

Note

Commitment by Agreement[†]

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Abstract

Commitments change games' outcomes, which vary depending on the conformation of the commitments. We consider commitment games that have a one-shot commitment stage that needs a unanimous agreement. Here, all feasible and individually rational payoffs of a two-player strategic game can be attained at the equilibria.

Key words: Commitment, Unanimous agreement, Folk theorem

1. Import of Bilateral Commitment Ability

Various forms of commitment and their impact on game outcomes have been widely studied. Our focus is on commitments that take effect only with the unanimous agreement of all players. Much of the existing research has centered on the import of unilateral commitment ability. However, the argument that commitments, by altering the rules of the game, require some form of social agreement is compelling. A typical example can be seen in the case of demolishing a shared resource such as a bridge: agreement among the owners is required. More generally, when organizing an event that imposes constraints on people's behavior, it is necessary to inform all relevant parties (such as local residents) in advance about whose actions will be restricted and in what way, and to obtain their consent. Furthermore, when commitments are enforced by restricting future actions—such as imposing penalties on those who deviate—some form of agreement is needed among all parties involved, such as through legal reforms, contracts, or terms of service. In this study, we consider a setting in which, prior to a two-player simultaneous-move game, both players simultaneously propose their own commitments, and then simultaneously indicate whether or not they agree to enact those commitments. Players are required to commit to the proposed actions, if and only if, both agree. In the context of pre-play communication, this corresponds to an intermediate situation between the two extremes of cheap-talk on one side and unilateral commitment on the other.

Renou (2009) and Dutta and Ishii (2016) demonstrate that when players unilaterally commit to restricting their own strategy sets, strategy profiles that are not Nash equilibria of the underlying

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† I am grateful to Tadashi Sekiguchi, Takanori Adachi, Haruo Imai and Takaomi Notsu for their comments and suggestions. I also thank seminar audiences at Kan-kita CURIOSCHOLA Workshop. This work was supported by JSPS KAKENHI Grant Numbers JP20K01551. Any remaining errors are my own responsibility.

game can nevertheless become subgame perfect equilibrium outcomes. Kalai et al. (2010) establish a commitment folk theorem by introducing “voluntary commitment devices” as strategies. Here, a “device” refers to a function from the profile of devices chosen by the other players to one’s own action. Since the set of available devices includes those that always choose the same action regardless of the opponent’s device, the commitment is termed “voluntary.” Xiong (2020) considers a setting where, before a perfect-information extensive-form game, players take turns declaring their actions at each of their decision nodes and then sequentially express approval or disapproval of the declared strategy profile. If all players approve, the declared profile is played; otherwise, the commitment phase is repeated for a number of rounds. Xiong (2020) concludes that the subgame perfect equilibrium is always Pareto efficient in his setting. In contrast, the model in this study assumes a single-stage commitment phase in which decisions are made simultaneously rather than sequentially.

We consider a setting in which agreement on a conditionally committed action profile is binding. If no agreement is reached, the Nash equilibrium of the underlying game is played. In this sense, the commitment functions similarly to the “conditioning device” in Kalai et al. (2010). This parallel allows us to derive a commitment folk theorem.

2. Mutually Saying “Yes” Makes a Commitment

As an underling game, consider two-person finite strategic game, defined by a triple $G \equiv (N = \{1, 2\}, A = A_1 \times A_2, u = (u_1, u_2) : A \rightarrow \mathbb{R}^2)$. $N = \{1, 2\}$ is the set of players, each A_i is a non-empty finite set that describes the feasible strategies of player i , and each u_i is the payoff function of player i . We use the standard convention where for every player i , player $-i$ denotes the other player. To ease the discussion, we abuse notation and denote mixed strategy profiles in G as $a = (a_1, a_2) \in \Delta A = \Delta A_1 \times \Delta A_2$.

We define the commitment game Γ as follows. In the commitment stage, both players $i (= 1, 2)$ simultaneously propose an action $a_i \in A_i$. After observing the proposed action profile $a = (a_1, a_2)$, both players simultaneously choose either “yes” or “no.” The action profile a is played if and only if both players choose “yes.” If either player chooses “no,” the underlying game G is played instead.

Observation The Nash equilibrium $a^* = (a_1^*, a_2^*) \in \Delta A$ of the game G constitutes a subgame perfect equilibrium outcome of the commitment game Γ .

Proof. Consider the following strategy. In the commitment stage, player i proposes an arbitrary action and chooses “no” regardless of the proposed action profile. Then, in any subgame G , player i chooses a_i^* . Since a^* is a Nash equilibrium of G , this constitutes a best reply in the subgame. In the stage where players choose between “yes” and “no,” player $-i$ selects “no,” which makes player i indifferent between choosing “yes” or “no.” As agreement will not be reached in any case, player i is indifferent about which action to propose in the commitment stage. Therefore, this strategy profile forms a subgame perfect equilibrium, a^*

is realized. ■

By employing strategies similar to trigger strategies in repeated games, a commitment folk theorem can be established.

Proposition An action profile $a^{**} = (a_1^{**}, a_2^{**})$ that weakly Pareto improves upon a Nash equilibrium $a^* = (a_1^*, a_2^*)$ of G constitutes a subgame perfect equilibrium outcome of Γ .

Proof. Consider the following strategy. In the commitment stage, player i proposes a_i^{**} . If the proposed action profile is a^{**} , then both players choose “yes”; otherwise, they choose “no.” Afterward, in the game G , player i plays a_i^* . Since a^* is a Nash equilibrium of G , this is a best reply in the subgame following disagreement. In the stage where players choose between “yes” and “no,” if the action profile a^{**} is proposed, and if player $-i$ were to choose “no,” it makes player i indifferent between the two options. On the other hand, if a^{**} is proposed and player $-i$ chooses “yes,” then player i receives the payoff $u_i(a^{**})$ by choosing “yes,” and $u_i(a^*)$ by choosing “no.” Since $u_i(a^{**}) \geq u_i(a^*)$, choosing “yes” is a best reply. Given that player $-i$ proposes a_{-i}^{**} , player i knows that if the proposed action profile is a^{**} , then the outcome will be a^{**} ; otherwise, the outcome will revert to a^* . Therefore, player i has an incentive to propose a^{**} , as it yields at least as much utility as a^* , and possibly more. Hence, this strategy profile constitutes a subgame perfect equilibrium of the commitment game Γ , and the action profile a^{**} is implemented. ■

3. Comments about Commitments by Agreements

We analyzed a game in which the implementation of commitments requires agreement. As a result, we were able to establish a commitment folk theorem. We found that if agreed-upon commitments are binding, they can serve as a solution to the one-shot prisoners’ dilemma.

Conflict of Interest: The author has no conflict of interest.

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