

Statistical closure using bulk properties for
turbulent shear flows

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Of a variety of turbulence models, the k- ϵ model is widely used for engineering purposes. Its importance is diminishing, however, from the viewpoint of scientific turbulence research. This fact is attributed to the deficiency that the k- ϵ model cannot predict anisotropy of turbulent intensities accurately.

Previously, the author studied the Reynolds stress with the aid of a two-scale direct-interaction approximation (TSDIA).¹ This formalism is founded on an asymptotic expansion based on a scale parameter distinguishing the slow variation of mean flows from the fast variation of turbulent fluctuations. Consequently, TSDIA may be regarded as a derivative expansion formalism incorporated with DIA^{2,3} by Kraichnan (for details about the structure of scale expansion in TSDIA, see Ref.4). For the Reynolds stress $R^{\alpha\beta}$, TSDIA gives

$$\begin{aligned} R^{\alpha\beta} &\equiv - \langle u'^{\alpha} u'^{\beta} \rangle \\ &= - \frac{2}{3} k \delta^{\alpha\beta} + \nu_e \left(\frac{\partial U^{\alpha}}{\partial x^{\beta}} + \frac{\partial U^{\beta}}{\partial x^{\alpha}} \right) + \tau^{\alpha\beta} \end{aligned} \quad (1)$$

($\langle \rangle$ denotes the ensemble mean). Here, $\vec{U}$ and $\vec{u}'$ are the

ensemble mean and fluctuation of velocity, k is the turbulent kinetic energy ($k \equiv \langle u'^a u'^a \rangle / 2$; the summation convention is applied to repeated superscripts), and $\delta^{\alpha\beta}$ is the Kronecker delta symbol. The first two terms on the right-hand side give the familiar eddy-viscosity representation. The appearance of isotropic eddy viscosity ν_e is due to an assumption about isotropy of the basic field or the lowest-order term in the two-scale expansion of the fluctuating field. On the other hand, $\tau^{\alpha\beta}$ expresses the deviation from the isotropic eddy-viscosity representation (the details are given in Ref.1). Equation (1) was incorporated into the k - ϵ model to be applied successfully to the study of anisotropic turbulent intensities in channel flows.⁵ This improvement of the k - ϵ closure for turbulent shear flows indicates a possibility that models of k - ϵ type as well as the second-order models can remain a useful method for simulating complicated flows such as recirculating flows.

At this time, it is a major concern about turbulence modeling (including the second-order models) that no statistical foundation has yet been given to a model equation for ϵ (the dissipation rate of turbulent kinetic energy), unlike a model equation for k . In reality, the structure of the transport equation for k is simple, and its modeling is not difficult. Some theoretical support can be given to the modeling by using turbulence theories such as DIA.⁶

On the other hand, the structure of the transport equation for ϵ is extremely complicated. As a result, the

ϵ equation in the k - ϵ model has not been constructed statistically so far on the basis of the exact transport equation for ϵ . This situation will become a big obstacle when we desire to incorporate anisotropic effects such as and buoyancy force into the ϵ equation. This point is a major motivation of present work.

In the present paper, we can make use of an analytical expression for k from TSDIA to show that a model ϵ equation is derived with the aid of the concept of transferability of model. The model equation is very similar to the counterpart in the k - ϵ model.

The details of this work are given in Ref.7.

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