

NONEXISTENCE OF STABLE INTEGRAL CURRENTS

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1. INTRODUCTION

In this paper, we discuss the nonexistence of stable integral p -currents in compact submanifolds of m -dimensional submanifolds in Euclidean spaces $\mathbf{R}^{m+1}$ and $\mathbf{R}^{m+2}$. The existence theorems due to Federer and Fleming in [3] state that for any compact Riemannian manifold M , any non-trivial integral homology class in $\mathcal{H}_p(M, \mathbf{Z})$ corresponds to a stable integral current, where $\mathcal{H}_p(M, \mathbf{Z})$ is the p th singular homology group with integer coefficients. The Federer-Fleming theorem and the calculus of variations were employed by Lawson and Simons [4] in the study of the topology and geometry of submanifolds of standard spheres. They proved the following:

Theorem LS1. *There are no stable integral p -currents in spheres of dimension m , where $0 < p < m$.*

In [4], Lawson and Simons conjectured the following:

Conjecture of Lawson and Simons. *There are no stable integral p -currents in any compact, simply-connected Riemannian manifold of dimension m which is $1/4$ -pinched, where $0 < p < m$.*

This conjecture is also open now. It is very well-known that any compact, simply-connected Riemannian manifold of dimension m which is $1/4$ -pinched is homeomorphic to sphere. On the other hand, any compact connected convex hypersurface in $\mathbf{R}^{m+1}$ is diffeomorphic to sphere. So we can consider the following problem:

Problem. *Do there exist stable integral p -currents in any compact connected convex hypersurface in $\mathbf{R}^{m+1}$.*

For this problem, the first author proved the following in [1]:

Theorem C. *Let M be an m -dimensional compact hypersurface with positive Ricci curvature in $\mathbf{R}^{m+1}$. Assume that M does not admit any point $x \in M$ at which M has only two distinct principal curvatures λ_1 and λ_0 such that*

$$\max\{\lambda_1, \lambda_0\} \geq (3 + 2\sqrt{2})\min\{\lambda_1, \lambda_0\}.$$

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Then for any $p \in (0, m)$, there are no stable integral p -currents in M and

$$\mathcal{H}_p(M, Z) = 0.$$

Corollary 1. Let M be an m -dimensional compact hypersurface in $\mathbf{R}^{m+1}$ with sectional curvature $K(M) > A\lambda^2$ ($A \geq \frac{1}{3+2\sqrt{2}}$ is constant). Then for any $p \in (0, m)$, there are no stable integral p -currents in M and

$$\mathcal{H}_p(M, Z) = 0.$$

Lawson and Simons [4] also studied the nonexistence of stable integral p -currents in compact submanifolds of standard spheres $S^m(c)$. They showed that

Theorem LS2. Let N be an n -dimensional compact submanifold of $S^m(c)$. Then, for given integer $p \in (0, n)$ there is no stable integral p -current in N and $\mathcal{H}_p(N, Z) = \mathcal{H}_{n-p}(N, Z) = 0$ if

$$\sum_{i, \alpha} [2\|h'(e_i, e_\alpha)\|^2 - \langle h'(e_i, e_i), h'(e_\alpha, e_\alpha) \rangle] < p(n-p)c$$

is satisfied for every $x \in N$ and any orthonormal basis $\{e_i, e_\alpha\}$ of $T_x N$, $i = 1, \dots, p$; $\alpha = p+1, \dots, n$. Here h' is the second fundamental form of N in $S^m(c)$.

We emphasize that their techniques can be applied in more general settings. This observation makes it possible to obtain further results on the non-existence of stable integral currents.

Throughout this paper let N be an n -dimensional compact Riemannian submanifold in an m -dimensional Riemannian manifold M which is a submanifold of $\mathbf{R}^{m+1}$ or $\mathbf{R}^{m+2}$. Let h' be the second fundamental form of N in M and $\{e_i, e_\alpha\}$ an orthonormal basis of the tangent space $T_x N$ for a point $x \in N$ where $1 \leq i \leq p$ and $p+1 \leq \alpha \leq n$. The purpose of this paper is to prove the following statement (A) for various ambient submanifolds of Euclidean spaces:

(A) For given integer $p \in (0, n)$ there is no stable integral p -current in N and $\mathcal{H}_p(N, Z) = \mathcal{H}_{n-p}(N, Z) = 0$.

The same notation and formulas as in [1] and [2] will be used throughout this paper. We first prove the generalized version of the Lawson-Simons theorem [4] as follows.

Theorem 1 (Cheng and Shiohama [2]). Let N be an n -dimensional compact submanifold of M^m which is an m -dimensional hypersurface of an $m+1$ -dimensional Euclidean space $\mathbf{R}^{m+1}$. Then the statement (A) is true if

$$(0) \quad \sum_{i, \alpha} [2\|h'(e_i, e_\alpha)\|^2 - \langle h'(e_i, e_i), h'(e_\alpha, e_\alpha) \rangle] < 2p(p-n)\lambda(x)^2$$

is satisfied for every $x \in N$ and any orthonormal basis $\{e_i, e_\alpha\}$ of $T_x N$, $i = 1, \dots, p$; $\alpha = p+1, \dots, n$. Here h' is the second fundamental form of N in M^m , and $\lambda(x)^2$ is the maximum at x of the square of principal curvatures of M^m .

Remark 1. If M^m is $\mathbf{R}^m$, then $\lambda = 0$. The condition (0) then becomes

$$\sum_{i, \alpha} [2\|h'(e_i, e_\alpha)\|^2 - \langle h'(e_i, e_i), h'(e_\alpha, e_\alpha) \rangle] < 0.$$

Hence Xin's result in [5] is a special case of our Theorem 1.

In addition to the assumptions of Theorem 1, if M has nonnegative Ricci curvature, then a simple computation shows that the sectional curvature of M is non-negative. In this case we prove

Theorem 2 (Cheng and Shiohama [2]). *Let N be an n -dimensional compact submanifold of M^m and M^n an m -dimensional hypersurface with nonnegative Ricci curvature of an $m+1$ -dimensional Euclidean space $\mathbf{R}^{m+1}$. Then the statement (A) holds if*

$$\sum_{i,\alpha} [2\|h'(e_i, e_\alpha)\|^2 - \langle h'(e_i, e_i), h'(e_\alpha, e_\alpha) \rangle] < \frac{1}{4}p(p-n)\lambda(x)^2$$

is satisfied for any $x \in N$ and any orthonormal basis $\{e_i, e_\alpha\}$ of $T_x N$, $i = 1, \dots, p$; $\alpha = p+1, \dots, n$. Here h' is the second fundamental form of N in M^m , and $\lambda(x)^2$ is the maximum at x of the square of principal curvatures of M^m .

As a corollary to Theorem 2 we obtain a slight extension of the Lawson-Simons theorem for product Riemannian manifolds:

Corollary 2, (Cheng and Shiohama [2]). *Let N be an n -dimensional compact submanifold of $\mathbf{R}^k \times S^{m-k}(c)$. Then the statement (A) is true if*

$$\sum_{i,\alpha} [2\|h'(e_i, e_\alpha)\|^2 - \langle h'(e_i, e_i), h'(e_\alpha, e_\alpha) \rangle] < 0$$

is satisfied for every $x \in N$ and any orthonormal basis $\{e_i, e_\alpha\}$ of $T_x N$, $i = 1, \dots, p$; $\alpha = p+1, \dots, n$. Here h' is the second fundamental form of N in $\mathbf{R}^k \times S^{m-k}(c)$.

Example (cf. Zhang [7]). We consider the ellipsoid

$$M = \{(x_1, \dots, x_{m+1}) \in \mathbf{R}^{m+1}; \sum_{i=1}^m x_i^2 + \frac{x_{m+1}^2}{c^2} = 1\}.$$

Let N be a compact submanifold of M . By direct computation, we can prove that if for every $x \in N$ and any orthonormal basis $\{e_i, e_\alpha\}$ of $T_x N$, $i = 1, \dots, p$; $\alpha = p+1, \dots, n$,

$$\sum_{i,\alpha} [2\|h'(e_i, e_\alpha)\|^2 - \langle h'(e_i, e_i), h'(e_\alpha, e_\alpha) \rangle] < n(n-p)c$$

then the statement (A) holds. In this case, h' is the second fundamental form of N in M .

We next discuss the case where M is a Riemannian product manifold: $M = M_1 \times M_2$, and each M_i ($i = 1, 2$) is an m_i -dimensional hypersurface in $\mathbf{R}^{m_i+1}$ of positive Ricci curvature.

Theorem 3 (Cheng and Shiohama [2]). Let N be an n -dimensional compact submanifold of $M = M_1^{m_1} \times M_2^{m_2}$, where $M_i^{m_i}$ ($i = 1, 2$) is an m_i -dimensional hypersurface with positive Ricci curvature of an $m_i + 1$ -dimensional Euclidean space $\mathbf{R}^{m_i+1}$. Then statement (A) is true if

$$\sum_{i,\alpha} [2\|h'(e_i, e_\alpha)\|^2 - \langle h'(e_i, e_i), h'(e_\alpha, e_\alpha) \rangle] < \frac{1}{4}p(p-n)(\lambda(x)^2 + \mu(x)^2)$$

is satisfied for every $x \in N$ and any orthonormal basis $\{e_i, e_\alpha\}$ of $T_x N$, $i = 1, \dots, p$; $\alpha = p+1, \dots, n$. Here h' is the second fundamental form of N in M and $\lambda(x)^2$ and $\mu(x)^2$ are the maximums at x of the square of principal curvatures of $M_1^{m_1}$ and $M_2^{m_2}$ respectively.

With the same notation as in Theorem 3, we prove the following:

Theorem 4 (Cheng and Shiohama [2]). Let $M_i^{m_i}$ ($i = 1, 2$) have distinct principal curvatures other than two. Then the statement (A) holds if

$$\sum_{i,\alpha} [2\|h'(e_i, e_\alpha)\|^2 - \langle h'(e_i, e_i), h'(e_\alpha, e_\alpha) \rangle] < 0$$

is satisfied for every $x \in N$ and any orthonormal basis $\{e_i, e_\alpha\}$ of $T_x N$, $i = 1, \dots, p$; $\alpha = p+1, \dots, n$. Here h' is the second fundamental form of N in M .

Corollary 3 (Cheng and Shiohama [2]). Let N be an n -dimensional compact submanifold of $M = S^{m_1}(c_1) \times S^{m_2}(c_2)$, where $S^{m_i}(c_i)$ ($i = 1, 2$) is an m_i -dimensional sphere of constant curvature c_i . Then the statement (A) holds if

$$\sum_{i,\alpha} [2\|h'(e_i, e_\alpha)\|^2 - \langle h'(e_i, e_i), h'(e_\alpha, e_\alpha) \rangle] < 0$$

is satisfied for every $x \in N$ and any orthonormal basis $\{e_i, e_\alpha\}$ of $T_x N$, $i = 1, \dots, p$; $\alpha = p+1, \dots, n$. Where h' is the second fundamental form of N in M .

Remark 2. When $c_1 = c_2 = 1$, Corollary 3 reduces to Zhang's Theorem 2 in [6].

PROOF OF THEOREMS

Let D be the Euclidean flat connection of $\mathbf{R}^{m+1}$. Since N is a submanifold of M^m , N is also a submanifold in $\mathbf{R}^{m+1}$. Let ∇' and ∇ denote the Levi-Civita connections of N with respect to M^m and $\mathbf{R}^{m+1}$, respectively. Also let $\chi(T^\perp(\mathbf{R}^{m+1}, N))$, $\chi(T^\perp(\mathbf{R}^{m+1}, M^m))$ and $\chi(T^\perp(M^m, N))$ be the respective spaces of normal vector fields. For any simple p -vector $\xi \in \wedge^p T_x N$ and for any vector field V tangent to N , let Ψ be the flow generated by V . Define

$$Q_\xi(V) = \frac{d^2 \|\Psi_{t*}\xi\|}{dt^2} \Big|_{t=0}.$$

Proof of Theorem 1. Since N is a submanifold of M^m , it is also a submanifold in $\mathbf{R}^{m+1}$. Let ∇' and ∇ denote the Levi-Civita connections of N with respect to M^m

and $\mathbf{R}^{m+1}$ respectively. The shape operator A_η determined by $\eta \in \chi(T^\perp(\mathbf{R}^{m+1}, N))$ is given by

$$(1) \quad -A_\eta Y = (D_Y \eta)^T,$$

where $Y \in \chi(TN)$. If $\eta \in \chi(T^\perp(M^m, N))$, then

$$(2) \quad \begin{aligned} A_\eta Y &= -(D_Y \eta)^T = -(\bar{\nabla}_Y \eta + \bar{h}(\eta, Y))^T \\ &= -(\bar{\nabla}_Y \eta)^T = A'_\eta Y, \end{aligned}$$

where $Y \in \chi(TN)$, $\bar{\nabla}$ and $\bar{h}$ are the Levi-Civita connection and the second fundamental form on M^m with respect to $\mathbf{R}^{m+1}$ and A'_η is the shape operator determined by $\eta \in \chi(T^\perp(M^m, N))$. If η is the normal to M^m , then

$$(3) \quad A_\eta Y = (\bar{A}_\eta Y)^T,$$

where $\bar{A}_\eta$ is the so-called shape operator determined by $\eta \in \chi(T^\perp(\mathbf{R}^{m+1}, M^m))$.

Let (S, ξ) be an oriented p -rectifiable set. For $x \in S$, we have a tangent p -space $T_x S \subset T_x N$. Choose an orthonormal basis $\{e_i, e_\alpha\}$ of $T_x N$ such that $\{e_i\}$ is an orthonormal basis of $T_x S$ and $\xi = e_1 \wedge \cdots \wedge e_p$. Suppose that $\{\eta_u\}$ is an orthonormal basis of $T_x^\perp(\mathbf{R}^{m+1}, N)$. Let $A_u = A_{\eta_u}$. Then $\{e_i, e_\alpha, \eta_u\}$ is an orthonormal basis of $\mathbf{R}^{m+1}$. Hence

$$(4) \quad \text{tr} Q_\xi = \sum Q_\xi(e_i) + \sum Q_\xi(e_\alpha) + \sum Q_\xi(\eta_u).$$

Making use of the proof given for the Theorem appearing in [1], we have

$$(5) \quad \text{tr} Q_\xi = \sum_{j, \alpha, u} [2\langle A_u(e_j), e_\alpha \rangle^2 - \langle A_u(e_\alpha), e_\alpha \rangle \langle A_u(e_j), e_j \rangle].$$

At a point $x \in N$, we take an orthonormal basis $\{\eta_1, \dots, \eta_{m-n}, \eta\}$ of $T_x^\perp(\mathbf{R}^{m+1}, N)$ so that $\{\eta_v\}$ and η are the orthonormal bases of $T_x^\perp(M^m, N)$ and $T_x^\perp(\mathbf{R}^{m+1}, M^m)$, respectively, and

$$(6) \quad \bar{A}_\eta(\tilde{e}_a) = -\lambda_a \tilde{e}_a \quad \text{for } a = 1, \dots, m,$$

where $\{\tilde{e}_a\}$ is an orthonormal basis of $T_x M^m$ and $\bar{A}_\eta$ is the so-called shape operator

determined by $\eta\epsilon\chi(T^\perp(\mathbf{R}^{m+1}, M^m))$. From (2), (3) and (5), we obtain

$$\begin{aligned}
 (7) \quad & \text{tr} Q_\xi \\
 &= \sum_{v=1}^{m-n} \sum_{j,\alpha} [2\langle A_v(e_j), e_\alpha \rangle^2 - \langle A_v(e_\alpha), e_\alpha \rangle \langle A_v(e_j), e_j \rangle] \\
 &+ \sum_{j,\alpha} [2\langle A_\eta(e_j), e_\alpha \rangle^2 - \langle A_\eta(e_\alpha), e_\alpha \rangle \langle A_\eta(e_j), e_j \rangle] \\
 &= \sum_{v=1}^{m-n} \sum_{j,\alpha} [2\langle A'_v(e_j), e_\alpha \rangle^2 - \langle A'_v(e_\alpha), e_\alpha \rangle \langle A'_v(e_j), e_j \rangle] \\
 &+ \sum_{j,\alpha} [2\langle \bar{A}_\eta(e_j), e_\alpha \rangle^2 - \langle \bar{A}_\eta(e_\alpha), e_\alpha \rangle \langle \bar{A}_\eta(e_j), e_j \rangle] \\
 &= \sum_{j,\alpha} [2\|h'(e_j, e_\alpha)\|^2 - \langle h'(e_\alpha, e_\alpha), h'(e_j, e_j) \rangle] \\
 &+ \sum_{j,\alpha} [2\langle \bar{A}_\eta(e_j), e_\alpha \rangle^2 - \langle \bar{A}_\eta(e_\alpha), e_\alpha \rangle \langle \bar{A}_\eta(e_j), e_j \rangle], \\
 (8) \quad & \sum_{j,\alpha} [2\langle \bar{A}_\eta(e_j), e_\alpha \rangle^2 - \langle \bar{A}_\eta(e_\alpha), e_\alpha \rangle \langle \bar{A}_\eta(e_j), e_j \rangle] \\
 &= \sum_{j,\alpha} \{2[\sum_a e_j^a e_\alpha^a \lambda_a]^2 - \sum_a (e_j^a)^2 \lambda_a \sum_a (e_\alpha^a)^2 \lambda_a\},
 \end{aligned}$$

where $e_j = \sum_a e_j^a \tilde{e}_a$, $e_\alpha = \sum_a e_\alpha^a \tilde{e}_a$. Since $\|e_j\| = \|e_\alpha\| = 1$ and $\langle e_j, e_\alpha \rangle = 0$, we have

$$\sum_a (e_j^a)^2 = 1, \quad \sum_a (e_\alpha^a)^2 = 1, \quad \sum_a e_j^a e_\alpha^a = 0.$$

Making use of assertion (1) in the following Lemma 1, we get

$$(9) \quad \sum_{j,\alpha} [2\langle \bar{A}_\eta(e_j), e_\alpha \rangle^2 - \langle \bar{A}_\eta(e_\alpha), e_\alpha \rangle \langle \bar{A}_\eta(e_j), e_j \rangle] \leq 2p(n-p)\lambda^2,$$

where λ^2 is the maximum of the square of principal curvatures of M^m at the corresponding point. According to (7), (9) and the assumption in Theorem 1, we conclude

$$\begin{aligned}
 & \text{tr} Q_\xi \\
 &= \sum_{j,\alpha} [2\|h'(e_j, e_\alpha)\|^2 - \langle h'(e_\alpha, e_\alpha), h'(e_j, e_j) \rangle] \\
 &+ \sum_{j,\alpha} [2\langle \bar{A}_\eta(e_j), e_\alpha \rangle^2 - \langle \bar{A}_\eta(e_\alpha), e_\alpha \rangle \langle \bar{A}_\eta(e_j), e_j \rangle] \\
 &= \sum_{j,\alpha} [2\|h'(e_j, e_\alpha)\|^2 - \langle h'(e_\alpha, e_\alpha), h'(e_j, e_j) \rangle] \\
 &+ 2p(n-p)\lambda^2 < 0.
 \end{aligned}$$

Hence

$$\operatorname{tr} Q_S = \sum_{n=1}^{\infty} \int_{S_n} n \operatorname{tr} Q_{\xi_n} d\mathcal{H}^p(x) < 0.$$

This implies that there are no stable integral p -currents in N . By Theorem FF in [1], we have

$$\mathcal{H}_p(N, Z) = \mathcal{H}_{n-p}(N, Z) = 0.$$

Lemma 1. Let $a_1, \dots, a_m, b_1, \dots, b_m$ be real numbers satisfying $\sum_j a_j^2 = \sum_j b_j^2 = 1$ and $\sum_j a_j b_j = 0$. Then we have the following:

(1) For given real numbers $\lambda_1, \lambda_2, \dots, \lambda_m$,

$$2\left(\sum_j \lambda_j a_j b_j\right)^2 - \sum_j \lambda_j a_j^2 \sum_j \lambda_j b_j^2 \leq 2\lambda^2.$$

(2) For given nonnegative real numbers $\lambda_1, \lambda_2, \dots, \lambda_m$,

$$2\left(\sum_j \lambda_j a_j b_j\right)^2 - \sum_j \lambda_j a_j^2 \sum_j \lambda_j b_j^2 \leq \frac{1}{4}\lambda^2.$$

Here λ^2 is the maximum of λ_j^2 .

Lemma 1 can be proved by the same method as the lemma in [1].

Proof of Theorem 2. Since M^m is a hypersurface in $\mathbf{R}^{m+1}$ with nonnegative Ricci curvature, we know that M^m is of nonnegative sectional curvature (see-[1]). Hence assertion (2) of Lemma 1 and the same arguments appearing in the proof of Theorem 1 imply that Theorem 2 holds.

Proof of Corollary 2. Since $\mathbf{R}^k \times S^{m-k}(c)$ is a hypersurface in $\mathbf{R}^{m+1}$ with nonnegative Ricci curvature and there are only two distinct constant principal curvatures 0 and $\sqrt{c}$, we have

$$\sum_{j,\alpha} [2\langle \bar{A}_\eta(e_j), e_\alpha \rangle^2 - \langle \bar{A}_\eta(e_\alpha), e_\alpha \rangle \langle \bar{A}_\eta(e_j), e_j \rangle] \leq 0.$$

Hence

$$\begin{aligned} & \operatorname{tr} Q_\xi \\ &= \sum_{j,\alpha} [2\|h'(e_j, e_\alpha)\|^2 - \langle h'(e_\alpha, e_\alpha), h'(e_j, e_j) \rangle] \\ &+ \sum_{j,\alpha} [2\langle \bar{A}_\eta(e_j), e_\alpha \rangle^2 - \langle \bar{A}_\eta(e_\alpha), e_\alpha \rangle \langle \bar{A}_\eta(e_j), e_j \rangle] \\ &= \sum_{j,\alpha} [2\|h'(e_j, e_\alpha)\|^2 - \langle h'(e_\alpha, e_\alpha), h'(e_j, e_j) \rangle] \\ &< 0. \end{aligned}$$

Thus

$$\operatorname{tr} Q_S = \sum_{n=1}^{\infty} \int_{S_n} n \operatorname{tr} Q_{\xi_n} d\mathcal{H}^p(x) < 0.$$

We obtain the result that there are no stable integral p -currents in N and

$$\mathcal{H}_p(N, Z) = \mathcal{H}_{n-p}(N, Z) = 0.$$

This proves Corollary 2.

Using proofs similar to that given for Theorem 1, the following Lemma 2 yields Theorems 3 and 4.

Lemma 2 (Cheng and Shiohama [2]). *Let a_i, b_i, d_j and e_j be real numbers ($i = 1, \dots, m_1; j = 1, \dots, m_2$) satisfying $\sum_i a_i^2 + \sum_j e_j^2 = \sum_i b_i^2 + \sum_j d_j^2 = 1$ and $\sum_i a_i b_i + \sum_j d_j e_j = 0$. Then the following holds:*

(1) *For given positive real numbers λ_i and μ_j ,*

$$(11) \quad 2[(\sum_i \lambda_i a_i b_i)^2 + (\sum_j \mu_j d_j e_j)^2] - \sum_i \lambda_i a_i^2 \sum_i \lambda_i b_i^2 - \sum_j \mu_j d_j^2 \sum_j \mu_j e_j^2 \leq \frac{1}{4}(\lambda^2 + \mu^2).$$

(2) *For given positive real numbers λ_i and μ_j , where neither the number of distinct λ_i nor the number of distinct μ_j is not two,*

$$(12) \quad 2[(\sum_i \lambda_i a_i b_i)^2 + (\sum_j \mu_j d_j e_j)^2] - \sum_i \lambda_i a_i^2 \sum_i \lambda_i b_i^2 - \sum_j \mu_j d_j^2 \sum_j \mu_j e_j^2 \leq 0,$$

where λ^2 and μ^2 are the maximum of λ_i^2 and μ_j^2 , respectively.

This Lemma 2 can be proved by making use of the method of Lagrange multipliers (see [2] for in details).

REFERENCES

1. Q. M. Cheng, *Nonexistence of stable currents*, Ann. Global Analysis and Geom., **13** (1995), 197-205.
2. Q. M. Cheng & K. Shiohama, *Nonexistence of stable currents II*, Kyushu J. Math., **51** (1997), 1-16.
3. H. Federer & M. Fleming, *Normal and integral currents*, Ann. of Math., **72** (1960), 458-520.
4. H. B. Lawson & J. Simons, *On stable currents and their application to global problems in real and complex geometry*, Ann. of Math., **98** (1973), 427-450.
5. Y. L. Xin, *An application of integral currents to the vanishing theorems*, Sci. Sinica, Ser. A, **27** (1984), 233-241.

6. X. S. Zhang, *Nonexistence of stable currents in submanifolds of a product of two spheres*, Bull. Austral. Math. Soc., **44** (1991), 325-336.
7. X. S. Zhang, *Geometry and topology of submanifolds immersed in space forms and ellipsoids*, Kodai Math. J., **17** (1994), 262-272.

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