

ON A CLASS OF SINGULAR DIFFERENTIAL OPERATORS

上智大理工 田原 秀敏
(Sophia Univ. Hidetoshi TAHARA)

In this note, the author will report some results for a class of non-Fuchsian singular hyperbolic operators including

$$L = (t\partial_t)^2 - \Delta_X + a(t, x)(t\partial_t) + \langle b(t, x), \partial_X \rangle + c(t, x).$$

1. CLASS OF OPERATORS.

Let $(t, x) = (t, x_1, \dots, x_n) \in \mathbb{R}_t \times \mathbb{R}_x^n$ and let

$$P = (t\partial_t)^m + \sum_{\substack{j+|\alpha| \leq m \\ j < m}} a_{j, \alpha}(t, x) (t\partial_t)^j \partial_X^\alpha,$$

where $m \in \{1, 2, \dots\}$, $\partial_t = \partial/\partial t$, $\partial_X = (\partial/\partial x_1, \dots, \partial/\partial x_n)$, $\alpha = (\alpha_1, \dots, \alpha_n) \in \{0, 1, 2, \dots\}^n$, $|\alpha| = \alpha_1 + \dots + \alpha_n$, $\partial_X^\alpha = (\partial/\partial x_1)^{\alpha_1} \dots (\partial/\partial x_n)^{\alpha_n}$, and the coefficients $a_{j, \alpha}(t, x)$ ($j+|\alpha| \leq m$ and $j < m$) are C^∞ functions defined in an open neighborhood U of $(0, 0)$ in $\mathbb{R}_t \times \mathbb{R}_x^n$. Our main assumption on P is as follows:

(A) All the roots of

$$\lambda^m + \sum_{\substack{j+|\alpha| = m \\ j < m}} a_{j, \alpha}(t, x) \lambda^j \xi^\alpha = 0$$

are real, simple and non-zero for any $(t, x, \xi) \in U \times (\mathbb{R}_\xi^n \setminus \{0\})$.

Remark 1. Note that P is not of Fuchsian type in t .

Remark 2. Recall that the typical model of Fuchsian hyperbolic operators in t is the following:

$$R = (t\partial_t)^m + \sum_{\substack{j+|\alpha|\leq m \\ j < m}} a_{j,\alpha}(t,x) (t\partial_t)^j (t^k \partial_x)^\alpha,$$

where $(t^k \partial_x)^\alpha = (t^k \partial/\partial x_1)^{\alpha_1} \dots (t^k \partial/\partial x_n)^{\alpha_n} (= t^{k|\alpha|} \partial_x^\alpha)$ and the following conditions are imposed on R .

(B-1) $k \in \mathbf{Z}$ and $k > 0$.

(B-2) All the roots of

$$\lambda^m + \sum_{\substack{j+|\alpha|=m \\ j < m}} a_{j,\alpha}(t,x) \lambda^j \xi^\alpha = 0$$

are real and simple for any $(t,x,\xi) \in U \times (\mathbb{R}_\xi^n \setminus \{0\})$.

(B-3) All the roots of

$$\rho^m + \sum_{j < m} a_{j,0}(0,0) \rho^j = 0$$

are non-integers.

2. SOME RESULTS.

Here, we want to consider the following problems (I)~(V) for P .

(I) Is $Pu=f$ solvable in C^∞ ?

(II) Is $Pu=f$ solvable in $\mathcal{D}'$?

(III) Can we construct a parametrix ?

(IV) Does the uniqueness of the C^∞ -solution hold ?

(V) Is every solution $u \in \mathcal{D}'(t > 0)$ of $Pu=0$ extendable to $\{t \leq 0\}$ as a distribution ?

In order to make clear our situation, we present the following table which compares the results for P with those for Fuchsian operators R.

operator	non-Fuchsian case		Fuchsian case	
	P	under (A)	R	under (B-1) ~ (B-3)
Problem (I)	Yes	[T,S]	Yes	[T]
Problem (II)	Yes	[T]	Yes	[B-L-P,T]
Problem (III)	Yes	[S]	Yes	[B-L-P-T]
Problem (IV)	No	[M]	Yes	[T,R,U]
Problem (I)	Conjecture		Yes	[P-T]
	No	*)		

In the above table, we quoted names by their initials: T=Tahara, S=Serra, B=Bove, L=Lewis, P=Parenti, M=Mandai, R=Roberts and U=Uryu.

As to *): the case $n=1$ is already proved; but the case $n \geq 2$ is still open (up to the date Nov. 14, 1988).

3. CONCLUSION.

By the results in Section 2, we may assert that our class of non-Fuchsian operators has an interesting nature and therefore it is worthy to investigate it.