

Bayesian Updating of Nonlinear Constitutive Models for Steel–Concrete Composite Girder Bridges

Using Large-Scale Load Testing Data

Debao Chen¹, Chul-Woo Kim^{2,*}, Xin Zhou³ and Alfred Strauss⁴

¹ MSc, PhD. Candidate, Department of Civil and Earth Resources Engineering, Graduate School of Engineering, Kyoto University, Kyoto 615-8540, Japan, chen.debao.66u@st.kyoto-u.ac.jp

² PhD., Professor, Department of Civil and Earth Resources Engineering, Graduate School of Engineering, Kyoto University, Kyoto 615-8540, Japan. kim.chulwoo.5u@kyoto-u.ac.jp (*Corresponding Author)

³ PhD., Researcher, School of Civil and Environmental Engineering, Nanyang Technological University, Singapore zhou.xin@ntu.edu.sg

⁴ PhD., Professor, Institute of Structural Engineering (IKI), University of Natural Resources and Life Sciences, Peter-Jordan-Straße 82, 1190 Vienna, Austria. alfred.strauss@boku.ac.at

Abstract: Establishing an accurate finite element (FE) model that faithfully replicates the nonlinear behavior of steel-concrete composite girder bridges is crucially important for effective model-based structural health monitoring (SHM) for this specific bridge type. While Bayesian model updating methodology has gained widespread acclaim for its application in data-driven FE models, apprehensions have emerged regarding the rationality of the updated model parameter values. These concerns predominantly stem from the use of oversimplified FE beam models in model updating studies. This study addresses these concerns by introducing nonlinear constitutive models for the steel–concrete composite girder bridge, as well as comprehensive and detailed three-dimensional (3D) FE model updates. Key parameters for nonlinear constitutive models were estimated based on structural responses obtained from a large-scale static loading test. Bayes' theorem was employed to infer posterior probability density functions (PDFs) for the model parameters. A Transitional Markov Chain Monte Carlo (TMCMC) sampler was used as a computational tool to generate samples for representing the posterior PDFs. A two-step model updating approach designed to achieve a balance between computational efficiency and simulation accuracy was proposed in the context of nonlinear model updating. Initially, deflection and neutral axis height data were used to update the linear segment of the constitutive model. Load–deflection curves were then used to update the nonlinear segment. Following the model updating with deflection and strain data, the nonlinear simulation results showed improved comparability to the measured data, indicating significant improvement of the model accuracy. Furthermore, the model

update effectiveness was cross-verified successfully by comparing load–strain curves of concrete and reinforcing bars obtained during the experiment. The updated model showcases its capability for structural performance evaluation and its potential application in the domain of SHM.

Author Keywords: Steel–concrete composite girder bridge, Bayesian model updating, Finite-element model, Nonlinear constitutive model, Sampling.

Introduction

Steel–concrete composite girder bridges are extensively applied in urban viaducts due to their favorable construction convenience (Martínez-Muñoz et al. 2020). The development of high-fidelity finite element (FE) models holds significant importance for enabling the monitoring and maintenance of this bridge type. The inclusion of both steel and concrete materials in the cross-section, along with the complex connections at the steel–concrete interface, poses a challenge for the simplified single-beam model to accurately capture the mechanical behavior of steel–concrete composite girder bridges, particularly in terms of nonlinear mechanical behaviors (Xu et al. 2018). With advancements in FE method and computer performance, researchers have increasingly employed three-dimensional (3D) structural models featuring shell or solid elements and sophisticated nonlinear material constitutive models for simulating and evaluating the structural performance of steel–concrete composite girders (Tang et al. 2023). Nevertheless, models based on design drawings and nonlinear constitutive models derived from material coupon tests may still exhibit errors in accurately simulating overall structural behavior (Simoen et al. 2015). For example, the elastic modulus and material strength obtained from standard material coupon tests represent nominal values that may not accurately reflect the actual properties of full structural members. Discrepancies between these values can result from factors like scale differences, variations in test conditions, and inherent material property variability. When measured structural responses are available, structural models can be updated to improve accuracy.

Model updating methods can be categorized into deterministic and probabilistic approaches, depending on whether

they incorporate the quantification of uncertainty. Deterministic methods (Ren et al. 2011; Ribeiro et al. 2012; Wang et al. 2016; Lin et al. 2021) are designed to identify the numerical model parameters that produce the best fit between the observed data and model output. Although deterministic methods have shown considerable progress, they still include limitations that can be overcome by probabilistic approaches (Beck 2010). A notable shortcoming of deterministic methods is their emphasis on seeking a single optimal solution, which can pose challenges to solving locally identifiable or unidentifiable inverse problems. In fact, because uncertainty is an inherent aspect of reality, addressing uncertainty is a valuable and potentially necessary undertaking in model updating (Song et al. 2018).

During the last two decades, a notable shift has occurred toward probability-based model updating, primarily because of the introduction of Bayesian model updating (Beck and Katafygiotis 1998). Bayesian model updating methods provide a logical solution for addressing the earlier described issues using Bayesian inference to estimate the posterior probability density function (PDF) of uncertain model parameters, considering the available measured data and modeling assumptions. Beck and Au (2002) used Markov Chain Monte Carlo (MCMC) simulations to improve the accuracy of structural response predictions and performance reliability. Beck and Yuen (2004) proposed a Bayesian probabilistic approach to ascertain the most plausible model class for a given structural or mechanical system from a set of available model classes. Muto and Beck (2008) implemented Bayesian model updating and model class selection for hysteretic models. Additional investigations of this topic can be found in many reports of the relevant literature (Mustafa and Matsumoto 2017; Betz et al. 2018; Zhou et al. 2022). In addressing real-world systems characterized by multiple parameters, the computation of joint and marginal PDFs requires the handling of high-dimensional integrals. Therefore, researchers frequently resort to approximate methodologies and sampling techniques to estimate posterior PDFs, with the MCMC method and its diverse variants emerging as prominent choices (Lye et al. 2021; Yang et al. 2022). These variants encompass but are not restricted to the Delayed-Rejection Adaptive Metropolis-Hastings MCMC, the Adaptive MCMC, and the Transitional MCMC (TMCMC) (Simoen et al. 2013).

Previous investigations into Bayesian model updating have predominantly relied on validation through numerical cases or simplistic structures with elastic model analyses (Shi et al. 2019; Malekghaini et al. 2023). However, limited attention

has been devoted to the exploration of large-scale and intricate bridge structures, particularly within the realm of nonlinear model updating (Xia et al. 2020; Zhang et al. 2021; Ramancha et al. 2022). In existing studies, the fine-tuning of nonlinear model parameters aims to establish an agreement between simulated responses and experimentally measured data (Wang et al. 2021; Xiao et al. 2023). Nevertheless, the oversimplification of FE models raises concerns regarding the rationale behind updated parameter values. The elastic modulus and the area moment of inertia are often combined in structural analysis, commonly referred to as flexural stiffness (EI). When a concrete structural member enters the nonlinear stress state, cracks begin to form, leading to a decrease in flexural stiffness. To avoid frequent adjustments to the cross-sectional shape during model updating, the elastic modulus is typically used as the updating parameter rather than the area moment of inertia. In the design of complex bridge structures according to design codes, FE models often employ conservative estimates of the cross-sectional properties of the main girders to ensure structural safety. In these simplified beam models, the cross-sectional properties of the main girders can be inaccurately represented. As a result, the posterior distribution of the elastic modulus estimated during the FE model update tends to capture various effects that are not explicitly modelled in the FE model, effectively compensating for the limitations of other fixed parameters. In such situations, the updated elastic modulus typically differs from the material's true value significantly (Vereecken et al. 2022). Given the frequent existence of multiple potential solutions in the inverse problem, the forced correction of model parameters may result in an FE model incapable of faithfully reproducing various other structural responses. For example, an FE model updated solely based on modal parameters may struggle to predict displacement responses accurately. Addressing these challenges requires a realistic representation of the structure's geometric features. Additionally, cross-validation of the calibrated FE model using data from multiple sources is essential to ensure its reliability.

Modeling a real-world bridge is highly complex, especially when incorporating a 3D FE model and nonlinear material constitutive models. Ensuring computational convergence is imperative for the nonlinear solution, and the time-consuming nonlinear analysis significantly amplifies the computational burden associated with Bayesian model updating. Therefore, the utilization of 3D refined FE models for Bayesian model updating constitutes a challenging yet valuable endeavor to improve the fidelity of the model analysis. The objective of this study is to update the nonlinear constitutive models for a

large-scale steel–concrete composite girder bridge. This updating will be accomplished using a Bayesian probabilistic approach along with the TMCMC sampler, using measurements of deflection and strain from the static loading test. To strike a balance between computational efficiency and simulation accuracy, a two-step model updating approach is proposed for the comprehensive 3D FE model. While the case study presented in this paper centers on the steel–concrete girder bridge, the outlined research framework is versatile. It can be applied to diverse structures, facilitating the creation of high-fidelity FE models. This paper comprises six sections, including the introduction in Section 1. Section 2 provides an overview of the Bayesian inference methodology and the TMCMC sampler. Section 3 introduces the experiment and the refined FE model used for the case study. Section 4 presents details of the procedure for updating the parameters of the linear model. Section 5 describes the updating process of the nonlinear segments within the constitutive models and the cross-verification of the updated FE model. Lastly, Section 6 provides concluding remarks.

TMCMC-based Bayesian Model Updating

Bayesian Inference

Given a forward model of a physical system $\mathbf{M}$, uncertain model parameter vector $\boldsymbol{\theta} \in \mathbf{R}^{N_\theta}$, and fixed model parameter vector $\mathbf{x}$, the system output vector with noise can be expressed as presented below.

$$\mathbf{D} = \mathbf{M}(\mathbf{x}, \boldsymbol{\theta}) + \mathbf{e} \quad (1)$$

where $\mathbf{e} \in \mathbf{R}^{N_d}$ denotes the error term which results from measurement errors and model uncertainties.

When a set of independent observations becomes available, the posterior PDF for the parameters of interest can be inferred using Bayes' theorem.

$$P(\boldsymbol{\theta}|\mathbf{D}) = \frac{P(\mathbf{D}|\boldsymbol{\theta}) \cdot P(\boldsymbol{\theta})}{P(\mathbf{D})} \quad (2)$$

In the equation, $P(\mathbf{D})$ represents the evidence of system response $\mathbf{D}$. Also, $P(\mathbf{D}) = \int P(\mathbf{D}|\boldsymbol{\theta}) P(\boldsymbol{\theta}) d\boldsymbol{\theta}$, $P(\boldsymbol{\theta})$ denotes the prior PDF, and $P(\mathbf{D}|\boldsymbol{\theta})$ represents the likelihood function. In this study, a uniform distribution is assumed for the prior PDF of model parameters.

Under the assumptions that the measurements D_k (for $k=1, \dots, n$) are independently and identically distributed (iid), and that the error $\mathbf{e}$ between the observation and the model follows a zero-mean Gaussian distribution (normal distribution) with fixed variance, the likelihood function for model updating is expressed as shown below (Zhang et al. 2017).

$$P(\mathbf{D}|\boldsymbol{\theta}) = \prod_{k=1}^n P(D_k|\boldsymbol{\theta}) = c_g e^{-J_g(\boldsymbol{\theta})/2\sigma^2} \quad (3)$$

Therein, $J_g(\boldsymbol{\theta})$ denotes the goodness-of-fit function used to minimize errors, σ^2 stands for the variance of Gaussian distribution, and c_g denotes a normalized constant.

Transitional Markov Chain Monte Carlo (TMCMC) Sampler

The principle of Transitional Markov Chain Monte Carlo (TMCMC) involves the generation of intermediate PDFs in a series that progresses from the prior PDF to the posterior (target) PDF. This approach is adopted because of the considerable difficulty of direct sampling from complex target distributions (Betz et al. 2016). TMCMC offers improved efficiency for handling sampling difficulties associated with higher-dimensional PDFs. The transitional distributions are defined specifically as presented below.

$$P^j \propto P(\mathbf{D}|\boldsymbol{\theta})^{\beta_j} \cdot P(\boldsymbol{\theta}) \quad (4)$$

where j denotes the transition step number, and β_j signifies the tempering parameter, which takes values such that $\beta_0 = 0 < \beta_1 < \beta_2 < \dots < \beta_m = 1$. It is noteworthy that $P^0 = P(\boldsymbol{\theta})$ denotes the prior distribution and $P^m = P(\boldsymbol{\theta}|\mathbf{D})$ denotes the posterior distribution.

A crucially important aspect of the TMCMC sampler is the determination of the transition step size, denoted as $\Delta\beta_j = \beta_j - \beta_{j-1}$, at each transitional step j . To identify the optimal value of $\Delta\beta_j$, Ching and Chen (2007) proposed maintaining the coefficient of variation (COV) of the value set $P(\mathbf{D}|\boldsymbol{\theta}_{j,k})^{\Delta\beta_j}$ as close to 100% as achievable. The COV of $P(\mathbf{D}|\boldsymbol{\theta}_{j,k})^{\Delta\beta_j}$ is defined as shown below.

$$COV(\beta_j) = \frac{\sigma(P(\mathbf{D}|\boldsymbol{\theta}_{j,k})^{\Delta\beta_j})}{\mu(P(\mathbf{D}|\boldsymbol{\theta}_{j,k})^{\Delta\beta_j})} \quad (5)$$

In that equation, $\sigma(P(\mathbf{D}|\boldsymbol{\theta}_{j,k})^{\Delta\beta_j})$ and $\mu(P(\mathbf{D}|\boldsymbol{\theta}_{j,k})^{\Delta\beta_j})$ respectively denote the standard deviation and mean of the value set $P(\mathbf{D}|\boldsymbol{\theta}_{j,k})^{\Delta\beta_j}$ (for $j=1, \dots, m, k=1, \dots, N$). Once $COV(\beta_j)$ is obtained, the value of β_j can be determined from β_{j-1} using the argument which minimizes the absolute difference between $COV(\beta_j)$ and v_t (i.e. 100%), as shown in Eq. (6).

$$\beta_j = \operatorname{argmin}_{\beta_j} \{|COV(\beta_j) - v_t|\} \quad (6)$$

The statistical weight function which describes the importance weight assigned to each sample $\boldsymbol{\theta}_{j,k}$ in the current iteration is defined as presented below.

$$w(\boldsymbol{\theta}_{j,k}) = \frac{P(\mathbf{D}|\boldsymbol{\theta}_{j,k})^{\Delta\beta_j}}{\sum_{k=1}^N P(\mathbf{D}|\boldsymbol{\theta}_{j,k})^{\Delta\beta_j}} \quad (7)$$

Then, $\boldsymbol{\theta}_{j,k}$ is resampled from the sample set obtained in the earlier transitional step with probability of $w(\boldsymbol{\theta}_{j,k})$.

The decrease in the number of distinct samples in $\boldsymbol{\theta}_{j,k}$ which occurs as the transitional step number increases because of resampling is undesirable. To address this issue, the samples undergo perturbation using the Metropolis–Hastings (MH) sampler, wherein the stationary PDF is set to $P(\boldsymbol{\theta}|\mathbf{D})^{j+1}$. More precisely, with a probability of $w(\boldsymbol{\theta}_{j,k})$, a Markov chain sample is generated in the i th chain (the chain with $\boldsymbol{\theta}_{j,k}$ as the leader) using a Gaussian proposal PDF centered at the current sample of the i th chain.

Fig. 1 presents an illustration of the framework of the proposed TMCMC-based Bayesian model updating algorithm. Within each iteration of the loop, the model parameters assume diverse values. ABAQUS is used to obtain the corresponding model responses. Subsequently, the likelihood function is calculated to infer the posterior distribution of the model parameters. Furthermore, using the TMCMC sampler, samples are generated to estimate the posterior distribution of the model parameters. The primary objective of Bayesian model updating revolves around the minimization of errors between the simulated and experimentally measured model responses. This minimization leads the samples generated by the TMCMC sampler to converge gradually toward the posterior distributions of the model parameters as the

loop iterations continue. For this study, Python scripts are developed and executed using ABAQUS, facilitating the smooth implementation of the Bayesian model updating framework. This approach provides various benefits such as direct access to the ABAQUS model and resulting data, and the utilization of Python's built-in functions and external libraries for efficient data manipulation. Additionally, ABAQUS offers reliable and efficient solvers for solving the governing equations of a nonlinear system, along with user-friendly interfaces that enable engineers to configure and conduct simulations quickly.

Experiment and Finite Element Model

To validate the proposed framework for model updating of steel–concrete composite girder bridges, a case study was conducted for a large-scale model test of a steel–concrete composite girder bridge. The prototype bridge investigated for this study exemplifies a continuous steel–concrete composite girder design commonly employed in the construction of highway and city viaducts. To facilitate the experimentation, a scaled-down model of a real 4×40 m steel–concrete composite girder bridge was used, with a reduction ratio of 1:3. The structural layout and cross-sectional dimensions are portrayed in Fig. 2. The test girder consisted of two spans with unequal lengths: a longer span measuring 13.14 m and a shorter span measuring 10.66 m. The composite girder comprised a steel girder and a concrete slab, interconnected by shear stud connectors. Its total section height measured 731 mm, with the steel girder itself being 626 mm in height and a partially haunched concrete slab ranging in thickness from 85 mm to 105 mm. As illustrated in Fig. 3, the testing procedure involved the application of concentrated loads using four hydraulic jacks with subsequent static destructive testing. Data collected during the tests included loads, bearing reactions, structural deflections, and sectional strains. Pressure cells, linear variable differential transformers (LVDTs), and strain gauges were used, respectively, to measure and record the reaction forces, deflections, and strains. The locations of the sensors are visible in Fig. 2, where “D” and “S” respectively stand for displacement transducer (or LVDT) and the cross-section for installing strain gauges.

A 3D FE model of the test girder was developed using ABAQUS, incorporating all relevant structural characteristics. As illustrated in Fig. 4, the steel girder and stiffeners were modeled using shell elements, whereas the concrete slab and

plan bracings were represented respectively by solid elements and beam elements. To simulate shear studs along different directions, three spring elements were used. The axes of the spring elements aligned with the same orientation as the overall coordinate system. The shear stiffness values in the longitudinal and transverse directions of the bridge were represented respectively by k_{sz} and k_{sx} , whereas the uplift stiffness was represented by k_{sy} . For simplicity, both k_{sx} and k_{sz} adopted shear stiffness k_s (reference value 2.2×10^5 N/mm) obtained from push-out testing of headed studs. Similarly, k_{sy} used the uplift stiffness k_u (reference value 2.4×10^5 N/mm) acquired from pull-out testing of headed studs. Contact and friction were defined to simulate the interaction between the concrete and steel flanges. Additionally, coupling constraints were established between the reference points and the loading surfaces followed by the application of concentrated loads to the reference points.

For this study, a two-step approach was employed for the FE model updating to improve the parameter estimation efficiency, as depicted in Fig. 5. By reducing the parameter dimension in each step, this approach is aimed at alleviating the computational burden associated with the TMCMC sampler, which depends on the parameter dimensions. The initial step involved constructing the goodness-of-fit function using deflection and strain data followed by model updating to estimate critical linear model parameters that characterize the linear segment of the constitutive models. Static simulations were performed using the “Static, General” procedure in ABAQUS, which is based on the Newton–Raphson method. In the subsequent step, the most probable values (MPVs) of the estimated parameters obtained from the first step were used. The parameters of interest at this step correspond to the nonlinear segments of the constitutive model. Load–deflection curves were employed to establish the goodness-of-fit function. Nevertheless, when dealing with complex nonlinear problems, the “Static, General” procedure becomes time-consuming and might give rise to convergence issues. In consequence, in the second step, quasi-static simulations were conducted using the “Dynamic, Explicit” procedure in ABAQUS. Unlike the “Static, General” procedure, the “Dynamic, Explicit” procedure determines the solution without iteration by explicitly advancing the kinematic state from the previous increment. A key requirement for achieving a quasi-static solution in ABAQUS/Explicit is to maintain a small ratio of kinetic energy to the internal energy of the control system (usually controlled within 5%) (Dassault 2021). It is noteworthy that the quasi-static solution in ABAQUS/Explicit

is slightly less accurate than the iterative solution provided by ABAQUS/Standard for static problems, but its computational accuracy is acceptable for most engineering problems.

In this study, the linear model update in the initial step and the subsequent nonlinear model update took 38 hours and 282 hours, respectively (processor information: i9-9900KF CPU@3.60GHz). The computational time for the Bayesian model update process is contingent on both the solution time of a single simulation and the number of samples taken. As the dimensionality of model parameters increases, an increased number of samples becomes necessary. Consequently, the two-step model updating approach employed improves efficiency by reducing the dimensionality of the model parameters in each model update step.

Update of the Linear Model Parameters

Goodness-of-fit Function for Model Updating

The initial step of the model updating process entails estimation of the linear model parameters which mainly pertain to material properties and steel–concrete connections, given their strong effect on structural mechanical behavior. As depicted in Fig. 2(a), the process of updating the parameters of the linear model involved using observations from 13 deflection sensors (D1 to D13) and longitudinal strain data obtained from two sections (S1 and S2). Notably, among the 13 sensors, D1, D7, and D13 sensors were positioned at the support locations. These sensors played a crucially important role in calibrating the measurements obtained from the other 10 sensors. The calibration process was aimed at elimination of any potential displacement caused by the rigid body displacement of the supports during the test. To estimate the important property of the steel–concrete composite girder, specifically the equivalent neutral axis height, the strain data for the section in the linear state, as depicted in Fig. 2(c), was used. Accordingly, in the process of model updating, only the calibrated deflection data from these ten sensors and the neutral axis heights acquired from two cross-sections were employed. In this step, the goodness-of-fit function was established using the experimentally measured data obtained under a 50 kN load for each jack, considering the linear elastic state exhibited by the structure under this load condition.

To assess the effects of model updating with different data types, two groups of data types were examined: Group 1 relied solely on deflection data, whereas Group 2 integrated both deflection and neutral axis height data.

For Group 1, the goodness-of-fit function to be minimized is expressed as shown below.

$$J_{g1}(\boldsymbol{\theta}) = \sum_{i=1}^{N_p=10} \left[\frac{\delta_i(\boldsymbol{\theta}) - \delta_i^*}{\delta_i^*} \right]^2 \quad (8)$$

For Group 2, the goodness-of-fit function to be minimized is expressed as presented below.

$$J_{g2}(\boldsymbol{\theta}) = \sum_{i=1}^{N_p=10} \left[\frac{\delta_i(\boldsymbol{\theta}) - \delta_i^*}{\delta_i^*} \right]^2 + \sum_{j=1}^{N_q=2} \left[\frac{h_j(\boldsymbol{\theta}) - h_j^*}{h_j^*} \right]^2 \quad (9)$$

where $\delta_i(\boldsymbol{\theta})$ and $h_j(\boldsymbol{\theta})$ respectively denote the deflection and neutral axis height, derived from the FE model with uncertainty in the multidimensional vector $\boldsymbol{\theta}$ which represents the model parameters to be updated. The corresponding measured values are represented as δ_i^* and h_j^* . Due to the absence of explicit information regarding the weights assigned to errors associated with deflection and strain, equal weights are assigned to these two sources of measured data in Eq. (9).

Selection of Updating Parameters

The material properties which exert the most significant influence are the elastic modulus of concrete (E_c), the elastic modulus of reinforcing bar (E_r), and the elastic modulus of steel (E_s). Moreover, the behavior of the steel–concrete connection is influenced by the shear stiffness (k_s) and the uplift stiffness (k_u) of stud connectors. Consequently, these five parameters were chosen initially as potential candidates for updating parameters. This study encompasses a thorough examination of sensitivity on the initial FE model to scrutinize the diverse effects of individual parameters on response indicators. Given spatial constraints, only the sensitivity analysis conclusions are presented in this section. The sensitivity analyses unveiled a notable discrepancy in sensitivity between the elastic modulus of steel and reinforcing bars, with the former displaying considerably higher sensitivity. Furthermore, the sensitivity analysis revealed that the shear stiffness of studs exhibited greater sensitivity when contrasted with the uplift stiffness of studs. As a result, the selected parameters for the update comprise the elastic modulus of concrete (E_c), the elastic modulus of steel (E_s), and the shear stiffness of stud connectors (k_s).

Results and Discussion

The samples obtained during the last sampling step form a posterior distribution of model parameters. By analyzing the posterior distribution, the model parameters associated with the highest probability, representing the global minimum of the goodness-of-fit function, are identifiable as the optimal values. These optimal model parameters are commonly known as the most probable values (MPVs).

A comparison between the MPVs of model parameters and their corresponding reference values is presented in Table 1. Additionally, Table 2 offers a summary incorporating the measured responses from the experiment, simulated responses derived from the initial model, and updated responses obtained through the application of the MPVs of model parameters. Fig. 6 and Fig. 7 exhibit histograms depicting the samples for each model parameter, thereby illustrating the corresponding marginal posterior PDF. This approach is chosen to address the difficulty in visualizing the joint posterior PDF directly within high-dimensional spaces.

From Table 1, it is readily apparent that Group 1 estimated the elastic modulus of steel to be 0.83 times the reference value, whereas the elastic modulus of concrete was determined to be 1.79 times the reference value. Conversely, Group 2 deduced the elastic modulus of steel and concrete to be 0.91 and 1.11 times their respective reference values. Additionally, both Group 1 and Group 2 demonstrated the shear stiffness of studs aligned with the reference value in terms of their order of magnitude.

According to the data presented in Table 2, the initial FE model exhibited deflection and neutral axis height values that were smaller than the corresponding experimentally obtained values. However, after updating the model, a considerable decrease in the discrepancy between the simulated and experimentally obtained deflections was observed for both Group 1 and Group 2. More specifically for Group 2, the deflections predicted by the model for the longer span (D2 to D6) tended to be slightly lower than the corresponding experimentally obtained values. However, for the shorter span (D8 to D12), they tended to be slightly higher, with the exception of sensor D8. It is noteworthy that comparison of Group 1 and Group 2 showed a noticeable deviation of 8.38% in the calculated neutral axis height for Group 1 from its corresponding experimentally obtained value. Conversely, after considering the error in the neutral axis height in the goodness-of-fit

function, the simulated neutral axis height for Group 2 after updating differed from the experimentally obtained value by only -0.04%, as expected.

The trend which can be observed in Fig. 6 and Fig. 7 demonstrated that the posterior PDFs for the elastic modulus of steel, the elastic modulus of concrete, and the shear stiffness of studs in Group 1 portrayed a wider distribution in contrast to those in Group 2. This finding suggested the existence of numerous combinations of model parameters with comparable frequencies within the samples resulted in error-reduced values of the goodness-of-fit function. This observation underscored that exclusive reliance on deflection data was inadequate for discerning the individual stiffnesses of steel and concrete in the linear elastic state of the composite girder. Instead, it primarily reflected the collective stiffness of the composite section. However, by adding information about the height of the neutral axis to the data, the ill-conditioned problem can be constrained properly, and the modulus of elasticity of steel and concrete can be identified independently. In Group 2, the slight reduction in the updated modulus of elasticity of steel compared to the reference value can be attributed to the potential impact of initial defects. The thin steel plates used, particularly the thinnest web plate with a thickness of only 4 mm, were susceptible to residual deformation during the welding process. These defects may have contributed to a reduction in the flexural stiffness of the steel beam. Given the heterogeneous nature of concrete and its considerable variability in elastic modulus, a marginally higher updated elastic modulus than the reference value is deemed acceptable.

Update of the Nonlinear Segment of Constitutive Models

Once the linear model parameters have been updated, the MPVs extracted from the posterior distributions in Group 2 are inserted into the corresponding linear sections of the constitutive model for the elastic modulus of steel, concrete, and shear stiffness of studs. These models are subsequently used in the second step of the model updating process.

Nonlinear Material Constitutive Models

The concrete damaged plasticity (CDP) model, which is widely recognized for its robustness in analyzing concrete behavior, is commonly used in related studies (Minh et al. 2021). For this study, the CDP model was used to characterize

the nonlinear behavior of concrete. The stress–strain curves from *fib* Model Code 2010 (MC2010) were employed to develop the constitutive model for concrete. Fig. 8 depicts the damaged plasticity model of concrete under compression and tension. The constitutive laws are visualized as thick solid lines, whereas the unloading and reloading branches are presented as thin dashed lines. In these figures, f_{cm} and f_{tm} respectively denote compressive and tensile strengths, whereas ε_{cm} and ε_{tm} represent their corresponding strains. The secant modulus, also known as Young’s modulus, is represented by E_0 . Additionally, the variables d_c and d_t respectively correspond to compressive and tensile damage. In Fig. 8(a), ε_c^{in} and ε_{0c}^{el} respectively represent the crushing and elastic undamaged components of strain, whereas ε_c^{pl} and ε_c^{el} represent the plastic and elastic damaged components. Similarly, In Fig. 8(b), ε_t^{ck} and ε_{0t}^{el} respectively denote the cracking and elastic undamaged components of strain, whereas ε_t^{pl} and ε_t^{el} respectively signify the plastic and elastic damaged components. The parameters of the yield function and flow potential in the CDP model are summarized in Table 3.

The uniaxial compressive stress–strain law of concrete is expressed by Eq. (10) to Eq. (13).

$$\sigma_{c(1)} = E_0 \varepsilon_c, \quad \varepsilon_c \leq f_{c0}/E_0 \quad (10)$$

$$\sigma_{c(2)} = \frac{E_{cl} \frac{\varepsilon_c}{f_{cm}} - \left(\frac{\varepsilon_c}{\varepsilon_{cm}} \right)^2}{1 + \left(E_{cl} \frac{\varepsilon_{cm}}{f_{cm}} - 2 \right) \frac{\varepsilon_c}{\varepsilon_{cm}}} f_{cm}, \quad f_{c0}/E_0 < \varepsilon_c \leq \varepsilon_{cm} \quad (11)$$

$$\sigma_{c(3)} = \left(\frac{2 + \gamma_c f_{cm} \varepsilon_{cm}}{2 f_{cm}} - \gamma_c \varepsilon_c + \frac{\varepsilon_c^2 \gamma_c}{2 \varepsilon_{cm}} \right)^{-1}, \quad \varepsilon_c > \varepsilon_{cm} \quad (12)$$

$$\gamma_c = \frac{\pi^2 f_{cm} \varepsilon_{cm}}{2 \left[\frac{G_{ch}}{l_{eq}} - 0.5 f_{cm} \left(\varepsilon_{cm} (1-b) + b \frac{f_{cm}}{E_0} \right) \right]^2}, \quad b = \frac{\varepsilon_c^{pl}}{\varepsilon_c^{in}} \quad (13)$$

In terms of uniaxial tensile behavior, the ratio between tensile stress $\sigma_t(w)$ (for crack width w), and maximum tensile strength f_{tm} can be expressed as

$$\frac{\sigma_t(w)}{f_{tm}} = \left[1 + \left(c_1 \frac{w}{w_c} \right)^3 \right] e^{-c_2 \frac{w}{w_c}} - \frac{w}{w_c} (1 + c_1^3) e^{-c_2} \quad (14)$$

$$\varepsilon_t = \varepsilon_{tm} + \frac{w}{l_{eq}} \quad (15)$$

$$w_c = 5.14 \frac{G_F}{f_{tm}} \quad (16)$$

Additionally, the ratio between crushing and fracture energies can be assumed as $G_{ch} = \left(\frac{f_{cm}}{f_{tm}}\right)^2 G_F$. For a detailed understanding of the interpretation and value of each parameter in the formulas above, readers may turn to the references.

Alfarah et al. (2017) introduced a novel methodology for ascertaining the evolution of damage variables in the CDP model. That study also provided closed-form expressions for the compressive and tensile damage variables, derived from the concrete constitutive laws described earlier. To implement this methodology, an iterative algorithm was developed using Python scripts. Upon achieving convergence, the resultant output curves illustrating the correlation between compressive/tensile stress and damage variables in relation to crushing/cracking strain, were used for Bayesian model updating.

The uniaxial stress–strain curve of steel and reinforcing bars was characterized using a simplified version of the quadratic plastic flow model (Shi et al. 2011), as illustrated in Fig. 9.

$$\sigma = \begin{cases} E_s \varepsilon & \varepsilon \leq \varepsilon_y \\ f_y & \varepsilon_y < \varepsilon \leq k_1 \varepsilon_y \\ k_3 f_y + \frac{E_s(1 - k_3)}{\varepsilon_y(k_2 - k_1)^2} (\varepsilon - k_2 \varepsilon_y)^2 & k_1 \varepsilon_y < \varepsilon \leq k_2 \varepsilon_y, k_3 = \frac{f_u}{f_y} \end{cases} \quad (17)$$

Within this model, ε_y denotes the yield strain, E_s represents the elastic modulus, and f_y and f_u respectively stand for the yield strength and the ultimate strength. Parameters k_1 , k_2 , and k_3 are used to control the shape of the constitutive curve, with specific values of $k_1=4.5$ and $k_2=45$.

The shear force–relative slip curve of stud connectors, as depicted in Fig. 10, was characterized by the simplified expression proposed in an earlier report (Ollgaard et al. 1971). This expression was derived from push-out test results of headed studs as presented below.

$$V = V_u(1 - e^{-0.71s})^{0.4} \quad (18)$$

where V represents the shear force, s denotes the relative slip (in millimeters), and V_u signifies the ultimate bearing capacity of the studs.

Goodness-of-fit Function for Model Updating

In the nonlinear stress state of a composite girder, the assumption that the sectional strain conforms to the plane section no

longer holds. First-order linear regression of strain to determine the height of the neutral axis becomes implausible. Moreover, the presence of plasticity within the section introduces significant variations in the measured strain patterns at different heights. Even for the same section height, there might exist a notable dispersion of strain data at symmetrical section locations. In consequence, for updating the nonlinear model, load–deflection curves obtained from static loading tests were used to establish the goodness-of-fit function, instead of relying on strain measurements. Specifically, regarding deflection information, it is possible to ascertain the individual stiffnesses of steel and concrete under nonlinear stress conditions while considering the different nonlinear constitutive models associated with the respective materials.

The goodness-of-fit function used for updating the nonlinear model is defined as shown below.

$$J_g(\boldsymbol{\theta}) = \sum_{i=1}^{N_s} \sum_{j=1}^{N_d} \left(1 - \frac{\delta_{i,j}^{FEM}(\boldsymbol{\theta})}{\delta_{i,j}^{Test}} \right)^2 \quad (19)$$

where $\delta_{i,j}^{FEM}(\boldsymbol{\theta})$ represents the deflection obtained from the FE model under an uncertain parameter vector $\boldsymbol{\theta}$. Also, $\delta_{i,j}^{Test}$ denotes the measured deflection. N_s represents the number of sensors. N_d denotes the number of scatter points for a load–deflection curve.

Selection of Updating Parameters

The selected constitutive model of concrete reveals that the nonlinear segments of compressive behavior are affected by both compressive strength and crushing energy, whereas the nonlinear segments of tensile behavior are affected by tensile strength and fracture energy. Moreover, once the concrete strength grade is established, correlation can be found between compressive and tensile strength, and between crushing energy and fracture energy. Therefore, accurate determination of the stress–strain relation for concrete requires the updating of two parameters: tensile strength and fracture energy.

Similarly, determining the nonlinear segment of the steel’s constitutive model requires consideration of two parameters: yield strength and ultimate strength. Because the shear force–relative slip relation of the stud was based on an empirical formula, it can be used directly by adjusting its linear segment with shear stiffness derived from the first step of model updating. As a result, the initial candidates for updating parameters in the second step of model updating include the tensile

strength (f_{tm}) and the fracture energy (G_f) of concrete, the yield strength (f_{sy}) and the ultimate strength (f_{su}) of steel, as well as the yield strength (f_{ry}) and the ultimate strength (f_{ru}) of reinforcing bars.

The initial reference material properties of concrete and steel are summarized in Table 4. It is noted that the data highlighted in bold in Table 4 represent values obtained from material coupon tests, while the remaining data were extracted from design codes. The choice to use the MC 2010 code values instead of the directly tested material property values for concrete was driven by limitations in the test conditions. Furthermore, individual measurements from the material coupon tests were omitted and the design code values were used instead, as the variability in the elastic moduli of steel specimens was minimal.

The sensitivity analysis results indicated that the ultimate strength of steel, as well as the yield strength and the ultimate strength of the reinforcing bar, had only a minimal effect on the load–deflection curves. On the other hand, other parameters such as the tensile strength and fracture energy of concrete, along with the yield strength of steel, exhibited greater sensitivity. Therefore, they took priority in the updating process. Consequently, the final set of parameters selected for the nonlinear model updating includes the tensile strength of concrete (f_{tm}), the fracture energy of concrete (G_f), and the yield strength of steel (f_{sy}).

Results and Discussion

Fig. 11 portrays histograms depicting the samples of nonlinear model parameters, offering a visual representation of their posterior distributions. Table 5 presents a comprehensive analysis by comparing the MPVs obtained from the samples with their corresponding reference values.

The results indicated that the tensile strength of the concrete decreased to around 34.5% of its initial value, whereas the yield strength of the steel was approximately 92.2% of its initial value. The fracture energy of concrete demonstrated no discernible changes compared to the initial value. Fig. 12 and Fig. 13 visually present the updated nonlinear constitutive models of concrete and steel by adopting the MPVs of model parameters. Importantly, Bayesian model updating yields

parameter estimations in the form of distributions, rather than a singular optimal value. Accordingly, substituting different parameter samples engenders the generation of a diverse range of constitutive models, each with varying probabilities.

Material tests were conducted to find the compressive strength and modulus of deformation of concrete using prism specimens with dimensions of 150 mm × 150 mm × 300 mm. The obtained values for the compressive strength and modulus of deformation were 37.9 MPa and 35,300 MPa, respectively, both of which were below the reference values specified in the design code. Fig. 12 also presents the constitutive model of concrete derived from these material test values according to MC 2010. The notable decline in the updated concrete strength value can be attributed to the fact that the compressive strength of concrete specified in the design code is a target strength. The standard compressive strength of cylinders tested in experiments would vary from the target strength naturally due to material variability. Additionally, the standard compressive strength values obtained from material testing of a cylinder would not have the same strength observed for the full member. Moreover, the combined effect of the steel–concrete composite cross-section led to the restraint of concrete shrinkage by the steel beam. This caused the concrete slab to undergo tensile forces, making it susceptible to developing shrinkage cracks during the curing process. The existence of initial microcracks may have diminished the tensile properties of the concrete in the negative moment zone, consequently causing a substantial reduction in the updated nominal tensile strength of concrete. However, as shown in Fig. 13, the difference in constitutive models of steel before and after model updating was small compared to those in the constitutive model of concrete. The model update results discussed above revealed that diverse initial defects occurring during the construction of complex bridge structures may result in a decrease in model parameter values. The phenomenon of strength reduction in full members is also observed in many other experiments. Therefore, the material properties of a concrete cylinder should not be directly applied to member strength but with a reduction in strength.

Fig. 14 presents results of a comparative analysis of load–deflection curves obtained from two sensor locations (as depicted in Fig. 2(a)) before and after model updating. The legend differentiates between the simulation results before and after model updating, labeled as “initial” and “updated” respectively, whereas the measured load–deflection curves from

the experiment are denoted as “test”. Furthermore, the load–deflection curve is flanked by two scatter plots on its left and right sides, visually illustrating the relative error between the measured deflection and the corresponding simulated deflection after and before model updating at each load level. The red-filled area and the green-filled area represent relative errors within $\pm 10\%$. Table 6 presents the average relative errors of the load–deflection curves at various sensor locations, both before and after model updating.

The analyses presented in Fig. 14 and Table 6 clarified that the initial FE model had more error initially to represent the plastic behavior of the structure accurately, particularly for the longer span. It is noteworthy that the simulated deflection values of the plastic section in the load–deflection curve, as derived from the initial model, exhibited discrepancies exceeding -10% when compared to the experimentally obtained deflection values. Furthermore, these errors tended to diverge with increasing load levels in Fig. 14. However, following the nonlinear model updating process, the simulated load–deflection curves closely mirrored the measured data, highlighting remarkable improvement in the model accuracy. Moreover, the relative errors between the measured and updated deflection values remained consistently below $\pm 10\%$ at each load level throughout the nonlinear stage, thereby underscoring the updated model’s ability to forecast the behavior of the structure more closely. Whereas certain sensor locations within the shorter span of the specimen displayed a slight increase in errors after model updating in Table 6, the cumulative error across all sensor locations decreased considerably for the longer span, indicating marked improvement of the overall model accuracy.

In Fig. 14, it is readily apparent that the relative error between the simulated load–deflection curves and the measured curves was significant in the linear section. This discrepancy stemmed from the large ratio of kinetic energy to the internal energy of the system during the initial phase, caused by the collision effect when solving for the quasi-static process using ABAQUS/Explicit. The kinetic and internal energy values were both small during this phase, which led to a large deviation in the initial stiffness of the simulated load–deflection curve. To address this deviation and ensure simulation accuracy, ABAQUS/Standard was used for the present study to conduct static simulations for updating the linear model parameters in the first step.

Model Verification

To evaluate the effectiveness of implementing the model update for assessment of structural performance, the updated FE model discussed earlier was used to simulate load–strain curves of both concrete and reinforcing bars. Subsequently, a comparison was made between these simulated results and the measurements obtained from static loading tests. Fig. 15 presents a visual comparison of load–strain curves of concrete at cross-sections S1 and S2, obtained both before and after model updating, with the corresponding strain gauge positions labeled in Fig. 2(c). Similarly, Fig. 16 depicts a comparison of load–strain curves of reinforcing bars before and after model updating at the cross-section 730 mm from the intermediate support: S1 is the strain in the cross-section of the longer span; S2 is the strain in the cross-section of the shorter span. The simulated load–strain curves for both the concrete and reinforcing bars demonstrated improved accuracy and similarity to the measured data following model updating. Fig. 15 unveils that although the general trend of the experimentally obtained data from strain gauges positioned symmetrically across the deck width in cross-section (i.e. S1-C1-test and S1-C2-test) remained consistent, the strain values measured by the two strain gauges were not exactly identical. Unlike deflection, which provides an overview of the structure’s mechanical behavior, strain reflects localized mechanical behavior at specific points in the cross-section. Because of its inherent nature, strain values often exhibit significant fluctuations leading to unsatisfactory symmetry between strain values at symmetrically installed sensors. Drawing from the considerations presented earlier, the produced goodness-of-fit function for nonlinear model updating only accounted for the deflection error between simulated and experimentally obtained data, while neglecting the strain error. Observations from Fig. 16 indicated that the reinforcing bars in the S1 and S2 cross-sections had not yet reached yield. Both experimental and FE simulation results showed that once plasticity developed at the cross-section of the intermediate support, the composite beam reached its ultimate stress state due to local buckling of the steel beam, which restricted further plasticity development. This explains why the reinforcing bars in the S1 and S2 cross-sections did not reach yield.

Conclusions

For this study, we proposed a two-step Bayesian model updating approach to specifically examine the update of nonlinear constitutive models for the steel–concrete composite girder bridge using static responses. The primary objective was to strike a balance between computational efficiency and simulation accuracy in updating complex FE models. The first step involved construction of a goodness-of-fit function based on deflection and neutral axis height data. Following that, model updating was conducted to estimate the essential linear model parameters which describe the linear segment of the constitutive models. In the subsequent step, the MPVs of the estimated parameters obtained from the first step were used. Load–deflection curves were employed to construct the goodness-of-fit function for estimating the key parameters which characterize the nonlinear segments of the constitutive models. The feasibility of this approach was verified by implementing a framework in ABAQUS for developing a 3D FE model of a large-scale steel–concrete composite girder bridge. Based on findings from the case study conducted for this investigation, the following conclusions can be drawn.

1. The updated parameter results revealed that exclusive reliance on deflection data was inadequate to differentiate between the individual stiffnesses of steel and concrete in a linear elastic state of the composite girder. However, by augmenting the data with knowledge about the neutral axis height, it was possible to distinguish the modulus of elasticity of steel and concrete separately. Given that solving inverse problems can often yield nonunique or even unidentifiable solutions, deterministic model updating methods, which specifically identify a single solution while disregarding other potential solutions, have limitations when compared to probability-based methods.
2. Limitations of the initial FE model in accurately capturing the plastic behavior of the structure were evident. Nonetheless, using Bayesian model updating, the simulated load–deflection curves exhibited a close alignment with the measured data, highlighting their marked improvement in model accuracy. The discrepancy between the simulated and measured load–deflection curves for the linear section can be attributed to the large ratio of kinetic energy to internal energy caused by the initial collision effect of the quasi-static process solution. Therefore, the two-step model updating

approach, which emphasized updating the linear model parameters in the first step, became crucially important for addressing this issue.

3. The load–strain curves simulated by the updated FE model demonstrated improved accuracy and a closer resemblance to the measured data for both the concrete and reinforcing bars. These results affirm the effectiveness of the two-step model updating approach, indicating that the updated model can represent the static structural response of the steel–concrete composite girder accurately. Moreover, this approach holds promise for broader application in structural model updates for various types of structures, provided nonlinear stress state data are available.
4. Results of a comprehensive case study confirmed the proposed two-step Bayesian model updating approach as effective for improving the assessment of structural performance of real-world bridge structures. With continued advances in computation performance, the use of a 3D FE model incorporating the updating of sophisticated nonlinear constitutive models shows great potential for application to model-based structural health monitoring and near real-time assessment of structural performance.
5. However, the proposed method still has limitations in obtaining structural response data for the material’s nonlinear regime. This is because actual structures primarily operate in a linear elastic stress state. Nevertheless, even if only data from the linear elastic regime is available, it can still be used to update the model parameters. Once updated, the model can then be used to help predict nonlinear structural responses.

Data Availability Statement

All data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request.

Acknowledgments

This study was partially sponsored by the Japanese Society for the Promotion of Science (JSPS) Grant-in-Aid for Scientific Research (B) under project No. 22H01576. That financial support is gratefully acknowledged. The first author expresses sincere gratitude for the financial support received from the China Scholarship Council under Grant No. 202106260039.

References

- Alfarah, B., F. López-Almansa, and S. Oller. 2017. "New methodology for calculating damage variables evolution in Plastic Damage Model for RC structures." *Eng. Struct.* 132: 70-86.
- Beck, J. L. 2010. "Bayesian system identification based on probability logic." *Struct. Control Health Monit.* 17(7): 825-847.
- Beck, J. L., and S. K. Au. 2002. "Bayesian updating of structural models and reliability using Markov Chain Monte Carlo simulation." *J. Eng. Mech.* 128(4): 380-391.
- Beck, J. L., and L. S. Katafygiotis. 1998. "Updating models and their uncertainties. I: Bayesian statistical framework." *J. Eng. Mech.* 124(4): 455-461.
- Beck, J. L., and K. V. Yuen. 2004. "Model selection using response measurements: Bayesian probabilistic approach." *J. Eng. Mech.* 130(2): 192-203.
- Betz, W., I. Papaioannou, J. L. Beck, and D. Straub. 2018. "Bayesian inference with Subset Simulation: Strategies and improvements." *Comput. Methods Appl. Mech. Eng.* 331: 72-93.
- Betz, W., I. Papaioannou, and D. Straub. 2016. "Transitional Markov Chain Monte Carlo: Observations and improvements." *J. Eng. Mech.* 142(5): 04016016.
- Ching, J. Y., and Y. C. Chen. 2007. "Transitional Markov Chain Monte Carlo method for Bayesian model updating, model class selection, and model averaging." *J. Eng. Mech.* 133(7): 816-832.
- Dassault Systèmes SIMULIA. 2021. Abaqus 2021 User Documentation. Providence, RI.
- Fib Model Code for Concrete Structures 2010. Ernst & Sohn, Wiley, 2013.
- Lin, K., Y.-L. Xu, X. Lu, Z. Guan, and J. Li. 2021. "Collapse prognosis of a long-span cable-stayed bridge based on shake table test and nonlinear model updating." *Earthq. Eng. Struct. Dyn.* 50(2): 455-474.
- Lye, A., A. Cicirello, and E. Patelli. 2021. "Sampling methods for solving Bayesian model updating problems: A tutorial." *Mech. Syst. Signal Process.* 159: 107760.
- Malekghaini, N., F. Ghahari, H. Ebrahimian, M. Bowers, E. Ahlberg, and E. Taciroglu. 2023. "A two-step FE model updating approach for system and damage identification of prestressed bridge girders." *Buildings.* 13(2): 420.
- Martínez-Muñoz, D., J. V. Martí, and V. Yepes. 2020. "Steel-concrete composite bridges: Design, life cycle assessment, maintenance, and decision-making." *Adv. Civ. Eng.* 2020: 8823370.
- Minh, H. L., S. Khatir, M. A. Wahab, and C. L. Thanh. 2021. "A concrete damage plasticity model for predicting the effects of compressive high-strength concrete under static and dynamic loads." *J. Build. Eng.* 44: 103239.
- Mustafa, S., and Y. Matsumoto. 2017. "Bayesian model updating and its limitations for detecting local damage of an existing truss bridge." *J. Bridge Eng.* 22(7): 04017019.
- Muto, M., and J. L. Beck. 2008. "Bayesian updating and model class selection for hysteretic structural models using stochastic simulation." *J. Vib. Control.* 14(1-2): 7-34.
- Ollgaard, J. G., R. G. Slutter, and J. W. Fisher. 1971. "Shear strength of stud connectors in lightweight and normal-weight concrete." *Engineering Journal-American Institute of Steel Construction Inc.* 8(2): 55-64.
- Ramancha, M. K., R. Astroza, R. Madarshahian, and J. P. Conte. 2022. "Bayesian updating and identifiability assessment of nonlinear finite element models." *Mech. Syst. Signal Process.* 167: 108517.
- Ren, W. X., S. E. Fang, and M. Y. Deng. 2011. "Response surface-based finite-element-model updating using structural static responses." *J. Eng. Mech.* 137(4): 248-257.
- Ribeiro, D., R. Calçada, R. Delgado, M. Brehm, and V. Zabel. 2012. "Finite element model updating of a bowstring-arch railway bridge based on experimental modal parameters." *Eng. Struct.* 40: 413-435.
- Shi, Y. J., M. Wang, and Y. Q. Wang. 2011. "Experimental and constitutive model study of structural steel under cyclic loading." *J. Constr. Steel Res.* 67(8): 1185-1197.
- Shi, Z., Y. Hong, and S. L. Yang. 2019. "Updating boundary conditions for bridge structures using modal parameters." *Eng. Struct.* 196: 109346.
- Simoen, E., G. De Roeck, and G. Lombaert. 2015. "Dealing with uncertainty in model updating for damage assessment:

- A review.” *Mech. Syst. Signal Process.* 56-57: 123-149.
- Simoen, E., C. Papadimitriou, and G. Lombaert. 2013. “On prediction error correlation in Bayesian model updating.” *J. Sound Vib.* 332(18): 4136-4152.
- Song, M. M., L. Renson, J. P. Noël, B. Moaveni, and G. Kerschen. 2018. “Bayesian model updating of nonlinear systems using nonlinear normal modes.” *Struct. Control Health Monit.* 25(12): e2258.
- Tang, J., Q. T. Su, H. Su, and J. R. Casas. 2023. “Experimental and numerical study on the mechanical behavior of prestressed continuous composite I-girder bridges with partial connection.” *J. Bridge Eng.* 28(3): 04022154.
- Vereecken, E., A. Slobbe, A. Rózsás, W. Botte, G. Lombaert, and R. Caspeele. 2022. “Efficient Bayesian model selection and calibration using field data for a reinforced concrete slab bridge.” *Struct. Infrastruct. Eng.* 20(5): 741-759.
- Wang, J. J., C. Liu, X. Nie, J. S. Fan, and Y. J. Zhu. 2021. “Nonlinear model updating algorithm for biaxial reinforced concrete constitutive models of shear walls.” *J. Build. Eng.* 44: 103215.
- Wang, Z. C., Y. Xin, and W. X. Ren. 2016. “Nonlinear structural joint model updating based on instantaneous characteristics of dynamic responses.” *Mech. Syst. Signal Process.* 76-77: 476-496.
- Xia, Q., Y. Xia, H. P. Wan, J. Zhang, and W. X. Ren. 2020. “Condition analysis of expansion joints of a long-span suspension bridge through metamodel-based model updating considering thermal effect.” *Struct. Control Health Monit.* 27(5): e2521.
- Xiao, J. L., X. Nie, Y. F. Liu, and J. S. Fan. 2023. “Structural performance assessment of RC flexural members through crack-based fibre beam-column model updating.” *Comput. Struct.* 281: 107029.
- Xu, L. Y., X. Nie, and M. X. Tao. 2018. “Rational modeling for cracking behavior of RC slabs in composite beams subjected to a hogging moment.” *Constr. Build. Mater.* 192: 357-365.
- Yang, J. H., H. F. Lam, and Y. H. An. 2022. “Development of a two-phase adaptive MCMC method for efficient Bayesian model updating of complex dynamic systems.” *Eng. Struct.* 270: 114836.
- Zhang, F. L., Y. C. Ni, and H. F. Lam. 2017. “Bayesian structural model updating using ambient vibration data collected by multiple setups.” *Struct. Control Health Monit.* 24(12): e2023.
- Zhang, J., K. Maes, G. De Roeck, and G. Lombaert. 2021. “Model updating for a large multi-span quasi-periodic viaduct based on free wave characteristics.” *J. Sound Vib.* 506: 116161.
- Zhou, X., C. W. Kim, F. L. Zhang, and K. C. Chang. 2022. “Vibration-based Bayesian model updating of an actual steel truss bridge subjected to incremental damage.” *Eng. Struct.* 260: 114226.

List of Figures

Fig. 1. Framework of the proposed method.

Fig. 2. Design drawing of the test girder and sensor locations (unit: mm): (a) structural layout and displacement transducer locations; (b) cross-sectional dimensions; (c) strain gauge positions.

Fig. 3. Loading procedures and devices: (a) static loading test; (b) loading locations.

Fig. 4. Three-dimensional FE model: (a) element types for different components; (b) simulation of connections and contacts within the interface.

Fig. 5. Flowchart of model updating.

Fig. 6. Histograms of samples for linear model parameters (Group 1): (a) elastic modulus of steel; (b) elastic modulus of concrete; (c) shear stiffness of stud.

Fig. 7. Histograms of samples for linear model parameters (Group 2): (a) elastic modulus of steel; (b) elastic modulus of concrete; (c) shear stiffness of stud.

Fig. 8. Concrete damaged plasticity model: (a) compressive behavior; (b) tensile behavior.

Fig. 9. Uniaxial constitutive model of steel.

Fig. 10. Shear force–relative slip curve of studs.

Fig. 11. Histogram of samples for nonlinear model parameters: (a) Ratio of f_{tm} ; (b) Ratio of G_f ; (c) Ratio of f_{sy} . (Note: The horizontal axis shows the ratio of updated value (θ'')/reference value (θ').)

Fig. 12. Comparison of CDP models (tensile stress vs. tensile strain) before and after model updating.

Fig. 13. Comparison of constitutive models of steel before and after model updating.

Fig. 14. Comparison of load–deflection curves before and after model updating: (a) Sensor D4; (b) Sensor D10.

Fig. 15. Comparison of load–strain curves of concrete before and after model updating: (a) cross-section S1; (b) cross-section S2.

Fig. 16. Comparison of load–strain curves of reinforcing bars before and after model updating: (a) cross-section S1; (b) cross-section S2.

Table 1. Comparison between MPVs and reference values.

Model parameter	Reference value	Group 1		Group 2	
		Updated	Ratio	Updated	Ratio
E_s (MPa)	2.06×10^5	1.72×10^5	0.83	1.87×10^5	0.91
E_c (MPa)	3.45×10^4	6.17×10^4	1.79	3.83×10^4	1.11
k_s (N/mm)	2.20×10^5	2.03×10^5	0.99	1.43×10^5	0.96

Note: Ratio for E_s and E_c is updated value (θ^u)/reference value (θ^r)

Ratio for k_s is $\log_{10}\theta^u/\log_{10}\theta^r$

Table 2. Comparison between measured and simulated responses (Unit: mm).

Observation	Measured	Initial Analysis		Group 1		Group 2	
		Simulated	Error	Updated	Error	Updated	Error
D2	-2.10	-1.86	-11.24%	-2.00	-4.52%	-2.02	-3.70%
D3	-3.57	-3.29	-7.94%	-3.55	-0.75%	-3.58	0.08%
D4	-4.13	-3.79	-8.07%	-4.11	-0.54%	-4.13	0.14%
D5	-3.11	-2.81	-9.75%	-3.05	-2.02%	-3.06	-1.62%
D6	-1.29	-1.11	-14.34%	-1.21	-6.61%	-1.20	-6.84%
D8	-0.59	-0.46	-20.99%	-0.53	-10.50%	-0.52	-12.21%
D9	-1.47	-1.39	-5.77%	-1.54	4.69%	-1.53	4.30%
D10	-1.93	-1.88	-2.39%	-2.08	7.69%	-2.08	7.82%
D11	-1.73	-1.60	-7.71%	-1.76	1.41%	-1.76	1.76%
D12	-0.96	-0.92	-3.75%	-1.01	5.48%	-1.01	5.98%
D_Avg.	—	—	-9.19%	—	-0.57%	—	-0.43%
h	542.41	531.75	-1.96%	587.85	8.38%	542.20	-0.04%

Note: Sensors D1, D7, and D13 were installed at the supports, measuring rigid body displacements requiring calibration. As a result, the calibrated deflection value was exactly zero. h represents the distance from the bottom of the lower flange.

The average relative error of deflection $\bar{e} = \frac{1}{N_p} \sum_{i=1}^{N_p} \frac{\delta_i(\theta) - \delta_i^*}{\delta_i^*}$, where $\delta_i(\theta)$ and δ_i^* respectively represent the deflection derived from the FE model and the corresponding measured deflection; Here, N_p denotes the number of displacement transducers, with $N_p = 10$.

Table 3. The parameters of the yield function and flow potential in the CDP model.

Dilation Angle	Eccentricity	σ_{b0}/σ_{c0}	K_c	Viscosity Parameter
30°	0.1	1.16	0.667	0.0001

Note: σ_{b0}/σ_{c0} is the ratio of initial equibiaxial compressive yield stress to initial uniaxial compressive yield stress; K_c is the ratio of the second stress invariant on the tensile meridian.

Table 4. Initial reference values for material properties.

Material	Grade	E (kN/mm ²)	f_{cm} (N/mm ²)	f_{tm} (N/mm ²)	G_f (N/mm)	f_y (N/mm ²)	f_u (N/mm ²)
Concrete	C50	35.3	37.9	3.53	0.147	—	—
Steel ($\delta = 13$ mm)	Q420	206	—	—	—	441.2	603.6
Rebar ($d = 12$ mm)	HRB400	200	—	—	—	399.8	602.5

Note: Data shown in bold represent measurements obtained from material coupon tests, while the remaining are extracted from the design codes.

E , Elastic modulus; f_{cm} , Compressive strength; f_{tm} , Tensile strength; G_f , fracture energy; f_y , yield strength; f_u , ultimate strength.

C50 denotes a target compressive strength of concrete cubes at 50 N/mm², while Q420 and HRB400 represent target yield strengths of 420 N/mm² and 400 N/mm², respectively.

Table 5. MPVs of nonlinear model parameters.

Scenario	Tensile strength of concrete (N/mm ²)	Fracture energy of concrete (N/mm)	Yield strength of steel (N/mm ²)
Initial	3.528	0.121	440.0
Updated	1.218	0.122	405.7

Table 6. Average relative errors at different sensor locations (%).

Location	Initial	Updated	Difference
D2	14.73	4.67	10.06↓
D3	13.77	3.39	10.38↓
D4	14.53	3.59	10.94↓
D5	15.12	3.92	11.20↓
D6	23.27	11.02	12.25↓
D8	13.16	7.97	5.19↓
D9	7.23	8.82	1.59↑
D10	6.64	8.51	1.87↑
D11	7.04	8.18	1.14↑
D12	6.03	11.41	5.38↑

Note: Average relative error $\bar{e}_i = \frac{1}{N_d} \sum_{j=1}^{N_d} \sqrt{\left(1 - \frac{\delta_{i,j}^{FEM}}{\delta_{i,j}^{Test}}\right)^2}$, where N_d denotes the number of scatter points for a load–deflection curve.

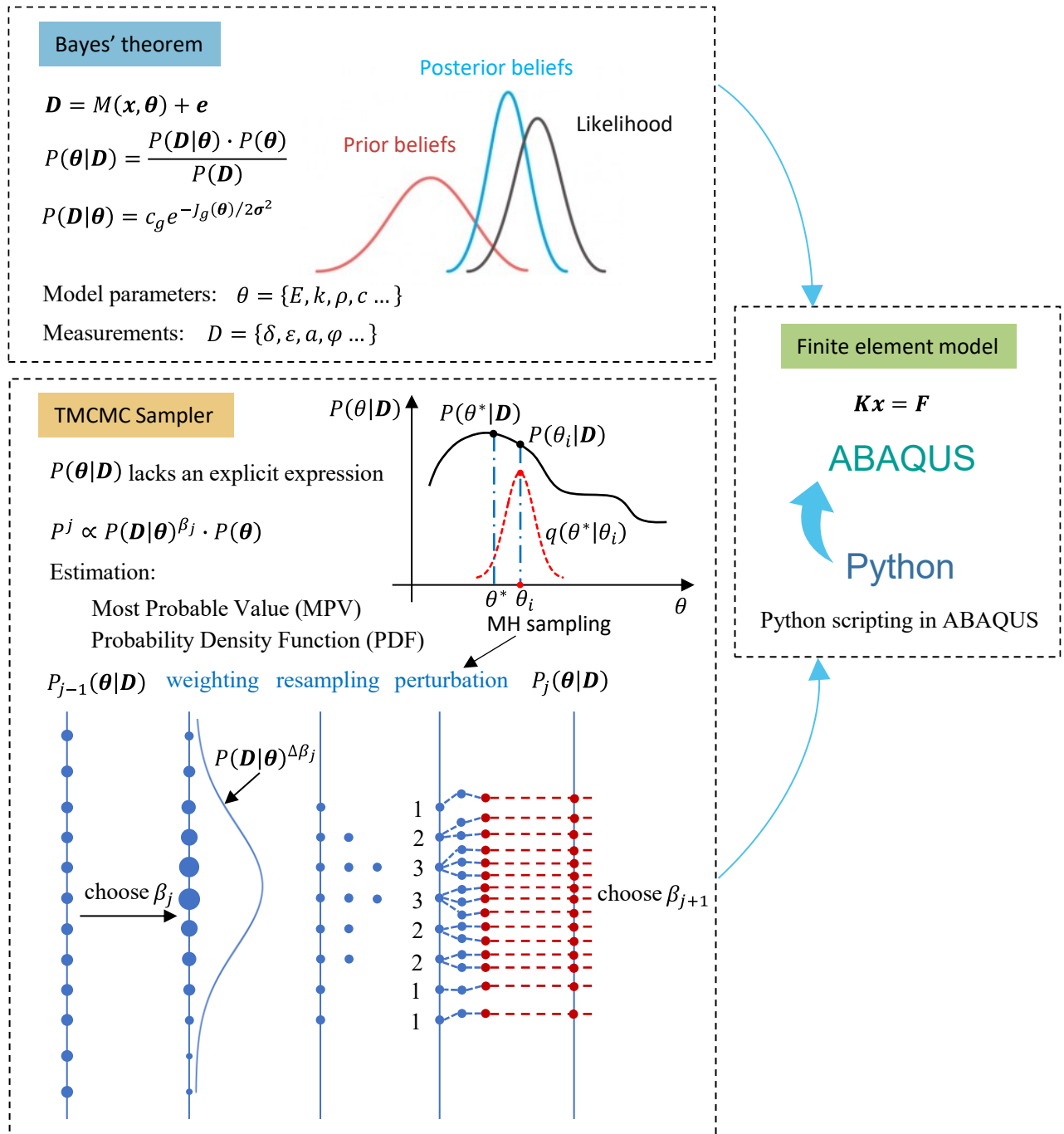
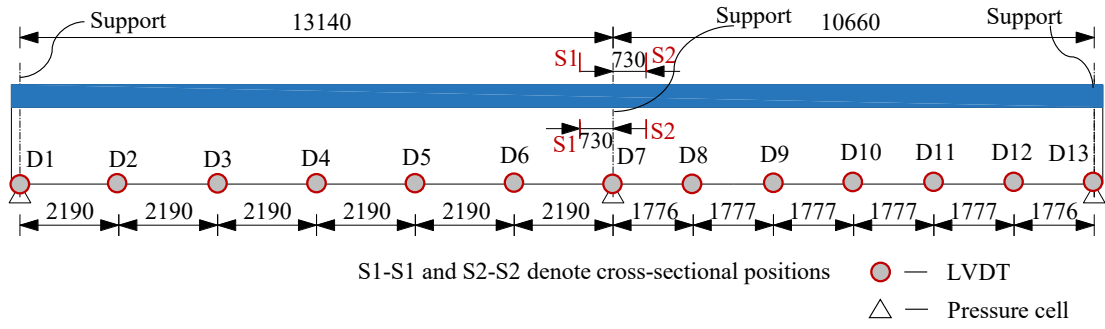
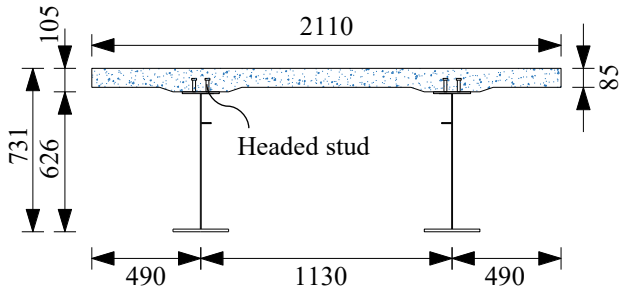


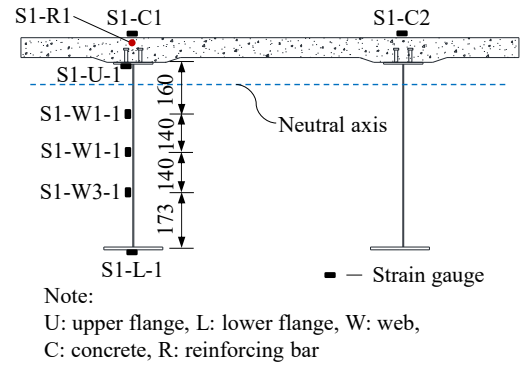
Fig. 1. Framework of the proposed method.



(a) structural layout and displacement transducer locations

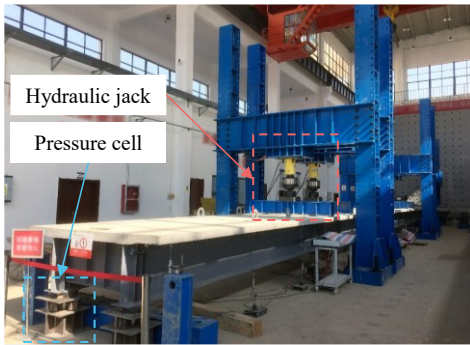


(b) cross-sectional dimensions

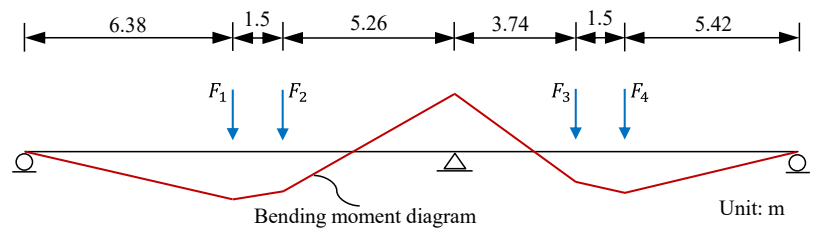


(c) strain gauge positions

Fig. 2. Design drawing of the test girder and sensor locations (unit: mm).

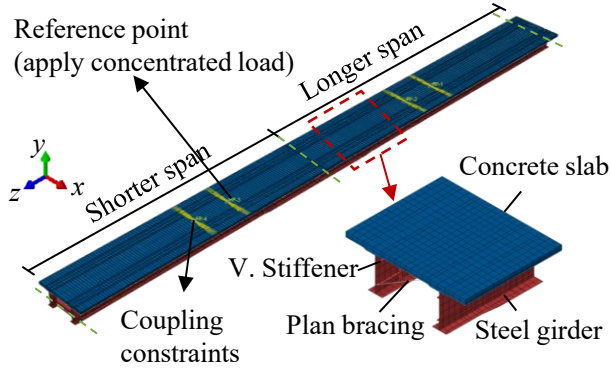


(a) static loading test



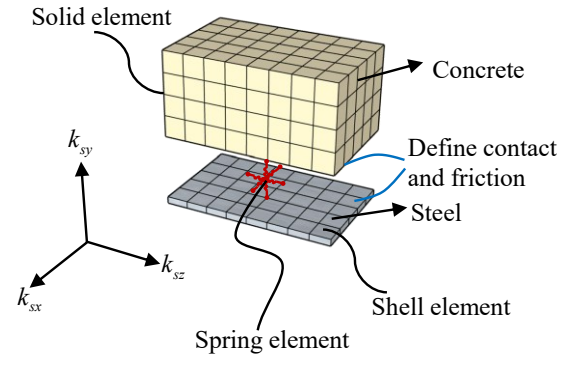
(b) loading locations

Fig. 3. Loading procedures and devices.



Note: L. = Longitudinal, V. = Vertical --- Support

(a) element types for different components



(b) simulation of connections and contacts within the

interface

Fig. 4. Three-dimensional FE model.

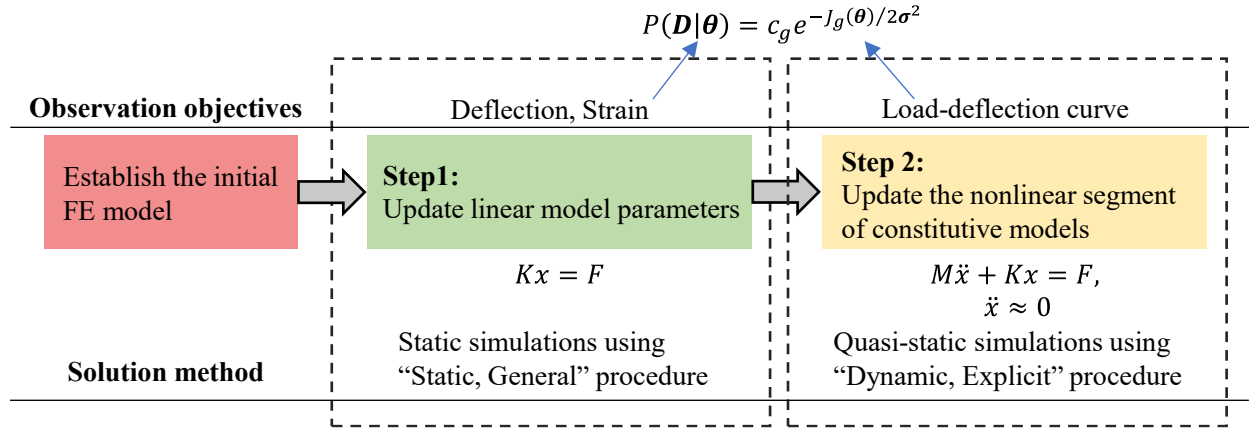


Fig. 5. Flowchart of model updating.

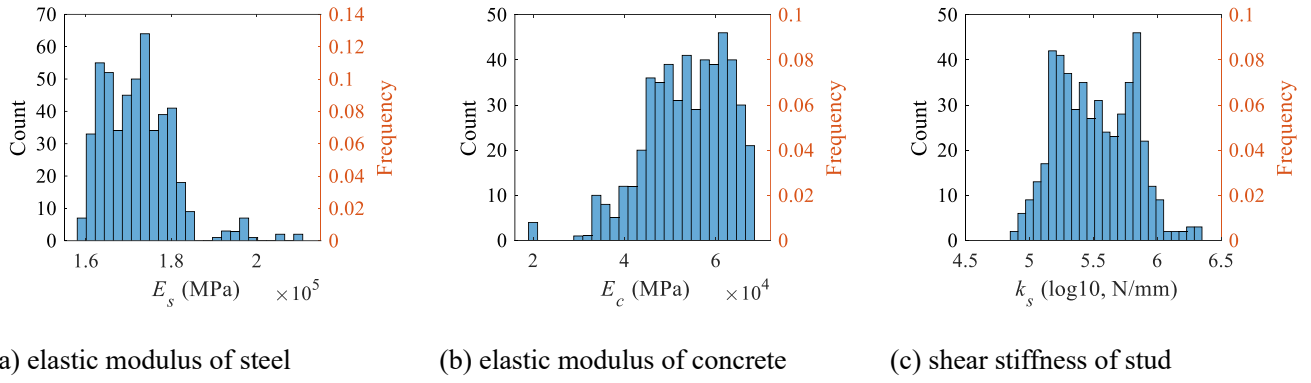


Fig. 6. Histograms of samples for linear model parameters (Group 1).

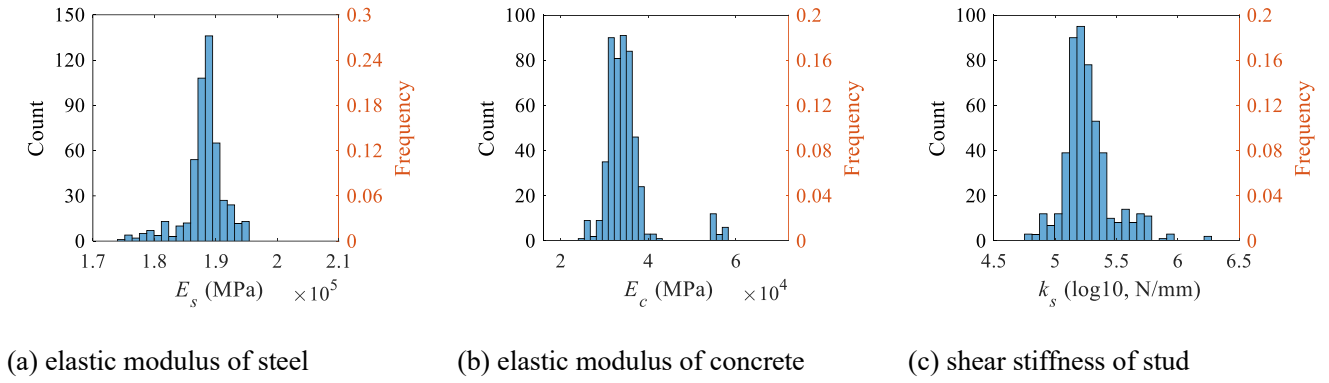


Fig. 7. Histograms of samples for linear model parameters (Group 2).

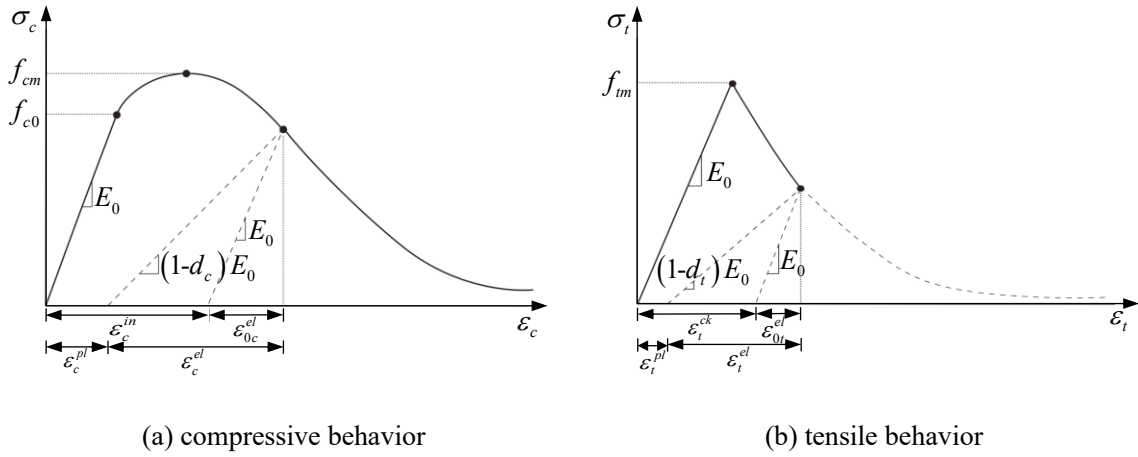


Fig. 8. Concrete damaged plasticity model.

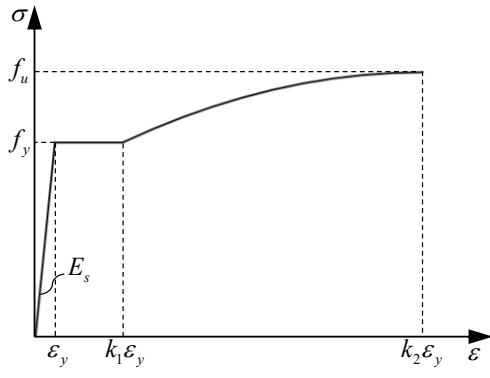


Fig. 9. Uniaxial constitutive model of steel.

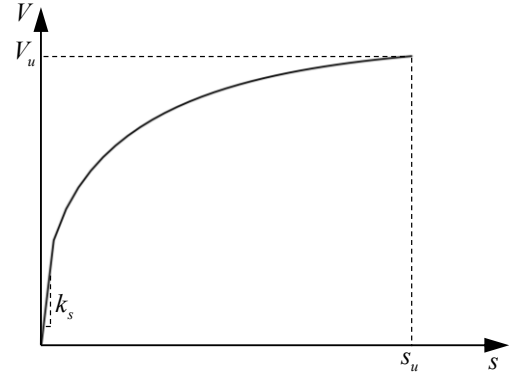
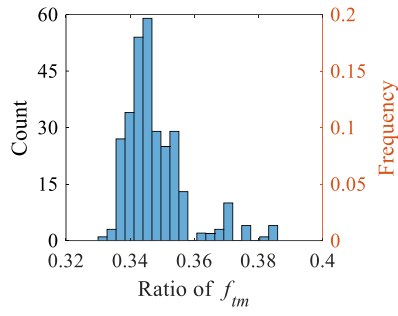
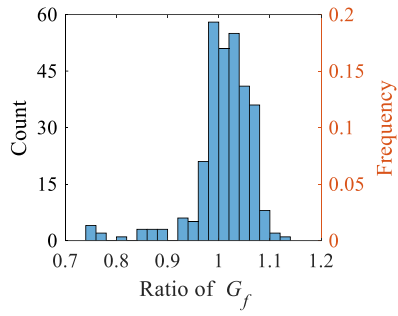


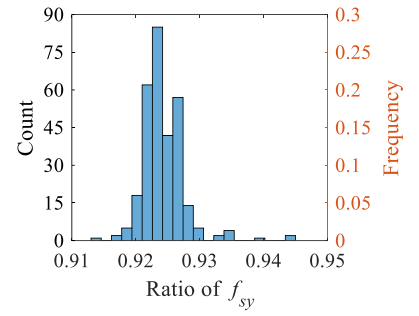
Fig. 10. Shear force–relative slip curve of studs.



(a)



(b)



(c)

Fig. 11. Histogram of samples for nonlinear model parameters (Note: The horizontal axis shows the ratio of updated value (θ^u)/reference value (θ^r)).

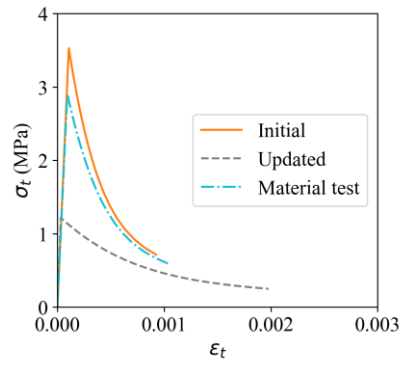


Fig. 12. Comparison of CDP models (tensile stress vs. tensile strain) before and after model updating.

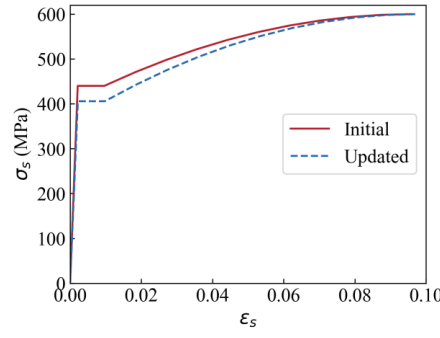
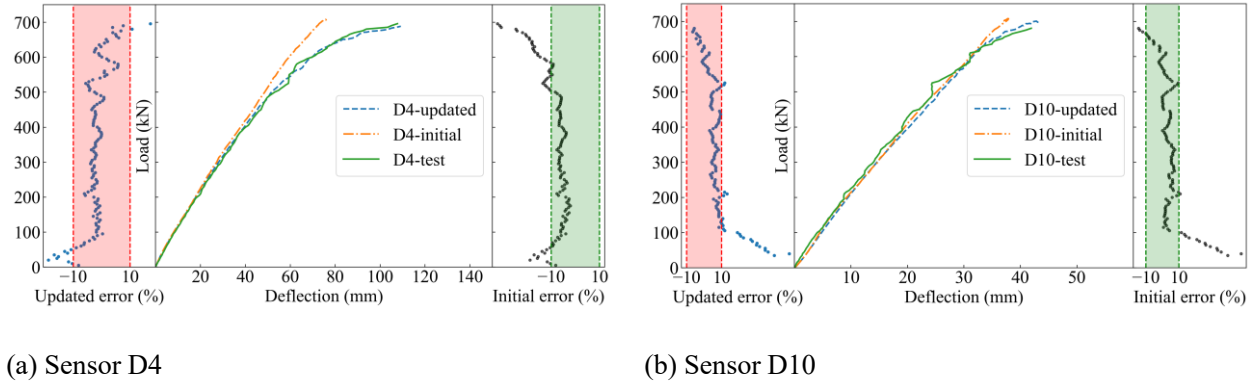


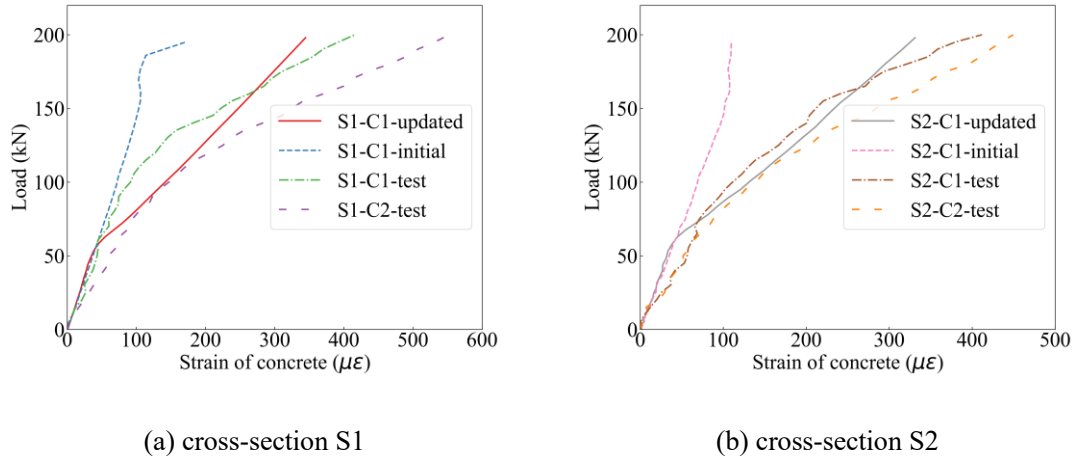
Fig. 13. Comparison of constitutive models of steel before and after model updating.



(a) Sensor D4

(b) Sensor D10

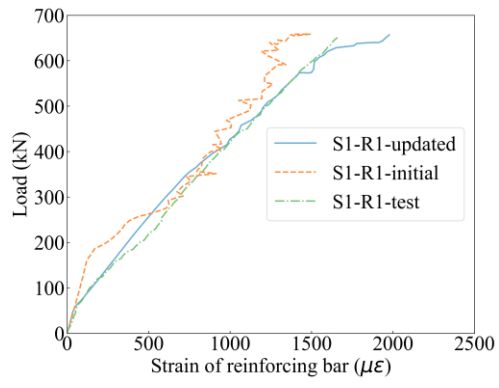
Fig. 14. Comparison of load-deflection curves before and after model updating.



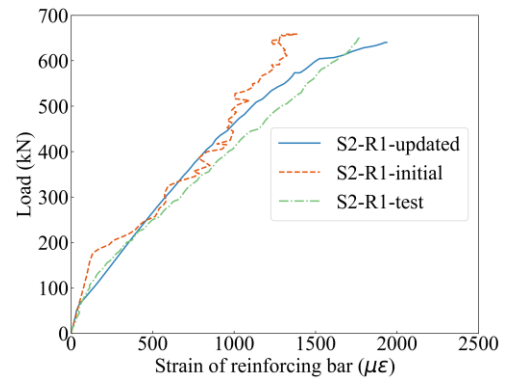
(a) cross-section S1

(b) cross-section S2

Fig. 15. Comparison of load-strain curves of concrete before and after model updating.



(a) cross-section S1



(b) cross-section S2

Fig. 16. Comparison of load-strain curves of reinforcing bars before and after model updating.