

Finite groups with Sylow 2-subgroups of type A_{16}

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A 2-group is said to be of type X if it is isomorphic to a Sylow 2-subgroup of the group X . If G is a group with a Sylow 2-subgroup S of type X , we say that G has the involution fusion pattern of X if for some isomorphism θ of S onto a Sylow 2-subgroup of X , two involutions a, b of S are conjugate in G if and only if the involutions $\theta(a), \theta(b)$ of $\theta(S)$ are conjugate in X . Also we say that a group G is fusion-simple if $G = O^2(G)$ and $O(G) = Z(G) = 1$.

Now we have obtained the following:

THEOREM A. Let G be a fusion-simple finite group with Sylow 2-subgroups of type A_{16} . Then one of the following holds:

- (1) $G \cong A_{16}$ or A_{17} ,
- (2) $G \cong A_9 \cdot E_{256}$, the split extension of an elementary abelian group E_{256} of order 256 by A_9 with the action afforded by the 8-dimensional irreducible $GF(2)$ -representation, or
- (3) G has the involution fusion pattern of $\Omega_9(3)$.

Here $\Omega_9(3)$ denotes the orthogonal commutator group of degree 9 over the field of 3-elements and A_m the alternating group on m -letters.

In the process of proving Theorem A we obtain the following characterization.

THEOREM B. Let G be a finite group with Sylow 2-subgroups of type A_{16} . If G has the involution fusion pattern of A_{16} , then $G/O(G) \cong A_{16}$ or A_{17} .

Proof of the Theorem A is obtained in the following way which appears to be rapidly becoming standard(cf. Gorenstein-Harada[5], [6], Solomon[9]). Let S be a Sylow 2-subgroup of G and A be the unique elementary abelian subgroup of S of order 256. At first we show that the fusion of elements of S is controlled by $N_G(A)$ and $N_G(Z_2(S))$ where $Z_2(S)$ is the second center of S , using results of Alperin[1] and Goldschmidt[2] on conjugation family. Since S/A is of type A_8 , the structure of $N_G(A)/C_G(A)$ which is isomorphic to a subgroup of $GL(8,2)$ is determined by theorems of Harada[7] and Gorenstein-Harada[5],[6]. Then the fusion possibilities of involutions follow immediately. Here we can prove that if A is strongly closed in S with respect to G , then $G = N_G(A) \cong A_9 \cdot E_{256}$ by a recent result of Goldschmidt [4]. Characterization theorems of Gorenstein-Harada[5],[6] and Solomon[9] permit the determination of $C_G(a)/O(C_G(a))$ for all involution a in S . Now O is an A -signalizer functor and a signalizer functor theorem[3] implies that $W_A = \langle O(C_G(a)); a \in A^\# \rangle$ has odd order. It follows that $N_G(W_A)$ is strongly embedded in G provided $W_A \neq 1$. Since G has more than one conjugacy class of involutions, $W_A = 1$. Therefore $O(C_G(a)) = 1$ and Kondo's characterization theorem[8] implies that $G \cong A_{16}$ or A_{17} .

References

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