [20]. This will likely cause unfair restriction in customer consumption, leading to further distrust of DLC programs. Hence, the performance degradation on fairness caused by distributed management should be carefully investigated. However, it has been confirmed only in numerical ways in these papers but has not yet been investigated theoretically.

1.2 Contributions and comparisons with existing papers

The present paper addresses a distributed DLC problem involving power systems over network communication of EMCs to maximize customer welfare. The dynamics and communication intervals of the EMCs are first modeled as a linear discrete-time system using a *distributed and uniform controller*. Under distributed control, each EMC can use only information about the neighbors that are directly connected to it via network links. Moreover, uniform controllers are employed for fair and easy implementation. Next, a set of distributed and uniform controllers is derived to stabilize the entire system and converge the power imbalance to zero at an assigned rate. Here, the controller that maximizes customer welfare is found in the derived set. Finally, numerical examples involving the consumption data of actual residential consumers demonstrate the effectiveness of the proposed method.

The main motivations of this work are (i) to find a fair controller for distributed DLC that guarantees the optimality and stability of the entire system and (ii) to investigate the relations between the performances on fairness, network topology, and customers' properties to

help the design of distributed DLC programs. Due to the assumption of a quick response by the DLC, we can derive an equation to represent this relation explicitly, which indicates that the performance can be degraded by distributed management and that the degradation depends on the variation in customer consumption and the network topology of the EMCs. Consequently, we can enhance the distributed DLC programs by considering customer properties and network design.

A common mathematical approach for establishing DR program algorithms is to formulate the social and individual requirements in power systems with an optimization problem that maximizes customers' utility under constraints, including power balance [21–24], as surveyed in [25]. Following this approach, distributed DR algorithms have been proposed in [26, 27]. Safdarian *et al.* minimized customers' monetary expenses while flattening the total load profile in their recent study [26]. A problem was considered to simultaneously minimize the aggregate cost and dissatisfaction and maximize the retailer profit and was solved with an adaptive diffusion-Stackelberg algorithm by Latifi *et al.* [27]. However, the efficiency of their methods was only confirmed in a numerical manner, not in a theoretical one. Hence, neither the stability nor the optimality of the algorithms was ensured.

Conventional approaches to solving optimization problems in a theoretically rigorous manner introduce Lagrangian multipliers as penalties for the constraints [22, 24]. Lagrangian multipliers are common to all customers, whereas customers cannot share common variables in distributed systems. Hence, to establish distributed DR programs, a method must be developed to individually estimate the Lagrangian multipliers. For this purpose, consensus controllers are employed to ensure that variables corresponding to the Lagrangian multipliers are in agreement among the customers [28, 29]. The stability of the distributed management systems that use consensus-based algorithms has been analyzed in a theoretical manner [30–32]. In particular, Chen *et al.* developed a consensus-based distributed DLC program over a two-layer communication network [32]; a similar structure will be also used in the present paper. Nevertheless, previous studies have not derived the optimal controllers obtainable under distributed management.

In contrast to the existing papers, several notable contributions are presented in this study. First, most of the existing works have considered a control or regulation of power generation, i.e., from the perspective of generators. They implemented control of the network by changing the quantity of power generated to meet the supply and demand balance. However, in this work, we effect control of the network by equally emphasizing the generation side and demand side, and both the quantity generated and demanded are controlled to maximize the optimality while maintaining supply-demand balance. Second, in ordinary distributed optimization, multiple Lagrangian parameters and some subsidiary parameters need to be updated, whereas there are fewer parameter updates in this work; only one parameter (control signal) is updated. Third, this work is the first to design an optimal controller that maximizes customer welfare over a given network communication while considering the system stability and convergence rate.

Comparison with the latest literature on optimization methods for DR is given as follows. The authors [29] proposed a consensus-based distributed optimization method. Wang *et al.* [33] and Tsai *et al.* [34] proposed a consensus-based distributed optimization, using the technique of alternating direction method of multipliers (ADMM). Diekerhof *et al.* [35] proposed a robust optimization method based on ADMM. These papers neither consider the stability nor convergence. Moreover, they have no discussion on the relations between the performance on fairness, network topology, and customers' properties. In contrast, the present paper guarantees not only the stability but also the convergence with an assigned rate and declares these relations in a theoretically explicit form.

A part of this work was presented as a conference paper [36] by the corresponding author. The updated points are as follows: (i) while the conference paper deals with a mathematical problem of aggregate state control, the present paper focuses on its application to power systems from a practical perspective. (ii) All proofs of lemmas and theorems that are omitted in the conference paper due to space

Table 1 Notation

$\mathcal{C} = \{1, 2, \dots, n\}$	Set of the customers
$x_i(t) \in \mathbb{R}$	Electricity supplied or consumed by customer i
$u_i(t) \in \mathbb{R}$	Control input to customer i
$y(t) \in \mathbb{R}$	Power imbalance (sum of all $x_i(t)$ for $i \in \mathcal{C}$)
$x_{*i} \in \mathbb{R}$	Desired amount of $x_i(t)$
$\mu_{x_*} \in \mathbb{R}$	Expectation of x_{*i} for $i \in \mathcal{C}$ ($= E(x_{*i})$)
$\sigma_{x_*}^2 \in \mathbb{R}$	Variance of x_{*i} for $i \in \mathcal{C}$ ($= E((x_{*i} - \mu_{x_*})^2)$)
$V(k) \in \mathbb{R}$	Expected total dissatisfaction ($=$ minus welfare)
$w_i \in \mathbb{R}$	Weight of customer i in $V(k)$
$\mu_w \in \mathbb{R}$	Expectation of w_i for $i \in \mathcal{C}$ ($= E(w_i)$)
$\sigma_w^2 \in \mathbb{R}$	Variance of w_i for $i \in \mathcal{C}$ ($= E((w_i - \mu_w)^2)$)
$\mathcal{N}_i \subset \mathcal{C}$	Set of the neighbors of $i \in \mathcal{C}$
$n_i \in \{0, 1, \dots, n\}$	Number of the neighbors of $i \in \mathcal{C}$
$\mathcal{E} \subset \mathcal{C}$	Set of the customer pairs neighboring each other
$G = (\mathcal{C}, \mathcal{E})$	Graph of network derived by $\mathcal{N}_i$
$L_G \in \mathbb{R}^{n \times n}$	Graph Laplacian of G
$\lambda_i \in \lambda(L_G)$	Eigenvalues of L_G ($i \in \{1, 2, \dots, n\}$)
$v_i \in \mathbb{R}^n$	Eigenvectors of L_G corresponding to λ_i
$k = (k_a, k_b, k_c, k_d)$	Parameters of distributed and uniform controller
$\gamma \in (0, 1)$	Assigned convergence rate of $y(t)$
$\mathcal{K} \subset \mathbb{R}^4$	Set of the parameters k which stabilize the entire system and make $y(t)$ converge to 0 with rate γ

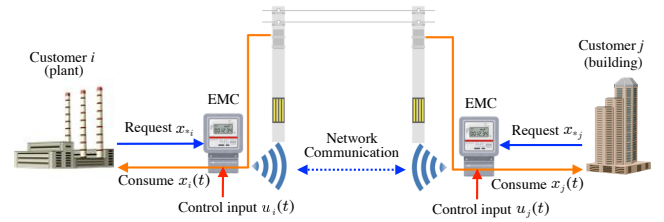


Fig. 1: Functions of EMCs.

limitation are completed. (iii) Consumption data of actual consumers are used to conduct more realistic simulations.

1.3 Organization

The rest of this paper is organized as follows. The dynamics and control input of EMCs and policies for a DLC program are described in Section 2. In Section 3, the class of the control gains that stabilize the system and maintain the power balance are derived, and the best gain that maximizes customer welfare is obtained. A numerical example illustrates the effectiveness of the proposed method in Section 4, and the paper is concluded in Section 5.

2 Problem setting

Let $\mathbb{R}$ be the set of real numbers and $\mathbb{Z}_+$ be the set of nonnegative integers. The identity matrix is represented by I , and $\mathbf{1}$ represents a vector whose components are all ones. For a matrix, $M \in \mathbb{R}^{n \times n}$, the set of all of its eigenvalues is denoted as $\lambda(M)$. The expectation of a random variable, x is designated as $E(x)$. A function, $f: \mathbb{R} \rightarrow \mathbb{R}$ is said to be of class K if it is strictly increasing and $f(0) = 0$. For a set, $\mathcal{S}$, $\text{cl}(\mathcal{S})$ denotes the closure of $\mathcal{S}$. Other notations used in this paper are summarized in Table 1.

2.1 Target system

A power system managed by a system operator is considered, whose customers are commercial and industrial facilities and aggregators

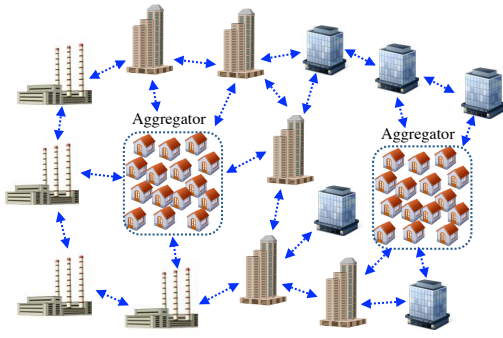


Fig. 2: The communication network of EMCs.

managing groups of residences (prosumers) and suppliers. Each of the facilities and aggregators is assumed to be equipped with an EMC that has two functions, as illustrated in Fig. 1: (i) First, EMCs exchange control signals with other EMCs over a communication network. All EMCs are controlled in a uniform manner using only local information obtained through the aforementioned communication. (ii) Second, each EMC controls (curtails) the loads of the customer(s) using its own control scheme. The EMCs that manage a group of residences will choose appropriate customer(s) to implement the target curtailment. This work focuses on the appropriate determination of the control signal over the network communication between the EMCs.

We assume that this network contains n EMCs; the set of the EMCs is denoted by $\mathcal{C} = \{1, 2, \dots, n\}$. With a function (i) of the EMCs, a large-scale communication network is composed, as shown in Fig. 2. The topology of this network is modeled by a graph, $G = (\mathcal{C}, \mathcal{E})$ with an edge set, $\mathcal{E}$. Hence, EMCs $i \in \mathcal{C}$ and $j \in \mathcal{C}$ are directly connected over the network if and only if $\{i, j\} \in \mathcal{E}$. The set of neighbors of EMC $i \in \mathcal{C}$ is defined as $\mathcal{N}_i = \{j \in \mathcal{C} : \{i, j\} \in \mathcal{E}\}$. Let $n_i \in \mathbb{Z}_+$ ($n_i \leq n$) represent the number of the elements of $\mathcal{N}_i$. We assume that G is undirected and time-invariant, and that there exists a pair of EMCs $i, j \in \mathcal{C}$ ($i \neq j$) with different numbers of neighbors; namely, $n_i \neq n_j$.

Each EMC is under the DLC program implemented by the system operator and directly controls the supply and consumption load of its customers in real time with a function (ii). Within this framework, the system operator must maintain the power system effectively under a control policy approved by all the customers.

2.2 Regulation of EMCs

The system of EMCs is assumed to consist of a regulation system and a distributed controller, as explained below.

The quantity of electricity supplied or consumed by the customers of an EMC $i \in \mathcal{C}$ at time $t \in \mathbb{Z}_+$ is denoted by $x_i(t) \in \mathbb{R}$. If $x_i(t) < 0$, electricity is consumed; otherwise, electricity is supplied. The desired supply or consumption requested by the customers of EMC i is designated as $x_{*i} \in \mathbb{R}$, and the control signal from the EMC is represented by $u_i(t) \in \mathbb{R}$. The quantity of electricity supplied or consumed is regulated by the *regulation system*, defined by the following equation:

$$x_i(t) = x_{*i} + u_i(t). \quad (1)$$

Here, when the control signal is zero, the customers of EMC i can supply or consume as they like; namely, $x_i(t) = x_{*i}$ is realized. Otherwise, their supply or consumption is controlled.

Remark 1. Eq. (1) can be considered as an approximate model of customers' behavior for supply and consumption. This approximation is valid under the DLC as follows. The customers' behavior is nonlinear in practice and modeled as

$$x_i(t+1) = x_i(t) + \frac{\partial U_i}{\partial x_i}(x_i(t)) + u_i(t) \quad (2)$$

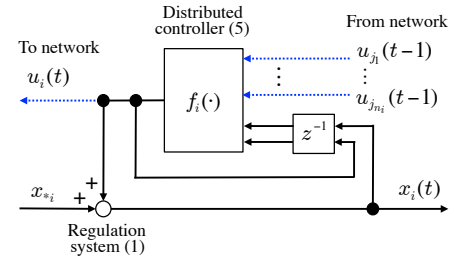


Fig. 3: Block diagram of an EMC system with a distributed controller. The solid arrows denote internal signal transmission in the EMCs, and the dotted lines denote communication transmission to/from other EMCs.

with a concave function $U_i(x_i)$, which is a utility function that describes the customer's preference determining his action. The function, $U_i(x_i)$ takes a peak value at the desired quantity, x_{*i} , and its shape around the peak can approximate to the quadratic function $-|x_i - x_{*i}|^2/2$. As a result, (2) is reduced to (1). Under the assumption of DLC usage, we can control customers' behavior to this approximation through sufficient incentives. $\square$

The desired quantities, x_{*i} of the customers of the EMCs are assumed to be uncorrelated random values with expectation, $\mu_{x_*} \in \mathbb{R}$ and variance, $\sigma_{x_*}^2 \geq 0$. Thus, the following expressions are obtained:

$$E(x_{*i}) = \mu_{x_*}, \quad E((x_{*i} - \mu_{x_*})^2) = \sigma_{x_*}^2, \quad (3)$$

$$E((x_{*i} - \mu_{x_*})(x_{*j} - \mu_{x_*})) = 0 \quad (i \neq j). \quad (4)$$

EMC i is assumed to be able to decide the value of x_{*i} , which is unknown to any other EMCs or the system operator. The statistical values, μ_{x_*} and $\sigma_{x_*}^2$ are unknown but are later used for performance analysis.

The control input $u_i(t)$ is determined by EMC i that is able to receive its own signals and those of its neighbors through the network. The variables of the previous time are available at time $t \in \mathbb{Z}_+$, namely $x_i(t-1)$, $u_i(t-1)$, $u_{j_1}(t-1)$, $u_{j_2}(t-1)$, $\dots$, $u_{j_{n_i}}(t-1)$, where $\{j_1, j_2, \dots, j_{n_i}\} = \mathcal{N}_i$. However, the $x_j(t-1)$ of neighbors $j \in \mathcal{N}_i$ are unavailable because they include private information (e.g., quantity consumed) and thus cannot be sent to others. The control input is then generated from the *distributed controller*, as shown in the following equation:

$$u_i(t) = f_i(x_i(t-1), u_i(t-1), u_{j_1}(t-1), \dots, u_{j_{n_i}}(t-1)) \quad (5)$$

with some function $f_i : \mathbb{R}^{2+n_i} \rightarrow \mathbb{R}$. The EMC system consists of the regulation system defined by (1) and the distributed controller defined by (5), as shown by the block diagram in Fig. 3. The communication network comprising multiple EMC systems is shown in Fig. 4.

For fairness to the customers and simplicity of implementation, a linear distributed controller with uniform independence of the index i in the gains $k_a, k_b, k_c, k_d \in \mathbb{R}$ is employed:

$$u_i(t) = k_a x_i(t-1) + k_b u_i(t-1) + \sum_{j \in \mathcal{N}_i} (k_c u_i(t-1) + k_d u_j(t-1)). \quad (6)$$

This is called a *distributed and uniform controller*. There are several advantages of using this controller: First, because it is uniform, namely, its gains are independent of the index i , the same rule is applied to all EMCs, which is fair to all customers. Second, even if the network changes when an EMC j_{new} joins (j_{old} leaves),

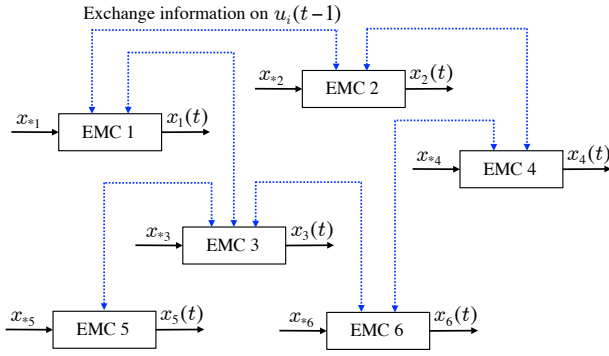


Fig. 4: Communication network composed of the EMC systems in Fig. 3. The solid and dotted arrows denote the internal signal and communication transmission between the EMCs, respectively.

the controller still works by just putting $k_c u_{j_{\text{new}}}(t)$ to (6) (removing $k_c u_{j_{\text{old}}}(t)$ from (6)). In this sense, the controller is tolerant to network changes.

The system operator establishes the control policies to maintain the power system while maximizing the welfare of the customers. Following these policies, the control gains k_a, k_b, k_c, k_d of the distributed and uniform controller [Eq. (6)] are determined; the collection of the gains is defined as $k = (k_a, k_b, k_c, k_d) \in \mathbb{R}^4$.

2.3 Control policies

The system operator has two control policies as follows.

First, the power system must be maintained by balancing the supply and consumption. Power imbalance, $y(t) \in \mathbb{R}$ is defined as

$$y(t) = \sum_{i=1}^n x_i(t). \quad (7)$$

Eq. (7) is then required to converge to zero at the assigned convergence rate $\gamma \in (0, 1)$, as

$$|y(t)| \leq \alpha(|y(0)|)\gamma^t \quad (8)$$

with some function $\alpha : \mathbb{R} \rightarrow \mathbb{R}$ of class K. In addition, EMC systems (1) and (6) are required to be stable. Let $\mathcal{K}$ be the set of the control gains, $k \in \mathbb{R}^4$ with which these two requirements are fulfilled for any $x_{*i}, u_i(0) \in \mathbb{R}$ ($i \in \mathcal{C}$). Then, we have to choose k from $\mathcal{K}$.

Second, the expected total curtailment by the EMCs at the terminal time, defined as

$$V(k) = \mathbb{E} \left(\sum_{i=1}^n w_i \left(\lim_{t \rightarrow \infty} x_i(t) - x_{*i} \right)^2 \right) \quad (9)$$

with weights $w_i \in \mathbb{R}$, should be minimized. In (9), $(x_i(t) - x_{*i})^2$ indicates the squared amount of curtailment for each EMC. As the curtailment of electricity causes inconvenience to customers, their welfare increases as the amount of curtailment decreases. Thus, the minimization of the sum of curtailments, as given in Eq. (9), maximizes the welfare of the customers.

The weights, w_i are assumed to be random, uncorrelated values with x_{*j} for any $j \in \mathcal{C}$. The expectation and variance of w_i are given by $\mu_w > 0$ and $\sigma_w^2 \geq 0$, respectively. Thus, the following expressions hold for any $i \in \mathcal{C}$:

$$\mathbb{E}(w_i) = \mu_w, \quad \mathbb{E}((w_i - \mu_w)^2) = \sigma_w^2, \quad (10)$$

$$\mathbb{E}((w_i - \mu_w)(w_j - \mu_w)) = 0 \quad (i \neq j), \quad (11)$$

$$\mathbb{E}((w_i - \mu_w)(x_{*j} - \mu_{x_*})) = 0 \quad (j \in \mathcal{C}). \quad (12)$$

The function in the expectation of Eq. (9) depends on the random variables, $x_{*1}, x_{*2}, \dots, x_{*n}, w_1, w_2, \dots, w_n \in \mathbb{R}$.

We will find the best control gain, k of the distributed and uniform controller (6) in the sense that the objective function, $V(k)$ is minimized for all $k \in \mathcal{K}$:

$$\min_{k \in \mathcal{K}} V(k). \quad (13)$$

3 Main results

3.1 Derivation of constraint set

Theorem 1, shown below, provides the necessary and sufficient condition for the stability of the entire system with the assignment of the convergence to $y(t)$.

Theorem 1. A system consisting of (1) and (6) for all $i \in \mathcal{C}$ is stable, and $y(t)$ in (7) satisfies (8) with some function, $\alpha : \mathbb{R} \rightarrow \mathbb{R}$ of class K for a given $\gamma \in (0, 1)$ and any $x_{*i}, u_i(0) \in \mathbb{R}$ ($i \in \mathcal{C}$) if and only if $k = (k_a, k_b, k_c, k_d) \in \mathcal{K}$ is satisfied for the set:

$$\mathcal{K} = \{(k_a, k_b, k_c, k_d) \in \mathbb{R}^4 : k_b = 1, k_c = -k_d, |k_a + 1| \leq \gamma, |k_a + 1 + k_c \max \lambda(L_G)| < 1\}, \quad (14)$$

where $L_G \in \mathbb{R}^{n \times n}$ is the graph Laplacian of G . $\square$

There are three main roles of the expressions in (14): (i) The conditions $k_b = 1$ and $k_c = -k_d$ guarantee that $y(t)$ converges to zero if it converges. (ii) From $|k_a + 1| \leq \gamma$, the convergence rate of $y(t)$ is assigned to γ . (iii) The stability of the entire system is ensured by $|k_a + 1 + k_c \max \lambda(L_G)| < 1$. Items (i), (ii), and (iii) are proved below in Lemmas 1, 2, and 3, respectively.

A system that consists of (1) and (6) for all $i \in \mathcal{C}$ is equivalent to a linear discrete-time system

$$u(t) = (k_{ab}I + K_{cd})u(t-1) + k_a x_* \quad (15)$$

where $u(t) = [u_1(t) \ u_2(t) \ \dots \ u_n(t)]^\top$ is the state, $x_* = [x_{*1} \ x_{*2} \ \dots \ x_{*n}]^\top$ is the reference, $k_{ab} = k_a + k_b$, and the (i, j) th component of $K_{cd} \in \mathbb{R}^{n \times n}$ is given as

$$(K_{cd})_{ij} = \begin{cases} n_i k_c & \text{if } j = i, \\ k_d & \text{if } j \in \mathcal{N}_i, \\ 0 & \text{otherwise.} \end{cases} \quad (16)$$

If all the absolute eigenvalues of the matrix, $k_{ab}I + K_{cd}$ are less than one, system (15) will be stable, and the solution, $u(t)$ of (15) is calculated as follows:

$$\begin{aligned} u(t) &= (k_{ab}I + K_{cd})^t u(0) + \sum_{\ell=0}^{t-1} (k_{ab}I + K_{cd})^\ell k_a x_* \\ &= (k_{ab}I + K_{cd})^t u(0) \\ &\quad + (I - (k_{ab}I + K_{cd}))^{-1} (I - (k_{ab}I + K_{cd})^t) k_a x_*, \end{aligned} \quad (17)$$

as derived by the law of geometric series.

Under the assumption of stability, $y(t)$ must converge to zero to achieve (8), and the following lemma is derived.

Lemma 1. Assume that (15) is stable. Then, $\lim_{t \rightarrow \infty} y(t) = 0$ holds for any $x_{*i}, u_i(0) \in \mathbb{R}$ ($i \in \mathcal{C}$) if and only if $k_a \neq 0$ and

$$k_b = 1, \quad k_c = -k_d. \quad (18)$$

Proof: The terminal value of $y(t)$ is first calculated from the assumption of stability, where $\lim_{t \rightarrow \infty} (k_{ab}I + K_{cd})^t = 0$ holds. From

this and from (1), (7), and (17), the following equation is derived:

$$\begin{aligned}\lim_{t \rightarrow \infty} y(t) &= \lim_{t \rightarrow \infty} \mathbf{1}^\top (u(t) + x_*) \\ &= \mathbf{1}^\top (I - (k_{ab}I + K_{cd}))^{-1} k_a x_* + \mathbf{1}^\top x_*. \quad (19)\end{aligned}$$

Next, the condition of parameters (k_a, k_b, k_c, k_d) to achieve $\lim_{t \rightarrow \infty} y(t) = 0$ is derived. Eq. (19) is zero for any $x_* \in \mathbb{R}^n$ if and only if

$$\mathbf{1}^\top (I - (k_{ab}I + K_{cd}))^{-1} k_a + \mathbf{1}^\top = 0 \quad (20)$$

holds. For (20), $k_a \neq 0$ is necessary. Hence, (20) is equivalent to

$$\mathbf{1}^\top k_a + \mathbf{1}^\top (I - (k_{ab}I + K_{cd})) = 0,$$

which can be reduced to

$$\mathbf{1}^\top (I - (k_bI + K_{cd})) = 0. \quad (21)$$

From the assumption that G is undirected, K_{cd} in (16) is symmetric. Then, the i th component of $\mathbf{1}^\top K_{cd}$ is calculated as

$$\begin{aligned}(\mathbf{1}^\top K_{cd})_i &= \sum_{j=1}^n (K_{cd})_{ji} = \sum_{j=1}^n (K_{cd})_{ij} = n_i k_c + \sum_{j \in \mathcal{N}_i} k_d \\ &= n_i k_c + n_i k_d\end{aligned}$$

is obtained, with which Eq. (21) is reduced to

$$1 - k_b - n_i k_c - n_i k_d = 0. \quad (22)$$

The conditions in (18) are sufficient for (22). To show necessity, consider two EMCs $i, j \in \mathcal{C}$ such that $n_i \neq n_j$, and Eq. (22) is reduced to

$$1 - k_b - n_i(k_c + k_d) = 0, \quad 1 - k_b - n_j(k_c + k_d) = 0. \quad (23)$$

By subtracting the first equation in (23) from the second one, $(n_i - n_j)(k_c + k_d) = 0$ is obtained, leading to the second equation of (18) from $n_i \neq n_j$. Eq. (23) then leads to the first equation of (18). $\square$

Under (18), the system matrix of (15) is reduced to

$$k_{ab}I + K_{cd} = (k_a + 1)I + k_c L_G. \quad (24)$$

To analyze this matrix, the properties of the graph Laplacian, $L_G \in \mathbb{R}^{n \times n}$ of G must be investigated. Let $\lambda_i \in \lambda(L_G)$ ($i \in \mathcal{C}$) be the eigenvalues of L_G and let $v_i \in \mathbb{R}^n$ be the corresponding eigenvectors that are orthogonal to each other. Without loss of generality, two expressions

$$\lambda_1 = 0, \quad v_1 = \frac{1}{\sqrt{n}} \quad (25)$$

hold from the properties of the graph Laplacian [37]. From this, the necessary and sufficient condition of (8) can be achieved as shown in Lemma 2.

Lemma 2. Assume that (15) is stable. Then, (8) holds for a given $\gamma \in (0, 1)$ and any $x_{*i}, u_i(0) \in \mathbb{R}$ ($i \in \mathcal{C}$) if and only if (18) and the following are satisfied:

$$|k_a + 1| \leq \gamma. \quad (26)$$

Proof: The value of $y(t)$ is first calculated. From Lemma 1, $k_a \neq 0$ and (18) are necessary for (8), which allows (20) and (24) to hold. From (1), (17), (20), and (24), $y(t)$ in (7) is reduced to

$$\begin{aligned}y(t) &= \mathbf{1}^\top (x_* + u(t)) \\ &= \mathbf{1}^\top x_* + \mathbf{1}^\top (k_{ab}I + K_{cd})^t u(0) \\ &\quad + \mathbf{1}^\top (I - (k_{ab}I + K_{cd}))^{-1} (I - (k_{ab}I + K_{cd})^t) k_a x_* \\ &= \mathbf{1}^\top (k_{ab}I + K_{cd})^t (x_* + u(0)) \\ &= \mathbf{1}^\top ((k_a + 1)I + k_c L_G)^t (x_* + u(0)) \\ &= \mathbf{1}^\top \sum_{i=1}^n (k_a + 1 + k_c \lambda_i)^t v_i v_i^\top (x_* + u(0)) \\ &= \mathbf{1}^\top (x_* + u(0)) (k_a + 1)^t = y(0) (k_a + 1)^t, \quad (27)\end{aligned}$$

where the second-to-last equation is derived from (25) and the orthogonality of v_i .

Next, a condition of parameters (k_a, k_b, k_c, k_d) is derived to achieve (8). From (27), (26) is the necessary and sufficient condition for (8) under $k_a \neq 0$ and (18). If (26) is satisfied, then $k_a \neq 0$ holds. Thus, (18) and (26) are necessary and sufficient for (8). $\square$

The condition for the stability of system (15) is next derived.

Lemma 3. Assume that (18) and (26) hold. Then, system (15) is stable if and only if

$$|k_a + 1 + k_c \max \lambda(L_G)| < 1. \quad (28)$$

Proof: From (18), (24) holds. From this, system (15) is stable (i.e., the absolute eigenvalues of $k_{ab}I + K_{cd}$ are less than one) if and only if

$$|k_a + 1 + k_c \lambda_i| < 1 \quad \forall i \in \mathcal{C} \quad (29)$$

holds. From (25) and (26), the inequality in (29) holds for $i = 1$. From the assumption that G is undirected, $L_G \in \mathbb{R}^{n \times n}$ is symmetric, and thus its eigenvalues are real. Thus, (29) holds for any $i \in \mathcal{C}$ if and only if it is satisfied by the maximum λ_i , $\max \lambda(L_G)$. Then, (28) is obtained. $\square$

Proof of Theorem 1: A system consisting of (1) and (6) for all $i \in \mathcal{C}$ is equivalent to system (15). To show sufficiency, it is assumed that $k \in \mathcal{K}$ holds for the set, $\mathcal{K}$ in (14). Then, $k_a \neq 0$, and (18), (26), and (28) are satisfied. Thus, Lemma 3 guarantees that the system is stable, and Lemma 2 guarantees that (8) holds for a given $\gamma \in (0, 1)$ and any $x_{*i}, u_i(0) \in \mathbb{R}$ ($i \in \mathcal{C}$).

To show necessity, we assume that $k \notin \mathcal{K}$ for the set, $\mathcal{K}$ in (14). One or more of (18), (26), or (28) then do not hold. If (28) does not hold, the system is not stable, or either (18) or (26) does not hold, as shown by Lemma 3. Eq. (8) does then not hold, or the system is not stable, as shown by Lemma 2. $\square$

3.2 Solution of the optimization problem

The main result of this paper is attained as follows.

Theorem 2. For a given $\gamma \in (0, 1)$,

$$\begin{aligned}&\inf_{k \in \mathcal{K}} V(k) \\ &= \mu_w \left(n \mu_{x_*}^2 + \sigma_{x_*}^2 \sum_{\hat{\lambda} \in \lambda(L_G)} \left(1 + \frac{1 + \gamma}{1 - \gamma} \frac{\hat{\lambda}}{\max \lambda(L_G)} \right)^{-2} \right) \quad (30)\end{aligned}$$

holds. The infimum is achieved for the following $k = (k_a, k_b, k_c, k_d) \in \text{cl}(\mathcal{K})$:

$$k_a = -(1 - \gamma), \quad k_b = 1, \quad k_c = -k_d = -\frac{1 + \gamma}{\max \lambda(L_G)}. \quad (31)$$

□

Eq. (30) indicates the best performance obtainable with a distributed and uniform controller of the form (6) over a network topology given by G . The properties of the customers of EMCs (i.e., μ_{x_*} and $\sigma_{x_*}^2$) affect the performance, which can be enhanced by choice of $\gamma \in (0, 1)$ and the network topology, in terms of the eigenvalues of L_G , through the second term in parentheses in (30). This term corresponds to the performance degradation caused by distributed management. If the optimization problem (13) was solved in a centralized manner, the solution, $\min_k V(k) = n\mu_w\mu_{x_*}^2$ would have been obtained, corresponding to the first term in (30).

A system with these gains is marginally stable because k in (31) is on the boundary of $\text{cl}(\mathcal{K})$. From (28), the gains k_c, k_d in (31) can be chosen as

$$k_c = -k_d = -\frac{1 + \gamma}{(1 + \varepsilon) \max \lambda(L_G)}$$

to stabilize the system, with some $\varepsilon > 0$. As ε is smaller, the objective function $V(k)$ approximates more to the infimum in (30). From (6), the resultant controller of the EMC is given as

$$\begin{aligned} u_i(t) &= u_i(t-1) - (1 - \gamma)x_i(t-1) \\ &\quad - \frac{1 + \gamma}{(1 + \varepsilon) \max \lambda(L_G)} \sum_{j \in \mathcal{N}_i} (u_i(t-1) - u_j(t-1)), \end{aligned} \quad (32)$$

where $\gamma \in (0, 1)$ and $\varepsilon > 0$. Practically, the last term of (32) attempts to equalize the values of $u_i(t)$ and to decrease $V(k)$, whereas the second term of (32) works to restrain $x_i(t)$ for the convergence of $y(t)$. Hence, although the last term of (32) is equivalent to consensus controllers [38], the values of $u_i(t)$ cannot completely be in agreement because of the second term.

Remark 2. The value of $\max \lambda(L_G)$ in (32) is difficult to obtain in large-scale systems. To solve this issue, we have to limit the number n_i of the neighbors. Let $\bar{n}$ be the upper bound of n_i , and $\max \lambda(L_G) \leq 2 \max_{i \in \mathcal{C}} n_i \leq 2\bar{n}$ hold [37]. Then, $\max \lambda(L_G)$ can be replaced by $2\bar{n}$ in (32), which maintains the stability of the system though the performance might be degraded. □

Proof of Theorem 2: Assume that the parameters $k = (k_a, k_b, k_c, k_d)$ belong to the set $\mathcal{K}$, as given by (14). The value of $V(k)$ given in (9) is first calculated for these parameters. From $k \in \mathcal{K}$, Theorem 1 guarantees that system (15) is stable and that (18) holds, which leads to (24). From the stability of system (15) and from (17) and (24), the following equation can be developed:

$$\begin{aligned} \lim_{t \rightarrow \infty} u(t) &= (I - (k_a I + K_{cd}))^{-1} k_a x_* \\ &= -(k_a I + k_c L_G)^{-1} k_a x_* \\ &= -\sum_{i \in \mathcal{C}} \left(1 + \frac{k_c \lambda_i}{k_a}\right)^{-1} v_i v_i^\top x_*. \end{aligned} \quad (33)$$

From (3) and (4), the following equation is obtained:

$$\begin{aligned} \sigma_{x_*}^2 I &= E((x_* - \mu_{x_*} \mathbf{1})(x_* - \mu_{x_*} \mathbf{1})^\top) \\ &= E(x_* x_*^\top) - 2E(x_*) \mu_{x_*}^\top \mathbf{1} + \mu_{x_*}^2 \mathbf{1} \mathbf{1}^\top \\ &= E(x_* x_*^\top) - \mu_{x_*}^2 \mathbf{1} \mathbf{1}^\top. \end{aligned} \quad (34)$$

From (1), (10), (12), (33), and (34), $V(k)$ in (9) is calculated as

$$\begin{aligned} V(k) &= E \left(\sum_{i \in \mathcal{C}} w_i \lim_{t \rightarrow \infty} (u_i(t))^2 \right) \\ &= \mu_w E \left(\lim_{t \rightarrow \infty} \|u(t)\|^2 \right) \\ &= \mu_w E \left(\sum_{i \in \mathcal{C}} \left(1 + \frac{k_c \lambda_i}{k_a}\right)^{-2} (v_i^\top x_*)^2 \right) \\ &= \mu_w \sum_{i \in \mathcal{C}} \left(1 + \frac{k_c \lambda_i}{k_a}\right)^{-2} v_i^\top E(x_* x_*^\top) v_i \\ &= \mu_w \sum_{i \in \mathcal{C}} \left(1 + \frac{k_c \lambda_i}{k_a}\right)^{-2} v_i^\top (\mu_{x_*}^2 \mathbf{1} \mathbf{1}^\top + \sigma_{x_*}^2 I) v_i \\ &= \mu_w \left(n\mu_{x_*}^2 + \sigma_{x_*}^2 \sum_{i \in \mathcal{C}} \left(1 + \frac{k_c \lambda_i}{k_a}\right)^{-2} \right). \end{aligned} \quad (35)$$

Next, the infimum of $V(k)$ for $k \in \mathcal{K}$ is then derived. From $k_a < 0$, the two inequalities in (14) are equivalent to

$$-\frac{1}{1 - \gamma} \leq \frac{1}{k_a} \leq -\frac{1}{1 + \gamma}, \quad -1 < \frac{k_c \max \lambda(L_G)}{k_a} < -1 - \frac{2}{k_a},$$

which leads to

$$\frac{k_c}{k_a} < \frac{1}{\max \lambda(L_G)} \left(-1 + \frac{2}{1 - \gamma} \right) = \frac{1}{\max \lambda(L_G)} \frac{1 + \gamma}{1 - \gamma}. \quad (36)$$

Thus, the infimum of k_c/k_a that satisfies (14) is given by the right-hand side of (36), which is achieved for k_a, k_c in (31). From (35), minimizing $V(k)$ with k_a and k_c is equivalent to maximizing k_c/k_a . The supremum of k_c/k_a is obtained with k_a and k_c in (31). The gains in (31) are obtained with the conditions in (14) for k_b and k_d . By replacing (31) in (35), (30) is achieved. □

3.3 Possibility of application to time-varying graphs

Even for a time-varying graph, the most important policy (8) for DLC that balances supply and consumption is still achieved by using gains $(k_a, k_b, k_c, k_d) \in \mathcal{K}$, where $\mathcal{K}$ is given in (14) in Theorem 1, as follows. Consider a time-varying graph $G(t)$, and the distributed and uniform controller (6) is reduced to

$$\begin{aligned} u_i(t) &= k_a x_i(t-1) + k_b u_i(t-1) \\ &\quad + \sum_{j \in \mathcal{N}_i(t)} (k_c u_i(t-1) + k_d u_j(t-1)), \end{aligned}$$

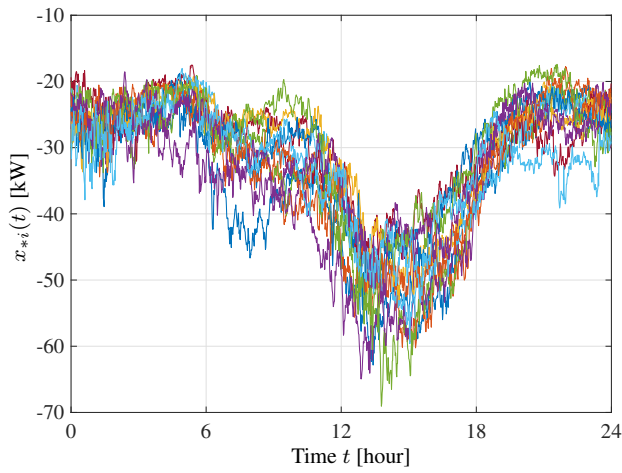


Fig. 5: Desired consumption: $x_{*i}(t)$ from $i = 1$ to 12.

where the set $\mathcal{N}_i(t)$ of neighbors is time-varying. From (1), (7), and (14), the output $y(t)$ is driven as

$$\begin{aligned} y(t) &= \sum_{i=1}^n x_i(t) = \sum_{i=1}^n (u_i(t) + x_{*i}) \\ &= \sum_{i=1}^n (k_a x_i(t-1) + u_i(t-1)) \\ &\quad + k_c \sum_{j \in \mathcal{N}_i(t)} (u_i(t-1) - u_j(t-1)) + x_{*i}) \\ &= \sum_{i=1}^n (k_a x_i(t-1) + u_i(t-1) + x_{*i}) \\ &\quad + k_c \sum_{i=1}^n \sum_{j \in \mathcal{N}_i(t)} (u_i(t-1) - u_j(t-1)) \end{aligned} \quad (37)$$

$$= \sum_{i=1}^n (k_a + 1) x_i(t-1) = (k_a + 1) y(t-1), \quad (38)$$

where the second term of (37) is zero from the undirectedness of graph $G(t)$. Since $|k_a + 1| < \gamma$ from (14), $y(t)$ is governed by (38) and satisfies (8).

Conversely, the second policy for DLC that minimizes the expected total curtailment (9) is not guaranteed because the relevant result in Theorem 2 totally relies on the eigenvalues of the graph Laplacian, L_G , which are not defined for time-varying graphs.

4 Numerical examples

4.1 Simulation setting

Numerical examples are employed to illustrate the effectiveness of the proposed method, even when applied to fluctuating demands. The consumption data of 540 residences in Pal Town Josai-no-Mori, Ota City, Gunma Prefecture, Japan were collected from August 1 to 3, 2007 by the New Energy and Industrial Technology Development (NEDO), a Japanese government organization [39]. In this example, there are 12 aggregators, each of which manages 45 residences. The desired consumption, $x_{*i}(t)$ ($i = 1, 2, \dots, 12$) of each aggregator is given by the sum of the total consumption of its managing 45 residences, as depicted in Fig. 5.

The supplier is regarded as customer $i = 13$ and is expected to supply the total desired consumption while maintaining a supply

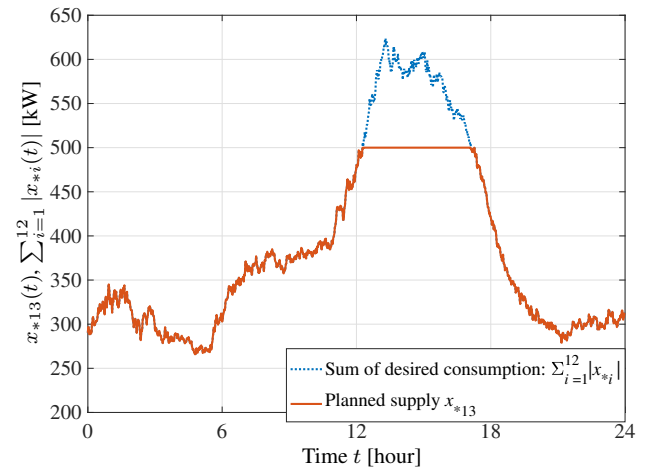


Fig. 6: Planned supply: $x_{*13}(t)$ (red line), total absolute desired consumption: $\sum_{i=1}^{12} |x_{*i}(t)|$ (blue line).

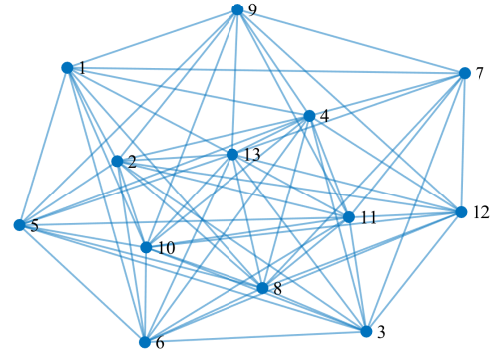


Fig. 7: Graph G_1 , representing the communication network among the aggregators and supplier in the simulation.

capacity of 550. Then, the supply, $x_{*13}(t)$ is determined as

$$x_{*13}(t) = \begin{cases} \sum_{i=1}^{12} |x_{*i}(t)| & \text{if } \sum_{i=1}^{12} |x_{*i}(t)| < 500 \\ 500 & \text{otherwise,} \end{cases} \quad (39)$$

which saturates at 500 for the supply capacity. Fig. 6 shows the planned supply, $x_{*13}(t)$ as a red line and the total desired consumption, $\sum_{i=1}^{12} |x_{*i}(t)|$ as a blue line.

There exist $n = 13$ customers in the system. The communication network for their EMCs is made into a graph, G_1 , as shown in Fig. 7. The system of each EMC consists of (1) and (32) with an assigned convergence rate, $\gamma = 0.95$ and the parameter, $\varepsilon = 0.1$. From Theorem 2, $V(k)$ in (9) is minimized, the system is stabilized, and the power imbalance, $y(t)$ (i.e., the sum of $x_i(t)$) converges to zero at a rate, $\gamma = 0.95$. The sampling time for the EMCs is 1 min.

4.2 Simulation results

Figs. 8 and 9 show the respective time transitions of the consumption, $x_i(t)$ and control input, $u_i(t)$ from $i = 1$ to 12 (all of the consumers). The consumption shown in Fig. 8 is similar to the desired consumption shown in Fig. 6, although differences arise due to the control input, as shown in Fig. 9. This observation is more clearly described in Fig. 10, in which the consumption, $x_1(t)$, desired consumption, $x_{*1}(t)$, and control input, $u_1(t)$ of customer $i = 1$ are plotted as a red, blue, and green line, respectively. Here, $x_1(t)$ differs from $x_{*1}(t)$, whose magnitude is equal to the value of $u_1(t)$. Due to the fact that the difference is small, the effect of the

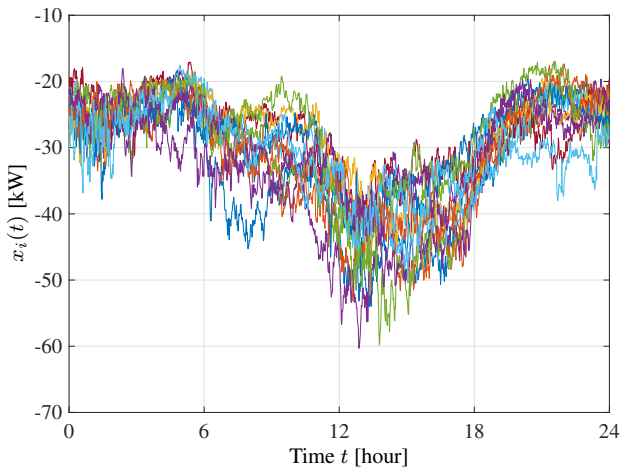


Fig. 8: Customer consumption: $x_i(t)$ from $i = 1$ to 12.

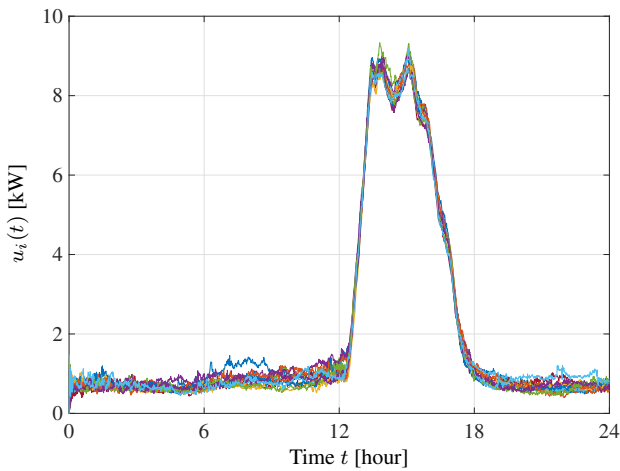


Fig. 9: Control input: $u_i(t)$ from $i = 1$ to 12.

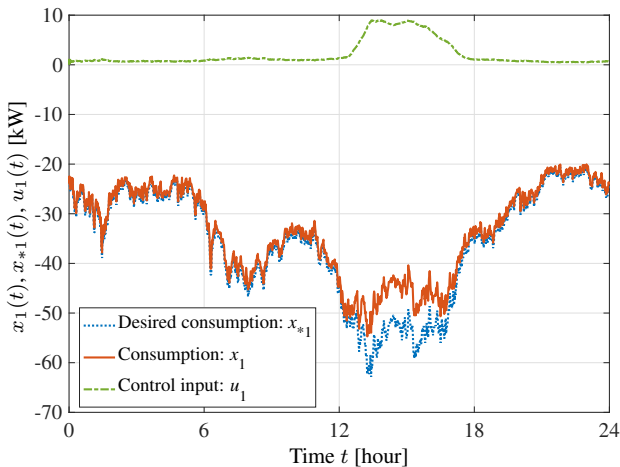


Fig. 10: Consumption: $x_i(t)$ (red line), desired consumption: $x_{*i}(t)$ (blue line), and control input: $u_i(t)$ (green line) of customer $i = 1$.

DLC on each customer is small. The fluctuations of $x_i(t)$ in Fig. 8 are caused by those of the desired amounts $x_{*i}(t)$ shown in Fig. 5.

Fig. 11 depicts the supply, $x_{13}(t)$ and the total of consumption, $\sum_{i=1}^{12} |x_i(t)|$ by a red line and a blue line, respectively. A small difference is seen between the supply and the total consumption, resulting from the supply capacity. However, the difference is

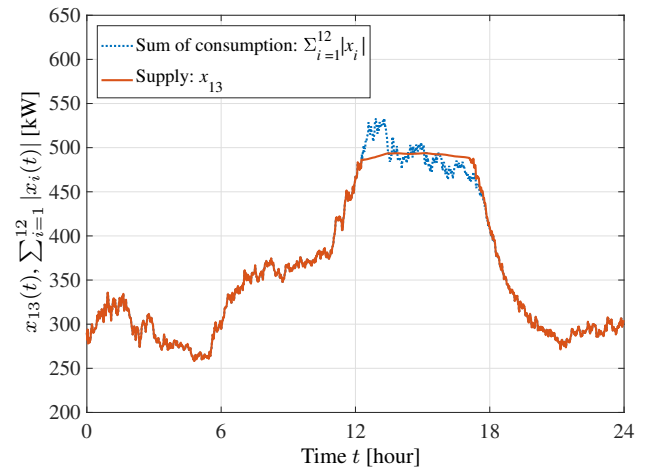


Fig. 11: Supply: $x_{13}(t)$ (red line), total of absolute consumption: $\sum_{i=1}^{12} |x_i(t)|$ (blue line).

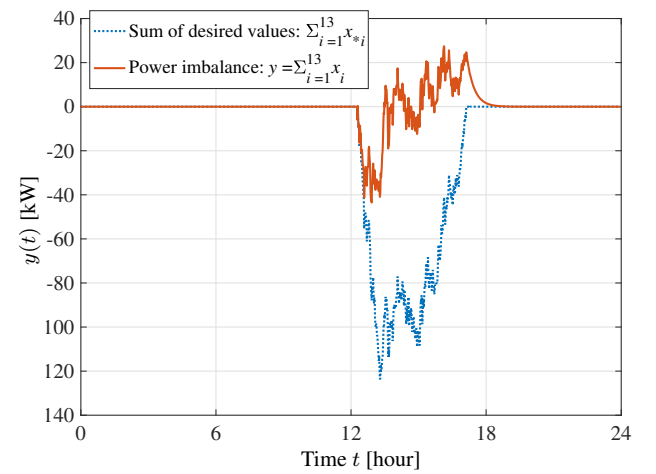


Fig. 12: Power imbalance: $y(t)$ (red line), sum of desired values: $\sum_{i=1}^{13} x_{*i}(t)$ (blue line).

smaller than that of the planned supply and the total desired consumption, shown in Fig. 6. The power imbalance, $y(t)$ and the sum of the desired quantities, $\sum_{i=1}^{13} x_{*i}(t)$ are shown in Fig. 12 by a red line and a blue line, respectively. The magnitude of $y(t)$ is smaller than that of $\sum_{i=1}^{13} x_{*i}(t)$, indicating the effectiveness of the proposed method.

The power imbalance, $y(t)$ in Fig. 12 is caused by the supply capacity and the time transition of the desired consumption. They could be removed in cooperation with reference shaping of the desired supply, $x_{13}(t)$ and faster power control. These two methods can be realized using a reference governor [40] and load frequency control [41].

4.3 Discussion concerning control performance

The time transition of $E(t)$, defined as

$$E(t) = \sum_{i=1}^n |u_i(t)|^2 = \sum_{i=1}^n |x_i(t) - x_{*i}(t)|^2,$$

is plotted by a red line in Fig. 13. Here, $E(t)$ is the total curtailment by the EMCs at time t , and it indicates the control performance in terms of customer welfare. Now, we confirm Theorem 2, that implies that the plotted $E(t)$ describes nearly the best performance obtainable under the distributed management and that $E(t)$ depends on (i) the mean, $\mu_{x_{*i}}(t)$ and variance, $\sigma_{x_{*i}}^2(t)$ of the desired quantities, $x_{*i}(t)$ and (ii) the network topology of G .

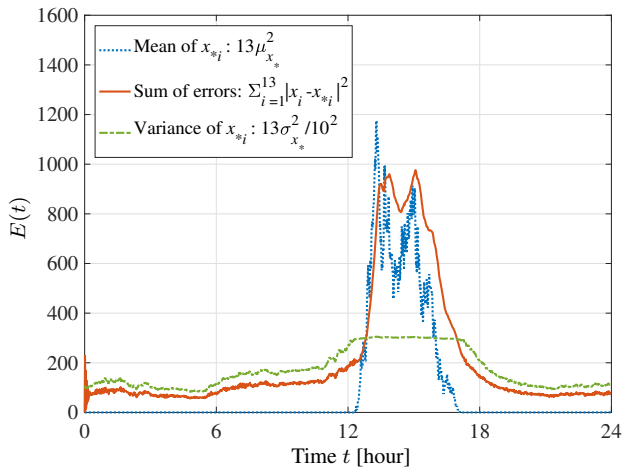


Fig. 13: Sum of squared errors given with G_1 : $E(t)$ (red line), 13 times squared mean: $13\mu_{x_*}^2(t)$ (blue line), and 13 times variance divided by 10^2 : $13\sigma_{x_*}^2(t)/10^2$ (green line).

First, Fig. 13 presents the time transition of $E(t)$ as a red line with $13\mu_{x_*}^2(t)$ and $13\sigma_{x_*}^2(t)/10^2$ as a blue and a green line, respectively. $E(t)$ constantly fluctuates because of $\sigma_{x_*}(t)$ and rapidly increases between $t = 12.5$ and 17 owing to $\mu_{x_*}(t)$, as implied by (30) in Theorem 2. The variance, $\sigma_{x_*}^2(t)$ persists, caused by the nonzero $u_i(t) = x_i(t) - x_{*i}(t)$ as shown in Fig. 9, even though the power is balanced without DLC, i.e., $\mu_{x_*}(t) = 0$ holds. This indicates the performance limitation of the distributed management because this value of $|u_i(t)|$ is almost the smallest under distributed management, as shown in Theorem 2. Therefore, to enhance the control performance, the variance of the consumer properties should be reduced.

Second, a simulation with graph G_2 depicted in Fig. 14 is performed under the same setting as the previous simulation except for the network topology. Fig. 15 shows the time transition of $E(t)$ as a red line. This line is larger than the red line in Fig. 13, $E(t)$ with G_1 , indicating that the control performance with G_2 is worse than that with G_1 . This observation can be explained by Theorem 2. The eigenvalues of the graph Laplacians of G_1 and G_2 are calculated as

$$\lambda(L_{G_1}) = \{0, 7.80, 8.49, 9.20, 10.33, 11.32, 11.61, 12.00, 12.00, 12.57, 12.68, 13.00, 13.00\} \quad (40)$$

$$\lambda(L_{G_2}) = \{0, 0.86, 1.66, 2.27, 3.26, 3.56, 3.90, 5.24, 5.63, 5.95, 7.00, 7.96, 8.71\}. \quad (41)$$

The eigenvalues of L_{G_1} in (40) are then more uniform than those of L_{G_2} in (41). Hence, from (30) in Theorem 2, the control performance achieved with G_1 is better than that achieved with G_2 , corresponding to the simulation results. Thus, to enhance control performance, a dense network of EMCs should be constructed for the graph Laplacian to have as uniform eigenvalues as possible.

5 Conclusions

In this paper, the distributed DLC of a power system for the network communication of EMCs was investigated with the aim of guaranteeing fair welfare maximization. The dynamics and communication intervals of the EMCs with a distributed and uniform controller were considered. A set of all the control gains that stabilize the entire system and converge the power imbalance to zero at an assigned rate was then derived. Furthermore, the best among these gains was found for maximizing the customer welfare. As a result, the performance degradation caused by distributed management was derived and was shown to depend on the variance of the desired consumption and the network topology of the EMCs. These results suggest ways of improving the operation of DLC programs from the perspective of

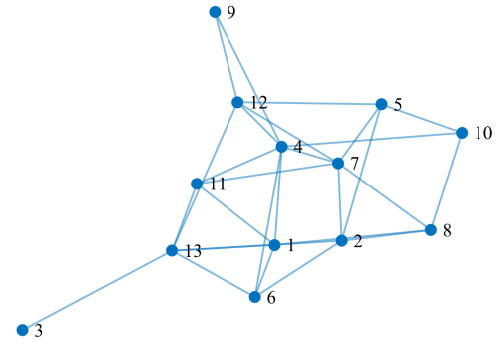


Fig. 14: Graph G_2 of the communication network in the second simulation.

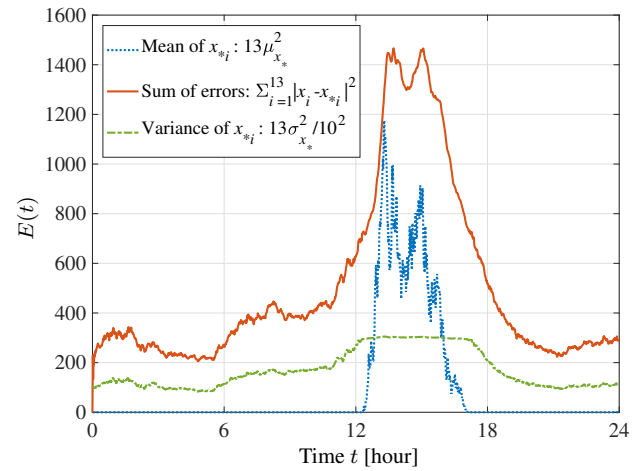


Fig. 15: Sum of squared errors given with G_2 : $E(t)$ (red line), 13 times squared mean: $13\mu_{x_*}^2(t)$ (blue line), and 13 times variance divided by 10^2 : $13\sigma_{x_*}^2(t)/10^2$ (green line).

contracts with customers and the network design of EMCs. Finally, simulations were performed using the consumption data of actual consumers to demonstrate the effectiveness of the proposed method. The results obtained revealed the potential of distributed DLCs for practical use.

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