

Isomorphisms of the Fourier Algebras in Crossed Products

by

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Abstract.

Let $(\mathcal{A}, G, \alpha)$, $(\mathcal{B}, H, \beta)$ be W^* -systems, $F_\alpha(G; \mathcal{A})$ and $F_\beta(H; \mathcal{B})$,
their Fourier algebras defined in [2]. The main result is that $F_\alpha(G; \mathcal{A})$
and $F_\beta(H; \mathcal{B})$ are isometrically isomorphic as Banach algebras if and
only if either G and H are topologically isomorphic (denoted by I) as
groups and $\mathcal{A}$ and $\mathcal{B}$ are isomorphic (denoted by θ) such that
 $\beta_{I(g)} \circ \theta = \theta \circ \alpha_g$ for all $g \in G$, or G and H are topologically anti-isomorphic
and $\mathcal{A}$ and $\mathcal{B}$ are anti-isomorphic such that $\beta_{I(g)^{-1}} \circ \theta = \theta \circ \alpha_g$ for all
 $g \in G$.

1. Introduction.

For locally compact abelian groups G, H , Pontryagin's duality theorem mentions that $L^1(G)$ and $L^1(H)$ are isometrically isomorphic if and only if G and H are topologically isomorphic as groups. T. Kawada [4] and J.G. Wendel [11] proved the above statement for arbitrary locally compact groups.

G is a locally compact abelian group, then $L^1(G)$ is isometrically isomorphic to Fourier algebra $A(G)$ in [7]. Therefore $A(G)$ and $A(H)$ are isometrically isomorphic as Banach algebras if and only if G and H are topologically isomorphic as abelian groups.

P. Eymard [1], on the other hand, defined the Fourier algebra $A(G)$ of a locally compact group G and showed that $A(G)$ is isometrically isomorphic to the predual $m(G)_*$ of the von Neumann algebra $m(G)$ generated by the left regular representation of G .

So that, M.E. Walter [10] showed that $A(G)$ and $A(H)$ are isometrically isomorphic as Banach algebras if and only if G and H are topologically isomorphic as groups for arbitrary locally compact groups.

Recently for W^* -system $(\mathcal{H}, G, \alpha)$, the Fourier space $F_\alpha(G; \mathcal{H}_*) \subset C_0(G; \mathcal{H}_*)$ was defined in [8] H. Takai such that $F_\alpha(G; \mathcal{H}_*)$ is isometrically isomorphic to the predual of the crossed product $G \rtimes_\alpha \mathcal{H}$ as Banach spaces.

M. Fugita [2] quite recently defined the Banach algebra structure in Fourier space $F_\alpha(G; \mathcal{H}_*)$ and all characters $\widehat{F_\alpha(G; \mathcal{H}_*)}$ is topologically isomorphic to G as groups and defined and investigated the support of the operators in $G \rtimes_\alpha \mathcal{H}$.

In this paper we generalize a Walter's result for W^* -system $(\mathcal{H}, G, \alpha)$ and show that the Banach algebra structure in $F_\alpha(G; \mathcal{H}_*)$ is essential in a sense.

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2. Notations and Preliminaries.

Let $\mathcal{A}$ be a von Neumann algebra on a Hilbert space $\mathcal{H}$ and G be a locally compact group. The triple $(\mathcal{A}, G, \alpha)$ is said a W^* -system if the mapping α of G into the group $\text{Aut}(\mathcal{A})$ of all automorphisms of $\mathcal{A}$ is a homomorphism and the function $g \mapsto \omega \circ \alpha_g(x)$ is continuous on G for all $x \in \mathcal{A}$ and $\omega \in \mathcal{A}_*$ ($\mathcal{A}_*$ is the predual of $\mathcal{A}$).

$G \otimes_{\alpha} \mathcal{A}$ is the von Neumann algebra generated by the family of the operators $\{\pi_{\alpha}(x), \lambda_G(g) ; x \in \mathcal{A}, g \in G\}$;

$$(\pi_{\alpha}(x)\xi)(h) = \alpha_h^{-1}(x)\xi(h)$$

$$(\lambda_G(g)\xi)(h) = \xi(g^{-1}h)$$

for $\xi \in L^2(G; \mathcal{H})$.

Each element ω of the predual $(G \otimes_{\alpha} \mathcal{A})_*$ of $G \otimes_{\alpha} \mathcal{A}$ may be regarded as an element u_{ω} of $C^b(G, \mathcal{A}_*)$;

$$u_{\omega}[g](x) = \langle \pi_{\alpha}(x)\lambda(g), \omega \rangle$$

for all $x \in \mathcal{A}$, $g \in G$ where $C^b(G; \mathcal{A}_*)$ is the space of all bounded continuous functions. And the new norm $\| \cdot \|$ on $F_{\alpha}(G; \mathcal{A}_*)$ is defined ;

$$\| u_{\omega} \| = \| \omega \|$$

such that $\| u \|_{\infty} \leq \| u \|$ for all $u \in F_{\alpha}(G; \mathcal{A}_*)$ where

$$F_{\alpha}(G; \mathcal{A}_*) = \{u_{\omega} ; \omega \in (G \otimes_{\alpha} \mathcal{A})_*\} \subset C^b(G; \mathcal{A}_*) .$$

We define the product on $F_{\alpha}(G; \mathcal{A}_*)$ by ;

$$(u * v)[g](x) = u(g)(x)v(g)(1)$$

for all $u, v \in F_{\alpha}(G; \mathcal{A}_*)$, $x \in \mathcal{A}$, $g \in G$. Then $F_{\alpha}(G; \mathcal{A}_*)$ became a Banach

algebra ([2] Theorem 3.5). So that we can define the product with an operator T in $G \otimes \mathcal{A}$ and an element u in $F_\alpha(G; \mathcal{A}_*)$;

$$\langle uT, v \rangle = \langle T, v * u \rangle$$

$$\langle Tu, v \rangle = \langle T, u * v \rangle$$

for all $v \in F_\alpha(G; \mathcal{A}_*)$ ((3.7), (3.9) in [2]).

Let T be an operator in $G \otimes \mathcal{A}$. Then the support $\text{supp}(T)$ of T is the set of all $g \in G$ satisfying the condition that $\lambda_g(g)$ belongs to the σ -weak closure of $\text{TF}_\alpha(G; \mathcal{A}_*)$ [See [2] Proposition 4.1].

Theorem. Let $(\mathcal{A}, G, \alpha)$, $(\mathcal{B}, H, \beta)$ be W^* -systems and $F_\alpha(G; \mathcal{A}_*)$, $F_\beta(H; \mathcal{B}_*)$ their Fourier algebras. Let ϕ be an isometric isomorphism of $F_\alpha(G; \mathcal{A}_*)$ onto $F_\beta(H; \mathcal{B}_*)$ as Banach algebras.

Then we get five elements (k, p, q, I, θ) with the following properties;

(1) k is an element of G such that $\lambda_G(k) = {}^t_\phi(\lambda_H(e))$, where ${}^t_\phi$ is the transposed map of ϕ , e is the identity of H ,

(2) I is either an isomorphism or anti-isomorphism of H onto G as locally compact groups,

(3) p (resp. q) is a projection of $\mathcal{B} \cap \mathcal{A}^G$ (resp. $\mathcal{B} \cap \mathcal{B}^H$),

(4) θ is a isometric linear map of $\mathcal{B}$ onto $\mathcal{A}$ such that,

θ is an isomorphism of $\mathcal{B}_q$ onto $\mathcal{A}_p$,

θ is an anti-isomorphism of $\mathcal{B}_{1-q}$ onto $\mathcal{A}_{1-p}$,

(5) $\phi(u)[h](y) = ({}_k u)[I(h)](\theta(y)p) + ({}_k u)[I(h)](\alpha_{I(h)}(\theta(y))(1-p))$

for all $y \in \mathcal{B}$, $h \in H$ and $u \in F_\alpha(G; \mathcal{A}_*)$, where $({}_k u)[g](y) = u[kg](\alpha_k(y))$,

(6) $\theta[\beta_k(y)] = [\alpha_{I(h)} \circ \theta(y)]p + [\alpha_{I(h)}^{-1} \circ \theta(y)](1-p)$

for all $y \in \mathcal{B}$, $h \in H$.

Corollary. Let $(\mathcal{H}, G, \alpha)$, $(\mathcal{B}, H, \beta)$ be W^* -systems, the two actions α and β are ergodic on their centers (ie. $\mathcal{Z}_{\mathcal{H}} \cap \mathcal{H}^G = \mathcal{Z}_{\mathcal{B}} \cap \mathcal{B}^H = \mathbb{C}$).

The following statements are equivalent ;

(i) $F_{\alpha}(G; \mathcal{H}_*) \cong F_{\beta}(H; \mathcal{B}_*)$ in the sense of Banach algebras,

(ii) there exist either an isomorphism I of H onto G , an isomorphism θ of $\mathcal{B}$ onto $\mathcal{H}$ such that $\theta \circ \beta_h = \alpha_{I(h)} \circ \theta$ for all $h \in H$, or an anti-isomorphism I of H onto G , an anti-isomorphism θ of $\mathcal{B}$ onto $\mathcal{H}$ such that $\theta \circ \beta_h = \alpha_{I(h^{-1})} \circ \theta$ for all $h \in H$.

[The proof of Theorem]. The transposed map ${}^t\phi$ of ϕ is an isometric linear map of $H \otimes_{\beta} \mathcal{B}$ onto $G \otimes_{\alpha} \mathcal{H}$. Using [3] Theorem 7, 10, we get ;

$${}^t\phi = {}^t\phi(\lambda_H(e))(\gamma_I + \gamma_A)$$

where γ_I is an isomorphism of $(H \otimes_{\beta} \mathcal{B})_z$ onto $(G \otimes_{\alpha} \mathcal{H})_z$, γ_A is an anti-isomorphism of $(H \otimes_{\beta} \mathcal{B})_{(1-z')}$ onto $(G \otimes_{\alpha} \mathcal{H})_{(1-z)}$, z (resp. z') is a central projection of $G \otimes_{\alpha} \mathcal{H}$ (resp. $H \otimes_{\beta} \mathcal{B}$).

For all $u, v \in F_{\alpha}(G; \mathcal{H}_*)$, $h \in H$, we obtain ;

$$\begin{aligned} \langle {}^t\phi(\lambda_H(h)), u * v \rangle &= \langle \lambda_H(h), \phi(u * v) \rangle \\ &= \langle \lambda_H(h), \phi(u) * \phi(v) \rangle \\ &= \langle \lambda_H(h) \otimes \lambda_H(h), \phi(u) \otimes \phi(v) \rangle \\ &= \langle {}^t\phi(\lambda_H(h)), u \rangle \langle {}^t\phi(\lambda_H(h)), v \rangle. \end{aligned}$$

Therefore ${}^t\phi(\lambda_H(h))$ is a character of $F_{\alpha}(G; \mathcal{H}_*)$ for all $h \in H$, which implies ${}^t\phi(\lambda_H(H)) = \lambda_G(G)$ since the character space $F_{\alpha}(G; *)$ is isomorphic to G ([2] theorem 3.14).

We denote ${}^t\phi(\lambda_H(e))$ by $\lambda_G(k)$.

By the quite same argument in [10] Theorem 2 we get that

$$\gamma \equiv {}^t_{\phi(\lambda_H(e))} {}^{-1} {}^t_{\phi} = \gamma_I + \gamma_A$$

is C^* -isomorphism in Kadison's sense [3] and $\gamma(\lambda_H(h_1)\lambda_H(h_2))$ is either $\gamma(\lambda_H(h_1))\gamma(\lambda_H(h_2))$ or $\gamma(\lambda_H(h_2))\gamma(\lambda_H(h_1))$, moreover we put $\gamma(\lambda_H(h)) = \lambda_G(I(h))$, so that I is either an isomorphism or an anti-isomorphism of H onto G as locally compact groups.

The transposed map ψ of γ is also an isometric isomorphism of $F_{\alpha}(G; \mathcal{K}_*)$ onto $F_{\beta}(H; \mathcal{B}_*)$. Then we get ;

$$\begin{aligned} \langle \gamma(\pi_{\beta}(y)), u * v \rangle &= \langle \pi_{\beta}(y), \psi(u * v) \rangle \\ &= \langle \pi_{\beta}(y), \psi(u) * \psi(v) \rangle \\ &= \langle \pi_{\beta}(y) \otimes 1, \psi(u) \otimes \psi(v) \rangle \\ &= \langle \gamma(\pi_{\beta}(y)) \otimes 1, u \otimes v \rangle \end{aligned}$$

for all $y \in \mathcal{B}$, $u, v \in F_{\alpha}(G; \mathcal{K}_*)$.

By [5] proposition 2.3, we obtain $\gamma(\pi_{\beta}(y))$ is an element of $\pi_{\alpha}(\mathcal{K})$, so that we can define a isometric surjective linear map θ of $\mathcal{B}$ onto $\mathcal{K}$ by $\theta = \pi_{\alpha}^{-1} \circ \gamma \circ \pi_{\beta}$.

Since γ is a Jordan isomorphism,

$$\gamma(T)\gamma(z') + \gamma(z')\gamma(T) = \gamma([T, z']) = 2\gamma(T z')$$

for all $T \in H \otimes_{\beta} \mathcal{B}$, therefore we get $\gamma(T z') = \gamma(T)z$.

Hence
$$\gamma(\pi_{\beta}(x y))z = \gamma(\pi_{\beta}(x))\gamma(\pi_{\beta}(y))z$$

for all $x, y \in \mathcal{B}$.

Since z is a central projection of $G \otimes_{\alpha} \mathcal{K}$, z is also an projection

of $\pi_\alpha(\mathcal{H})'$, so that $\gamma(\pi_\beta(xy))p = \gamma(\pi_\beta(x))\gamma(\pi_\beta(y))p$ for all $x, y \in \mathcal{B}$ where p is the central support of z in the von Neumann algebra $\pi_\alpha(\mathcal{H})'$.

We denote by q the central support of z' in the von Neumann algebra $\pi_\beta(\mathcal{B})'$, then $\gamma(q)z = \gamma(qz') = \gamma(z') = z$, implies that $\gamma(q)p = p$, similarly we also obtain $\gamma^{-1}(p)q = q$ so that $\gamma(q) = \gamma(\gamma^{-1}(p)q) = \gamma(\gamma^{-1}(p))\gamma(q)p = p\gamma(q)p = p$.

Hence θ is an isomorphism of $\mathcal{B}_q$ onto $\mathcal{H}_p$, moreover by the quite same argument, θ is an anti-isomorphism of $\mathcal{B}_{1-q}$ onto $\mathcal{H}_{1-p}$.

Since $\pi_\alpha(\mathcal{H})' = \lambda_G(g)\pi_\alpha(\mathcal{H})'\lambda_G(g)^*$, $\lambda(g)z\lambda(g)^* = z$ for all $g \in G$, we can prove easily that p is a G -invariant projection of $\mathcal{H}$, similarly q is a H -invariant projection of $\mathcal{B}$.

Now we have already proved (1) $\sim$ (4) and the statements (5) and (6) still remain.

For all $y \in \mathcal{B}$, $h \in H$, we get

$$\begin{aligned} \{\pi_\alpha \circ \theta(\beta_h(y))\}z &= \gamma(\lambda_H(h)\pi_\beta(y)\lambda_H(h)^*z') \\ &= \lambda_G(I(h))\pi_\alpha \theta(y)\lambda_G(I(h)^{-1})z \\ &= \pi_\alpha \alpha_{I(h)} \theta(y)z. \end{aligned}$$

Hence we get $\theta \circ \beta_h = \alpha_{I(h)} \circ \theta$ on $\mathcal{B}_q$, and similarly

$$\theta \circ \beta_h = \alpha_{I(h)^{-1}} \circ \theta \text{ on } \mathcal{B}_{1-q} \text{ for all } h \in H.$$

Therefore we get ;

$$\theta \circ \beta_h(y) = \alpha_{I(h)} \circ \theta(y)p + \alpha_{I(h)^{-1}} \circ \theta(y)(1-p)$$

for all $y \in \mathcal{B}$ and $h \in H$.

To prove the statement (5), we shall show first ;

$$\text{supp } \gamma(\pi_\beta(y) \lambda_H(h)) = \{I(h)\} .$$

For $u \in F_\alpha(G; \mathcal{H}_*)$, since $(\gamma(\pi_\beta(y) \lambda_H(h)))u = \gamma(\pi_\beta(y) \lambda_H(h)\psi(u))$ and ψ is surjective, we get ;

$$\begin{aligned} & [\gamma(\pi_\beta(y) \lambda_H(h)) F_\alpha(G; \mathcal{H}_*)]^{\sigma-W} \\ &= \gamma[\pi_\beta(y) \lambda_H(h) F_\beta(H; \mathcal{B}_*)]^{\sigma-W} , \end{aligned}$$

therefore $[\pi_\beta(y) \lambda_H(h) F_\beta(H; \mathcal{B}_*)]^{\sigma-W} \cap \lambda_H(H) = \mathcal{C} \lambda_H(h)$ because of $\text{supp } \pi_\beta(y) \lambda_H(h) = \{h\}$, so that we obtain ;

$$[\gamma(\pi_\beta(y) \lambda_H(h)) F_\alpha(G; \mathcal{H}_*)]^{\sigma-W} \cap \lambda_G(G) = \mathcal{C} \lambda_G(I(h)) ,$$

that is $\text{supp } \gamma(\pi_\beta(y) \lambda_H(h)) = \{I(h)\}$.

By [2] Theorem 4.4 or [6] Proposition 6.1, there exists an element x of $\mathcal{H}$ such that $\gamma(\pi_\beta(y) \lambda_H(h)) = \pi_\alpha(x) \lambda_G(I(h))$.

$$\begin{aligned} \text{On the other hand, } & \pi_\alpha(x) \lambda_G(I(h))z \\ &= \gamma(\pi_\beta(y) \lambda_H(h))z \\ &= \gamma(\pi_\beta(y)) \gamma(\lambda_H(h))z \\ &= \pi_\alpha(\theta(y)) \lambda_G(I(h))z \end{aligned}$$

therefore, we get $x p = \theta(y)p$, similarly we obtain $x(1-p) = \alpha_{I(h)} \circ \theta(y)(1-p)$, hence $x = \theta(y)p + \alpha_{I(h)} \circ \theta(y)(1-p)$,

$$\gamma(\pi_\beta(y) \lambda_H(h)) = \pi_\alpha(\theta(y)p) \lambda_G(I(h)) + \pi_\alpha(\alpha_{I(h)} \circ \theta(y)(1-p)) \lambda_G(I(h)) .$$

By the definition of Fourier algebras, the above equation and $\phi(u) = \psi_k(u)$ for all $u \in F_\alpha(G; \mathcal{H}_*)$, we can get the statement (5) easily.

[Proof of Cor.] Suppose ϕ is an isometric isomorphism of $F_\alpha(G; \mathcal{H}_*)$ onto $F_\beta(H; \mathcal{B}_*)$ and we use the same notations in the proof of the Theorem. The projection p in the Theorem must be zero or 1 by the conditions in the corollary, therefore θ must be either an isomorphism or an anti-isomorphism of $\mathcal{B}$ onto $\mathcal{H}$.

When G is a locally compact abelian group, (which implies that H is also a locally compact abelian group), I can be regarded as both an isomorphism and an anti-isomorphism as we like, therefore the Theorem says that θ is either an isomorphism of $\mathcal{B}$ onto $\mathcal{H}$ such that I is an isomorphism of h onto G and $\alpha_{I(h)} \circ \theta = \theta \circ \beta_h$ for all $h \in H$, or an anti-isomorphism of $\mathcal{B}$ onto $\mathcal{H}$ such that I is anti-isomorphic and $\alpha_{I(h)}^{-1} \circ \theta = \theta \circ \beta_h$ for all $h \in H$. Hence we may assume that G is non-abelian. When I is an anti-isomorphism of H onto G , the projection $(1-z)$ appearing in the proof of the Theorem must be non-zero. For, if the projection z is an identity operator in $G \otimes_\alpha \mathcal{H}$ then γ is an isomorphism of $H \otimes_\beta \mathcal{B}$ onto $G \otimes_\alpha \mathcal{H}$, so that the argument in the construction of the anti-isomorphism I tell us that I is isomorphic [See [10] Theorem 2]. Then I is both anti-isomorphic and isomorphic, which implies that G is an abelian group, which is a contradiction. Then we have gotten the projection $(1-z)$ is non-zero. Instead of considering the central support p of z in the proof of the Theorem, we may take the central support of $(1-z)$ in the von Neumann algebra $\pi_\alpha(\mathcal{H})'$, hence θ must be anti-isomorphic such that $\alpha_{I(h)^{-1}} \circ \theta = \theta \circ \beta_h$ for all $h \in H$. If I is an isomorphism of H onto G , we similarly get the conclusion that θ is isomorphic such that $\alpha_{I(h)} \circ \theta = \theta \circ \beta_h$ for all $h \in H$.

Conversely, suppose I is an isomorphism of H onto G such that $\theta \circ \beta_h = \alpha_{I(h)} \circ \theta$ for all $h \in H$. [9] proposition 3.4 says that there exists

an isomorphism γ of $H \otimes_{\beta} B$ onto $G \otimes_{\alpha} \mathcal{A}$ such that $\gamma(\pi_{\beta}(y)) = \pi_{\alpha}(\theta(y))$ for all $y \in B$, $\gamma(\lambda_H(h)) = \lambda_G(I(h))$ for all $h \in H$.

Then the transposed map ϕ of γ is an isometric isomorphism of $F_{\alpha}(G; \mathcal{A}_*)$ onto $F_{\beta}(H; B_*)$.

Suppose I is an anti-isomorphism of H onto G such that $\theta \circ \beta_h = \alpha_{I(h^{-1})} \circ \theta$ for all $h \in H$. By considering the opposite von Neumann algebra $\mathcal{A}^0$ of $\mathcal{A}$, the isomorphism J of H onto G by $J(h) = I(h^{-1})$ for all $h \in H$, similarly above, there exists an isomorphism γ of $H \otimes_{\beta} B$ onto $G \otimes_{\alpha} \mathcal{A}^0$ such that $\gamma(\pi_{\beta}(y)) = \pi_{\alpha}(\theta(y))$ for all $y \in B$, $\gamma(\lambda_H(h)) = \lambda_G(J(h))$ for all $h \in H$. On the other hand $G \otimes_{\alpha} \mathcal{A}^0$ is isomorphic to $G \otimes_{\alpha} \mathcal{A}$ as Banach spaces, therefore there exists an isometric linear map γ of $H \otimes_{\beta} B$ onto $G \otimes_{\alpha} \mathcal{A}$ with the above properties. Then it is quite clear that the transposed map ϕ of γ is an isometric isomorphism of $F(G; \mathcal{A}_*)$ onto $F_{\beta}(H; B_*)$.

Remark 1. This theorem is a kind of the generalization of [10]

Theorem 2.

Remark 2. Let $(\mathcal{A}, G, \alpha), (\mathcal{A}, G, \beta)$ be W^* -systems. Then the algebraic tensor product $A(G) \odot \mathcal{A}_*$ with the Fourier algebra $A(G)$ of G and the predual $\mathcal{A}_*$ of $\mathcal{A}$ is naturally imbedded in both the Fourier algebras $F_{\alpha}(G; \mathcal{A}_*)$, $F_{\beta}(G; \mathcal{A}_*)$, moreover $A(G) \odot \mathcal{A}_*$ is dense in these, therefore if the identity map i of $A(G) \odot \mathcal{A}_* \subset F_{\alpha}(G; \mathcal{A}_*)$ onto $A(G) \odot \mathcal{A}_* \subset F_{\beta}(G; \mathcal{A}_*)$ can be extended isometrically from $F_{\alpha}(G; \mathcal{A}_*)$ onto $F_{\beta}(G; \mathcal{A}_*)$, the two actions α, β are quite same in a sense. The algebraic structure of $F_{\alpha}(G; \mathcal{A}_*)$ determines the group structure of G and the norm in $F_{\alpha}(G; \mathcal{A}_*) \subset C_0(G; \mathcal{A}_*)$ which is quite different from the sup-norm in $C^b(G; \mathcal{A}_*)$ determines the action α of G .

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