

Optimum Pole Shape for Sextupole Magnet Having Circular Pole Tips

Tetsumi TANABE* and Masaaki KOBAYASHI**

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Harmonic field components in sextupole magnets having circular pole tips are numerically obtained by using a two dimensional scalar potential under assumed boundary conditions. Optimum radii of pole tips are found for various depths of coil slots.

I. INTRODUCTION

In the high resolution momentum analyzer system for particle beam, dipole, and quadrupole magnets are used to analyze the momentum of beam and to converge beam flux respectively. In addition to their fundamental functions, the dipole and quadrupole magnets also cause undesirable higher-order aberrations. The aberration often becomes a serious problem when a high resolution is required for the analyzer system. The second order aberration which affects significantly the resolution can be eliminated or at least reduced by employing sextupole magnets at appropriate locations in the beam optical system. It is desirable for these sextupole magnets that the sextupole component is dominant by suppressing the higher order components to as small as possible amount.

For the above purpose, the profile of the pole piece must be a part of a curve of a hyperbola in order to assure rigorous r^2 proportionality of the magnetic field up to the vicinity of pole surface, where r is the radial distance from the center axis of the magnet. However, precise fabrication of hyperbolic pole tips is rather difficult. The circular pole tips are simple to fabricate in place of hyperbolic ones. It is then important to estimate the harmonic field contents for magnets having circular pole tips in order to determine the optimum pole shape. This paper gives a numerical calculation of harmonic field contents for sextupole magnets having circular pole tips and discusses the optimum pole shape designs.

Calculations of magnetic field have been carried out for several pole shapes by using conformal transformations. Sonoda *et al.*¹⁾ treated the magnetic field for a trapezoidal pole profile and showed that the deviation from r^2 proportionality is negligibly small for $r < 0.6r_0$ where r_0 is the minimum distance of the pole tip from the center. Audoin²⁾ calculated the magnetic field for the case where the poles show a cross section, whose profile consists of an arc of a circle tangential to the sides of an angle whose apex is on the axis of the lens. These pole shapes seem not necessarily adequate for the correction lens of the second order aberration, because the higher order harmonic components

* 田辺徹美: Institute for Nuclear Study, University of Tokyo, Tokyo.

** 小林正明: National Laboratory for High Energy Physics, Oho-machi, Tsukuba-gun, Ibaraki.

than sextupole should be suppressed up to the radius as large as possible.

In this paper, the harmonic field contents are calculated for a pole shape as shown in Fig. 1 by using a formulation of two dimensional scalar potential as given by one of the authors (M. K.)³⁾ and Bellendir and Lari⁴⁾ for quadrupole magnet. The optimum radii of circular pole tips are searched for various locations of coil, L_{cut} . Calculations for a modified coil shape as shown in Fig. 2 are also given.

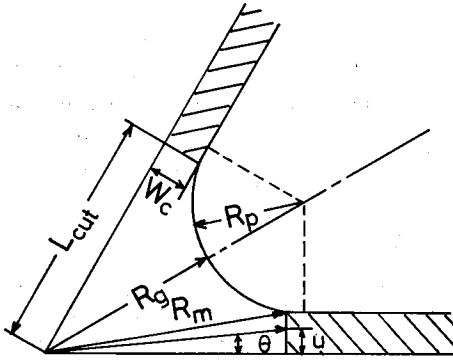


Fig. 1. Circular pole shape with the coil truncation at an arbitrary distance L_{cut} . Shaded areas show coil space.

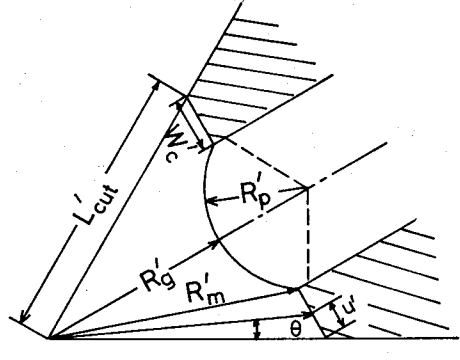


Fig. 2. The same as in Fig. 1 except for a different coil. The coil edge lies on a line perpendicular to the symmetry axis of pole.

II. METHOD OF CALCULATION

In the region of zero current density, the scalar magnetic potential, Ψ , should satisfy the Laplace's equation

$$\nabla^2 \Psi(r, \theta) = 0. \quad (1)$$

The general solution of Eq. (1) is given in the polar co-ordinate (r, θ) by

$$\begin{aligned} \Psi(r, \theta) = & \sum_{n=1}^{\infty} (A_n \sin n\theta + B_n \cos n\theta) \\ & \times (C_n r^n + D_n r^{-n}) + (E + F\theta)(G + H \ln r), \end{aligned} \quad (2)$$

where $A \sim H$ are arbitrary constants. Boundary conditions and symmetry conditions for symmetric sextupole magnet are given as follows:

- (1) 2π periodicity in θ ,
- (2) finiteness at the origin,
- (3) $\pi/3$ antisymmetry,
- (4) mirror symmetry with respect to $\theta = \pi/6$.

Applying these conditions to Eq. (2), the general solution is reduced to

$$\Psi(r, \theta) = \sum_{n=3, 9, 15, \dots}^{\infty} A_n r^n \sin n\theta. \quad (3)$$

Radial and azimuthal components of magnetic field are

$$B_r(r, \theta) = -\frac{\partial \Psi}{\partial r} = -\sum_{n=3,9,15,\dots}^{\infty} n A_n r^{n-1} \sin n\theta, \quad (4)$$

$$B_\theta(r, \theta) = -\frac{1}{r} \frac{\partial \Psi}{\partial \theta} = -\sum_{n=3,9,15,\dots}^{\infty} n A_n r^{n-1} \cos n\theta. \quad (5)$$

If the potential ϕ_i is given at M points, the coefficients A_n can be determined by fitting $\Psi(r, \theta)$ to ϕ_i 's as follows:

$$\partial \sigma / \partial A_p = 0 \quad (p=3, 9, 15, \dots), \quad (6)$$

$$\sigma = \sum_{i=1}^M (\Psi(r_i, \theta_i) - \phi_i)^2 w_i, \quad (7)$$

where w_i is the weighting factor for the i -th point. The coefficients A_n 's can be found by solving the following N simultaneous linear equations obtained from Eq. (6),

$$\sum_{n=3,9,15,\dots}^{6N-3} A_n \left(\sum_{i=1}^M \rho_{ni} \rho_{pi} \right) = \sum_{i=1}^M \phi_i \rho_{pi}, \quad (p=3, 9, 15, \dots, 6N-3), \quad (8)$$

where $\rho_{pi} = r_i^p \sin p\theta$. Here, higher harmonic components than the $(6N-3)$ -th are neglected from the summation. The strength of the n -th harmonic field components at the radius of the pole gap is given by

$$(B_n)_{R_g} = n A_n R_g^{n-1}. \quad (9)$$

The potential is assumed to be constant on the pole surface and to decrease linearly along the coil edge to zero on the axis. That is, the potential can be put as follows:

$$\phi_i = \begin{cases} 1, & \text{on the pole tip,} \\ u/W_c \text{ (or } u'/W'_c), & \text{on the coil edge,} \end{cases}$$

where the notations are given in Figs. 1 and 2. The widths of coils W_c (for the pole shape of Fig. 1) and W'_c (Fig. 2) are given as follows:

$$W_c = (R_g + R_p) \sin \frac{\pi}{6} - \left\{ R_p^2 - \left[(R_g + R_p) \cos \frac{\pi}{6} - L_{\text{cut}} \right]^2 \right\}^{1/2}, \quad (10)$$

$$W'_c = L'_{\text{cut}} \sin \frac{\pi}{6} - \left\{ R_p'^2 - \left[(R'_g + R'_p) - L'_{\text{cut}} \cos \frac{\pi}{6} \right]^2 \right\}^{1/2}. \quad (11)$$

The present calculations were carried out over the angular range from 0° to 60° for symmetric sextupole magnets. The potentials were given at 100 points with the constant angular step $\Delta\theta = \pi/300$. The weighting factor w_i was set to be unity for simplicity. The maximum number of harmonic components N was taken to be 8. Numerical calculations were carried out by using the double precision modes of electronic computers, TOSBAC-3400 (96 bits) and HITAC-5020E (64 bits).

III. RESULTS

The ratio of each harmonic field component to the sextupole one, B_n/B_3 , for the pole shape of Fig. 1 is shown in Figs. 3~6 as a function of R_p/R_g for various values of

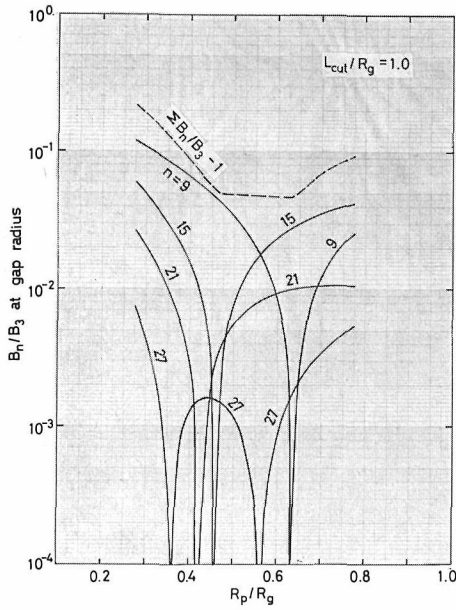


Fig. 3. Harmonic field contents for circular poles given in Fig. 1. The value of n in the sum $\Sigma B_n/B_3 - 1$ is from 3 to 45 in steps of 6. The higher harmonics than $n=27$ are almost less than 10^{-4} .

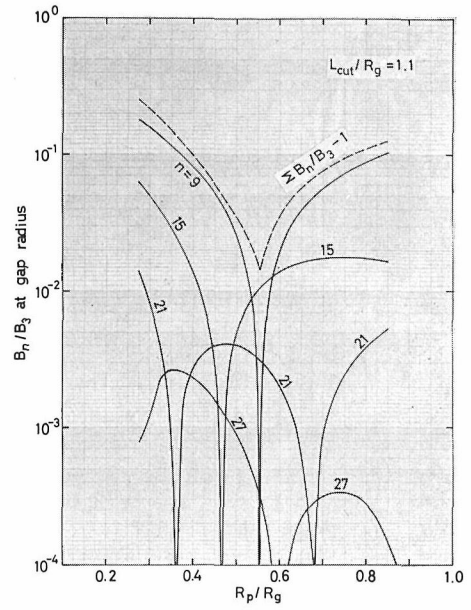


Fig. 4. The same as in Fig. 3. except for different values of L_{cut} .

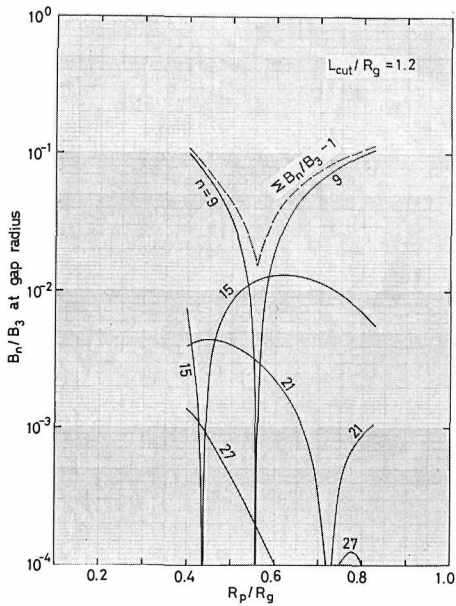


Fig. 5. The same as in Fig. 3 except for different values of L_{cut} .

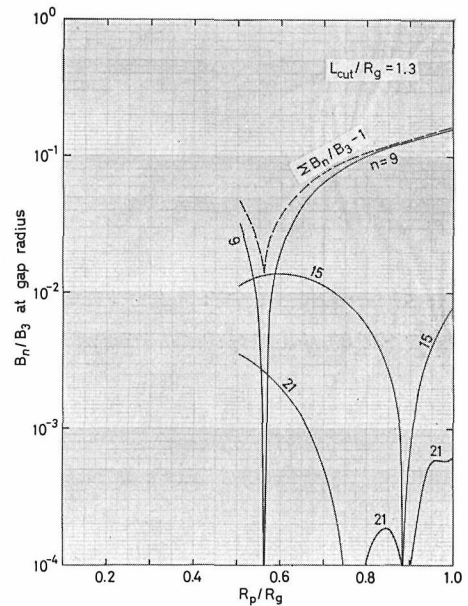


Fig. 6. The same as in Fig. 3 except for different values of L_{cut} .

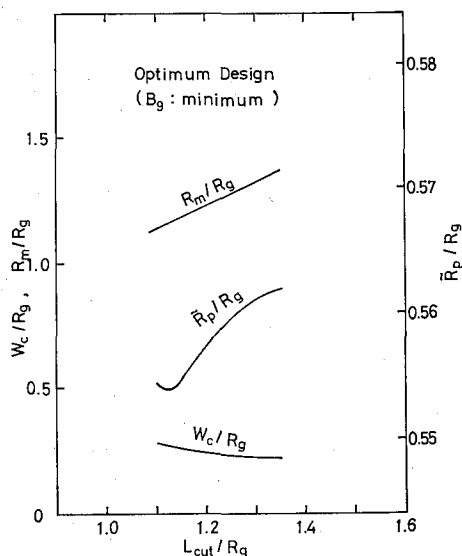


Fig. 7. Optimum radius $\tilde{R}_p$ of circular pole tip as a function of L_{cut}/R_g . Corresponding values of W_c/R_g and R_m/R_g are also shown.

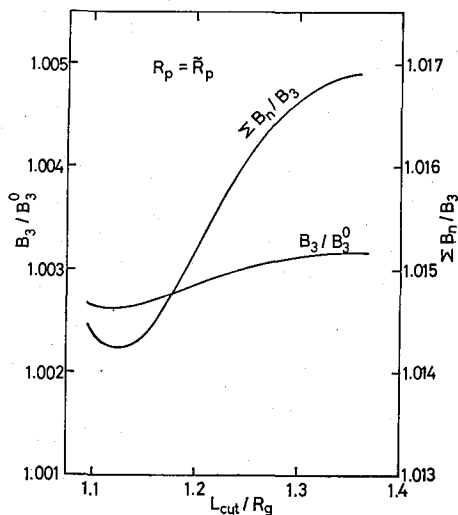


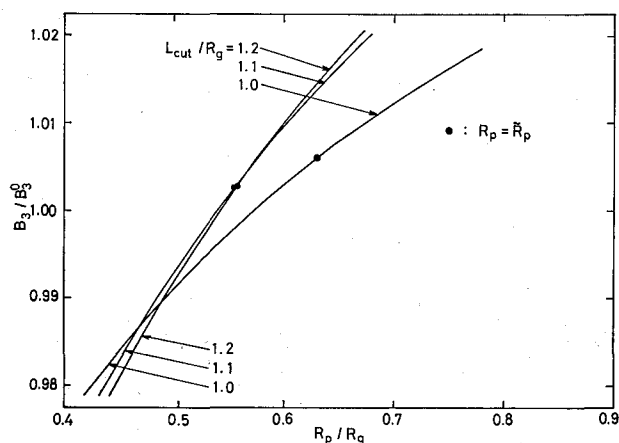
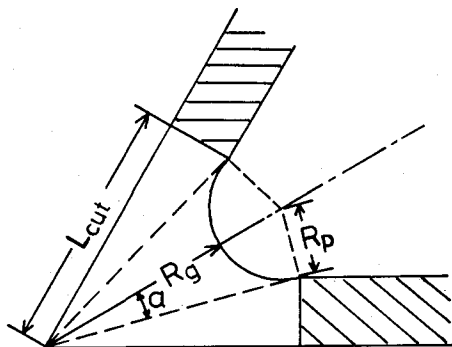
Fig. 8. Sum of harmonic field contents and the ratio of B_3 to B_3^0 , as a function of L_{cut}/R_g for the case of optimum pole tip design ($R_p = \tilde{R}_p$).

L_{cut}/R_g . The sums of higher components are also shown. Figure 7 gives $\tilde{R}_p/R_g$, W_c/R_g and R_m/R_g as a function of L_{cut}/R_g . Here the optimum radius of circular pole tip, $\tilde{R}_p$, is defined as the radius which minimizes the component B_9 . These radii are as follows, $\tilde{R}_p/R_g = 0.5544$ (for $L_{cut}/R_g = 1.10$), 0.5547 (1.15), 0.5573 (1.20), 0.5596 (1.25), 0.5611 (1.30), and 0.5619 (1.35) within an accuracy of ± 0.0001 . From the results presented in these figures we can conclude as follows:

- (1) For L_{cut}/R_g larger than or as large as 1.1, the main higher harmonic component is B_9 . Consequently both B_9 and the sum B_n/B_3 take their own minima for nearly the same value of R_p . On the other hand, for $L_{cut}/R_g = 1.0$, B_{15} is comparable with B_9 and the sum B_n/B_3 shows no sharp minimum.
- (2) The magnitude of B_9/B_3 depends significantly on R_p/R_g while not so significantly on L_{cut}/R_g .
- (3) The optimum radius $\tilde{R}_p/R_g$ is not so sensitive to L_{cut}/R_g .

Figure 8 shows the sum of harmonic field components as a function of L_{cut}/R_g for the pole profile having the optimum pole tip radius. The total deviation from the sextupole field, $\Sigma B_n/B_3 - 1$, is less than 1.7% and has a minimum at $L_{cut}/R_g \sim 1.13$. The deviation does not seriously depend on L_{cut}/R_g . Generally, it is expected for the optimum pole shape design that B_3 is very close to B_3^0 which is the flux density expected in an ideal sextupole magnet having a pole shape of a hyperbola. The magnitude of B_3/B_3^0 is nearly equal to unity as shown in the figure. Figure 9 gives B_3/B_3^0 for various L_{cut}/R_g as a function of R_p/R_g .

The results of present calculations are compared with those of Audoin.²⁾ Comparison is made for a geometry as shown in Fig. 10. The obtained ratio B_9/B_3 at the gap radius are shown in Table I for the pole shapes with various angles α . The values

Fig. 9. The ratio of B_3 to B_3^0 , as a function of R_p/R_g for various L_{cut} 's.Fig. 10. Circular pole shape chosen to compare the results of Audoin²⁾ with ours. Shaded areas show coil space.Table I. Comparison of the Ratios of B_3/B_3 at Gap Radius for Present Calculations and Audoin's Ones.

α	R_p/R_g	L_{cut}/R_g	present $(B_3/B_3)_{R_g}$	Audoin
10°	0.21014	1.11988	0.2644	0.2943
12.5°	0.27623	1.18832	0.2121	0.2187
15°	0.34920	1.25883	0.1479	0.1490
17.5°	0.43002	1.33152	0.0835	0.1088
20°	0.51980	1.40645	0.0243	0.0610

of $(B_3/B_3)_{R_g}$ in the two calculations nearly agree with each other. The small discrepancy will be due to a different geometry of Audoin from ours. In the case of Audoin, the pole shape slightly differs from circle and, moreover, the coil effect is not taken into account.

Harmonic field contents calculated for another pole shape of Fig. 2 are presented in Figs. 11 and 12 for $L'_{cut}/R'_g = 1.34$ and 1.44, respectively. The optimum pole tip

Optimum Pole Shape for Sextupole Magnet

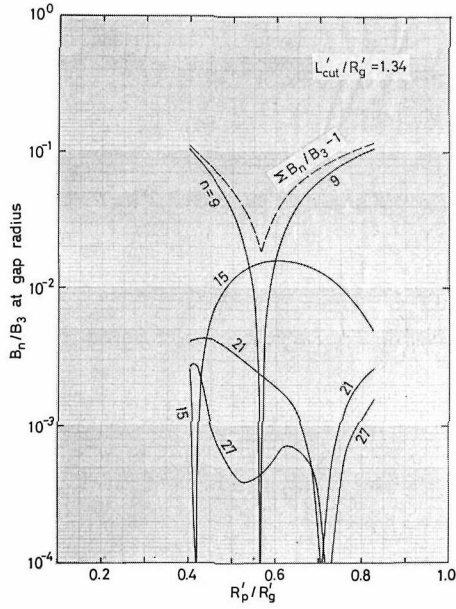


Fig. 11. Harmonic field contents for circular poles given in Fig. 2.

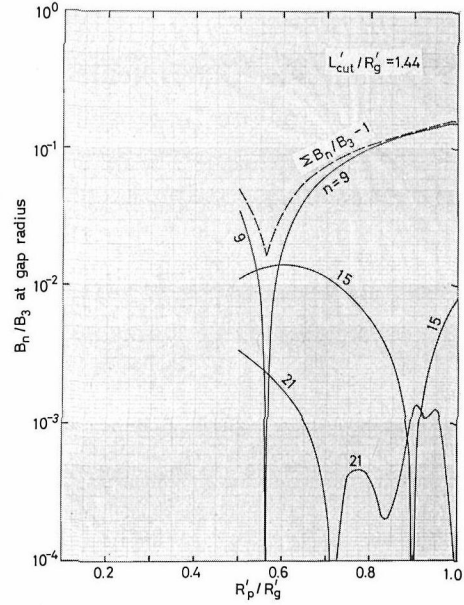


Fig. 12. The same as in Fig. 11 except for a different value of L'_{cut} .

radii are as follows, $R'_p/R'_g = 0.5651$ (for $L'_{cut}/R'_g = 1.34$) and 0.5623 (1.44) within an accuracy of ± 0.0001 .

From the present calculations, we can accurately obtain the optimum pole tip radius for a sextupole magnet, if the pole gap and the coil position are once fixed. In the coil design, L_{cut}/R_g should be larger than or as large as 1.1 , because much smaller L_{cut}/R_g gives considerably large amount of undesirable higher harmonic contents almost irrespectively of R_p/R_g .

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