

1 **Diagnostic Evaluation of Effects of Vertical Mixing on Meridional**
2 **Overturning Circulation in an Idealized Ocean**

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ABSTRACT

11 In this study, diagnostic equations are proposed to quantitatively evaluate
12 meridional overturning circulation (MOC) simulated in ocean general circu-
13 lation models (OGCMs). Applicability of the equations is illustrated by revis-
14 iting the MOC simulated in an idealized ocean. The simulations with surface
15 differential heating/cooling show that, for certain horizontal distribution of
16 vertical diffusivity, the stronger vertical mixing does not intensify the MOC
17 while it makes the deeper water less dense. This result, which is in marked
18 contrast to the widely accepted idea that the stronger vertical mixing promotes
19 upwelling and intensifies the MOC by making the deeper water less dense,
20 was investigated using the diagnostic equations. It was found that geostrophy
21 dominates the MOC, and the geostrophic flow normal to lateral boundaries
22 induced intense upwelling/downwelling along the boundaries. These results
23 indicate that the primary role played by the vertical mixing on the large-scale
24 MOC is to change hydrostatic pressure fields (geostrophic flow fields), rather
25 than to promote upwelling. The simulation with localized cooling on the other
26 hand showed that the ageostrophic flows significantly contribute to small-
27 scale features of the MOC, while the geostrophic flows determine large-scale
28 structure of the MOC. The proposed equations will thus be useful to quanti-
29 tatively diagnose the MOC dynamics in realistic OGCMs.

30 1. Introduction

31 Meridional overturning circulation (MOC), one of key ingredients for long-term variations in
32 the Earth climate, is known to be influenced by abyssal small-scale turbulent mixing (e.g., Wunsch
33 2002; Kuhlbrodt 2008). Numerical experiments of two-dimensional overturning circulation forced
34 by differential surface heating and cooling (e.g., Beardsley and Festa 1972; Rossby 1998) suggest
35 that, if the turbulent mixing was weak, a deeper ocean would be filled with colder water, and the
36 newly formed coldest surface water would have less (negative) buoyancy to sink vigorously into
37 the deeper ocean, hence making overturning circulation weaker in the deeper layer. Thus, stronger
38 mixing sustains the overturning circulation by transporting (positive) buoyancy to the deeper layer
39 and inducing buoyancy torque necessary to drive the MOC.

40 Intensity of the turbulent mixing can be estimated from the vertical temperature profile in the
41 real ocean. Based on a balance between advection and diffusion of potential temperature (T) in
42 the vertical direction

$$w \frac{\partial T}{\partial z} = \frac{\partial}{\partial z} \left(K_V \frac{\partial T}{\partial z} \right), \quad (1)$$

43 and a gross estimate of an upwelling velocity ($w \simeq 10^{-6} \text{ m s}^{-1}$), Munk (1966) pointed out that the
44 vertical eddy diffusivity K_V of $\mathcal{O}(10^{-4} \text{ m}^2 \text{ s}^{-1})$, which is much larger than the molecular diffu-
45 sivity of $\mathcal{O}(10^{-7} \text{ m}^2 \text{ s}^{-1})$, is necessary to explain the observed vertical profile of potential tem-
46 perature gradient ($\partial T / \partial z$). Thus, the turbulent mixing sustains the thermocline structure and the
47 MOC in the real ocean. This is also supported by numerical simulations of the three-dimensional
48 ocean general circulation model (OGCM) where the larger K_V corresponds to the more intense
49 MOC (e.g., Bryan 1987).

50 The large impact of the vertical mixing on the MOC shed light on small-scale turbulent mixing
51 in the abyssal ocean. Extensive efforts have been made to reveal magnitude and distribution of

K_V in the real abyssal ocean through direct and indirect measurements of the dissipation rates of
 turbulent kinetic energy (e.g., Waterhouse et al. 2014). The estimated K_V s in the abyssal ocean
 are found to range from $10^{-5} \text{ m}^2 \text{ s}^{-1}$ in the thermocline (e.g., Ledwell and Law 1993; Ledwell
 et al. 1998) to greater than $10^{-3} \text{ m}^2 \text{ s}^{-1}$ in the deep ocean above regions of rough topography (e.g.,
 Polzin et al. 1997; Ledwell et al. 2000). Large K_V s were also estimated at around 30° latitude (e.g.,
 Hibiya et al. 2007). These patterns can be interpreted in terms of the geography of internal wave
 generation, propagation, interactions and dissipation (e.g. MacKinnon et al. 2017). Noteworthy
 is that the estimated K_V s by these field measurements are overall smaller than $\mathcal{O}(10^{-4} \text{ m}^2 \text{ s}^{-1})$
 expected from the temperature gradient mentioned above (e.g., Munk and Wunsch 1998). Wind-
 induced mechanical upwellings in the Southern Ocean (e.g., Webb and Sugimotohara 2001) seem
 partly responsible for this gap. Another ingredient is inhomogeneity in K_V , as numerical studies
 (e.g., Tsujino et al. 2000; Jayne 2009) have shown that such spatial inhomogeneity in K_V has
 noticeable impact on the MOC.

Large impacts of the vertical mixing on the MOC described above are sometimes interpreted
 as “vertical mixing makes deeper water less dense, promoting upwelling” (e.g., Visbeck 2007)
 or vertical mixing “pulls” deeper water (e.g., Kuhlbrodt 2008). This interpretation is helpful to
 highlight the impact of one important ingredient for the MOC – the small-scale vertical mix-
 ing. However, another essential ingredient for large-scale ocean circulation, geostrophy, needs to
 be considered, as the geostrophic flows are dominant in the three-dimensional global overturn-
 ing circulations (e.g., Bryan 1987; Zhang et al. 1992; Marotzke 1997). Significant roles of the
 geostrophic flows as well as the vertical mixing in the MOC were demonstrated in numerical ex-
 periments of Marotzke (1997) and Scott and Marotzke (2002) where the vertical mixing along side
 boundaries was shown to have large impacts on east - west density differences and hence merid-
 ional geostrophic flows through the thermal wind balance. Convective mixing along boundaries,

76 which changes along-boundary buoyancy gradient and hence induces cross-boundary geostrophic
77 flows through the thermal wind relation, is also suggested to be related to the MOC (e.g., Spall
78 and Pickart 2001; Katsman et al. 2018). However, relative contributions from the geostrophic pro-
79 cess and ageostrophic process (that could be particularly large near the boundaries) in the MOC
80 remain unquantified. This leaves our understanding of the MOC dynamics and a role played by
81 the vertical mixing and the geostrophic flow in it vague.

82 The aim of this study is to quantitatively evaluate the geostrophic and ageostrophic processes
83 in the MOC using diagnostic equations derived in the present study. To this aim, we revisited the
84 three-dimensional thermohaline circulations in an idealized rectangular ocean that have been in-
85 vestigated in many previous studies (e.g., Bryan 1987; Zhang et al. 1992; Marotzke 1997; Park and
86 Bryan 2000; Scott and Marotzke 2002). In section 2, numerical model configuration is described.
87 The circulations driven by surface differential heating/cooling are presented in section 3 where
88 locally increased vertical diffusivity is found to slightly weaken MOC while making deeper water
89 less dense. This result, which is in marked contrast to the widely accepted idea of “pull by the
90 mixing”, was investigated quantitatively using a proposed diagnostic vorticity balance equation.
91 The vorticity equation highlights effects of the vertical mixing on large-scale hydrostatic pres-
92 sure field (geostrophic flows), rather than on the vertical velocity that could be directly impacted
93 through the advection - diffusion equation of temperature. The diagnostic equation for simulated
94 vertical velocity was also derived to decompose the velocity into three parts, the component due
95 to the planetary vorticity change, the component induced by ageostrophic advection and viscosity,
96 and the component caused by the geostrophic flow normal to the boundary, to show that the last
97 component dominantly shapes the large-scale structure of the MOC. Heat balances in the vertical
98 direction are also examined in this section. In section 4, the circulation driven by localized cooling

is shown to illustrate that the ageostrophic flows could be significant for the small-scale features of the MOC. Finally, concluding remarks and discussions are given in section 5.

2. Model Configuration

A simple rectangular ocean of dimension $L(= 6400 \text{ km})$ in both the zonal and the meridional directions and $D = (4000 \text{ m})$ in the vertical direction on a β -plane was considered. The ocean surface was assumed to be rigid. With the hydrostatic approximation, the governing equations for a Boussinesq ocean are given by

$$\frac{\partial u}{\partial t} + \mathcal{A}(u) - fv = -\frac{\partial}{\partial x} \frac{p}{\rho_0} + \mathcal{V}(u) \quad (2)$$

$$\frac{\partial v}{\partial t} + \mathcal{A}(v) + fu = -\frac{\partial}{\partial y} \frac{p}{\rho_0} + \mathcal{V}(v) \quad (3)$$

$$0 = -\frac{\partial}{\partial z} \frac{p}{\rho_0} + \alpha g T \quad (4)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (5)$$

$$\frac{\partial T}{\partial t} + \mathcal{A}(T) = \mathcal{D}(T) + F_T \quad (6)$$

$$\mathcal{A}(\bullet) = \frac{\partial u \bullet}{\partial x} + \frac{\partial v \bullet}{\partial y} + \frac{\partial w \bullet}{\partial z} \quad (7)$$

$$\mathcal{V}(\bullet) = A_H \left(\frac{\partial^2 \bullet}{\partial x^2} + \frac{\partial^2 \bullet}{\partial y^2} \right) + A_V \frac{\partial^2 \bullet}{\partial z^2} \quad (8)$$

$$\mathcal{D}(\bullet) = K_H \left(\frac{\partial^2 \bullet}{\partial x^2} + \frac{\partial^2 \bullet}{\partial y^2} \right) + K_V \frac{\partial^2 \bullet}{\partial z^2} \quad (9)$$

where x (eastward), y (northward) and z (upward) are Cartesian coordinates with the origin $(x, y, z) = (0, 0, 0)$ at the surface southwest corner, t is time, u , v and w are velocity in the x , y and z directions, respectively, p is pressure, T is potential temperature, $f (= \beta y)$ is the Coriolis parameter ($\beta = 2.0 \times 10^{-11} \text{ m}^{-1} \text{ s}^{-1}$), $\rho_0 (= 1000 \text{ kg m}^{-3})$ is a reference water density, $g (= 10 \text{ m s}^{-2})$ is the gravitational acceleration, and $\alpha (= 2.0 \times 10^{-4} \text{ K}^{-1})$ is a thermal expansion coefficient of water. Here the equation of state was linearized ($\rho/\rho_0 = 1 - \alpha T$) while neglecting

112 salinity effects on density (ρ) for simplicity. Horizontal and vertical eddy viscosities (A_H and A_V)
 113 were set as 4×10^4 and $1 \times 10^{-2} \text{ m}^2 \text{ s}^{-1}$, respectively. Horizontal eddy diffusivity (K_H) was set
 114 as $4 \times 10^2 \text{ m}^2 \text{ s}^{-1}$, and vertical eddy diffusivity (K_V) was set as $1 \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$ unless other-
 115 wise noted. Static stability was removed by convective adjustment, by increasing the vertical eddy
 116 diffusivity to $1 \times 10^{-2} \text{ m}^2 \text{ s}^{-1}$ wherever $\partial T / \partial z < 0$. The last term in Eq. (6) represents thermal
 117 forcing whose specific form is described in the following sections. The model ocean is forced
 118 only by this thermal forcing. (No wind forcing was imposed.) At the boundary, no normal flow,
 119 free-slip, and insulating conditions were applied.

120 The governing equations and boundary conditions were approximated with a second-order finite
 121 difference scheme with the Arakawa-C grid. Grid spacing was 100 km in the horizontal direction
 122 and 100 m in the vertical direction. Time integration was performed with the second-order Runge-
 123 Kutta scheme with time step of 5400 s. Other model configurations (e.g., specific form of F_T and
 124 initial condition) are described in the following sections.

125 3. Experiments With Surface Differential Heating/Cooling

126 In this section, the MOC driven by surface differential heating/cooling was investigated. For this
 127 purpose, the thermal forcing was set as

$$F_T = \gamma(SST - T)\delta_z, \quad (10)$$

$$SST = \Delta T (1 - y/L), \quad (11)$$

128 where $\delta_z = 1$ in the top surface grid box and $\delta_z = 0$ in the other subsurface grid boxes. The
 129 restoring time γ^{-1} was set as 10 days. Initially, the model ocean was at rest and the temperature
 130 was uniformly set at 0° C . The MOC, quantitative definition of which will be given later, reached
 131 an almost steady state by $t = 4000$ years. To illustrate how the vertical mixing alters the MOC, we

increased the vertical eddy diffusivity locally at and after $t = 4000$ years, and continued numerical integration until $t = 7000$ years; by then the MOCs influenced by the increased K_V reached another steady state. For comparison, we also continued the original (K_V unchanged) experiment until $t = 7000$ years. Results at $t = 7000$ years are presented below. Note that these simulations are meant to illustrate the potential impacts of the vertical mixing and the geostrophic flows on the MOC rather than to simulate realistic MOC influenced by complicated inhomogeneity in K_V .

a. Temperature and Velocity Fields

First, to overview the simulated circulation driven by the differential surface heating/cooling, temperature and velocity fields are briefly described. Note that they are essentially the same with those of the previous studies. Figure 1 shows horizontal fields of the temperature and flows in the experiment with constant K_V (referred to as a base experiment or B experiment). Due to the surface restoring forcing, zonal structure of the temperature is evident near the surface ($z = -150$ m) (Figure 1a). Slight deviations from the zonal structure reflect advection by the flow. The surface flow was eastward at $y \geq 3000$ km, which was in thermal wind balance with the meridional temperature (buoyancy) gradient. The eastward surface flow induced strong upwelling and downwelling at the western and eastern boundary, respectively (Figure 1c). The upwelling contributed to the surface water cooling near the western boundary. At $y \leq 1500$ km, the surface flow was westward, partly compensating the eastward flow to the north. These zonal currents and an intense northward western boundary current form a surface anti-cyclonic circulation in the surface layer. At $z = -1150$ m, (Figures 1b and d), warm temperature anomaly was apparent near the downwelling region along the eastern boundary. This temperature anomaly or lowered thermocline was propagated southwestward (Figure 1b). The circulation at greater depths was cyclonic, with an intense southward western boundary current.

Figure 2 shows vertical sections of the temperature. A meridional-vertical distribution of zonally averaged temperature (Figure 2a) shows thermocline formed above 500 m depth, with the temperature being less than 2 ° C below 800 m. A zonal vertical section of the temperature averaged over 4600 km $\leq y \leq$ 5000 km (Figure 2b) shows tilting (eastward deepening) of the thermocline.

The MOC was quantified by volume transport stream function Φ defined as

$$\int_0^L v dx = -\frac{\partial \Phi}{\partial z}. \quad (12)$$

The function, calculated by vertically integrating the above equation with $\Phi = 0$ at the bottom ($z = -D$), is shown in Figure 2c. The northward and southward transports are evident above and below the thermocline, respectively, which are supported by the eastward deepening thermocline (Figure 2b) through the thermal wind relation. The largest Φ (10.03 Sv) is found at around $y = 5000$ km, to the north (south) of which the thermocline is weakly (strongly) stratified (Figure 2a). All these features are consistent with the previous numerical experiments (e.g., Bryan 1987; Marotzke 1997; Spall and Pickart 2001; Zhang et al. 1992; Scott and Marotzke 2002).

To illustrate the impacts of the vertical mixing and geostrophy on Φ , we performed two experiments in which K_V was increased to be $10^{-2} \text{ m}^2 \text{ s}^{-1}$ (100 times larger than the original K_V) at 4600 $\leq y \leq$ 5000 km along the western ($x \leq 200$ km) and the eastern ($x \geq 6200$ km) boundaries, respectively. Note that area of the K_V intensified region (hereafter referred to as the intensified mixing region) and hence the area-averaged K_V were the same in these two experiments. Results of the experiment with increased K_V at the western boundary (hereafter referred to as W experiment) are shown in Figures 3 and 4. Horizontal temperature and flow fields in the W experiment (Figures 3a and b) were overall similar to those of the B experiment (Figure 1). Slight deviations from the B experiment, however, reveal effects of the increased K_V . The increased K_V decreased (increased) temperature at shallower (greater) depths in the intensified mixing region. In the W

177 experiment, the temperature decrease occurred at $z \geq -100$ m, and positive temperature anomaly
 178 from the B experiment was found at $z \leq -100$ m in the intensified mixing region (Figure 4b). The
 179 positive temperature anomaly corresponds to lowered thermocline, and this lowered thermocline
 180 propagated southward as the Kelvin wave along the western boundary (Figures 3a and b). This
 181 deepening of the thermocline along the western boundary resulted in reducing zonal tilt of the
 182 thermocline and consequently the northward (southward) western boundary current in the upper
 183 (lower) thermocline (Figures 3c and d).

184 The temperature anomaly was propagated cyclonically along the boundaries, and the thermo-
 185 cline in southern region was overall deepened (Figures 3a and b). This resulted in increasing
 186 eastward (westward) current in the upper (lower) thermocline (Figures 3c and d) through the
 187 thermal wind balance. As a consequence, downwelling velocity at around $y = 5500$ km at the
 188 eastern boundary was intensified, and the thermocline there was deepened (Figure 3b). The MOC
 189 (Figure 4c) was influenced by these processes. Due to the intensified downwelling at around
 190 $y = 5500$ km, Φ to the north of $y = 5000$ km was slightly increased. However, Φ to the south of
 191 $y = 5000$ km was greatly decreased, with the reduced northward western boundary current being
 192 responsible for this decrease. The decrease to the south of $y = 4600$ km was much larger than the
 193 increase to the north of $y = 5000$ km, and hence the largest Φ was 10.00 Sv, slightly smaller than
 194 that in the B experiment. Thus, the increased K_V did not intensify the MOC, despite the deeper
 195 water warming (Figure 4a). This is in marked contrast with the idea that vertical mixing promotes
 196 upwelling by making deeper water less dense.

197 Results of the experiment with increased K_V at the eastern boundary (hereafter referred to as E
 198 experiment) are shown in Figures 5 and 6. Horizontal temperature and flow fields (Figure 5) were
 199 similar to those of the B and W experiments, but deviations from the B experiment were different
 200 from W experiment. The temperature in the intensified mixing region decreased above (increases

below) $z = -800$ m (Figure 6b), and these deviations propagated northward along the eastern boundary as the Kelvin waves and southwestward as the Rossby waves. Eastward deepening of the thermocline became steeper, intensifying the northward (southward) geostrophic flow in the upper (lower) thermocline. This resulted in increased MOC (Figure 6c).

All these results are consistent with the results of Scott and Marotzke (2002); intensified vertical mixing along the eastern boundary causes larger MOC than the intensified mixing along the western boundary. Dynamical processes described above were also given by Scott and Marotzke (2002) in more details. The point to emphasize in this study is the fact that locally intensified vertical mixing (increased K_V), which makes deeper water less dense, does not always intensify global-scale MOC (promote upwelling). To quantitatively examine the mixing and the MOC on dynamical framework, a dynamical balance equation for the MOC is derived and diagnosed in the next subsection.

b. Vorticity Balance

Lee and Marotzke (1998) decomposed the MOC (Φ) into three components: a geostrophic shear component, an Ekman component, and a barotropic current component in a heuristic way, and discussed structure and variation of simulated MOC in the Indian ocean. Later studies (e.g., Hirschi and Marotzke 2007) used this decomposition to discuss the global MOC. Although this approach is successful at providing framework of the MOC dynamics, the dynamics remains less quantified because the decomposition was derived heuristically. In this study, in order to make more rigorous discussion on the MOC dynamics, a diagnostic equation for vorticity balance of the MOC is derived from the governing equations. In deriving the equation, arbitrary bottom topography is assumed, which will allow future application of the equation to more realistic simulations. We

use the equation to understand how the increased K_V changed MOC described in the previous subsection.

Cross differentiation of Eqs. (2) and (4) with respect to x and z and Eqs. (3) and (4) with respect to y and z respectively yields

$$\frac{\partial}{\partial t} \frac{\partial u}{\partial z} - f \frac{\partial v}{\partial z} = -\alpha g \frac{\partial T}{\partial x} - \frac{\partial \mathcal{A}(u) - \mathcal{V}(u)}{\partial z} \quad (13)$$

$$\frac{\partial}{\partial t} \frac{\partial v}{\partial z} + f \frac{\partial u}{\partial z} = -\alpha g \frac{\partial T}{\partial y} - \frac{\partial \mathcal{A}(v) - \mathcal{V}(v)}{\partial z} \quad (14)$$

Further manipulation ($\partial(14)/\partial t - f \times (13)$ and $\partial(13)/\partial t + f \times (14)$) results in

$$\left(\frac{\partial^2}{\partial t^2} + f^2 \right) \frac{\partial v}{\partial z} = \frac{\partial}{\partial t} \left[-\alpha g \frac{\partial T}{\partial y} - \frac{\partial \mathcal{A}(v) - \mathcal{V}(v)}{\partial z} \right] - f \left[-\alpha g \frac{\partial T}{\partial x} - \frac{\partial \mathcal{A}(u) - \mathcal{V}(u)}{\partial z} \right] \quad (15)$$

$$\left(\frac{\partial^2}{\partial t^2} + f^2 \right) \frac{\partial u}{\partial z} = \frac{\partial}{\partial t} \left[-\alpha g \frac{\partial T}{\partial x} - \frac{\partial \mathcal{A}(u) - \mathcal{V}(u)}{\partial z} \right] + f \left[-\alpha g \frac{\partial T}{\partial y} - \frac{\partial \mathcal{A}(v) - \mathcal{V}(v)}{\partial z} \right] \quad (16)$$

Zonal integration of Eq. (15) from the western boundary $x = X_W$ ($= 0$ in the present study) to the eastern boundary $x = X_E$ ($= L$) yields ¹

$$\begin{aligned} \left(\frac{\partial^2}{\partial t^2} + f^2 \right) \frac{\partial^2 \Phi}{\partial z^2} &= \frac{\partial}{\partial t} \left[\alpha g \int_{X_W}^{X_E} \frac{\partial T}{\partial y} dx + \int_{X_W}^{X_E} \frac{\partial \mathcal{A}(v) - \mathcal{V}(v)}{\partial z} dx \right] \\ &\quad - f \left[\alpha g (T_E - T_W) + \int_{X_W}^{X_E} \frac{\partial \mathcal{A}(u) - \mathcal{V}(u)}{\partial z} dx \right] \\ &\quad - \left(\frac{\partial^2}{\partial t^2} + f^2 \right) \left(\frac{\partial X_E}{\partial z} v_E - \frac{\partial X_W}{\partial z} v_W \right) \end{aligned} \quad (17)$$

where

$$\frac{\partial \Phi}{\partial z} = - \int_{X_W}^{X_E} v dx \quad (18)$$

is the meridional volume transport function (which reduces to Eq. (12) in the present model configuration), T_E (T_W) and v_E (v_W) are temperature and meridional velocity at the eastern (western) boundary $x = X_E$ (X_W), respectively. It should be noted that K_V (or $\mathcal{D}(T)$ in Eq. 6) does not appear in the above equation. It affects $\partial^2 \Phi / \partial z^2$ (and hence Φ) through $\partial T / \partial y$ and/or $T_E - T_W$. It should

¹ Meridional integration of Eq. (16) provides diagnostic equation of zonal overturning circulation.

be also noted that Eq. (17) is nonlinear in terms of Φ ; it is implicitly included in the right hand side of Eq. (17). Therefore, Eq. (17) was used to diagnose vorticity balance rather than to solve Φ itself.

To illustrate vorticity dynamics described in Eq. (17), let us first consider the MOC in the non-rotating ($f = 0$) flat ocean, referred to as horizontal convection (e.g., Hughes and Griffiths 2008). In this case, Eq. (17) reduces to

$$\frac{\partial}{\partial t} \frac{\partial^2 \Phi}{\partial z^2} = \alpha g \int_{X_W}^{X_E} \frac{\partial T}{\partial y} dx + \int_{X_W}^{X_E} \frac{\partial \mathcal{A}(v)}{\partial z} dx - \mathcal{V} \left(\frac{\partial^2 \Phi}{\partial z^2} \right), \quad (19)$$

where $\Phi = 0$ at $t = 0$ is assumed. This equation shows that Φ is forced by zonally integrated meridional torque associated with the temperature gradient ($\alpha g \partial T / \partial y$), damped by viscous diffusion ($\mathcal{V}(\partial^2 \Phi / \partial z^2)$), and modified by zonally-integrated vertically-differentiated advection of the meridional velocity ($\partial \mathcal{A}(v) / \partial z$). The advection term tends to shift the center of the circulation toward the sinking region (surface cooling region) and upward (e.g., Rossby 1998). The meridional temperature gradient, the driver of the MOC, is forced by the surface differential heating / cooling. The temperature distribution below the surface is determined by advection and diffusion of the temperature. The advection transports colder (warmer) water to greater (shallower) depths, while the diffusion or the vertical mixing K_V makes the surface temperature gradient to penetrate to greater depths. Without K_V , the advection makes deeper ocean filled with the cold water, and the temperature gradient (the meridional torque) and the MOC are weak in the deeper layer. Thus, K_V acts to penetrate the torque and strengthen the MOC at greater depths. This explains the overall mechanisms of the two-dimensional MOC (or horizontal convection), that is, push (surface cooling and advection) or pull (surface heating and diffusion) determine the intensity of the MOC.

In the rotating case, on the other hand, the Coriolis acceleration alters dynamical balances. In a steady state, Eq. (17) reduces to

$$f \frac{\partial^2 \Phi}{\partial z^2} = -\alpha g(T_E - T_W) - \int_{X_W}^{X_E} \frac{\partial \mathcal{A}(u) - \mathcal{V}(u)}{\partial z} dx - f \left(\frac{\partial X_E}{\partial z} v_E - \frac{\partial X_W}{\partial z} v_W \right) \quad (20)$$

A balance between the left-hand side term and the first term in the right hand side is the thermal wind balance. Deviation from the balance is caused by zonally-integrated vertically-differentiated advection and viscous diffusion of the zonal velocity (the second term on the right hand side), and terms associated with variable bottom topography (the last term on the right hand side). The bottom topography term corresponds to the barotropic component discussed in Lee and Marotzke (1998), which is nonzero if the bottom ocean is not flat. The Ekman component (Lee and Marotzke 1998) is forced by a zonal wind stress, which is represented by the $\mathcal{V}(u)$ term. The above equation is more rigorous than the equation used in Lee and Marotzke (1998) in that it is derived from the governing equations. The equation (20) enables us to make quantitative analysis of the MOC dynamics.

We evaluated each term in Eq. (17) in the B experiment with $X_W = 0$ and $X_E = L$ (Figure 7). In the figure, the second time derivative on the left-hand side was ignored, due to longer time scales of the MOC than the inertial period (f^{-1}). This approximation corresponds to the planetary geostrophic approximation that was validated for the large-scale MOC by Zhang et al. (1992). It is clear from this figure that the thermal wind balance dominated the vorticity balance; geostrophy dominates MOC (Φ). This is illustrated by geostrophic (Φ_g) and ageostrophic (Φ_a) volume transport functions shown in Figure 8, where the geostrophic velocity (u_g, v_g) and the ageostrophic velocity were defined as

$$u_g = -\frac{1}{\rho_0 f} \frac{\partial p}{\partial y}, v_g = \frac{1}{\rho_0 f} \frac{\partial p}{\partial x}, \quad (21)$$

$$u_a = u - u_g, \quad v_a = v - v_g, \quad (22)$$

275 and Φ_g and Φ_a were calculated by vertically integrating Eq. (12) with v_g and v_a instead of v , re-
 276 spectively. The volume transport stream function Φ was dominated by the geostrophic component
 277 (Φ_g , calculated from v_g) except near the southern boundary (equator) where small f amplified
 278 numerical differential errors. Correspondence between Φ and Φ_g simulated in a similar idealized
 279 ocean was also demonstrated by Zhang et al. (1992) and Hirschi and Marotzke (2007). In both
 280 the W and E experiments, vorticity balance was dominated by the thermal wind balance (Fig-
 281 ure 9). Thus in the present experiments, intensified vertical mixing does not alter dominance of
 282 the geostrophy.

283 These results confirm significance of the zonal tilts of the thermocline depths as the previous
 284 studies demonstrated (e.g., Zhang et al. 1992; Marotzke 1997; Park and Bryan 2000; Scott and
 285 Marotzke 2002). The (geostrophic) MOC was supported by the eastward deepening thermocline
 286 (or isopycnal surfaces), and intensity of the MOC was affected by processes that influence this
 287 thermocline tilt. In W experiment, the increased K_V (intensified mixing) deepened the thermocline
 288 at the western boundary, reducing the thermocline tilt and MOC. On the other hand, in the E
 289 experiment, the increased K_V deepened the thermocline at the eastern boundary, intensifying the
 290 thermocline tilt and MOC.

291 Note that horizontally uniform intensification of the vertical mixing makes the thermocline deep
 292 with less change in the thermocline tilt, resulting in deep northward geostrophic flows and in-
 293 tensified MOC as in the nonrotating case (e.g., Zhang et al. 1992; Park and Bryan 2000). If the
 294 intensification is horizontally nonuniform, however, the MOC, which depends greatly on the ther-
 295 mocline depth difference between the eastern and western boundaries, can be either intensified or
 296 weakened (e.g., Marotzke 1997; Scott and Marotzke 2002).

297 *c. Vertical Velocity*

298 The MOC is accompanied by upwelling/downwelling, while the geostrophic flows, the greatest
 299 component of the present MOC, are nondivergent to the first order (as the quasi-geostrophic theory
 300 shows) due to planetary vorticity constraints in the ocean interior (e.g., Spall 2000; Spall and
 301 Pickart 2001; Katsman et al. 2018). To evaluate “ageostrophic” upwelling/downwelling velocities
 302 involved in the MOC in our simulation in more quantitative manner, a diagnostic equation for the
 303 vertical velocity was derived in this study. Horizontal divergence of Eqs. (16) and (15) yields,
 304 together with (5),

$$\begin{aligned} \left(\frac{\partial^2}{\partial t^2} + f^2 \right) \frac{\partial^2 w}{\partial z^2} = \frac{\partial}{\partial t} \left\{ \alpha g \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) T + \frac{\partial}{\partial z} \left[\frac{\partial \mathcal{A}(u) - \mathcal{V}(u)}{\partial x} + \frac{\partial \mathcal{A}(v) - \mathcal{V}(v)}{\partial y} \right] \right\} \\ + f \frac{\partial}{\partial z} \left[\frac{\partial \mathcal{A}(v) - \mathcal{V}(v)}{\partial x} - \frac{\partial \mathcal{A}(u) - \mathcal{V}(u)}{\partial y} \right] + \frac{\partial f}{\partial y} \left[2f \frac{\partial v}{\partial z} - \alpha g \frac{\partial T}{\partial x} - \frac{\partial \mathcal{A}(u) - \mathcal{V}(u)}{\partial z} \right]. \end{aligned} \quad (23)$$

305 Assuming $\mathcal{A} = \mathcal{V} = \partial f / \partial y = 0$, this equation together with linearized buoyancy tendency equa-
 306 tion $\alpha g \partial T / \partial t + N^2 w = 0$ provides the dispersion relation of hydrostatic inertial-internal gravity
 307 waves. In a steady state, on the other hand, the above equation reduces to

$$\begin{aligned} w = -\frac{1}{f} \int_z^0 \left(\frac{\partial \mathcal{A}(v) - \mathcal{V}(v)}{\partial x} - \frac{\partial \mathcal{A}(u) - \mathcal{V}(u)}{\partial y} \right) dz - \frac{1}{f} \frac{\partial f}{\partial y} \int_z^0 v dz \\ = \int_z^0 \left(\frac{\partial u_a}{\partial x} + \frac{\partial v_a}{\partial y} \right) dz - \frac{1}{f} \frac{\partial f}{\partial y} \int_z^0 v_g dz. \end{aligned} \quad (24)$$

308 (In the above, the rigid-lid boundary condition ($w = 0$ at $z = 0$) was used.) Note that the above
 309 equations are applicable only if $f \neq 0$. (In nonrotating ($f = 0$) case, not w but $\partial^2 w / \partial t^2$ or $\partial w / \partial t$
 310 should be diagnosed from Eq. (23).) Equation (24) can also be derived directly from the steady
 311 state version of Eqs. (2), (3) and (5). The first term on the right hand side of Eq. (24) represents
 312 divergence of ageostrophic velocity $(u_a, v_a) = (-\mathcal{A}(v) + \mathcal{V}(v), \mathcal{A}(u) - \mathcal{V}(u)) / f$, while the sec-
 313 ond term represents planetary vorticity change that induces stretching/shrinking of water column
 314 ($\beta v_g = f \partial w / \partial z$). (Hereafter, the last term is referred to as the Sverdrup term.) Care should be

315 taken at lateral boundaries at which velocity normal to the boundary vanishes while the pressure
 316 gradient along the boundary (hence the geostrophic flow normal to the boundary) does not. For ex-
 317 ample, at the western or eastern boundary ($x = 0$ or L), $u = 0$ but $\partial p / \partial y = -\rho_0 f u_g$ that is generally
 318 nonzero. In the present study, $u_g \neq 0$ and $u_a = -u_g$ at the boundaries. Here we refer to this bound-
 319 ary ageostrophic flow that compensates the geostrophic flow normal to the boundary as a boundary
 320 ageostrophic flow. (The other ageostrophic flow is referred to as an interior ageostrophic flow.)
 321 The boundary ageostrophic flow is nonzero only on the boundary, while the interior ageostrophic
 322 flow is zero there (but nonzero in the interior). Horizontal divergence of the boundary ageostrophic
 323 flow (that is nonzero only within the one grid box adjacent to the boundaries) could be large and
 324 induce intense vertical velocity along the boundary. The vertical velocity due to the boundary
 325 ageostrophic flow was referred to as the mass-balance flow in Scott and Marotzke (2002). The
 326 pressure gradient along the boundary was induced by the surface differential cooling. Importance
 327 of the surface differential cooling along boundaries on the MOC were shown by Spall (2000) and
 328 Spall and Pickart (2001).

329 Figure 10 shows the diagnosed vertical velocity (the left-hand side term of Eq. (24)) as well
 330 as its three components, the interior and boundary ageostrophic components (the first term of the
 331 right hand side) and the Sverdrup component (the second term). The diagnosed vertical velocity
 332 agrees well with the simulated vertical velocity (Figure 1c), except at lower latitudes where numer-
 333 ical differential errors were amplified due to small f . Intense upwelling and downwelling were
 334 found along the western and eastern boundaries, respectively, which correspond to the bound-
 335 ary ageostrophic component. Thus, geostrophic flows (or pressure gradients along the boundary)
 336 induced intense vertical velocity along the boundary. Relatively intense vertical upwelling and
 337 downwelling were also found slightly away from the boundaries, which were driven by the vis-
 338 cous diffusion of (mostly geostrophic) horizontal flows $\mathcal{V}(u) \simeq \mathcal{V}(u_g)$ and $\mathcal{V}(v) \simeq \mathcal{V}(v_g)$ (not

shown) in the present simulation. Away from the boundary, the interior ageostrophic component was small and the Sverdrup component shaped the horizontal distribution of the vertical velocity which was negative (downwelling) in the northern region (where the flow above the thermocline base was northward) and positive (upwelling) in the southern region (where the flow was southward). These results, being consistent with the previous studies (e.g., Spall and Pickart 2001), show that Eq. (24) enables quantitative evaluations of the geostrophic and ageostrophic processes that derives the vertical velocity.

Figure 11 shows meridional distribution of the vertical velocity components (interior ageostrophic, boundary ageostrophic, and Sverdrup components) averaged over entire zonal - vertical ($x - z$) section. In the B experiment, the largest contribution was the boundary ageostrophic component that was positive (upwelling) in $y \leq 5500$ km and negative (downwelling) in $y \geq 5500$ km, in agreement with the volume transport function (Φ) distribution (Figure 8). The interior ageostrophic component was large only in $y \geq 5500$ km (and $y \leq 700$ km where errors in the geostrophic calculation were large) where its sign was opposite to that of the boundary ageostrophic component. The Sverdrup component was negative everywhere (downwelling) which reflected the northward flow (convergent geostrophic flow) in the upper layer and southward flow (divergent geostrophic flow) in the lower layer. These profiles of the vertical velocity components were similar in the W and E experiments (Figure 11b and c), although the interior ageostrophic upwelling was slightly intensified in the intensified mixing region. This is due to the intensified horizontal viscous diffusion of geostrophic flows around the intensified mixing region, where the geostrophic flows were distorted because of hydrostatic pressure anomaly caused by the vertical mixing. In this way, the vertical mixing can locally intensify the upwelling through the interior ageostrophic component ($\mathcal{A} - \mathcal{V}$) (e.g., Kawasaki and Hasumi 2010). Note that this locally intensified upwelling due to the interior ageostrophic component can contribute signifi-

363 cantly the MOC with horizontal scales smaller than the deformation radius, as will be shown in
364 the next section. In the present experiment, the contribution of this locally intensified upwelling to
365 the global-scale MOC was much weaker than that of the upwelling by the boundary ageostrophic
366 component.

367 *d. Heat Balance*

368 To see the impact of intensified vertical mixing on the vertical heat balance (Eq. 1) that directly
369 connects K_V and w , we examined the heat balance (Eq. 6) in the present experiments. Here,
370 the surface restoring forcing was included in the vertical diffusion term. At first, in order to
371 see typical heat balances in interior regions of the B experiment, the profiles in the southern and
372 northern central regions are shown in Figures 12a and b, respectively. In the southern central region
373 ($3000 \text{ km} \leq x \leq 3400 \text{ km}$ and $1400 \text{ km} \leq y \leq 1800 \text{ km}$), where the weak Sverdrup upwelling
374 occurred (Figure 10), a major heat balance was between cooling due to vertical advection (cold
375 water upwelling, represented by blue dashed line) and warming due to vertical diffusion (red
376 dashed line), as described by Munk (1966), though horizontal advection and diffusion (blue and
377 red dotted lines, respectively) also contributed to the cooling in the upper 500 m. In the northern
378 central region ($3000 \text{ km} \leq x \leq 3400 \text{ km}$ and $4600 \text{ km} \leq y \leq 5000 \text{ km}$) where the weak Sverdrup
379 downwelling occurred, on the other hand, the primary balance was between cooling due to the
380 horizontal diffusion and warming due to the horizontal advection. The vertical advection also
381 contributed to warming slightly, while the vertical diffusion cooled and warmed the upper ($z \geq$
382 -500 m) and lower ($z \leq -500 \text{ m}$) thermocline, respectively. Similar balances in the interior
383 regions were also found in both the W and E experiments (not shown). Note that in the real ocean,
384 the surface wind stress, which induces downwelling and upwelling in the subtropical and subpolar

regions through Ekman pumping and suction, respectively, may change direction of the interior vertical velocity and hence the above interior heat balance.

The heat balance in the boundary regions of the B experiment is shown in Figures 12c and d. In the western boundary region ($x \leq 200$ km and 4600 km $\leq y \leq 5000$ km, where the mixing was intensified in the W experiment), intense upwelling due to the boundary compensating flow caused intense cooling, which was balanced by the warming due to the horizontal diffusion. In the eastern boundary region ($x \geq 6200$ km and 4600 km $\leq y \leq 5000$ km, where the mixing was intensified in the E experiment) where the downwelling due to the geostrophic flow dominated, on the other hand, the balance was established by warming due to the horizontal advection and cooling due to the vertical diffusion.

The heat balance in the W and E experiments was almost the same as that in the B experiment, except in the intensified mixing region. In the intensified mixing region of the W experiment (Figure 12e), the intensified vertical diffusion transported surface heat downward (deeper layer warming), but the heating was limited above 2000 m depth. The intensified upwelling (Figure 3c) on the other hand cooled the water except very near the surface. Horizontally divergent flow also contributed to the cooling in the upper layer ($z \geq -1000$ m) and warming in the lower layer ($z \leq -1000$ m). The lower layer warming due to the horizontally divergent flow was balanced by the upwelling induced cooling. In the intensified mixing region of the E experiment, on the other hand, the heat balance was complicated. Combined effects of the horizontal and vertical advection warmed (cooled) the upper (lower) thermocline above (below) $z = -500$ m, which was balanced with the combined effects of diffusion. Near the surface, the geostrophic flows normal to the boundary transported the heat into this region, which was transported downward by the intensified downwelling and the vertical diffusion.

4. Experiment with Localized Cooling

In this section, the Stommel-Arons type thermohaline circulation (e.g., Stommel and Arons 1960) was investigated. For this purpose, the thermal forcing F_T in Eq. (6) was set as

$$F_T = \gamma(SST - T)\delta(z) + \begin{cases} \gamma(1 + \frac{z}{D})(DWT - T) & 400 \text{ km} < x < 800 \text{ km and } 5600 \text{ km} < y < 6000 \text{ km} \\ 0 & \text{otherwise} \end{cases} \quad (25)$$

where DWT (deep water temperature) = 0°C and $SST = 10^\circ\text{C}$. This forcing is meant to represent deeper water formation in a localized area near the northwest corner of the ocean. Initially the model ocean was at rest and temperature was set as 10°C . Other model configuration was the same as that in the B experiment. The integration was performed for 4000 years by which the MOC became steady. This experiment was referred to as SA experiment.

The circulation was spun up as described by Kawase (1987) (not shown), and the Stommel-Arons type thermohaline circulation established as shown by the velocity field at 3050 m depth (Figure 13) where an intense southward western boundary current (roughly at $x \leq 500$ km) and weak northward geostrophic flows in an interior region (roughly at $x \geq 1000$ km) are evident.

The MOC (Figure 14a) shows localized intense downwelling at the forcing latitude ($5600 \text{ km} < y < 6000 \text{ km}$) and broad weak upwelling at other latitudes. The geostrophic and ageostrophic component of the MOC (Figure 14b and c, respectively) shows that the localized downwelling is ageostrophic while large-scale MOC is geostrophic. Zonally averaged vertical velocity (Figure 15) shows that the interior ageostrophic component dominates the localized downwelling/upwelling at around the forcing latitude, while the boundary ageostrophic component (induced by geostrophic flow normal to the boundary) forms the large-scale upwelling. Note that the zonally averaged Sverdrup component was downward at all latitudes. Although the (non-averaged) Sverdrup com-

428 ponent away from the western boundary was upward due to the northward geostrophic flow as
429 assumed by the Stommel-Arons model (e.g., Figure 13b), the southward western boundary cur-
430 rent (mostly geostrophic) whose transport was larger than the northward interior geostrophic flow
431 made the Sverdrup component downward there, which compensated the weak upward Sverdrup
432 component in the interior.

433 The SA experiment illustrates importance of geostrophic flows not only for the horizontal struc-
434 ture of the Stommel-Arons thermohaline circulation but also for its vertical structure: the MOC.
435 In the SA experiment, the east-west temperature difference, a key for the geostrophic MOC (Φ_g),
436 was established primarily by the Kelvin waves which propagated cold temperature anomaly south-
437 ward along the western boundary, leaving the southward (mostly geostrophic) western boundary
438 current. (The Rossby waves on the other hand set the northward interior geostrophic flow.) Weaker
439 the Kelvin wave propagation, the MOC more localized near the forcing region (e.g., Greatbatch
440 and Lu 2003). In such cases, relative importance of the ageostrophic MOC (Φ_a) would increase.

441 **5. Concluding Remarks and Discussions**

442 This study evaluated the geostrophic and ageostrophic processes in the MOC using equations
443 derived to diagnose vorticity balance and vertical velocity. To illustrate applicability of the equa-
444 tions, we at first revisited the MOC in an idealized rectangular ocean forced by zonally uniform
445 differential surface heating and cooling. The diagnostic vorticity balance equation shows that the
446 simulated MOC was primarily geostrophic, as described in the previous numerical studies (e.g.,
447 Bryan 1987; Zhang et al. 1992; Marotzke 1997; Spall and Pickart 2001; Scott and Marotzke 2002).
448 The zonal tilt of the thermocline depth (or isopycnal surfaces) is thus a key for the MOC, and ver-
449 tical mixing, known as a key ingredient in sustaining the MOC, affects the MOC by changing the
450 zonal tilt (Zhang et al. 1992; Marotzke 1997; Scott and Marotzke 2002). Nonuniformly intensified

451 vertical mixing can either strengthen or weaken the tilt, and hence the MOC. The point to empha-
 452 size in this study is that stronger vertical mixing does not always intensify the MOC, though it
 453 always tends to warm the deeper water. Note that the vertical advection - diffusion balance (Eq. 1)
 454 was still valid in the interior upwelling dominated region; this balance primarily determines $T(z)$
 455 or thermocline depth with prescribed K_V and w (e.g., Bryan 1987; Zhang et al. 1992) rather than
 456 to control w with prescribed K_V and temperature profile $T(z)$. Intense upwelling/downwelling
 457 of the simulated MOC was found along lateral boundaries, as was shown by recent simulations
 458 (e.g., Spall and Pickart 2001; Katsman et al. 2018). The diagnostic equation for the vertical veloc-
 459 ity shows that the geostrophic flows normal to the lateral boundaries induced the upwelling and
 460 downwelling along the boundaries and were the largest component in the simulation. Neither the
 461 Sverdrup interior (geostrophic) component nor the interior ageostrophic component was strong
 462 enough to sustain the simulated large-scale MOC. The MOC driven by localized cooling on the
 463 other hand showed that the interior ageostrophic component was large at around the forcing lati-
 464 tude while the the geostrophic flows normal to the lateral boundaries induced upwelling at other
 465 latitudes. This clearly illustrates that the ageostrophic processes feature the small-scale MOC,
 466 while the geostrophic processes shape the large-scale MOC.

467 In the real ocean, variable bottom and coastal topography as well as spatially-varying sur-
 468 face forcing may enlarge the ageostrophic MOC. For example, realistic simulation of the Pacific
 469 (Kawasaki et al. 2021) shows that the simulated MOC is similar to the geostrophic MOC for large
 470 scales, while the deviation from the geostrophic MOC is evident for small scales (Figure 16).
 471 Each MOC should be evaluated if such small-scale overturnings are to be discussed. Katsman
 472 et al. (2018) argued the along-boundary downwelling simulated in their realistic OGCM in terms
 473 of the along-coast density gradient (that induces geostrophic flow normal to the boundary), based

474 on theoretical analysis by Spall and Pickart (2001). We believe that Eq. (24) will be useful to
475 diagnose the intense upwelling/downwelling along the boundaries even in realistic OGCMs.

476 The MOC transports heat and alters temperature field, and the altered temperature field may
477 change surface heat flux, stratification, and the vertical mixing. Thus, the entire processes between
478 the MOC and the vertical mixing in the real ocean are nonlinear and complicated. If the MOC is
479 found to be dominated by the geostrophy, then the question to be solved is the relation between
480 the hydrostatic pressure (temperature) field and the mixing. Adjustments by the Rossby waves
481 and the Kelvin waves will be one of key processes. Thus, diagnostic evaluation of the vorticity
482 balance of the MOC using Eq. (17) and vertical velocity using Eq. (24) will be a first step to resolve
483 such complicated processes. Such diagnostic evaluations may also be useful to reveal unexplored
484 effects of the vertical mixing on the MOC. For example, vertical variation in the vertical mixing
485 has been recently suggested to play an important role in abyssal overturning circulations (e.g.,
486 Ferrari 2014; McDougall and Ferrari 2017). The vertical eddy diffusivity is generally larger near
487 the bottom due to larger turbulence, indicating more mixing with deeper (more dense) water than
488 with shallower (less dense) water. This makes the deeper water more dense by turbulent mixing
489 and hence the deeper water tends to sink, except for the water very close to the bottom boundary
490 across which turbulent transport is negligible. Thus upwelling is expected to occur very close to
491 the bottom boundary. It should be noted that this upwelling is expected to be diagnosed as the
492 interior ageostrophic component. Thus, Eqs. (17) and (24) will be useful to quantitatively evaluate
493 how and where this process dominates. This will be our future study.

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- Fig. 10.** Diagnosed vertical velocities from Eq. (24): (a) total (the left hand side), (b) interior ageostrophic component (the first term of the right hand side evaluated at interior grid points), boundary ageostrophic component (the first term of the right hand side) evaluated at boundary grid points, and (d) Sverdrup component (the second term of the right hand side). Color and contour intervals are same as those in Figure 1c. 40
- Fig. 11.** Meridional profiles of the mean vertical velocity components (interior ageostrophic, boundary ageostrophic and Sverdrup components) averaged over zonal - vertical ($x - z$) section. (a) B experiment, (b) W experiment, and (c) E experiment. Large values at the southern end ($y = 0$) were errors due to small f 41
- Fig. 12.** Vertical profiles of temperature tendency terms. (a) Central southern region ($3000 \text{ km} \leq x \leq 3400 \text{ km}$ and $1400 \text{ km} \leq y \leq 1800 \text{ km}$), (b) central northern region ($3000 \text{ km} \leq x \leq 3400 \text{ km}$ and $6200 \text{ km} \leq y \leq 6600 \text{ km}$), (c) in the western boundary region ($x \leq 200 \text{ km}$ and $6200 \text{ km} \leq y \leq 6600 \text{ km}$, the intensified mixing region in the W experiment) and (d) in the eastern boundary region ($6200 \text{ km} \leq x$ and $6200 \text{ km} \leq y \leq 6600 \text{ km}$, the intensified mixing region in the E experiment) of the B experiment. (e) Same as (c) but for the W experiment. (f) Same as (d) but for the E experiment. Blue and red lines represent advection and diffusion terms,

618	respectively. Dotted and dashed lines are horizontal and vertical components, respectively,	
619	while solid lines are sum of the components.	42
620	Fig. 13. Horizontal distributions at $z = -3050$ m of (a) temperature and (b) vertical velocity with	
621	horizontal velocity vector in the SA experiment. Arrows in (b) is magnified by 10 times for	
622	better identifying weak interior flows.	43
623	Fig. 14. Meridional volume transport functions in the SA experiment. (a) Total (Φ). (b) Geostrophic	
624	component (Φ_g). (c) Ageostrophic component (Φ_a).	44
625	Fig. 15. Same as Figure 11 but for the SA experiment.	45
626	Fig. 16. MOCs in the Pacific simulated in the realistic OGCM of Kawasaki et al. (2021). (a) Total	
627	component (Φ). (b) Geostrophic component (Φ_g).	46

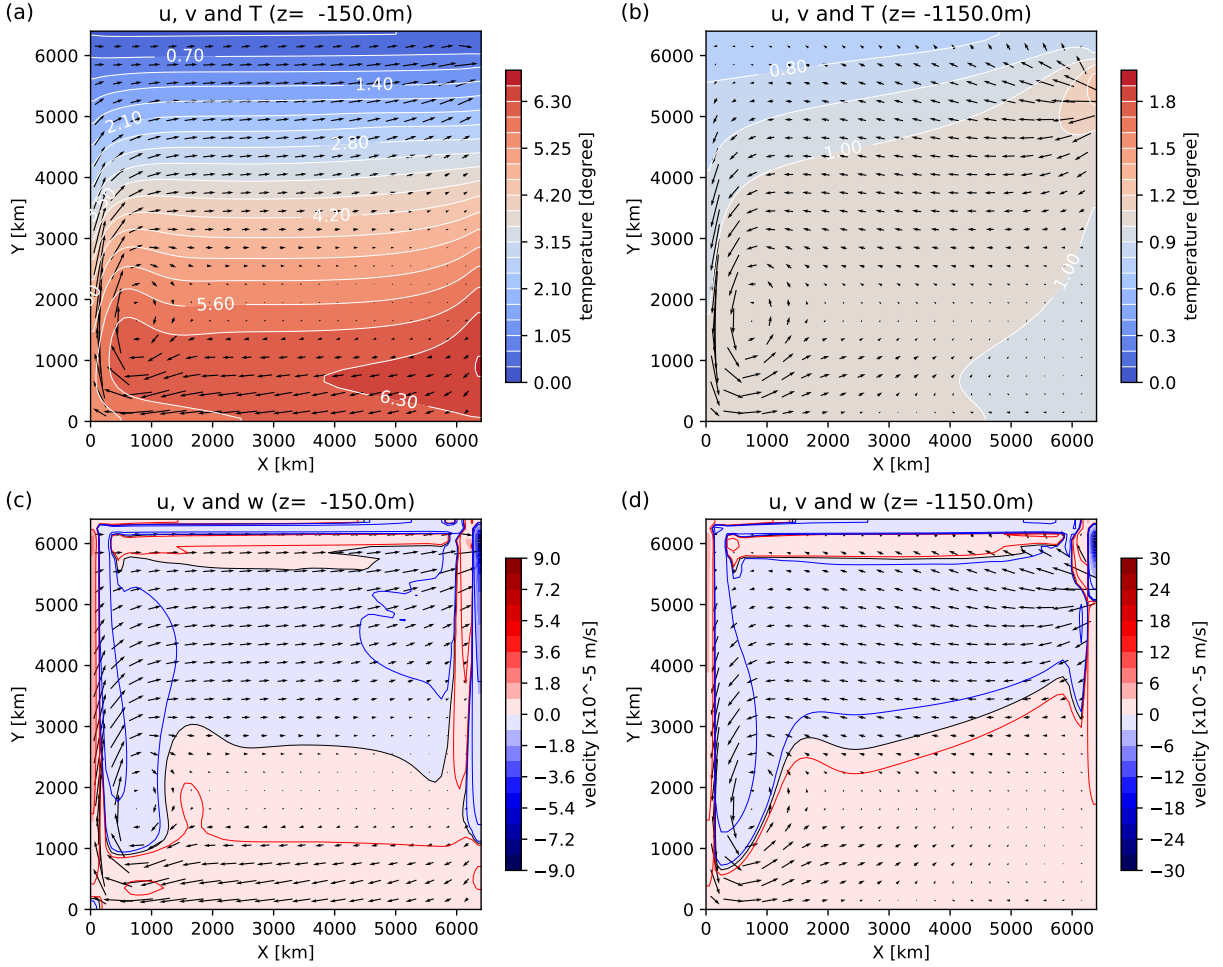


FIG. 1. (a, b) Temperature and (c, d) vertical velocity with horizontal velocity vectors at (a,c) $z = -150\text{ m}$ and (b,d) $z = -1150\text{ m}$ in the B experiment. In (c) and (d), colors are shaded in linear scale but contour lines are drawn in logarithmic scale ($10^{-7}, 10^{-6.5}, 10^{-6}, 10^{-5.5}, 10^{-5}, 10^{-4.5}\text{ m s}^{-1}$ in magnitude).

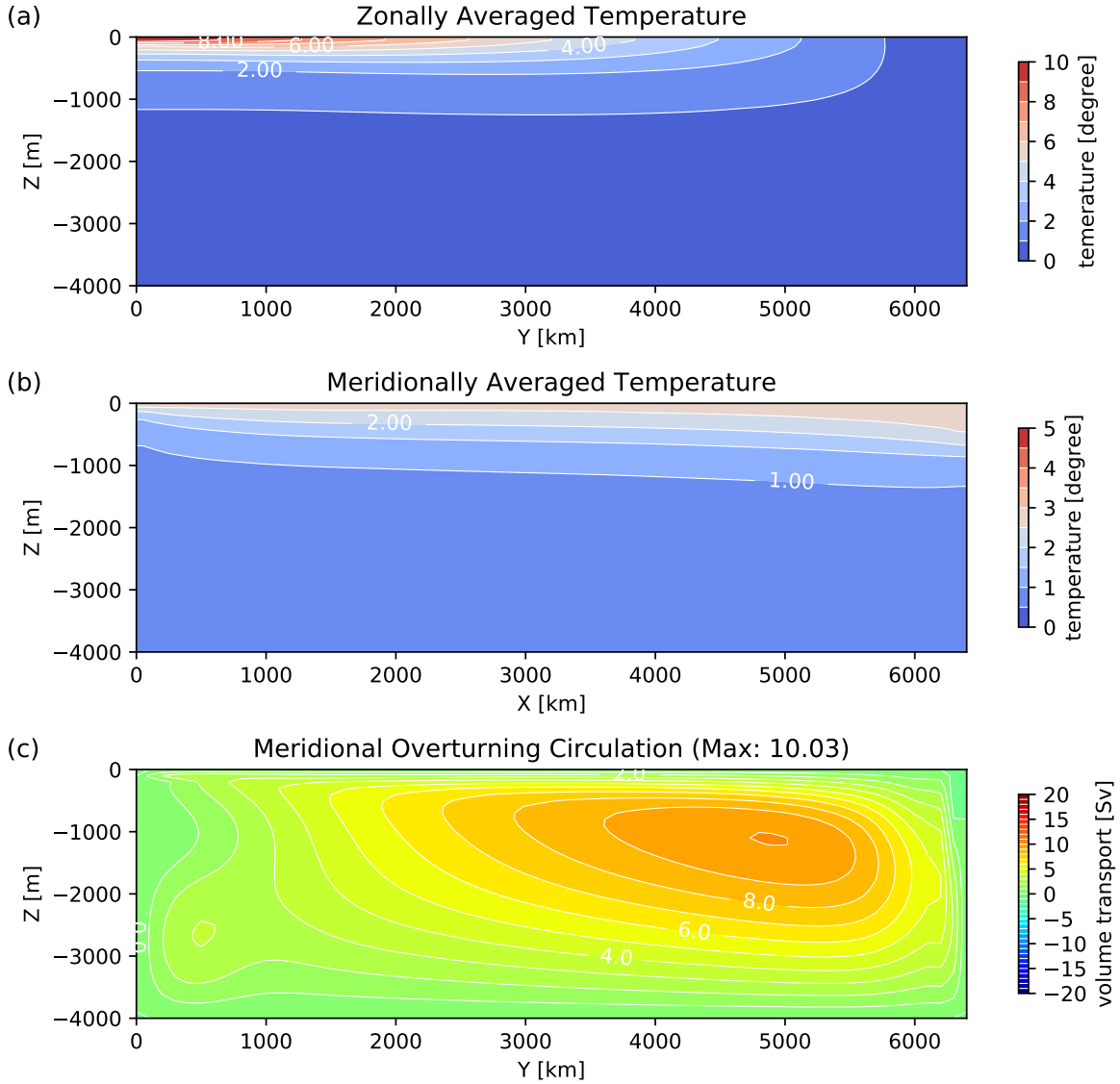


FIG. 2. (a) Meridional section of zonally averaged temperature. (b) Zonal section of temperature averaged over $4600 \text{ km} \leq y \leq 5000 \text{ km}$. (c) Meridional volume transport function (Φ). Results from the B experiment are shown.

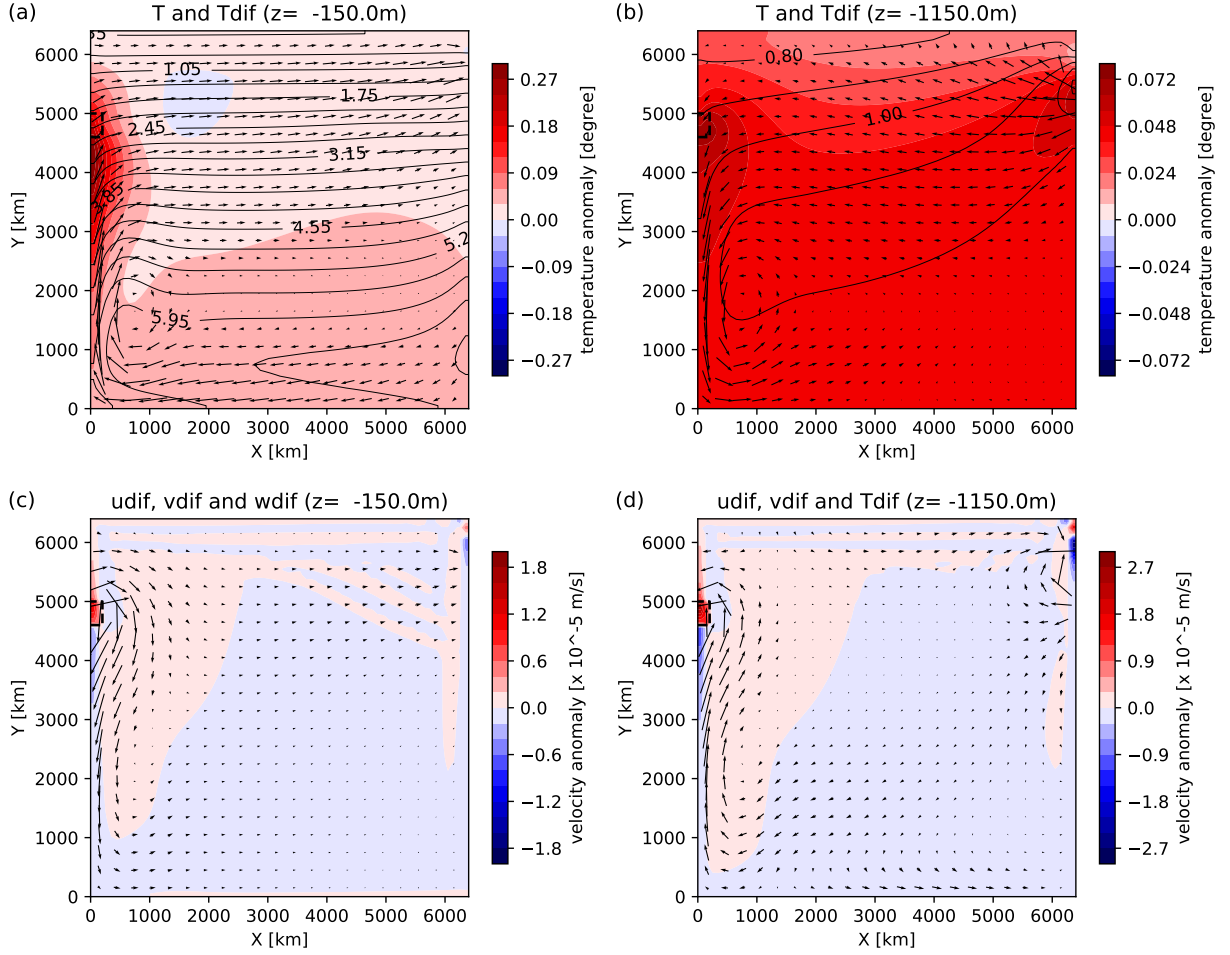


FIG. 3. (a, b) Temperature (contour) and horizontal velocity (arrows) in the W experiment at (a) $z = -150\text{ m}$ and (b) $z = -1150\text{ m}$. Color represents temperature anomaly from the B experiment (W experiment - B experiment). (c, d) Anomaly of horizontal velocity (arrows) and vertical velocity (color) in the W experiment from the B experiment at (c) $z = -150\text{ m}$ and (d) $z = -1150\text{ m}$.

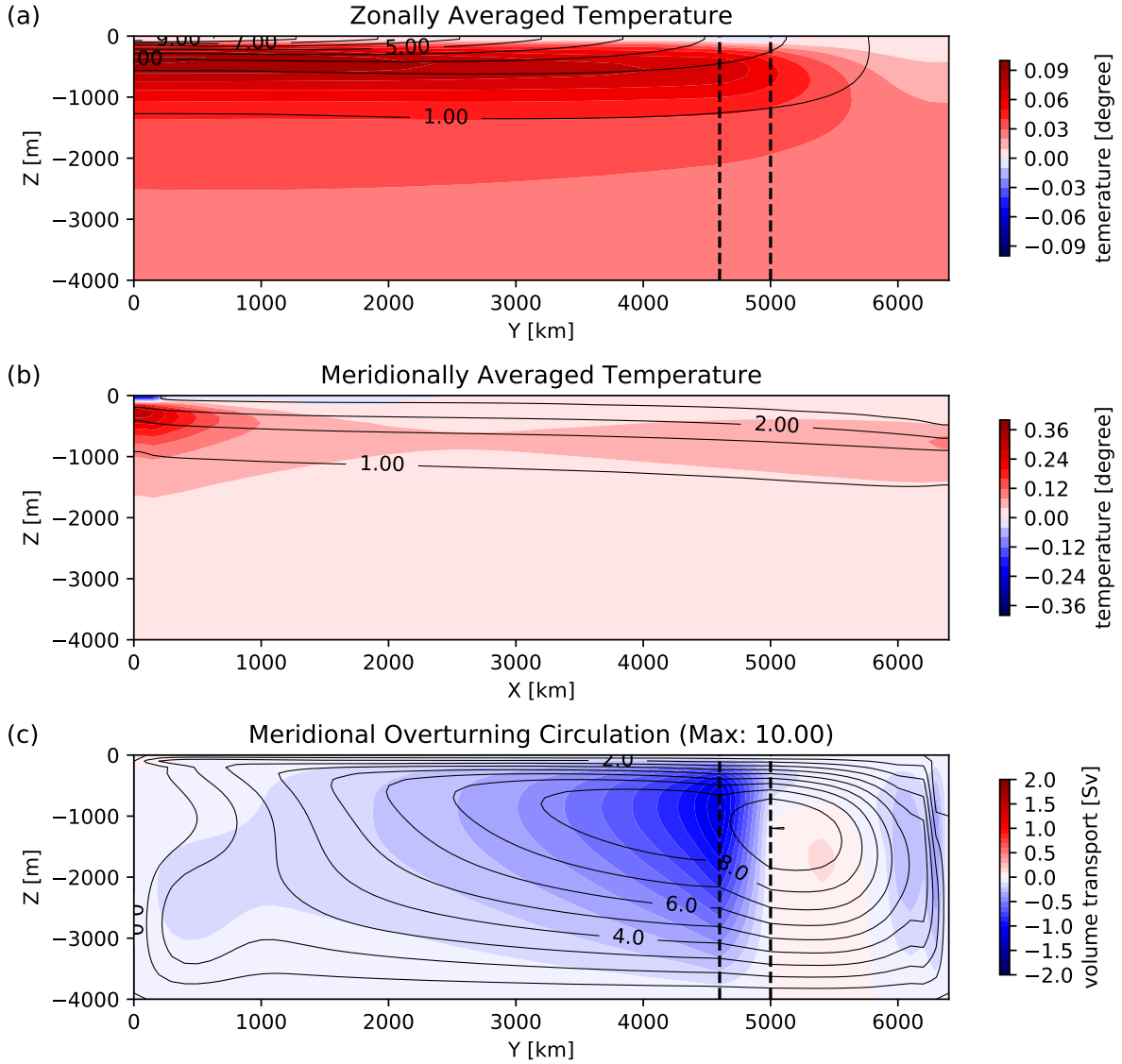


FIG. 4. Same as Figure 2 but for the W experiment. Color represents anomaly from the B experiment. Vertical dashed lines denote the intensified mixing region.

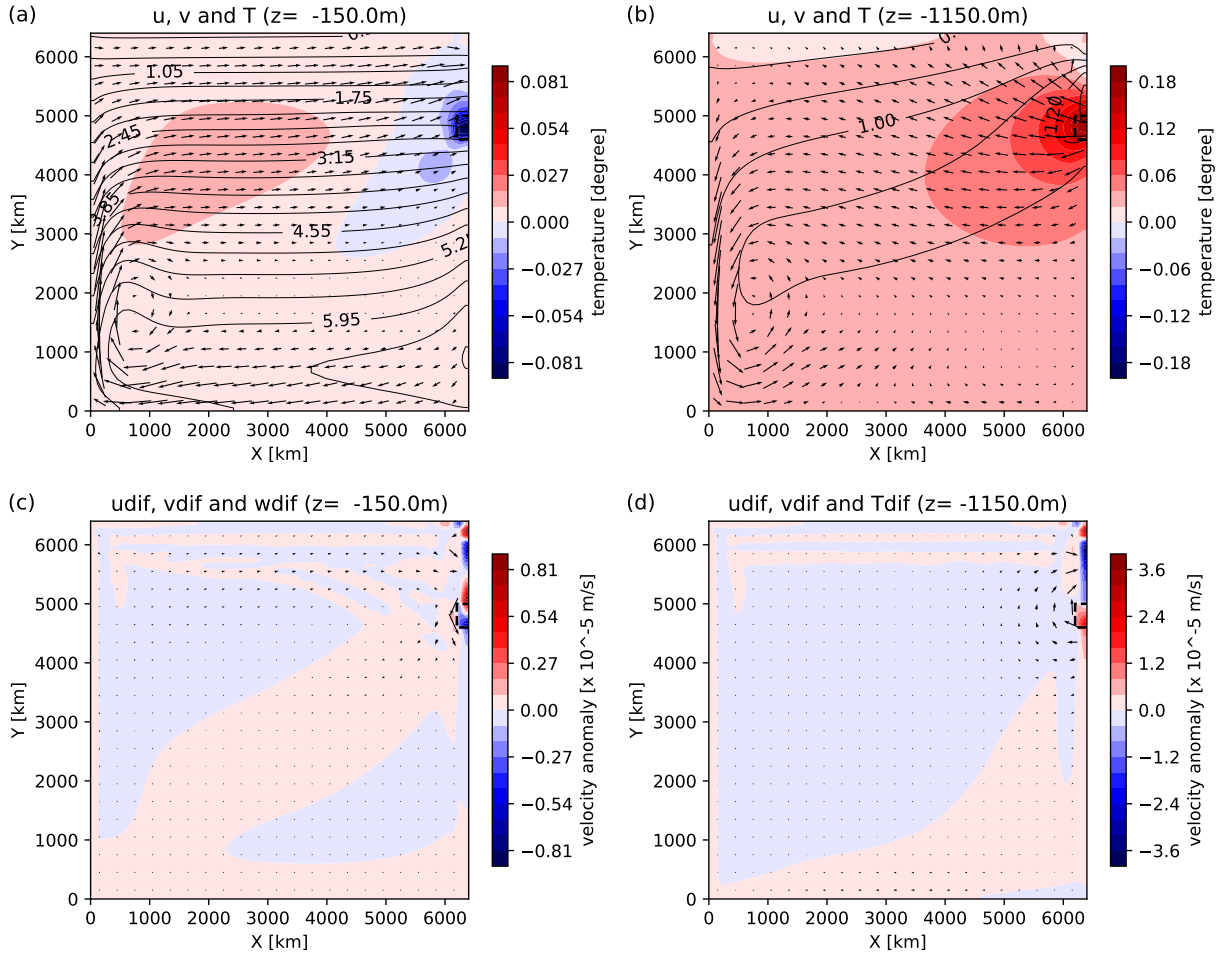


FIG. 5. Same as Figure 3 but for the E experiment.

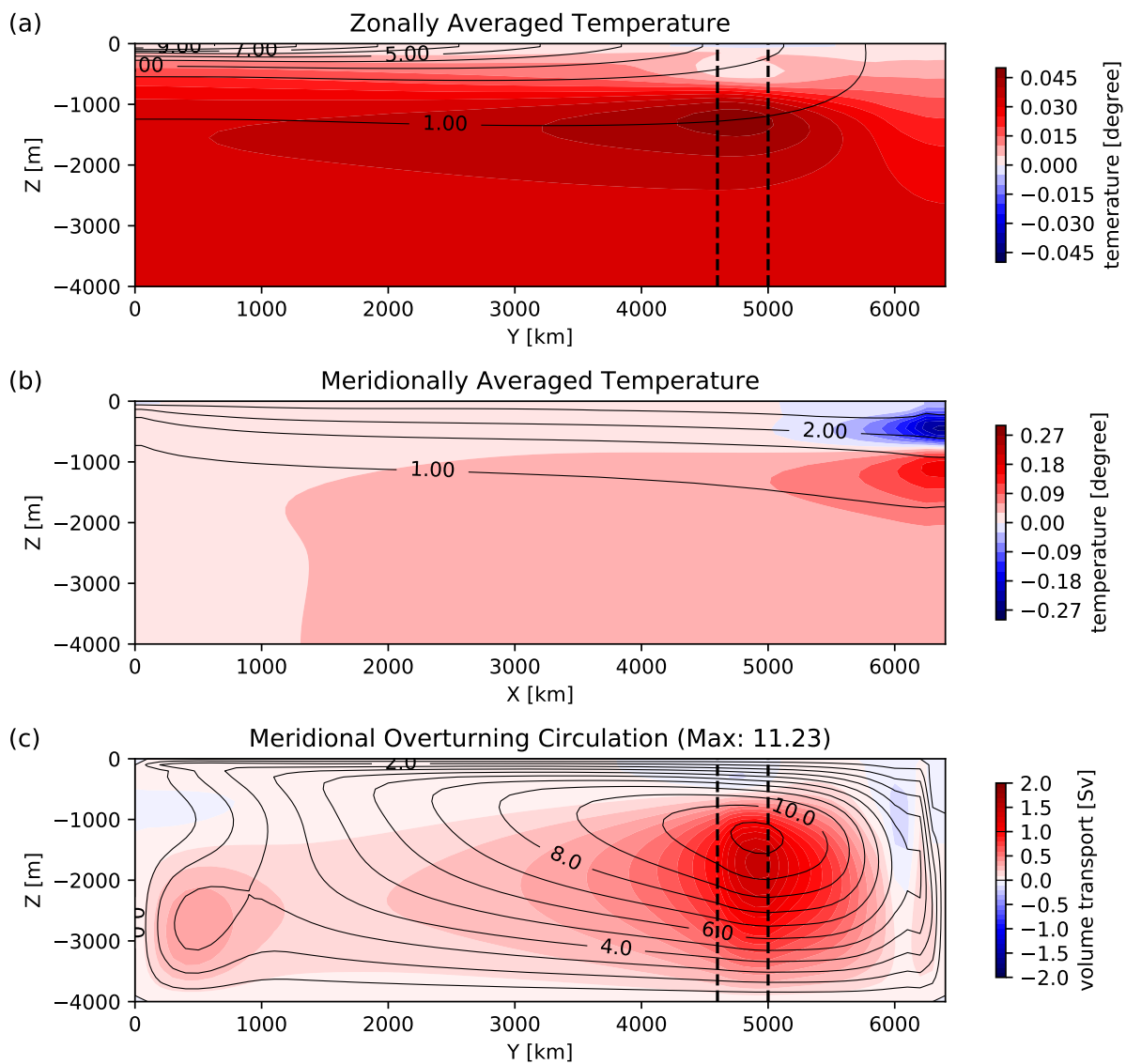


FIG. 6. Same as Figure 4 but for the E experiment.

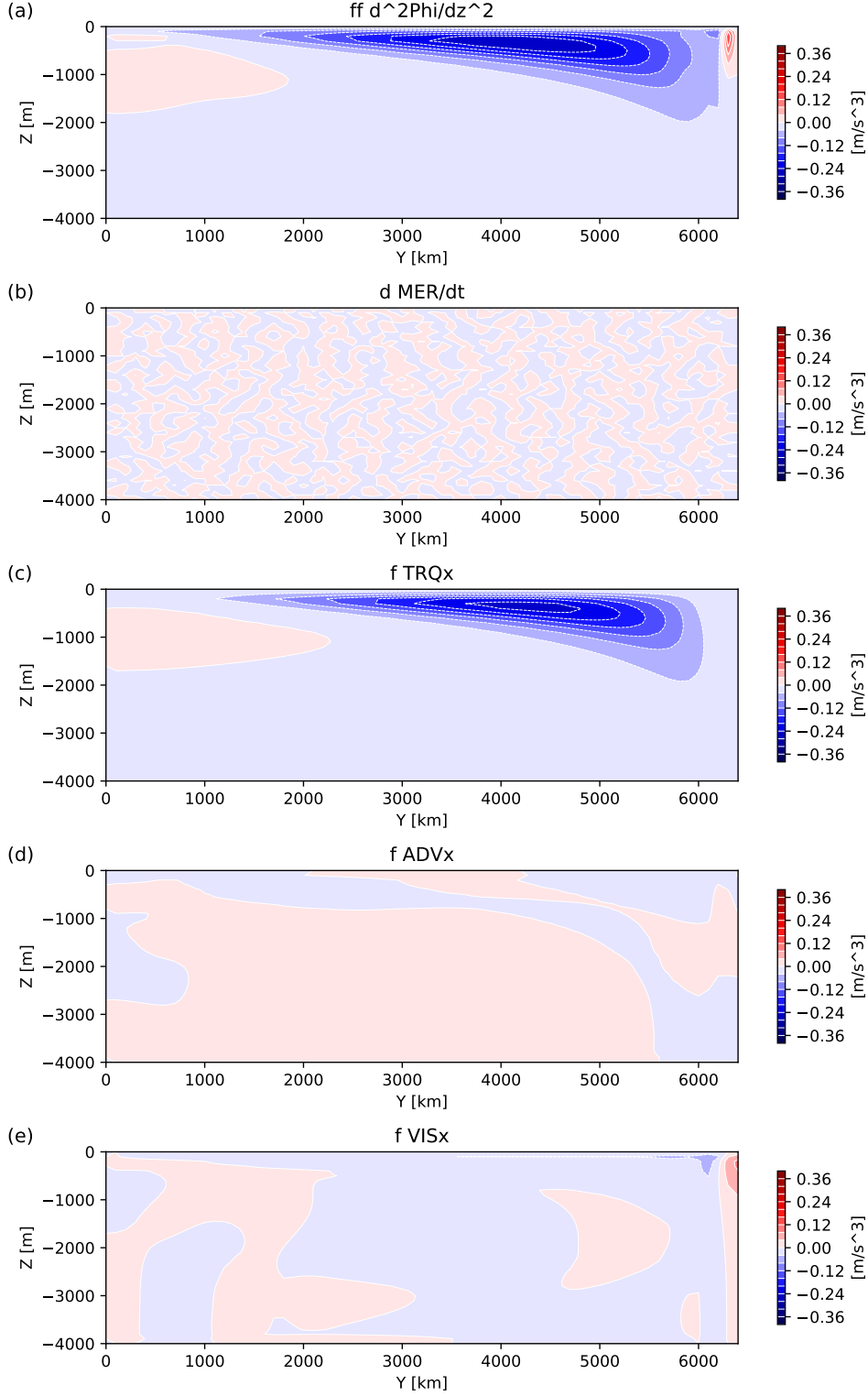


FIG. 7. Terms in Eq. (17): (a) the left-hand side term ($f^2 \partial \Phi / \partial z^2$, second time derivative ignored), (b) the first line in the right hand side, (c) the first term ($-f \alpha g(T_E - T_W)$), (d) the second term ($-f \int_0^L \partial \mathcal{A}(u) / \partial z dx$), and (e) the last term ($f \int_0^L \partial \mathcal{V}(u) / \partial z dx$) in the second line in the right hand side .

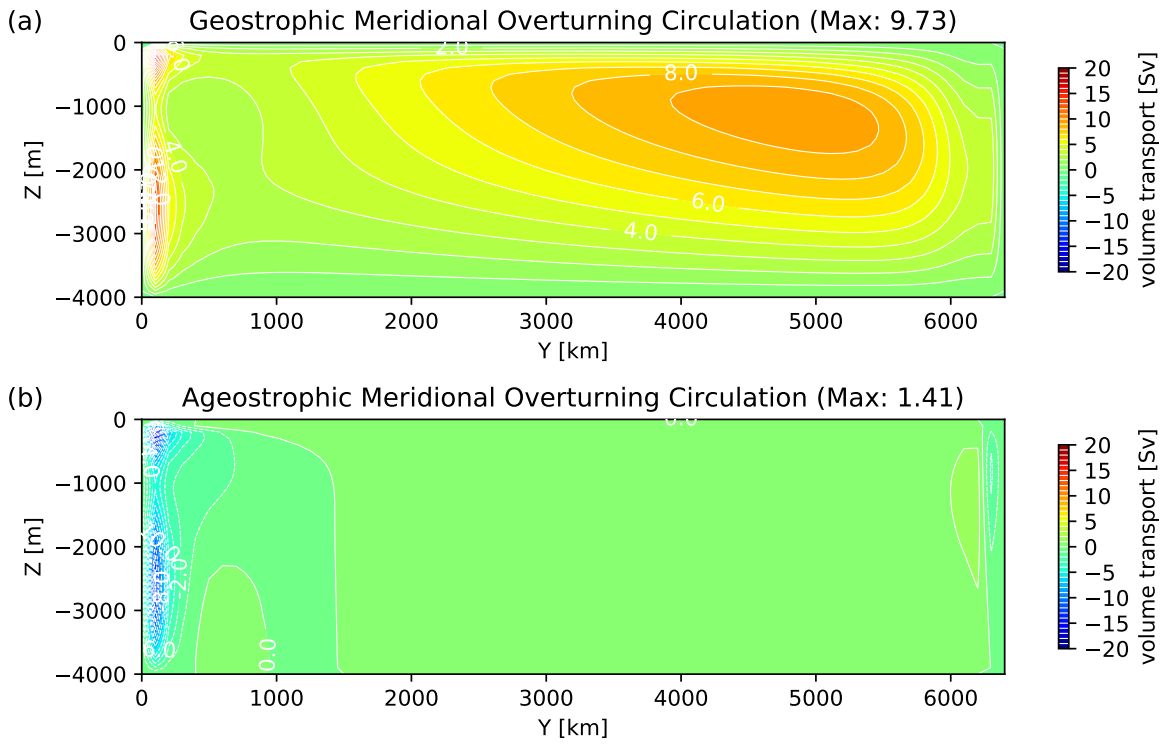


FIG. 8. Same as Figure 2c but for (a) geostrophic component (Φ_g) and (b) ageostrophic component (Φ_a).

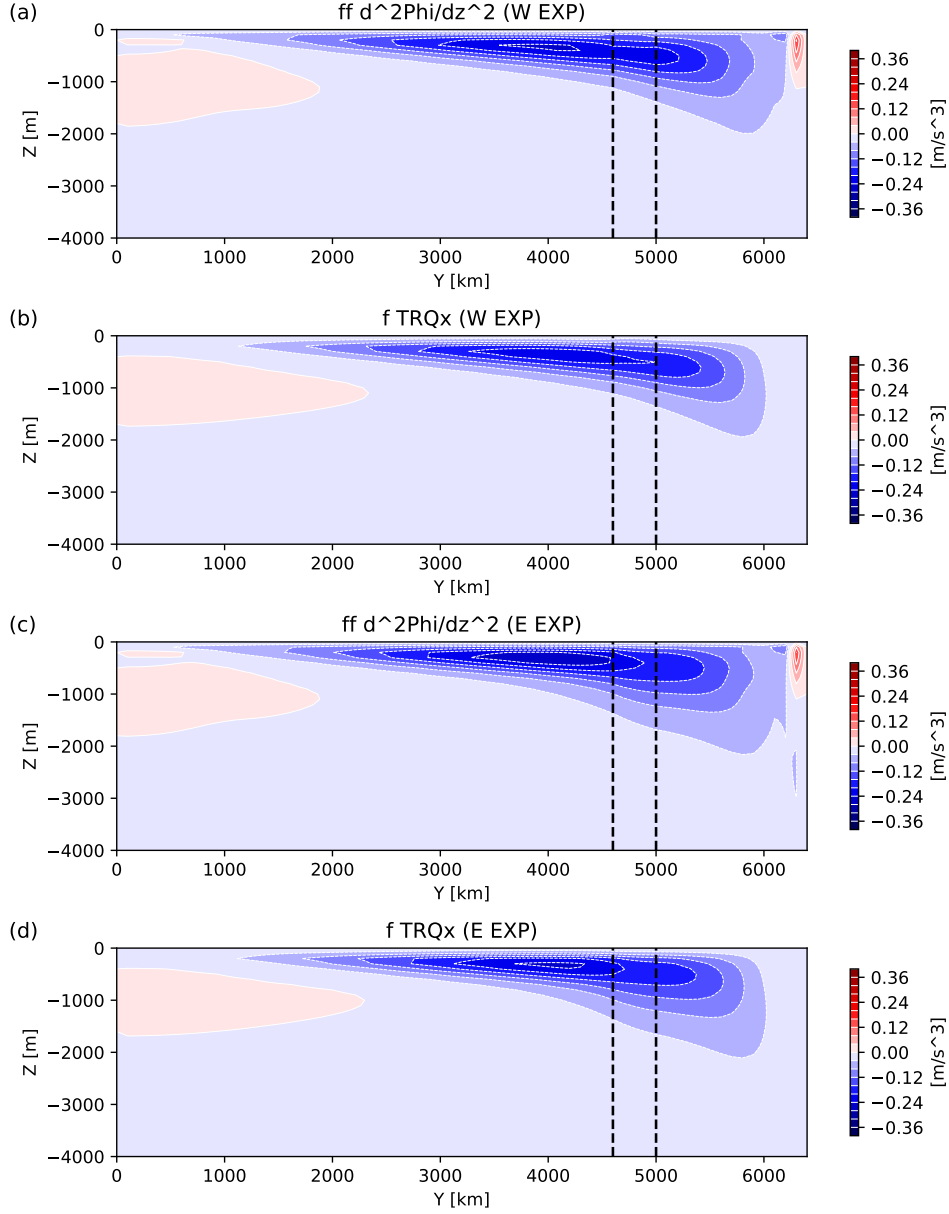


FIG. 9. Same as (a,c) Figure 7a and (b,d) Figure 7c but for (a,b) the W experiment and (c,d) the E experiment.

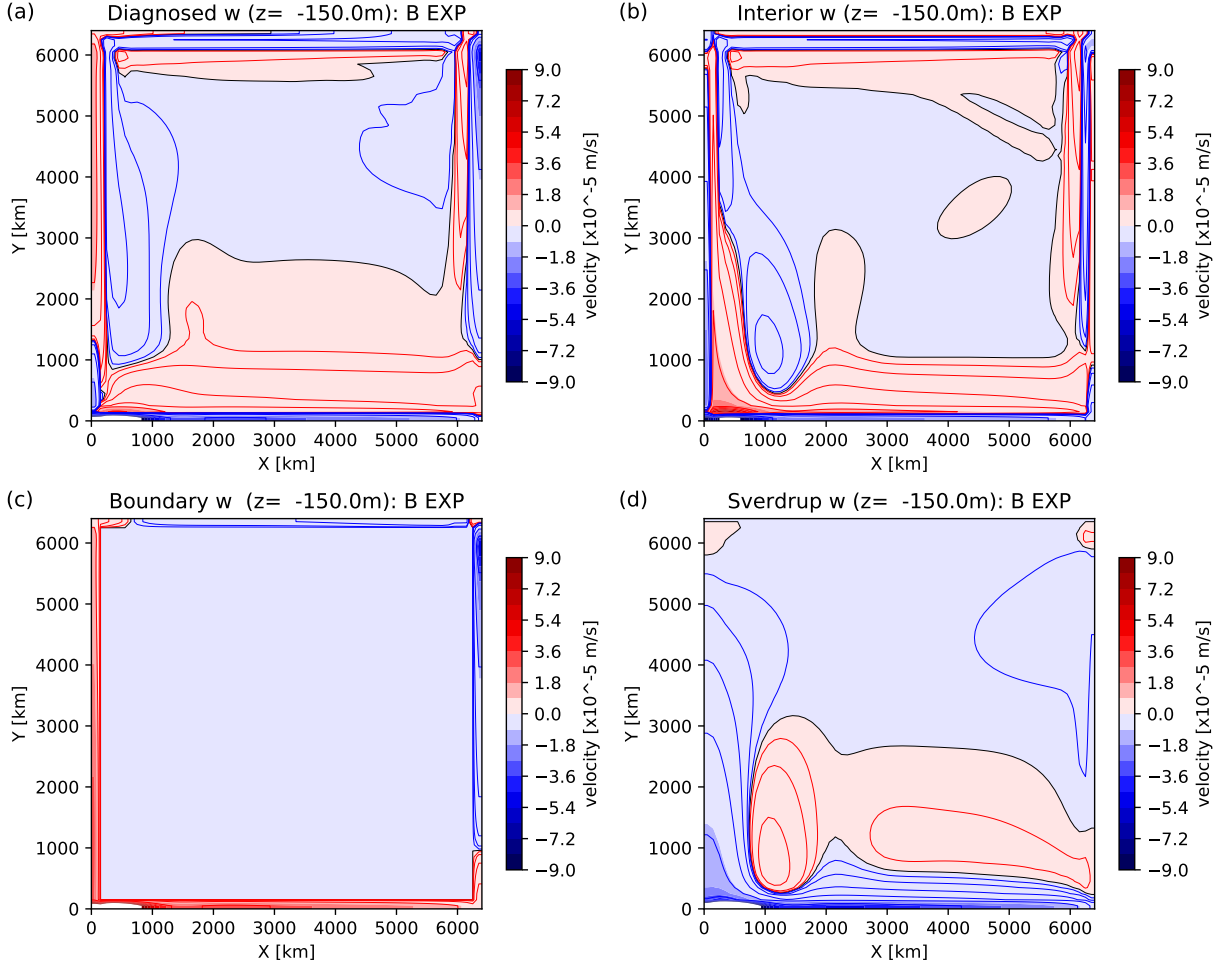


FIG. 10. Diagnosed vertical velocities from Eq. (24): (a) total (the left hand side), (b) interior ageostrophic component (the first term of the right hand side evaluated at interior grid points), boundary ageostrophic component (the first term of the right hand side) evaluated at boundary grid points, and (d) Sverdrup component (the second term of the right hand side). Color and contour intervals are same as those in Figure 1c.

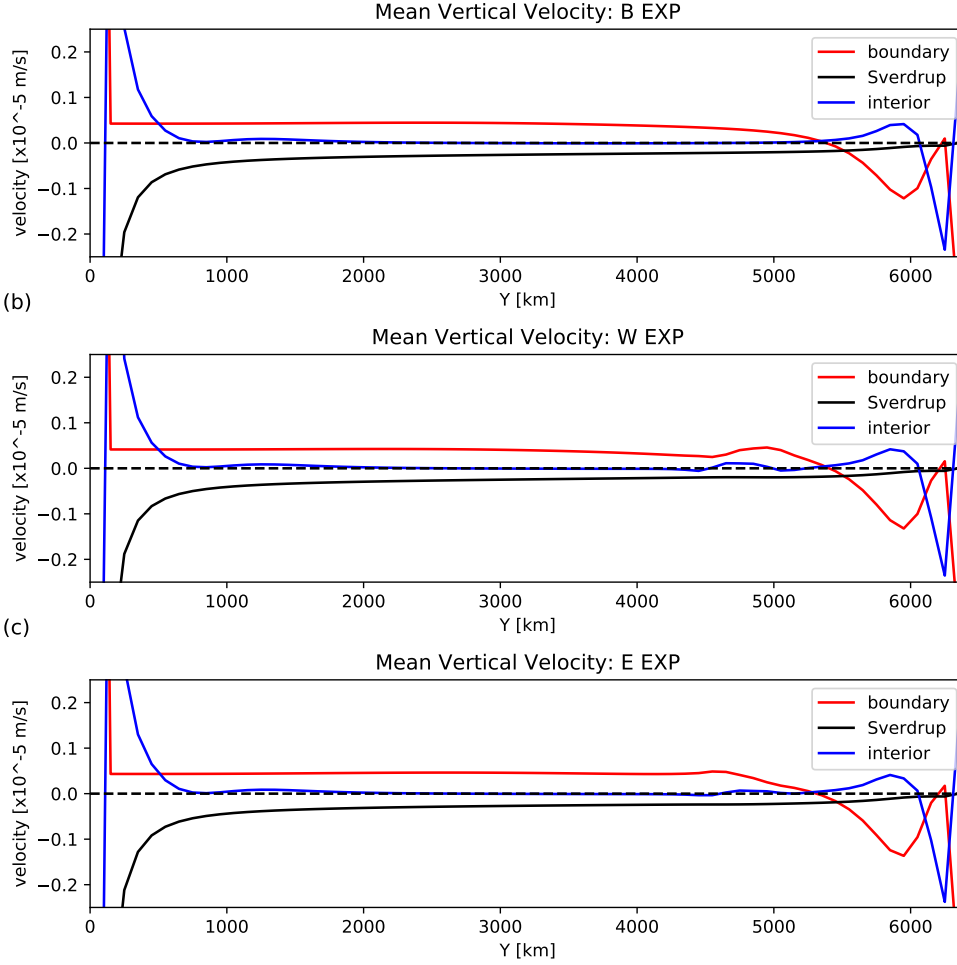


FIG. 11. Meridional profiles of the mean vertical velocity components (interior ageostrophic, boundary ageostrophic and Sverdrup components) averaged over zonal - vertical ($x - z$) section. (a) B experiment, (b) W experiment, and (c) E experiment. Large values at the southern end ($y = 0$) were errors due to small f .

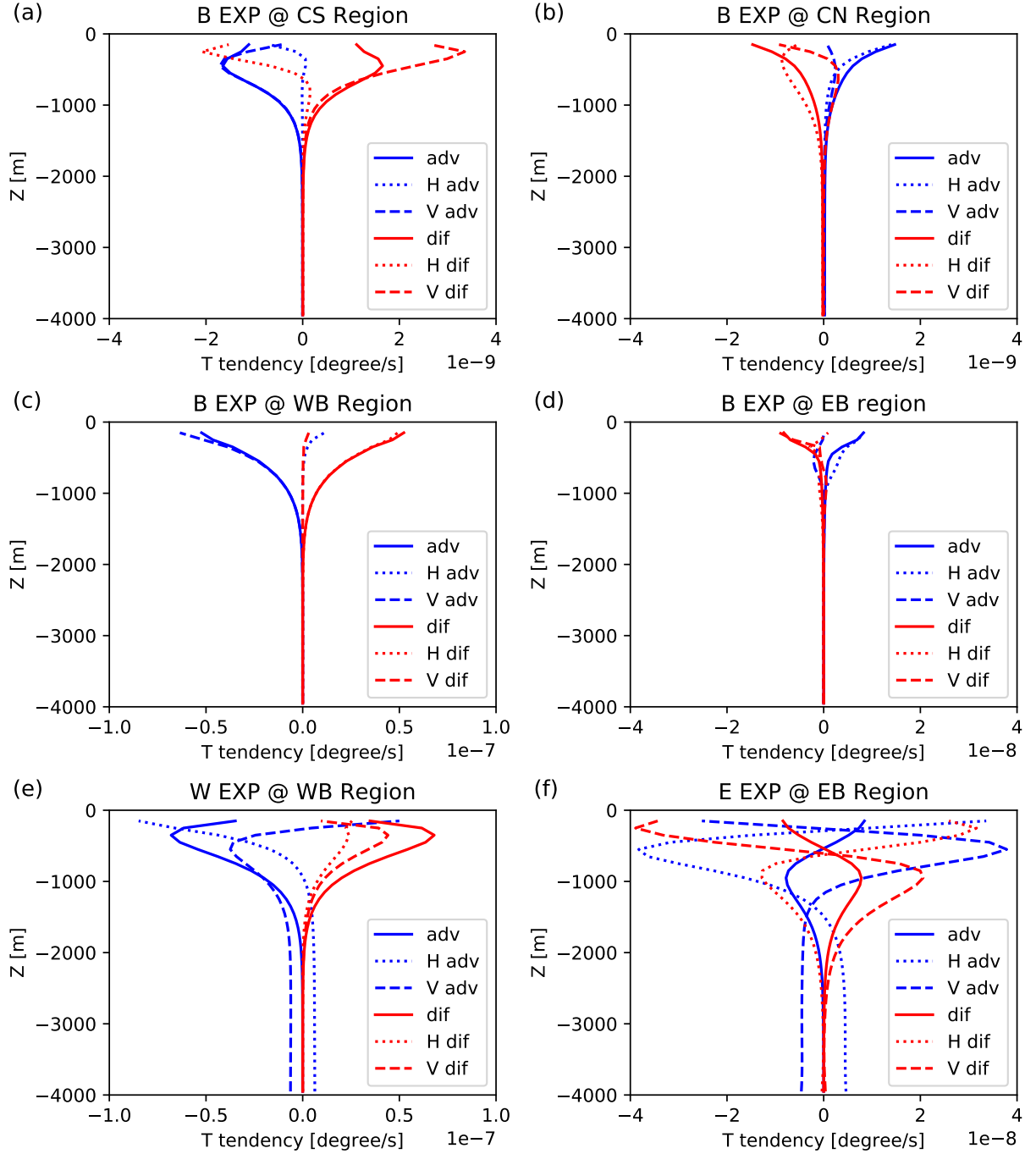


FIG. 12. Vertical profiles of temperature tendency terms. (a) Central southern region ($3000\text{ km} \leq x \leq 3400\text{ km}$ and $1400\text{ km} \leq y \leq 1800\text{ km}$), (b) central northern region ($3000\text{ km} \leq x \leq 3400\text{ km}$ and $6200\text{ km} \leq y \leq 6600\text{ km}$), (c) in the western boundary region ($x \leq 200\text{ km}$ and $6200\text{ km} \leq y \leq 6600\text{ km}$, the intensified mixing region in the W experiment) and (d) in the eastern boundary region ($6200\text{ km} \leq x$ and $6200\text{ km} \leq y \leq 6600\text{ km}$, the intensified mixing region in the E experiment) of the B experiment. (e) Same as (c) but for the W experiment. (f) Same as (d) but for the E experiment. Blue and red lines represent advection and diffusion terms, respectively. Dotted and dashed lines are horizontal and vertical components, respectively, while solid lines are sum of the components.

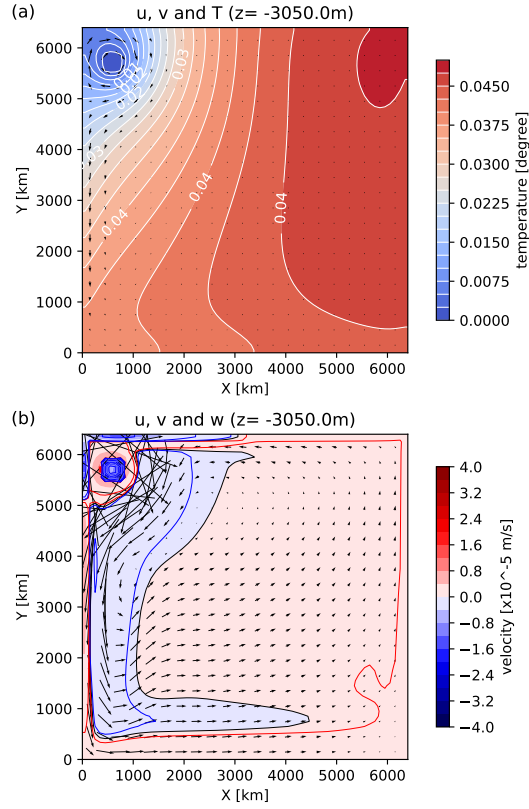


FIG. 13. Horizontal distributions at $z = -3050\text{ m}$ of (a) temperature and (b) vertical velocity with horizontal velocity vector in the SA experiment. Arrows in (b) is magnified by 10 times for better identifying weak interior flows.

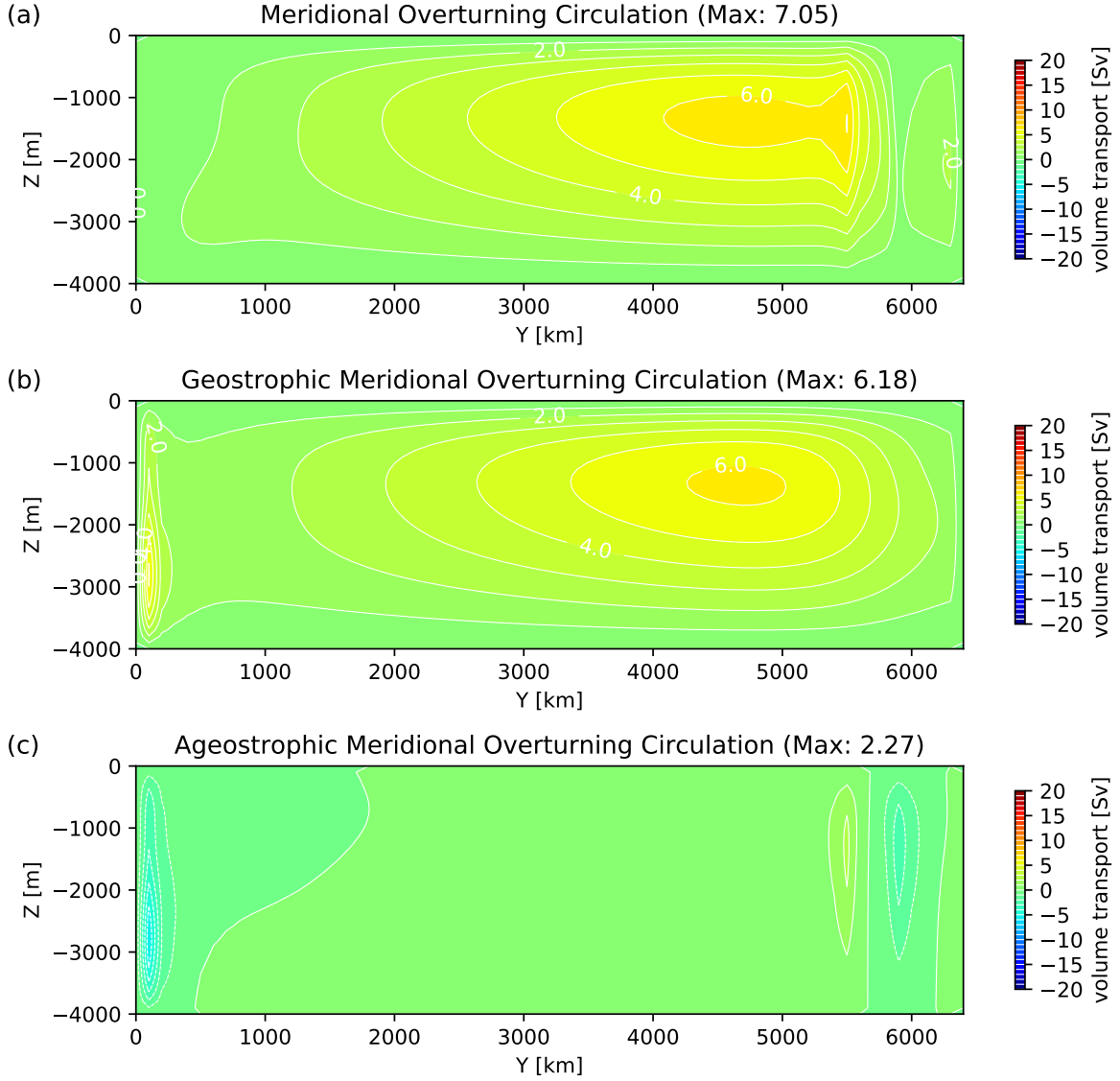


FIG. 14. Meridional volume transport functions in the SA experiment. (a) Total (Φ). (b) Geostrophic component (Φ_g). (c) Ageostrophic component (Φ_a).

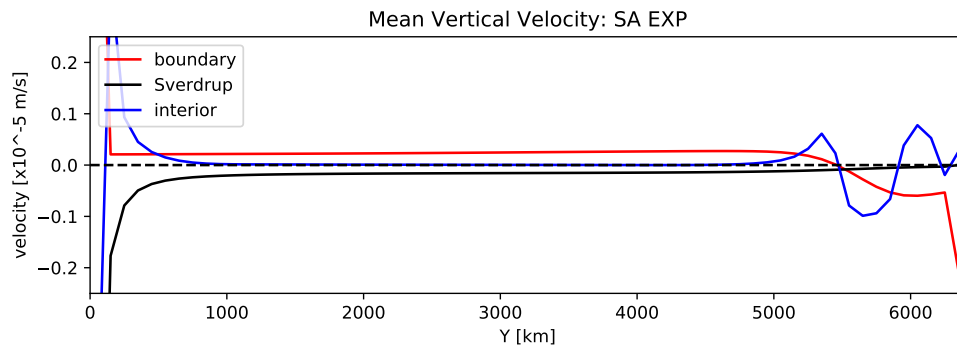


FIG. 15. Same as Figure 11 but for the SA experiment.

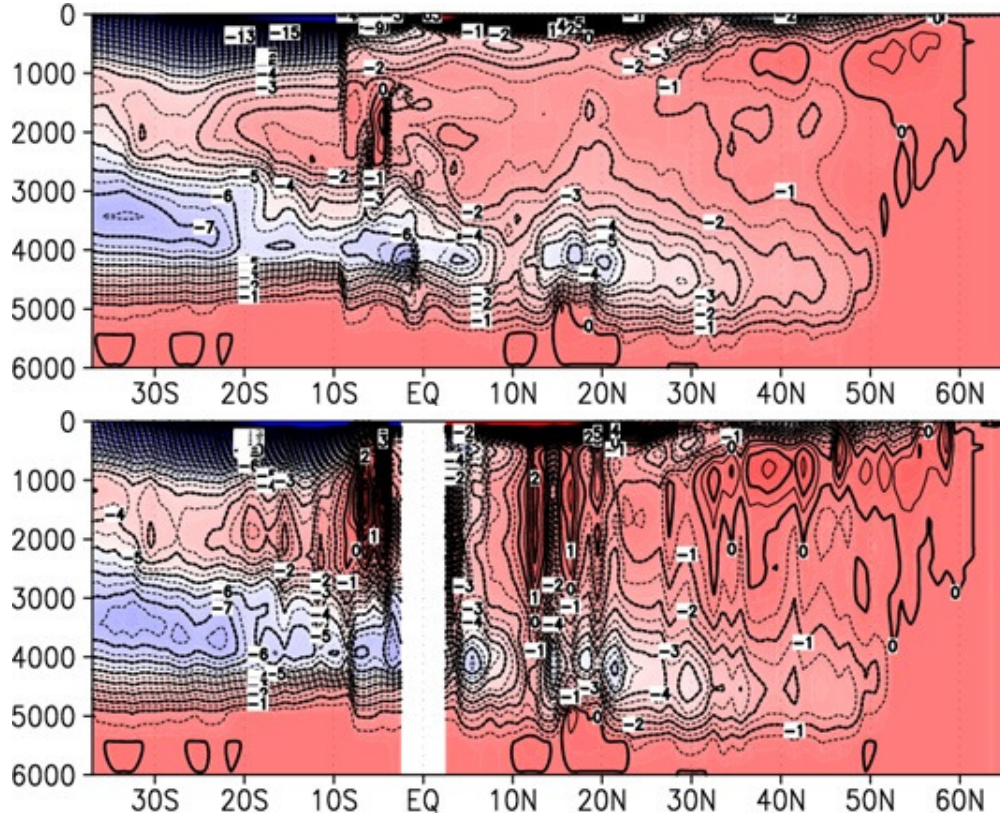


FIG. 16. MOCs in the Pacific simulated in the realistic OGCM of Kawasaki et al. (2021). (a) Total component
 (b) Geostrophic component (Φ_g).