

Harmonic analysis in new 2-microlocal Besov and Triebel-Lizorkin spaces

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I. Definition

Let $\mathcal{S} = \mathcal{S}(\mathbb{R}^n)$ be the space of all Schwartz functions on $\mathbb{R}^n$ and $\mathcal{S}'$ its dual.

Let ϕ be a Schwartz function satisfying

$$(1.1) \text{ supp } \hat{\phi} \subset \{\xi \in \mathbb{R}^n : \frac{1}{2} \leq |\xi| \leq 2\},$$

$$(1.2) \quad |\hat{\phi}(\xi)| \geq C > 0 \text{ if } \frac{3}{5} \leq |\xi| \leq \frac{5}{3}.$$

Let $\phi_j(x) = 2^{nj}\phi(2^jx)$, $j \in \mathbb{N}$.

We select a function $\phi_0 \in \mathcal{S}$ satisfying

$$(1.3) \quad \text{supp } \hat{\phi}_0 \subset \{\xi \in \mathbb{R}^n : |\xi| \leq 2\},$$

$$(1.4) \quad |\hat{\phi}_0(\xi)| \geq C > 0 \text{ if } |\xi| \leq \frac{5}{3}.$$

For $f \in \mathcal{S}'$ we define some sequences indexed by dyadic cubes P :

$$\begin{aligned} & B_{pq}^s(P) \\ &= \left(\sum_{j \geq (-\log_2 l(P)) \vee 0} \|2^{js} \phi_j * f\|_{L^p(P)}^q \right)^{1/q}, \\ & 0 < p, q \leq \infty, \text{ where } a \vee b = \max\{a, b\} \\ & F_{pq}^s(P) = \\ & \|(\sum_{j \geq (-\log_2 l(P)) \vee 0} (2^{js} |\phi_j * f|)^q)^{1/q}\|_{L^p(P)}, \\ & 0 < p < \infty, 0 < q \leq \infty \end{aligned}$$

Definition 1.1

Let $s, s', \in \mathbb{R}$, $\sigma \geq 0$ and let $x_0 \in \mathbb{R}^n$.

$$A^s(E_{pq}^{s'})_{x_0}^\sigma = \{f \in \mathcal{S}' : \|f\|_{A^s(E_{pq}^{s'})_{x_0}^\sigma} \equiv \sup_P (l(P) + |x_P - x_0|)^{-\sigma} l(P)^{-s} E_{pq}^{s'}(P) < \infty\}$$

where E_{pq}^s to denote either B_{pq}^s or F_{pq}^s .

II. Example.

Examples.

(i) $A^0(E_{pq}^{s'})_{x_0}^0 = E_{pq}^{s'}(\mathbb{R}^n)$ is the classical Besov space, or Triebel-Lizorkin space , [Triebel 10,11].

(ii) The Besov type spaces $B_{pq}^{s,\tau}(\mathbb{R}^n)$ and the Triebel-Lizorkin type spaces $F_{pq}^{s,\tau}(\mathbb{R}^n)$ introduced by [D. Yang and W. Yuan 12] and [D. Yang , W. Sickel and W. Yuan 13] , are contained in our definition as special cases that

$$E_{pq}^{s',s}(\mathbb{R}^n) = A^{ns}(E_{pq}^{s'})_0^0.$$

(iii)

The Morrey space is

$$\mathcal{M}_p^u = A^{n(\frac{1}{p}-\frac{1}{u})}(F_{p2}^0)_0^0 \text{ if } 1 < p \leq u < \infty.$$

(iv) The $\dot{B}_\sigma$ -Morrey spaces studied by [Y. Komori-Furuya et al. 3], are contained in our definition as special cases, that is, $\dot{B}_\sigma(L_{p,\lambda}) = A^{\lambda+\frac{n}{p}}(F_{p2}^0)_0^\sigma$, $1 < p < \infty$.

(v) The local Morrey spaces $LM_{p,\lambda}$ studied by [Ts. Batbold and Y. Sawano, 1] are realized in our cases as

$$LM_{p\lambda} = A^0(F_{p2}^0)_0^{\lambda/p}, \quad 1 < p < \infty.$$

(vi) $C_{x_0}^{s,s'}$ studied in [Y. Meyer, 4], is in our cases,

$$C_{x_0}^{s,s'} = A^0(B_{\infty\infty}^{s+s'})_{x_0}^{-s'}.$$

III. Harmonic analysis in new 2-microlocal Besov and Triebel-Lizorkin spaces

1. Characterization[K.Saka, 5,7]
 - (a) ϕ -transform in the sense of Fraizer-Jawerth[2]
 - (b) wavelet trnsform
 - (c) atomic and molecular decomposition
 - (d) defference characterization
 - (e) oscillation characterization
2. Boundedness [K. Saka, 5,7]
 - (a) Calderón-Zygmund operators
 - (b) psudodifferential operators
3. Local theory [K. Saka, 6]
4. Predual[K. Saka, 8]
 - (a) Zoro type

(b) Choquet integral

5. Pointwise multiplier[K. Saka, 9]

6. Interpolation

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