

FLOW ON A SINGLE ROCK FRACTURE IN THE SHEAR PROCESS AND THE VALIDITY OF THE CUBIC LAW EXAMINED THROUGH EXPERIMENTAL RESULTS AND NUMERICAL SIMULATIONS

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ABSTRACT

In order to discuss the flow through a rough surface rock fracture, direct shear and flow coupling tests on various rock fractures are carried out. Through the experimental results, the flow on the fracture and the validity of the cubic law are discussed and estimated. As a result, it is understood that the cubic law cannot sufficiently explain the experimental results, although it can partially take into account the experimental results in the Re range of 0.02 to 0.2. Moreover, a simulation by a 2D model, including the inertia term, has been carried out. The 2D model has been found to show a good performance of the fracture flow in comparison with the representative fracture flow model, namely, LCL (local cubic law). The flow on the fracture shows the channeling flow and a higher Re value. Consequently, the flow on the fracture is thought to be caused by the influence of the inertia term.

Key words: cubic law, direct shear test, hydraulic conductivity, numerical simulation, rock fracture/discontinuity (IGC: F4)

INTRODUCTION

The estimation of the fracture flow on rock masses is important for discussions in many areas of geo-science, such as oil recovery, geothermal energy extraction, groundwater flow, and the underground disposal of hazardous wastes and nuclear wastes. The later field requires a detailed understanding of the groundwater flow in order to evaluate the performance of planned underground structures and the modelling of the transport/immigration of underground pollutants. This flow dominantly occurs in the interconnected system of fractures. However, the theory of the flow through a rough surface rock fracture has not been fully developed, due to the difficulty in defining the geometry of the fracture and in understanding the flow mechanics through an individual fracture.

One of the most fundamental and serious problems is how the water behaves and is transported through a single fracture. Initially, flows in fractures were visualized conceptually as flows between parallel plates separated by an aperture. In the parallel plate model, an individual fracture is represented by two infinite smooth parallel plates, and the flow is assumed to be laminar with a parabolic velocity distribution across the aperture. This is the well-known cubic law (Snow, 1965; Iwai, 1976). The parallel plate representation is too idealized, however, since there are generally voids and contacts in the planes of real frac-

tures.

In considering the effect of aperture variation, the Reynolds equation is used to estimate the pressure distribution. Then, the cubic law is assumed to be void at local points in the fracture, and therefore, used to calculate the discharge. This is the local cubic law (LCL) (Walsh, 1981; Brown, 1987; Zimmerman and Bodvarsson, 1996; Renshaw, 1995; Brush and Thomson, 2003). The Reynolds equation is derived from the Navier-Stokes equation which assumes that most flows in real fractures are laminar and occur with a Reynolds number, namely, $Re < 1$ (Zimmerman and Bodvarsson, 1996). As a result, it is believed that inertia terms are negligible in comparison to the viscous terms. On the other hand, it is also assumed that the aperture variation occurs gradually, and therefore, the effect of the characteristic length of the aperture variation is also negligible. Furthermore, the pressure across the aperture is considered to be constant. In fact, the Reynolds equation only takes into account the effect of aperture variation. As a result, its applicability is limited.

In this study, direct shear and flow coupling tests on various rock fractures are carried out in order to discuss the flow through a rough surface rock fracture. The numerical simulation technique, including the inertia term, is also applied to the experimental conditions. Through the experimental and the numerical results, the flow of the rough surface fracture and the validity of the cubic

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law are discussed and estimated.

DIRECT SHEAR-FLOW COUPLING TESTS ON A SINGLE FRACTURE

Experimental Apparatus

In order to clarify the hydraulic characteristic for a single fracture under the shear process, a shear tester is employed (Ohnishi et al., 2000). It has been improved to add the flow system and a system for preventing water leakage around the specimen so as to enable the conducting of permeability tests on the fracture (Yano et al., 2005). Figure 1 shows the entire outline of the direct shear tester and Fig. 2 shows the shear box.

The shear unit of the apparatus in Fig. 1 is applied to an electric and oil pressure servo mechanism which includes a feedback system. The shear load is controlled by the servo mechanism and the computer. Four items, namely, shear (horizontal) load, shear displacement, normal (vertical) load, and normal displacement, are measured in the shear unit, and the obtained data are automatically recorded into the computer. The loads are measured by two types of load cells, MTS MODEL 661,238-01 by the vertical load and TCLU-20A by the horizontal load. In the experiments, the shear area on the single fracture is reduced by the shear process. So as to maintain the constant normal confining condition, the normal load is controlled by considering the reduction in shear area. An electric gap sensor, HA-162S-91108, is employed to measure both vertical and horizontal displacements.

On the other hand, the shear box shown in Fig. 2 has the functions of seepage and water leak prevention. In order to prevent water leakage, a gel sheet is applied in the space between the specimen and the shear box as a sealer. The sheet is designed in such a way as to prevent water leakage between the gap of both upper and lower shear boxes without disturbing the shear deformation, namely, without friction. Resolution of permeability test is 0.05 cc/mv.

Specimen

After choosing three kinds of natural joint surface roughness (Roughness *A*, *B*, and *C*), impressions are made of them. Using the impressions, reproduced plaster specimens are then created. Firstly, in order to make one side of each artificial specimen, cement mortar is poured into a mold from which an impression of the rough joint surface is taken. Using this side of the artificial specimen, an impression of the other side is made. Then, the sides of the other artificial specimens are created according to the same process. The specimens are cured in water for 28 days. The artificial specimens used in this research are rectangular prisms for which the cross section is 120 × 80 mm (80 mm is the flow direction) and the height is 120 mm. Each specimen contains a single joint, which is located at the center (lengthwise) of the specimen and is approximately aligned on the horizontal plane. Three types of reproduced specimens, representing three levels of

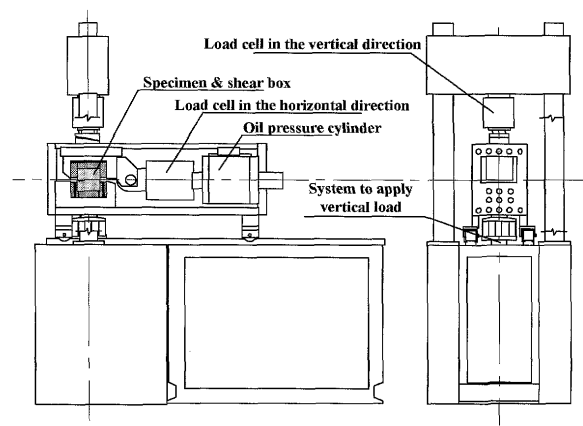


Fig. 1. Outline of the direct shear tester for a single fracture (Ohnishi et al., 2000)

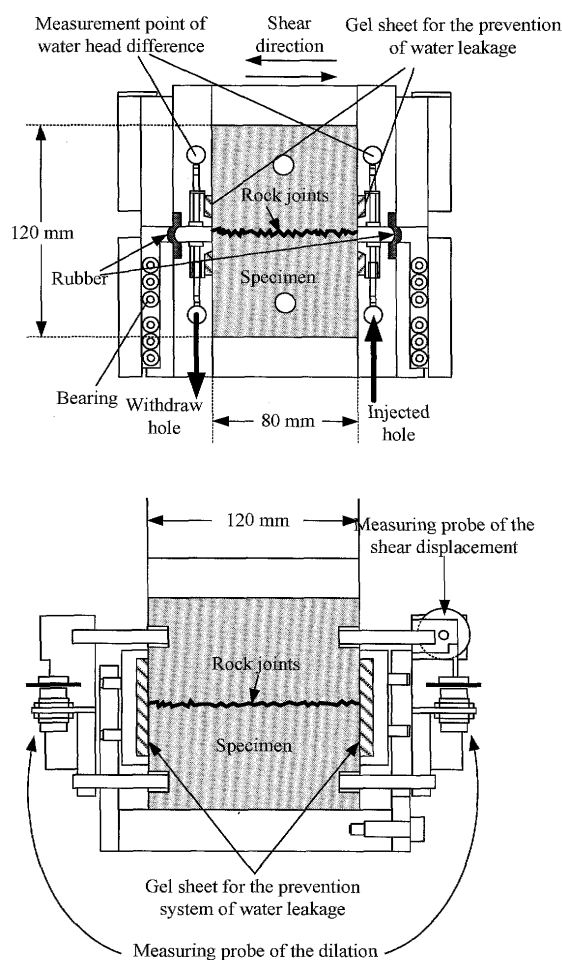


Fig. 2. Shear box

uniaxial compressive strength, are made by adjusting the amount of kaolin added. High- and early-strength Portland cement and silica sand number 6 are used. The combination ratio of cement, sand, and distilled water is 1:2:0.65. In one case, kaolin is not added at all. In the other two cases, the amounts of cement and kaolin added are as shown in Table 1 and more distilled water is added.

Table 1. Material properties of the specimens

Combination ratios for cement, sand, distilled water, and kaolin		1:2:0.65:0	1:2:0.917:0.5	1:2:1.184:1
Uniaxial compressive strength σ_c [MPa]		37.33	21.78	13.84
Basic friction angle ϕ_b [degree]		37.3	34.9	34.4
Specimen's name	Roughness A ($JRC=5.51$)	A-K	A-L	A-M
	Roughness B ($JRC=10.68$)	B-K	B-L	B-M
	Roughness C ($JRC=12.21$)	C-K	C-L	C-M

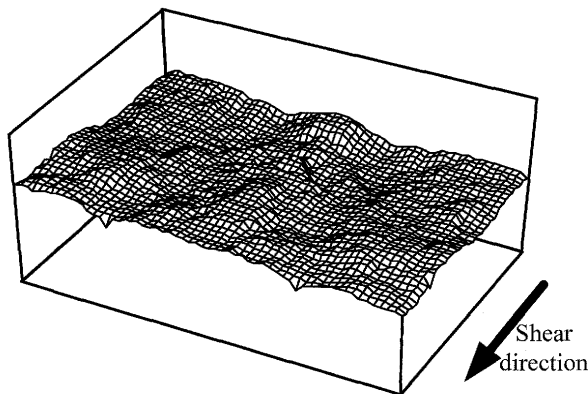


Fig. 3. Bird's eye view of Roughness A through the profiling data

All the combination ratios are presented in Table 1. The uniaxial compressive strength and the basic friction angle are also presented in this table.

Measuring the roughness and the contact points between the two surfaces is a very challenging yet an important task which is necessary for discussions on mechanical behaviour of rock joints. In this paper, after presenting the shear behaviour using the mechanical shear model (Kishida and Tsuno, 2001), the 3-D roughness profiling system (Ohnishi et al., 2000) is adopted to profile the joint surface roughness before and after performing the tests. This system consists of an X - Y positioning table, with a positioning accuracy of $\pm 15 \mu\text{m}$ and a reposting accuracy of $\pm 15 \mu\text{m}$, and a laser scan micrometer, with a maximum resolution of $0.5 \mu\text{m}$, a measurement allowance spot dimension of $4.0 \times 20 \mu\text{mm}^2$, and a measurement range of $\pm 8 \text{ mm}$. The rough joint surface is measured at 0.25 mm intervals. The profiling data is accumulated and fed into the computer as the digital format. A sample of an artificial specimen and its surface geometry (Roughness A) are shown in Fig. 3. Based on the digital data on the joint surface roughness, Barton's JRC (Joint Roughness Coefficient) (Barton, 1973; Barton and Choubey, 1977) for each specimen is calculated using the relationship between JRC and Z_2 (Tse and Cruden, 1979; Yu and Vayssade, 1991). The calculated JRC value for each specimen is 5.51 (A), 10.68 (B), and 12.21 (C), respectively.

Experimental Conditions

In this paper, nine kinds of specimens, namely, three

types of joint surface roughness and three kinds of material strength are employed in the laboratory direct shear and permeability coupling tests. Each specimen's name is shown in Table 1. The tests were performed at each predetermined shear displacement up to 3 mm , keeping the constant hydraulic head and the constant normal confining stress at 1.0 m and 1.0 MPa , respectively. The measured parameters in this experiment are the shear displacement, the shear stress, the normal displacement, the normal stress, the discharge, and the hydraulic pressure difference. Before starting the direct shear tests, cyclic loading and unloading tests, called roughness stiffness loading and unloading tests, were performed three times. The normal stress range of the cyclic loading and unloading is from 0.25 to 4.0 MPa . The purpose of performing the roughness stiffness loading and unloading tests is described as follows:

- At the initial set-up of the specimen, the joint surface roughness does not match perfectly. Therefore, the match of the joint surface roughness is improved in order to allow the application of the normal confining pressure.
- The aperture does not become zero, although several levels of normal confining pressure are applied. In order to estimate the aperture at the initial condition, normal cyclic loading and unloading tests are carried out. Then, the relation between the normal stress and the normal displacement of a single fracture is obtained. The shear speed of 0.1 mm/min is employed in the shear process of each specimen. And, the seepage direction is in agreement with the shear direction.

Estimation of the Initial Aperture

Considering the permeability of a single rock fracture, the aperture is one of the representative factors which control the observed discharge. Dilation, which is measured in the shear process on the rock joint, is the relative vertical displacement against the initial set-up condition of the upper specimen and is not the aperture. In this research work, the geometrical (mechanical) aperture is defined as the combined values of initial aperture and observed dilation. The initial aperture is determined by the contact condition between the upper specimen and the lower specimen under various normal confining conditions before starting the shear tests. Therefore, the determination of the initial aperture plays an important role in estimating the mechanical aperture in the shear process

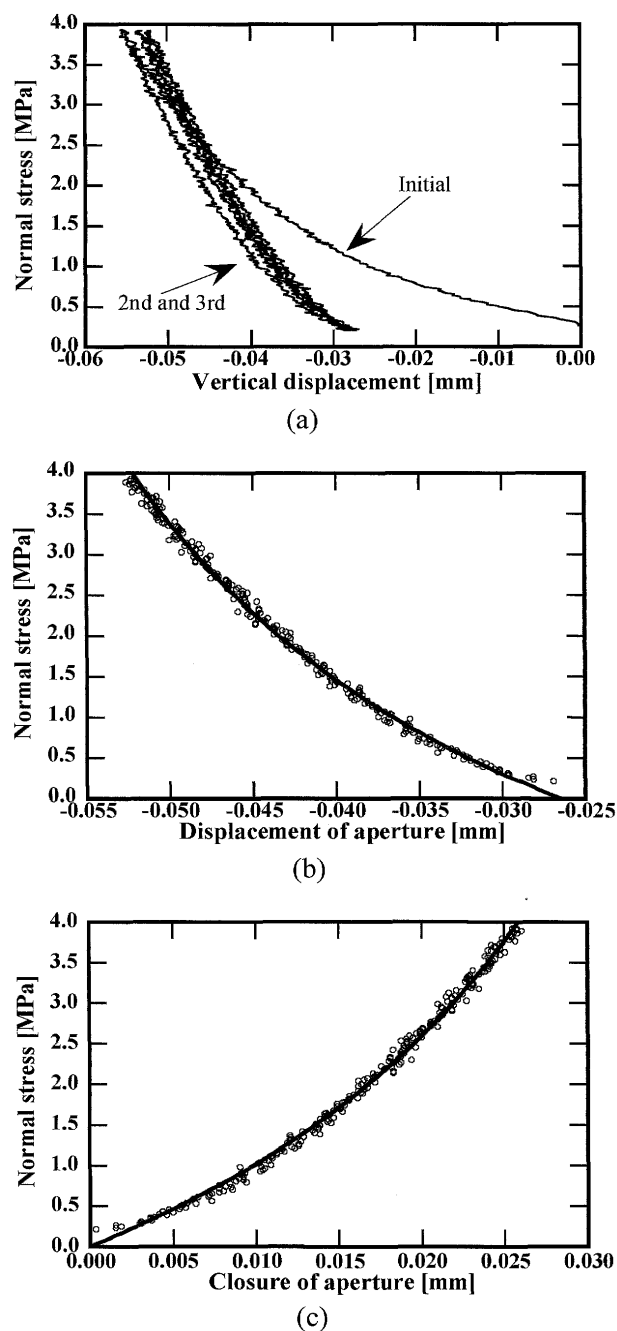


Fig. 4. Sample of cyclic loading-unloading experiments and the determination of the initial aperture: (a) Normal stress-vertical displacement relation through cyclic loading-unloading experiments (Specimens A-K), (b) 3rd loading-unloading curve obtained through (a) and an approximation of the hyperbolic curve and (c) Normal stress-closure of the aperture relation and applying Eq.(1)

with accuracy.

A sample of the roughness stiffness loading and unloading tests mentioned in *Experimental Conditions* is shown in Fig. 4(a). In this figure, with increments of normal stress, the normal displacement proceeds in a negative direction and the aperture is closed. In this research, it is assumed that the normal displacement is related to the changes in the aperture on the rock joint, and that the changes in the normal displacement are equal to the changes in the aperture. From Fig. 4(a), it can be con-

firmed that the initial loading and unloading process is different from the latter loading and unloading process. In the initial loading and unloading process, it is thought that the influence of matching the upper and the lower specimens is revealed. The aperture is determined using 3rd cycle of loading and unloading process without the initial contact error shown in Fig. 4(b).

The relationship between the changes in aperture and the normal stress can be approximated using the following expression (Bandis et al., 1983):

$$\sigma_n = \frac{v_j \cdot k_{ni} \cdot V_{mc}}{V_{mc} - v_j} \quad (1)$$

where, σ_n , v_j , k_{ni} , and V_{mc} are the normal stress, the closure of the aperture, the normal stiffness of the discontinuities, and the maximum closure of the aperture, respectively. Here, Bandis et al. did not apply to the changes in aperture. They applied the closure of the aperture and defined that the closure of the aperture was zero at a normal stress of zero.

EXPERIMENTAL RESULTS AND DISCUSSIONS

Shear Behaviour

Figure 5 shows the shear stress - shear displacement relations and the dilation - shear displacement relations for specimens with three types of joint surface roughness whose intact material is 37.33 MPa in uniaxial compressive strength. Figure 5(a) shows the relation between the shear stress and the shear displacement, while Fig. 5(b) shows the relation between the dilation and the shear displacement. In the dilation - shear displacement relation, a tendency can be found for the dilation to increase with increments in *JRC*. For the specimen with a large *JRC*, a relatively large number of asperities of the large dip angle exist. The shear behaviour controls the large dip angle's asperity, and it can be found that a large dilation is generated in the experiments. Figure 6(a) shows the peak shear strength or the residual stress at a shear displacement of 3 mm and the uniaxial compressive strength relation. Figure 6(b) also shows the peak shear strength or the residual stress at a shear displacement of 3 mm and the *JRC* relation. From these figures, a clear correlation could not be confirmed between the peak shear strength or the residual stress and the uniaxial compressive strength or *JRC*.

Permeability of Single Fracture under Shear Process

Figure 7 shows the relationship between transmissivity, T , and the shear displacement for three levels of uniaxial compressive strength. T is defined by the following equation:

$$T = \frac{Q \cdot l}{w \cdot \Delta H}, \quad (2)$$

where, Q is the discharge, l is the length of the specimen in the flow direction, w is the length of the specimen in the normal direction of flow, and ΔH is the potential head. It is found that the T of Roughness C, *JRC*=12.21, is large-

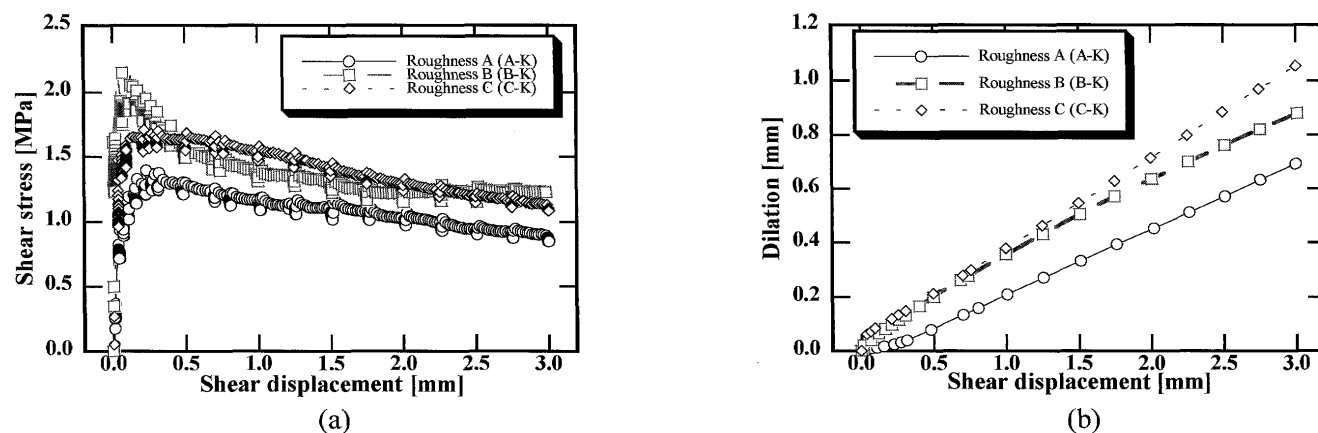


Fig. 5. Sample of the experimental results of the direct shear tests on rock joints (Uniaxial compressive strength of the material, σ_c , is 37.33 MPa): (a) Shear stress-shear displacement relation and (b) Dilation-shear displacement relation

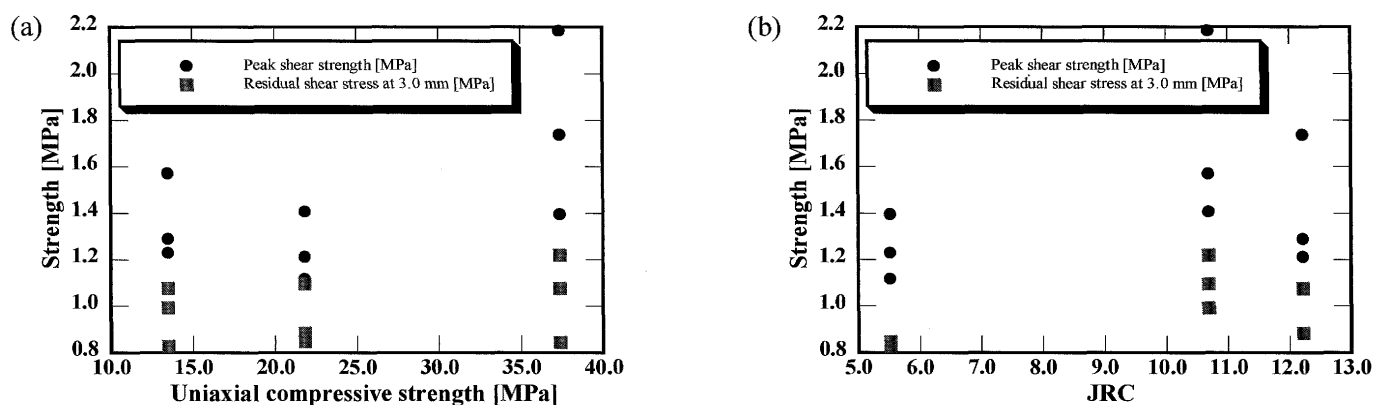


Fig. 6. Experimental results of direct shear tests on rock joints: (a) Peak and residual shear strength vs uniaxial compressive strength of the material and (b) Peak and residual shear strength vs joint roughness coefficient (JRC)

er than that of the other joint surface roughness. The relationship between Roughness A ($JRC=5.51$) and B (10.68) is only partially clear. However, it is basically thought that T increases with increments in JRC . As mentioned above and in Fig. 5, a larger dilation is generated in the case of a large JRC . With an increase in the dilation, the aperture becomes wide. Consequently, the large transmissivity of Roughness C can be confirmed. However, this tendency differs in the initial phase of the shear process. As mentioned in the sections on the experimental conditions and the estimation of the initial aperture, the contact of the joint surface roughness is inhomogeneous, although the cyclic loading and unloading process is applied before starting the shear process. Figure 4(c) shows the zero condition of the aperture. However, this aperture means the average aperture of the joint surface roughness. Actually, it is thought that the local opening apertures are inhomogeneous on the joint surface roughness and that they strongly affect the permeability of a single fracture in the initial shear process.

Figure 8 shows the relationship of T and the aperture, b . In Fig. 8, two types of apertures can be found. One is the mechanical aperture, b_m , which has been already ex-

plained in the previous section on the estimation of the initial aperture. The other is the hydraulic aperture, b_h , which will be explained in the next section. First of all, only the relation between T and b_m will be discussed. As compared with the T of each joint surface roughness in the same aperture, it can be found that T becomes small with increments in JRC . Many rough asperities exist on the joint surface roughness in the case of a large number of JRC . On the rough fracture, the flow is intercepted and changes suddenly. Finally, the flow path becomes longer than the length of the specimen. Moreover, with the complex geometry, it is thought that the loss of friction becomes large and that the hydraulic gradient becomes small. Furthermore, it is seen from Fig. 8 that T increases in each case with increments in b_m . In the b_m range of less than 0.08 mm, T does not change so much in most cases. In the b_m range of more than 0.2 mm, however, a constant increment in the T - b_m relation can be found. In the small aperture range, it is thought that the fracture flow is strongly affected by both joint surface roughness and the contact condition. As for increments in the aperture, the influences of joint surface roughness and the contact condition on the fracture flow become small.

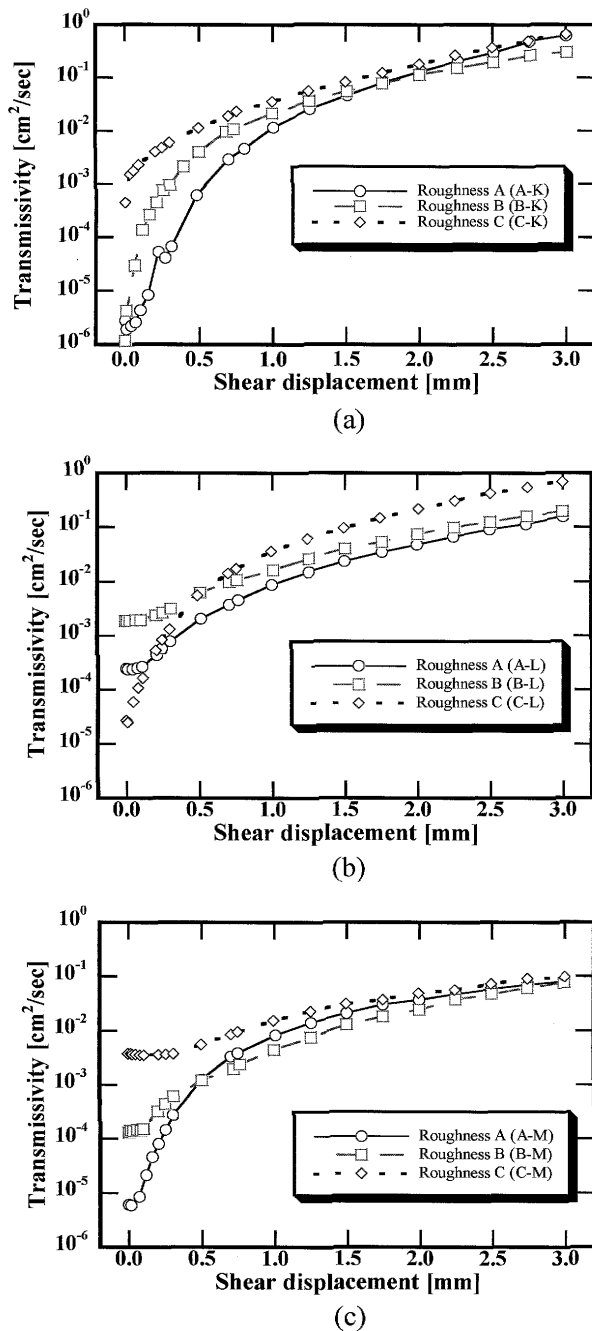


Fig. 7. Relationship between transmissivity and shear displacement: (a) σ_c is 37.33 MPa, (b) σ_c is 21.78 MPa and (c) σ_c is 13.48 MPa

Application of the Cubic Law and Its Validity

In this paper, the sum of the initial aperture which is determined by the above-mentioned method and the observed dilation is defined as the mechanical aperture. On the other hand, the hydraulic aperture is defined as the backward analytical estimation through the observed discharge and the well-known cubic law. The cubic law is based on the parallel plate model whereby an individual fracture is represented by two infinite smooth parallel plates separated by a constant distance. The flow is basically assumed to be laminar with a parabolic velocity profile across the aperture. The cubic law is expressed as follows (Snow, 1965; Iwai, 1976):

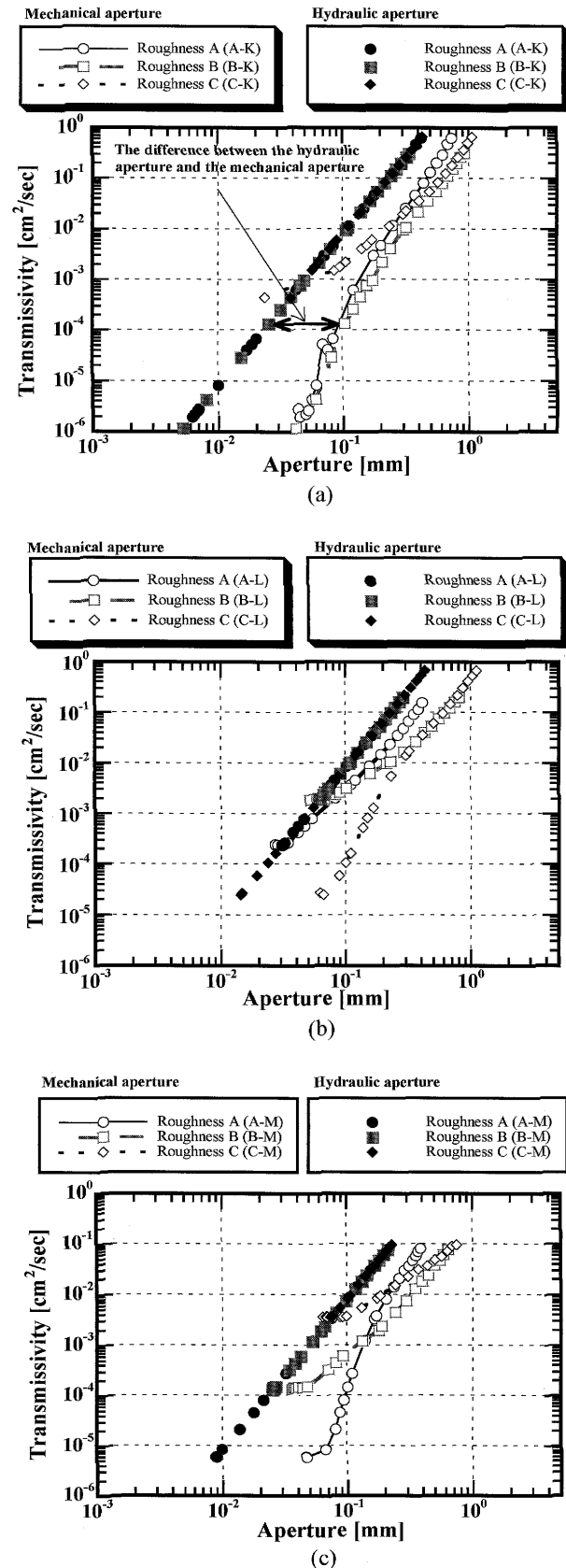


Fig. 8. Relationships between mechanical aperture and transmissivity, and hydraulic aperture and transmissivity: (a) σ_c is 37.33 MPa, (b) σ_c is 21.78 MPa and (c) σ_c is 13.48 MPa

$$\frac{Q}{\Delta H} = C \cdot b^3 \quad (3)$$

$$C = \frac{g}{12\nu} \cdot \frac{w}{l} \quad (4)$$

where b is the aperture, g is the gravitational acceleration, ν is the kinematic viscosity, w is the specimen's length in the normal direction of the flow, and l is the specimen's length in the flow direction, respectively. The other terms have been explained earlier. Using Eqs. (3) and (4), hydraulic aperture, b_h , is defined as follows:

$$b_h = \sqrt[3]{\frac{12\nu Q l}{g w \Delta H}} \quad (5)$$

Figure 8 also shows the relationship between T and b_h as well as between T and b_m . In the same transmissivity, as well as the discharge in these figures, the difference between b_m and b_h can be confirmed. In the b_m range of less than 0.1 mm, a tendency can be found for b_m to approach b_h in several specimens. It can also be found that the gradient of the T - b_m relation is quite equal to that of the T - b_h relation in the b_m range of more than 0.2 mm. In this range, b_m is basically larger than b_h .

From Figs. 5, 7, and 8, the close values between b_m and b_h can also be confirmed before the peak shear strength. In the post peak shear strength, where the range in increments in dilation as well as the range in aperture exists, it is confirmed that b_h is smaller than b_m . If b_m is equal to b_h , each transmissivity can be estimated using b_m and Eqs. (3) and (4). If the cubic law is satisfied for the fluid flow on a single fracture, b_m must be equal to b_h . In most of the experimental results, T is overestimated using the mechanical aperture and the cubic law. It is thought that the cubic law cannot necessarily be applied to the fluid flow on a single fracture.

Actually, the mechanical aperture is dilation. However, the mechanical aperture is not maintained on the whole fracture surface. There are several aperture conditions on the fracture, namely, one area has contact with the upper and lower surfaces, another area is located on a smaller aperture than the dilation, and another area is located on a wider aperture than the dilation. In fact, the aperture distribution on the fracture is inhomogeneous. Moreover, the irregular aperture distribution is not located on the flat and smooth plane, but on the rough-wall surface. As mentioned above and in Fig. 8, it is confirmed that the discharge estimated by the mechanical aperture is larger than the measured discharge. The mechanical aperture, that is, the average aperture on a fracture, cannot express the influence of the loss in fluid flow friction for the irregular distribution of the aperture. On the other hand, the hydraulic aperture cannot express kinematics information on the fracture. In fact, the information includes the energy loss of the fluid flow.

Using the artificial tension fracture, Witherspoon et al. (1980) clearly showed that the cubic law for fluid flows in a fracture, given by the following equation:

$$\frac{Q}{\Delta H} = \frac{C}{f} \cdot b^3, \quad (6)$$

is valid. f is the fracture surface characteristic factor. Their investigations covered apertures ranging from 250 to 4 μm and normal stress levels up to 20 MPa. They presented the idea that the cubic law seemed to hold regardless of the loading and unloading paths. They concluded that the hydraulic conductivity was uniquely defined by the fracture aperture and that the effects of deviations from the ideal parallel plate concept only caused an apparent reduction in flow and introduced factor f . Therefore, f is one of the significant indices which presents fracture surface roughness and the contact condition. In this paper, f is determined using the mechanical aperture b_m .

The law for the flow in fractures can be generalized in terms of the Reynolds number, Re , which is a common measure of the relative strength of inertial forces to viscous forces in flowing fluids. The Re for flow through a single fracture may be defined as:

$$Re = \frac{DV}{\nu} \quad (7)$$

where D is the characteristic length of the viscous forces, ν is the kinematic viscosity, and V is the characteristic velocity. As for D , there are a couple of concepts for the flow in fractures. One is that D is equal to 4 times the hydraulic radius, which is half of the fracture aperture (Lomitze, 1951; Romm, 1966; Louis, 1969; Witherspoon et al., 1980; Yano et al., 2005). The other is that D is the aperture of fractures (Zimmerman and Bodvarsson, 1996; Brush and Thomson, 2003). In this paper, the latter concept is applied to the discussion on the experimental results since it is considered that the aperture strongly affects the viscosity term of the fracture flow. Furthermore, the width of the fracture is much larger than the aperture, and the influence of the width of the fracture can be ignored when discussing the viscosity of a fracture flow. Substituting the continuity equation, Eq. (7) is expressed by:

$$Re = \frac{Q}{\nu w}, \quad \because V = \frac{Q}{bw}, \quad (8)$$

where Q is the discharge obtained through the experiments, b is the average aperture of a fracture, and w is the width of the specimen (120 mm). Consequently, Re can be obtained through the experimental results.

Figure 9 shows the relationship between fracture surface characteristic factor, f , and Reynolds number, Re . Except for the case of Roughness C (C - M), all cases show the same tendency for f to reach a constant value in the Re range of more than 1.0. A higher f can also be confirmed in the Re range of less than 1.0, except for the three cases of Roughness B (B - M), C (C - K) and C (C - M). In the local cubic law (LCL), which currently represents a useful simulation for the fluid flow in fractures, regarding the condition that inertia terms are negligible as long as Re is less than 1 (Zimmerman and Bodvarsson, 1996;

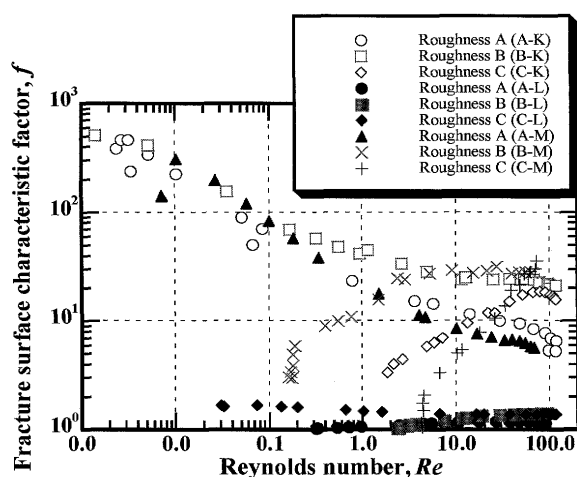


Fig. 9. Relationship between fracture surface characteristic factor f and Reynolds number Re

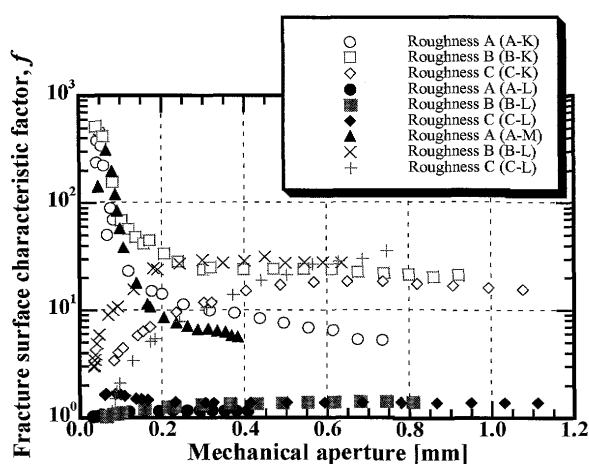


Fig. 10. Relationship between fracture surface characteristic factor f and mechanical aperture

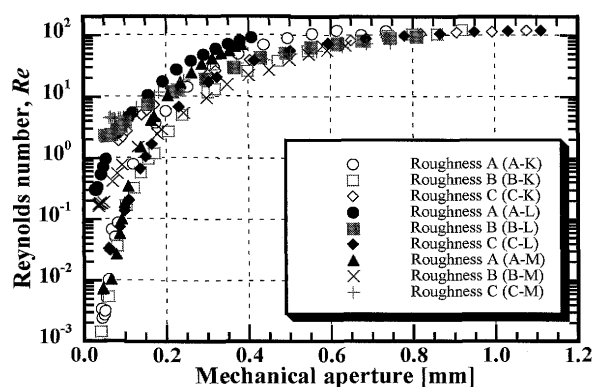


Fig. 11. Relationship between Reynolds number Re and mechanical aperture

Nicholl et al., 1999; Brush and Thomson, 2003), Re is utilized to discuss the fluid flow in fractures. From Fig. 9, it is confirmed that the roughness strongly affects the fluid flow in fractures in the Re range of less than 1.0.

Figure 10 shows the $f \sim b_m$ relation and Fig. 11 shows the $Re \sim b_m$ relation. In all cases, with increments in the mechanical aperture, that is, the progress in the shear process, it can be confirmed that both f and Re reach a constant value. As the shear progresses, it can also be confirmed that the aperture of the fracture increases and the influence of the surface roughness for the fluid flow decreases.

In general, the energy loss for friction occurs to the fluid flow in the rough wall. The energy loss for friction is defined as the Darcy-Weisbach formula, namely,

$$I = f_r \frac{1}{R} \frac{V^2}{2g}, \quad (9)$$

where I , R , g , and f_r are the hydraulic gradient, the hydraulic radius, the gravitational acceleration, and the friction factor, respectively. Using the width of the fracture, w , and the aperture of the fracture, b , R is defined as follows:

$$R = \frac{bw}{2(b+w)}. \quad (10)$$

Introducing the assumption of $w \gg b$, the following relation can be obtained:

$$R = \frac{b}{2}. \quad (11)$$

Substituting the continuity equation and Eq. (11) into Eq. (9), f_r is expressed as:

$$f_r = \frac{2gIb^3w^2}{Q^2}. \quad (12)$$

From Eq. (12), f_r can be obtained through the experimental results and conditions. Moreover, substituting Eq. (8) into Eq. (12), the f_r-Re relation assuming the cubic law can be obtained as follows:

$$f_r = \frac{12}{Re}. \quad (13)$$

Figure 12 shows the f_r-Re relation from the experimental results. With the increment of Re , f_r is found to decrease. As mentioned above, several previous research works referred to the Reynolds number. Zimmerman and Bodvarsson (1996) suggested that the inertial effects were not significant for flows at Re smaller than 1.0. Brush and Thomson (2003) concluded that LCL was valid provided that Re was less than 1.0 and the other kinematic criteria were satisfied. In our results, in an Re range of 0.02 to 1.0, a good agreement can be found between the experimental results and the theoretical relation using the assumption of the cubic law described in Eq. (13). Therefore, it has been confirmed through our experimental results that the cubic law is valid under the condition that Re is less than 1.0. From Fig. 11, an Re of less than 1.0 is equal to less than around 0.2 mm of the mechanical aperture. That means the cubic law may be applied before the peak shear strength and agrees to the interpretation of Figs. 5, 7, and 8.

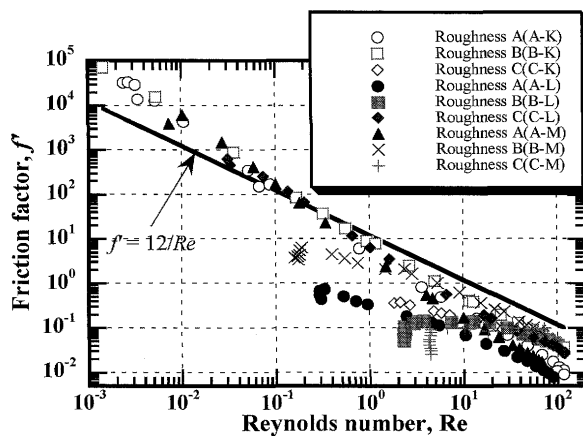


Fig. 12. Relationship between hydraulic friction factor f and Reynolds number Re

SIMULATION ON THE FLUID FLOW OF A SINGLE ROCK FRACTURE

Governing Equation

The local cubic law, LCL, is based on the Reynolds equation and is widely used for simulating the fluid flow in rough-walled fractures. Regardless of the fact that the solution of the Reynolds equation can simply be obtained in comparison to the full Navier-Stokes equations, its applicability is restricted because of the assumptions which govern its derivation. In fact, with other assumptions, the most restrictive ones are that the viscous terms must dominate the inertia terms and that the velocity distribution along the axis perpendicular to the fracture's plane is parabolic. In a previous study, Brush and Thomson (2003) carried out a comparison among Stokes, Navier-Stokes, and LCL for simulating the fluid flow in a fracture, and consequently, they proposed geometric and kinematic criteria which were satisfied with LCL. One of the proposed criteria is that Re must be kept at less than 1.0 in order for the viscous terms to dominate the inertia terms. On the other hand, for maintaining the parabolic velocity distribution across the aperture, Zimmermann and Bodvarsson (1996) suggested the wavelength of aperture variation $\lambda < 0.4$, while Brown (1987) suggested $\lambda < 0.03$. Furthermore, Mgaya et al. (2006) made a comparison between the depth-averaged flow model (2D-model) for a single fracture (Kishida et al., 2005) and the LCL, and the following conclusions could be made: (i) using the same hydraulic conditions, the effect of the inertia terms differs from one fracture to another, (ii) depending on the aperture distribution within the fracture, at $Re < 1.0$, the inertia terms could be important, and (iii) for the flow simulation of laboratory experiments, where Re cannot be kept much smaller than 1, and where the effect of the inertia terms was not guaranteed to be negligible, the depth-averaged flow model can improve the simulated results in comparison to the LCL model.

As mentioned in the previous chapter, the fracture flow obtained from the experiments shows several values for Re . In this paper, the 2D model will be applied to the ex-

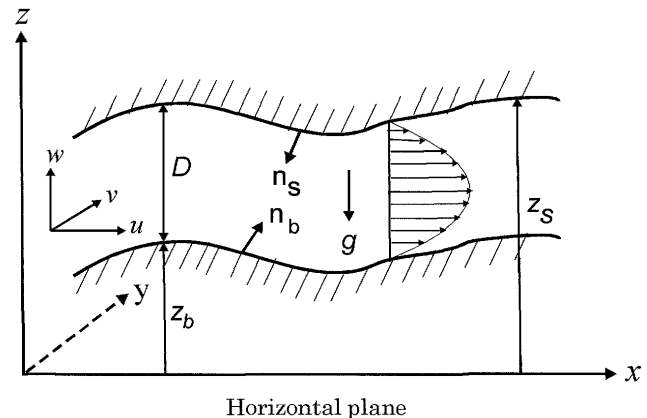


Fig. 13. Conceptual flow model in a single fracture

perimental works and to the discussion on the fluid flow on a single rock fracture.

The governing equations are derived based on the 3D continuity and Navier-Stokes equations, Eqs. (14) and (15), respectively, and the conceptual flow model shown in Fig. 13. The 3D Navier-Stokes equations are used with diffusion terms written in the form of viscous stress levels in tensor notation.

$$\nabla(\rho \mathbf{u}) = 0 \quad (14)$$

$$\rho \frac{D\mathbf{u}}{Dt} = \rho \mathbf{F} - \nabla P + \frac{\partial \tau_x}{\partial x} + \frac{\partial \tau_y}{\partial y} + \frac{\partial \tau_z}{\partial z} \quad (15)$$

where ρ is the fluid density, $\mathbf{u}$ is the velocity vector, $\mathbf{F}$ is the body force vector, P is the hydrodynamic pressure, and x , y , and z are the viscous stress vectors acting at a point in the fluid on planes which are normal to the x , y , and z directions, respectively. $\tau_x = \tau_{xx}\mathbf{i} + \tau_{xy}\mathbf{j} + \tau_{xz}\mathbf{k}$, with similar expressions for τ_y and τ_z .

The assumptions governing the derivation are: (i) the fluid is considered to be a Newtonian incompressible fluid and the flow is laminar, which means that non-turbulent conditions exist everywhere in the fracture; (ii) the fracture is free from any external forces such as compressive stress; (iii) neither slip nor flow occurs at the fracture walls (i.e., the upper and the lower fracture surfaces); (iv) the difference between true and apparent apertures is negligible, and therefore, the apparent aperture (i.e., the aperture measured perpendicular to the nominal fracture plane) is used; (v) the velocity distribution perpendicular to the nominal fracture plane (i.e., in the z -direction) is parabolic; (vi) the pressure distribution along the aperture is hydrostatic (i.e., not constant); and (vii) the inertia terms in the x - y plane are not negligible compared to the viscous terms.

Integrating Eqs. (14) and (15) from the lower wall to the upper wall, as shown in Fig. 13, and applying the Leibniz theorem, the following governing equations for the 2D model are obtained:

[Continuity equation]

$$\frac{\partial(UD)}{\partial x} + \frac{\partial(VD)}{\partial y} = 0 \quad (16)$$

[Momentum equation for the x-direction]

$$\begin{aligned} \frac{\partial(UD)}{\partial t} + \beta \frac{\partial(U^2D)}{\partial x} + \beta \frac{\partial(UVD)}{\partial y} = & -D \frac{\partial p_D}{\partial x} - gD \frac{\partial(Z_b + D)}{\partial x} \\ & - \frac{\tau_{bx}}{\rho} \sqrt{1 + \left(\frac{\partial Z_b}{\partial x}\right)^2 + \left(\frac{\partial Z_b}{\partial y}\right)^2} - \frac{\tau_{sx}}{\rho} \sqrt{1 + \left(\frac{\partial Z_s}{\partial x}\right)^2 + \left(\frac{\partial Z_s}{\partial y}\right)^2} \\ & + \frac{\partial}{\partial x} \left(\nu D \frac{\partial U}{\partial x} \right) + \frac{\partial}{\partial y} \left(\nu D \frac{\partial U}{\partial y} \right) \end{aligned} \quad (17)$$

$$\begin{aligned} \frac{\partial(VD)}{\partial t} + \beta \frac{\partial(UVD)}{\partial x} + \beta \frac{\partial(V^2D)}{\partial y} = & -D \frac{\partial p_D}{\partial y} - gD \frac{\partial(Z_b + D)}{\partial y} \\ & - \frac{\tau_{by}}{\rho} \sqrt{1 + \left(\frac{\partial Z_b}{\partial x}\right)^2 + \left(\frac{\partial Z_b}{\partial y}\right)^2} - \frac{\tau_{sy}}{\rho} \sqrt{1 + \left(\frac{\partial Z_s}{\partial x}\right)^2 + \left(\frac{\partial Z_s}{\partial y}\right)^2} \\ & + \frac{\partial}{\partial x} \left(\nu D \frac{\partial V}{\partial x} \right) + \frac{\partial}{\partial y} \left(\nu D \frac{\partial V}{\partial y} \right) \end{aligned} \quad (18)$$

where, U and V are the depth-averaged velocities in the x and the y -directions, respectively. D is the aperture ($D = Z_s - Z_b$). Z_s and Z_b are the lower and the upper wall location from the datum plane. p_D is the pressure on the upper wall and g is the gravitational acceleration. τ_s and τ_b are the shear stress on both upper and lower wall, respectively. ν is the kinematic viscosity and β is introduced here as a momentum correction factor for the laminar flow $\beta = 1.2$. The shear stress values on the fracture walls are calculated using the resistance law for laminar flows.

Procedure for Numerical Solution

Equations (16), (17), and (18) are discretized by the finite volume (FV) method with a staggered arrangement of hydraulic variables. The velocity components are defined on the cell faces and the scalar variables are defined at the cell centers. To calculate the pressure and the velocity field at the new time steps, the Highly Simplified Marker and Cell (HSMAC) method (Hirt and Cook, 1972) is applied. The boundary conditions are a constant hydraulic head (Dirichlet boundary conditions) along opposite sides of the fracture in the directions of flow and no flow (Neumann boundary conditions) across the other two sides, as shown in Fig. 14. At the initial condition, it is assumed that the hydraulic variables, $M(=UD)$ and $N(=VD)$, and pressure p_D at time $t = n\Delta t$ are known. Then, hydraulic variables M^* and N^* at time step $(n+1)\Delta t$ are calculated explicitly. This method uses the technique of pressure correction as follows:

i) The hydraulic variables are initially calculated using the following equations:

$$\begin{aligned} \frac{M^* - M^n}{\Delta t} = & -(\text{inertia term})^n - (\text{pressure term})^n \\ & - (\text{viscosity term})^n, \end{aligned} \quad (19)$$

$$\begin{aligned} \frac{N^* - N^n}{\Delta t} = & -(\text{inertia term})^n - (\text{pressure term})^n \\ & - (\text{viscosity term})^n. \end{aligned} \quad (20)$$

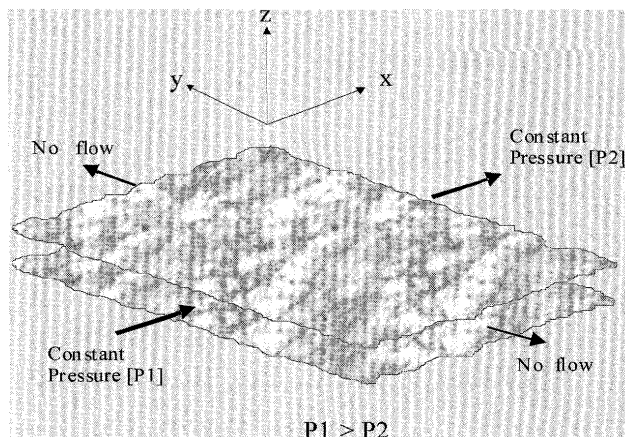


Fig. 14. Boundary conditions for the numerical solution

ii) Then, the pressure is corrected with the following equation:

$$p_{D_{ij}}^* = p_{D_{ij}}^n + \delta p_{D_{ij}}^*, \quad (21)$$

where,

$$\delta p_{D_{ij}}^* = - \frac{\omega \varepsilon_{ij}^*}{2gD_{ij}\Delta t \left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} \right)},$$

$$\varepsilon_{ij}^* = \frac{M_{i+1,j}^* - M_{ij}^*}{\Delta x} + \frac{N_{ij+1}^* - N_{ij}^*}{\Delta y} \text{ and } \omega \text{ is } 0.5 \text{ to } 1.0.$$

iii) Finally, M^* and N^* are recalculated using the new value for p_D^* . This process is repeated until the criterion for the error $|\varepsilon^*|$ is satisfied. Then, M^* and N^* are considered to be the hydraulic variables at the next step of $(n+1)\Delta t$ (i.e., M^{n+1} , N^{n+1}).

Model Development

The flow models in this research employ the finite volume method to approximate the differential governing equations on the single fracture. The accuracy of this simulation depends on the employed fracture geometry model and the aperture distribution.

The roughness geometry of the fracture walls and the aperture distribution of the rock fracture during the shearing process are obtained through the use of the shear mechanical model (Kishida and Tsuno, 2001). This model utilizes the discrete data of the surface roughness obtained through the profiling technique, as mentioned in the chapter on the experiments. The model is capable of simulating the shear behaviours of rock joints, such as expressing the variation in shear stress, the dilation angle, and the dilation of the entire specimen. Moreover, it can simulate the geometry of surface roughness at different stages of the shearing process.

Examples of the fracture surface roughness, the spatial variation in the fracture in the direction of the flow, and the aperture size distribution, based on the profiling data of surface roughness and the shear mechanical model, are shown in Fig. 15. It should be noted that the aperture distributions shown in Fig. 15 have similarities to those

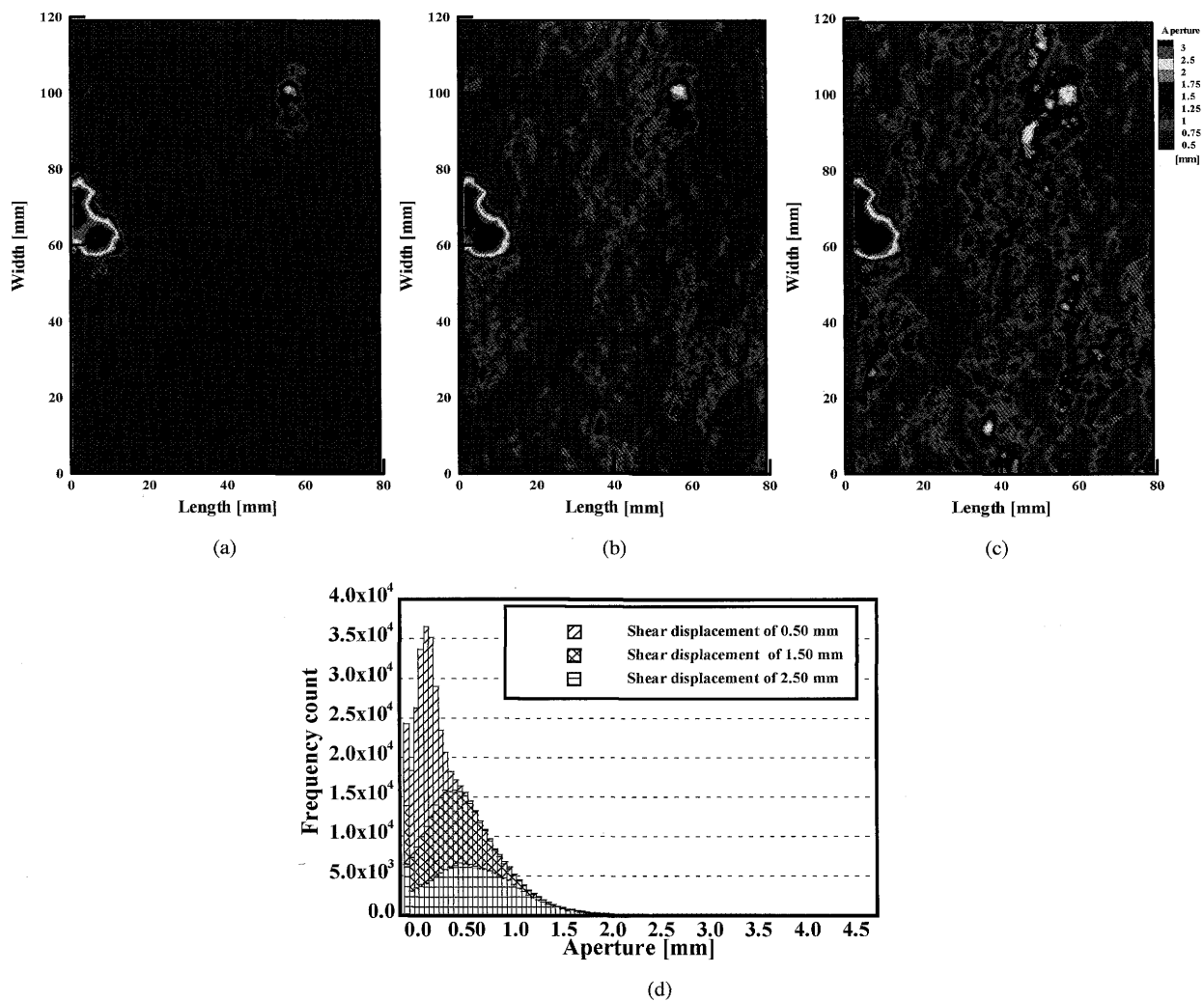


Fig. 15. Examples of aperture distribution at different stages of shear displacement based on the data obtained from the shear mechanical model (Specimens B-K): (a) Contour map of the aperture distribution at 0.50 mm of shear displacement, (b) Contour map of the aperture distribution at 1.50 mm of shear displacement, (c) Contour map of the aperture distribution at 2.50 mm of shear displacement and (d) Histogram of the aperture distribution

reported in the study by Oron and Berkowitz (1998). At a small shear displacement, the aperture distribution is skewed to small aperture sizes. As the shear displacement increases, the skewness of the aperture distribution moves towards larger aperture sizes.

Comparison of the Simulation and the Experimental Results

In this section, a comparison is made between the simulated hydraulic conductivity and the experimental results. Figure 16 shows the transmissivity - shear displacement relation. The transmissivity estimated by our 2D model and the experimental results are observed in several specimens. In addition, Fig. 16 shows the transmissivity estimated by LCL. This is based on the fact that during shearing the aperture size increases as a result of dilation, and therefore, the discharge trends to increase. The degree of disagreement between the 2D model and the LCL results increases with the advancement of the shear displacement with the LCL overestimating the transmissivity. In fact, since both models, namely, the 2D

model and LCL, use the same simulation conditions for the aperture fields by the mechanical shear model, it is believed that the difference is observed because of the terms included in the 2D model which are not included in the LCL model. Based on the investigation on the effect of the terms by Mgaya et al. (2006), it is evident that this disagreement is likely to have been caused by the inertia terms and the term which includes the effect of the variation in the fracture's surface wall. Mgaya et al. also reported that the effect of the inertia terms was dominant and increased with the increments in Re . A similar trend can be observed in Fig. 16 where the difference between the results for the 2D model and LCL diminishes with a decrease in transmissivity; this implies a decrease in Re . Based on the discussion through Fig. 16, therefore, it can be concluded that the difference between LCL and the 2D model is mainly due to the effect of the inertia forces. It can also be concluded that the 2D model shows a better agreement with the experimental results than the LCL model.

Regardless of the fact that the comparison between the

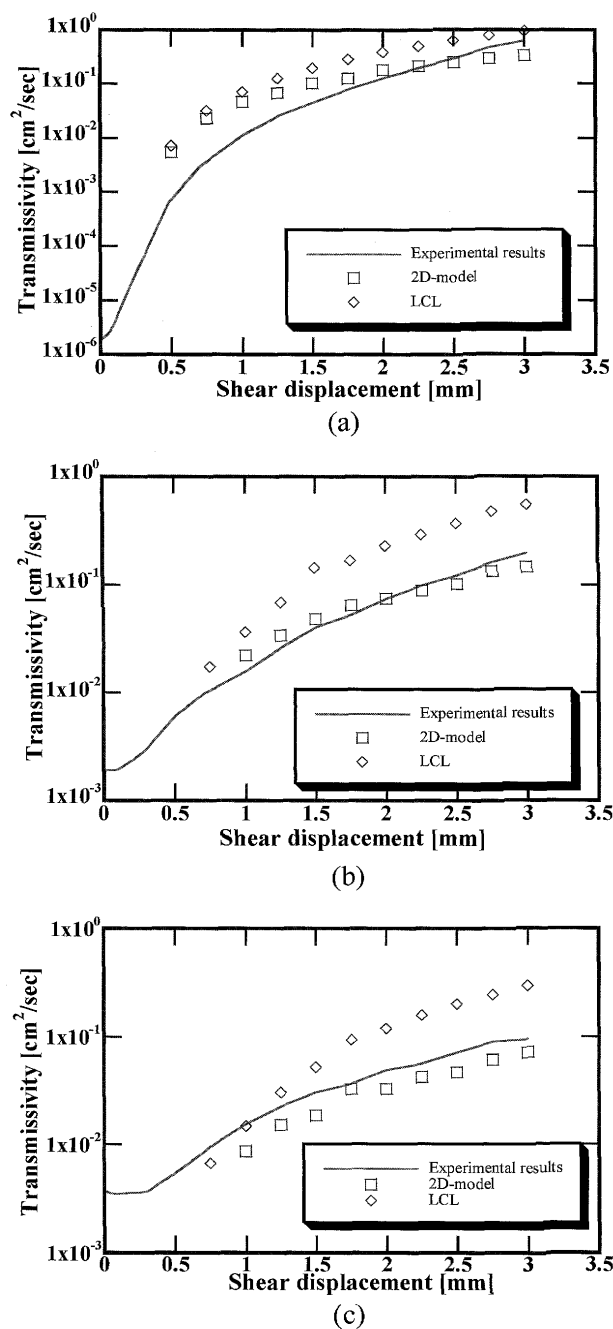


Fig. 16. Comparison of the hydraulic conductivity among experimental results, the 2D model, and LCL: (a) Specimens A-K, (b) Specimens B-L and (c) Specimens C-M

numerical simulations and the experimental results are affected by the errors that might have been introduced in the estimation of the aperture fields by the shear mechanical model, we can still gain insight into the effectiveness of the 2D model in comparison to LCL. In Fig. 16, it is seen that the results of the 2D model and LCL tend to converge to the same values of transmissivity at small shear displacements. This means that if we disregard the extra terms included in the 2D model, the results of both models will be the same for the entire range of shear displacements. Therefore, as can be observed from the results, the results of the 2D model tend to converge to

the experimental results when compared to the LCL results. This implies that the 2D model can generate a better estimate of the hydraulic conductivity than the LCL model.

Re Distribution on the Fracture

Fluids in fractures tend to flow easily in paths with larger aperture sizes. Figure 17 shows examples of the aperture distribution with corresponding flow vectors simulated by the 2D model. It is observed that the model has been able to simulate the channeling effect in the rock fracture of the experimental specimens with regard to aperture size distribution. This substantiates the ability of the 2D model to simulate the channeling effect of flows in fractures with variable apertures.

Figure 18 shows examples of the Re distribution at local points within the fracture. As for the estimation of Re at each local point, Eq. (7) is applied using the hydraulic radius, as shown in Eq. (10), as the characteristic length. It can be seen that the two fractures possess different distributions regardless of the fact that both have almost an equal global value for Re . This implies that different fractures have different distributions of the velocity gradient in the x - y plane, as shown in Fig. 17. In Fig. 18(a), a small aperture case, it can be found that a value of less than 10 for Re occupies majority of the fracture surface. On the other hand, it can be also found that the flow on the fracture is controlled by the area of the higher Re regions. When discussing the hydraulic conductivity for a range of specimens, small values for Re can be obtained relatively. In fact, the flow on the fracture shows the channeling in the local part. Then, the majority of the fracture flow is the higher Re flow. Consequently, Fig. 18 shows the distribution of different strength values for the inertia terms.

Figure 19 shows the relationship between the friction factor, f_r , and Re at each local point on the fracture in the shear process. The figure also shows the Eq. (13) relation. Re from 10 to 0.1 shows the correlation of Eq. (13) as well as the experimental results. However, it is also certain that the difference between the f_r - Re relation for each point and Eq. (13) is large. In the Re range of less than 0.1, on the other hand, the difference can be clearly confirmed. As the shearing progresses, it is found that Re increases. It is shown that the hydraulic conductivity increases with the opening of the aperture in the shear process.

CONCLUSIONS

In this study, direct shear - flow coupling tests on single fractures have been carried out for various types of fractures, and the applicability of the cubic law has been discussed through the experimental results. Under almost constant hydraulic gradient conditions, the cubic law can be applied to experimental results in the Re range from 0.02 to 1.0. It has been confirmed that the cubic law cannot take into account the fracture flow in most cases.

Moreover, two types of flow simulations in fractures,

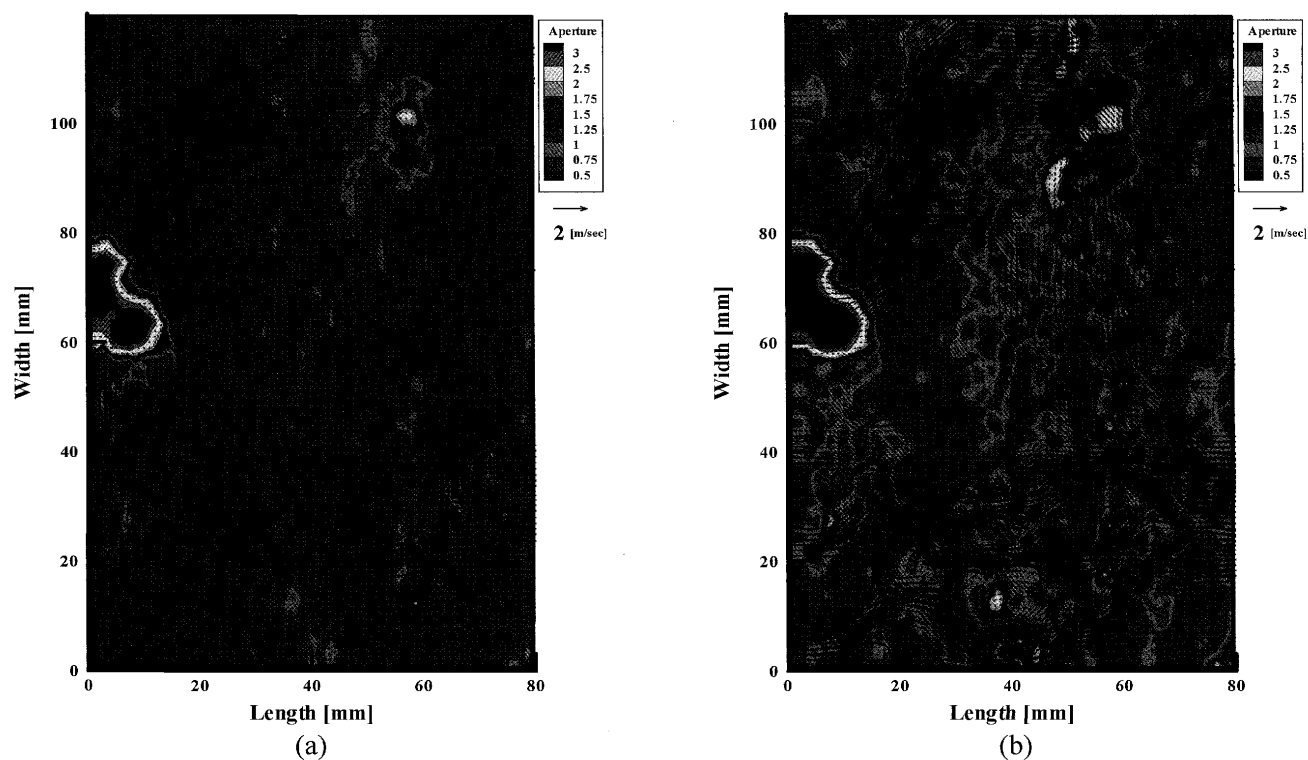


Fig. 17. Example of aperture distribution on the fracture and flow field distribution for Specimens B-K: (a) 1.00 mm of shear displacement and (b) 2.50 mm of shear displacement

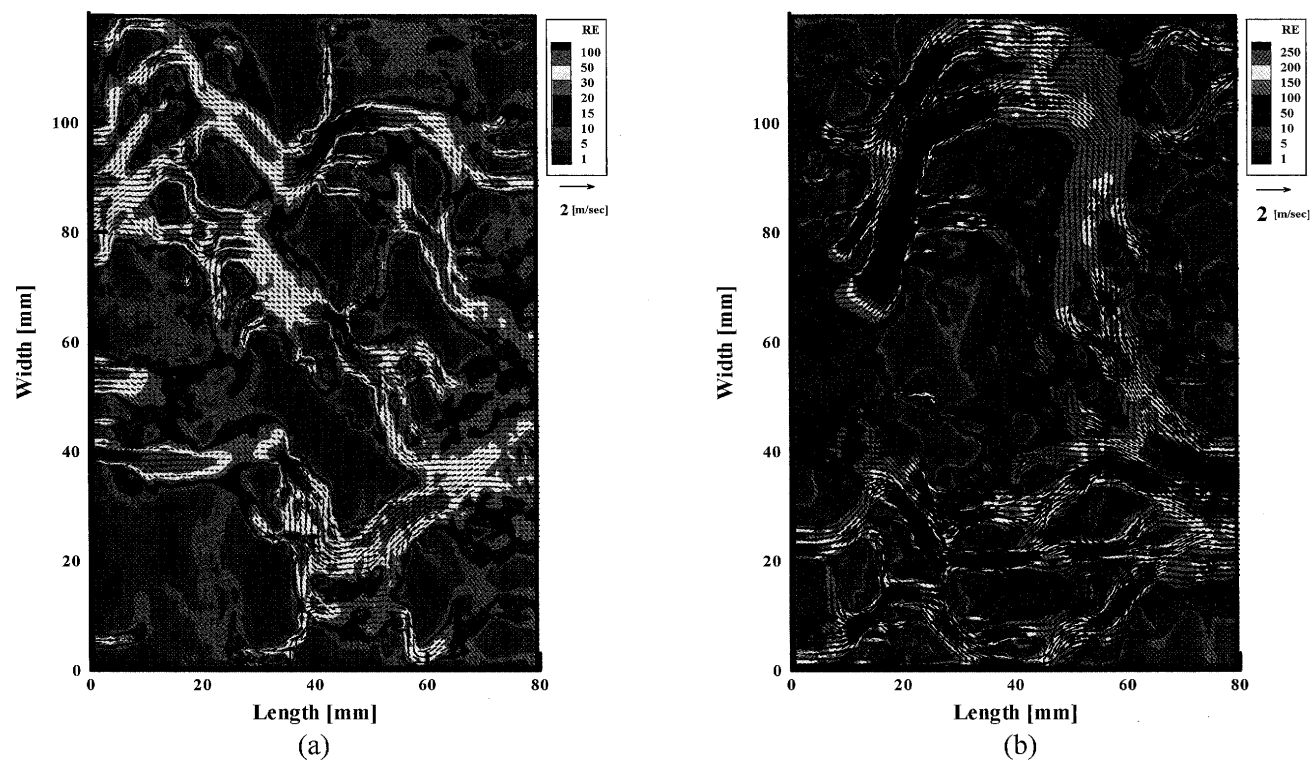


Fig. 18. Example of Reynolds number, Re , distribution and flow field distribution for Specimens B-K: (a) 1.00 mm of shear displacement and (b) 2.50 mm of shear displacement

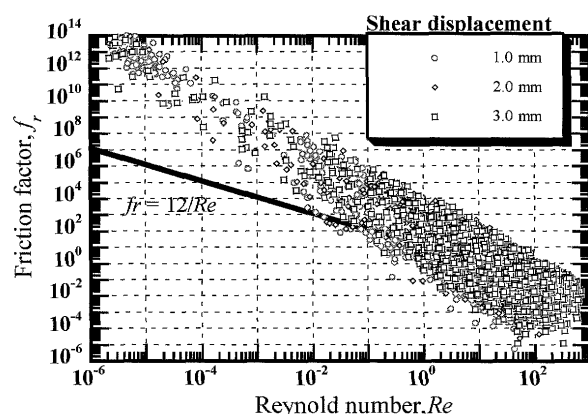


Fig. 19. Relationship between hydraulic friction factor, f_r , and Reynolds number, Re through the numerical simulation with Specimens B-K

LCL and the 2D model, which includes the inertia terms, have been carried out. As a result, it is seen that the 2D model can significantly improve the simulation results in comparison with LCL. In addition, the 2D model can clarify the flow phenomenon in a fracture. In total, the flow in the fracture shows a small Re . In fact, the flow in the fracture occupies a slight area and the channeling flow has been confirmed. The channeling flow basically shows a high Re , and it is believed that the cubic law cannot take into account the flow. Consequently, the influence of the inertia terms must be discussed when estimating the flow in a fracture.

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