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PROGRAM SCHEMAS WITHOUT GOTOS

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ABSTRACT

A programming system L for non-deterministic program schema is introduced. The principal features of L are as follows:

(1) All programs in L have two exits as subroutines in SNOBOL do.

(2) The branching function is realized by connectives • and +, and a duality is observed between them.

(3) The looping function is realized by recursive calls which is represented by a naming operator π.

(4) The fourth connective (-) has the exit-exchanging effect which has no equvalents in conventional programming languages.

(5) All predicate type operations in L may have side dffects.

In a sense, L is a proposal for goto-less programming. For example, two programs <u>if</u> p <u>then</u> α <u>else</u> β and <u>while</u> p <u>do</u> α are translated into L as follows: $p\alpha + \beta$ and $\pi x(p\alpha x + 1)$. A program $\pi x(pa + q + bx)$, however, has no equivalents in D-chart.

The meaning of a program α is defined from its computation $|\alpha|$, which is a pair of simple deterministic languages. Hence the equivalence problem in L is solvable.

0. INTRODUCTION

Ianov introduced an abstract model of computer programs and showed that the equivalence problem among them is solvable.[5] Ianov schemas permit, however, unlimitted use of GOTOs which are considered undesirable recently. In this paper we present a GOTO-less programming language system L in which loops are expressed by recursive calls.

In Section 1, we present the syntax of L and a computation $|\alpha|$ of a program α. It is easy to see that $|\alpha|$ is a pair of simple deterministic languges.[7]

The semantics is given in Section 2. Since The meaning of α is complerely determined by $|\alpha|$, the equivalence problem in L is solvable.

In appendices, the relations between our system and others are discussed.

1. PROGRAMS AND THEIR COMPUTATIONS

First, we introduce the syntax of L. We use three kinds of basic symbols and variables.

$A_0 = \{0\}$ is the singleton set of a null exit symbol.

$A_1 = \{1,a,b,c,\ldots\}$ is the set of single exit symbols.

$A_2 = \{p,q,r,\ldots\}$ is the set of double exit symbols.

$V = \{x,y,z,\ldots\}$ is the set of variables.

A program in L is a string constructed by basic symbols, variables, $\cdot$,+, π(naming operator) and parentheses:

(1) A basic symbol or a variable is a program.

(2) If $x \in V$ and α and β are programs, then so are $(\alpha \cdot \beta)$, $(\alpha + \beta)$, $(-\alpha)$ and $(\pi x\ \alpha)$.

(3) A string is a program only if it can be shown to be a program by (1) and (2).

In a program $(\pi x\ \alpha)$, the occurrence x is called a <u>name</u> and α a <u>scope of</u> the name. An occurrence of a variable x is said to be <u>bound</u> if it is a name or it is in a scope of the same name x; otherwise, <u>free</u>. A program is said to be <u>closed</u> if it has no free occurrences of variables. The notion of "normal form program" supports the definition above that a free variable in a scope is bound by a name.[1][3]

At this point, we stipulate some conventions to avoid the use of parentheses and connectives in writing programs. First, we may omit the outer pair of parentheses in a program. Second, the connectives are ordered as follows: $-$, π, $\cdot$, $+$. Third, $(-\alpha)$ may be written as $\bar{\alpha}$. Fourth, dots may be omitted. Then $p\bar{x} + a$ stands for $((p\cdot(-x)) + a)$. Fifth, $\alpha * \beta * \gamma$ denotes $((\alpha * \beta) * \gamma)$ for $* = \cdot$ or $+$.

Thereafter, $*_1, *_2, \ldots$ and $*$ stand for $\cdot$ or $+$.

In order to define the meaning of a program, we may construct an abstract machine with a push-down stack which executes non-deterministic computations under a specific interpretation. We adopt, however, another way because it is easier for us to utilize a well known result in formal language theory.

Two alphabets Σ and Σ_V denote the sets $\{a. \mid a \in A_1-\{1\}\}$ $\{p. , p_+ \mid p \in A_2\}$ and $\Sigma \cup \{x. , x_+ \mid x \in V\}$ respectively. The set of all words generated by an alphabet Z is denoted by Z^* and the empty word, λ. If $W_1, W_2 \subseteq Z^*$, then $W_1W_2 (\subseteq Z^*)$ denotes the set $\{w_1w_2 \mid w_1 \in W_1, w_2 \in W_2\}$. Let $W. , W_+ \subseteq \Sigma_V^*$, $W=(W. , W_+)$ and $w \in \Sigma_V^*$. Then $w[W/x]$ means the set of all words obtained from w by replacing each occurrence of $x.$ in w by some $w. \in W.$ and each occurrence of x_+ in w by some $w_+ \in W_+$; i.e., $w[W/x] =$ $\{v_0w_{*1}v_1w_{*2}\cdots w_{*k}v_k \mid v_0x_{*1}v_1x_{*2}\cdots x_{*k}v_k = w \wedge v_0,\ldots,v_k \in (\Sigma_V-\{x.,x_+\})^* \wedge *1,\ldots,*k \in \{\cdot,+\} \wedge w_{*1} \in W_{*1} \wedge \ldots \wedge w_{*k} \in W_{*k}\}$. Furthermore, if $W.', W_+' \subseteq \Sigma_V^*$ and $W' = (W.', W_+')$, then we stipulate that $W'[W/x] = (\bigcup_{w \in W.'} w[W/x], \bigcup_{w \in W_+'} w[W/x])$. If α is a program, then a <u>computation of</u> α, $|\alpha| = (|\alpha|., |\alpha|_+)$ is defined as follows ($|\alpha|.$ and $|\alpha|_+$ are called a <u>dot computation</u> and <u>plus computation of</u> α respectively.):

$$|0| = (\phi,\phi)$$

$$|1| = (\{\lambda\},\phi)$$

$$|a| = (\{a.\},\phi), \qquad \text{if } a \in A_1-\{1\}$$

$$|p| = (\{p.\},\{p_+\}), \qquad \text{if } p \in A_2$$

$$|x| = (\{x.\},\{x_+\}), \qquad \text{if } x \in V$$

$$|\alpha\cdot\beta| = (|\alpha|.|\beta|. , |\alpha|_+ \cup (|\alpha|.|\beta|_+))$$

$$|\alpha+\beta| = (|\alpha|. \cup (|\alpha|_+|\beta|.) , |\alpha|_+|\beta|_+)$$

$$|\bar{\alpha}| = (|\alpha|_+ , |\alpha|.)$$

$$|\pi x\alpha| = \bigcup_{n=0}^{\infty} |\alpha|_x^n, \quad \text{where} \quad \begin{cases} |\alpha|_x^0 = (\phi,\phi), \\ |\alpha|_x^{n+1} = |\alpha|[|\alpha|_x^n/x]. \end{cases}$$

Infact $|\alpha|., |\alpha|_+ \subseteq (\Sigma \cup \{x., x_+ \mid x \text{ is a free variable in } \alpha\})^*$. Hence, if α is closed, then $|\alpha|., |\alpha|_+ \subseteq \Sigma^*$.

Example 1.1 $|pa + b|. = \{p.a.\ , p_{+}b.\}$, $|pa + b|_{+} = \phi$.
$|pax + 1|. = \{p.a.x., p.a.x_{+}, p_{+}\}$, $|pax + 1|_{+} = \phi$.
$|\pi x(pax + 1)|. = \{p_{+}, p.a.p_{+}, p.a.p.a.p_{+},\ldots\}$, $|\pi x(pax + 1)|_{+} = \phi$.
$|px + q|. = \{p.x., p.x_{+}q., p_{+}q.\}$, $|px + q|_{+} = \{p.x_{+}q_{+}, p_{+}q_{+}\}$.
$|\pi x(px + q)|. = \{p_{+}q., p.p_{+}q., p.p_{+}q_{+}q., p.p.p_{+}q., p.p.p_{+}q_{+}q.,\ldots\}$,
$|\pi x(px + q)|_{+} = \{p_{+}q_{+}, p.p_{+}q_{+}q_{+}, p.p.p_{+}q_{+}q_{+}q_{+},\ldots\}$.

Korenjak and Hopcroft introduced the class of "simple deterministic languages" in their paper[7]. Now we adopt an extended definition that the singleton set $\{\lambda\}$ also is said to be simple deterministic.

Theorem 1.1 For any α, $|\alpha|.$ and $|\alpha|_{+}$ are simple deterministic.

Theorem 1.2 It is undecidable whether $|\alpha|_{*} \cap |\beta|_{*} = \phi$ for arbitrary α and β, for each $*$.

2. SEMANTICS

In this section, we describe how nondeterministic computations of a program go on a specific domain.

Let D be an arbitrary nonempty set and $\mathfrak{F}(D)$ the class of all partial functions: $D \to D$. An interpretation I in L is a pair (D, θ), where θ is a function: $\Sigma_V \to \mathfrak{F}(D)$. It is extended to $\Sigma_V{}^* \to \mathfrak{F}(D)$ as follows:

$$\begin{cases} \theta(\lambda) = \lambda uu \ (= \text{the identity function on D}), \\ \theta(wc) = \lambda u[\theta(c)(\theta(w)(u))], \quad \text{if } w \in \Sigma_V{}^* \text{ and } c \in \Sigma_V, \end{cases}$$

where $\theta(w)(u)$ = undefined implies $\theta(c)(\theta(w)(u))$ = undefined.

If $W \subseteq \Sigma_V{}^*$ and $u \in D$, then $\theta(W)(u)$ denotes the set $\{\theta(w)(u) \mid w \in W, \theta(w)(u) = \text{defined}\}$. We write $\alpha =_I \beta$ if $\theta(|\alpha|_*)(u) = \theta(|\beta|_*)(u)$ for any u and *. Furthermore we write $\models \alpha = \beta$ if $\alpha =_I \beta$ for any I.

Theorem 2.1 $\models \alpha = \beta$ iff $|\alpha| = |\beta|$.

Theorem 2.2 It is decidable whether $\models \alpha = \beta$ for any α and β.

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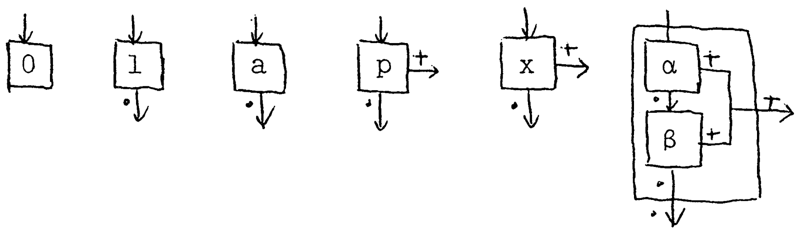
APPENDICES

A. TRANSLATION OF PROGRAMS INTO FLOWCHARTS

For any program α, its flowchart equivalent has zero, one or two exits as subroutines in SNOBOL do.[4]

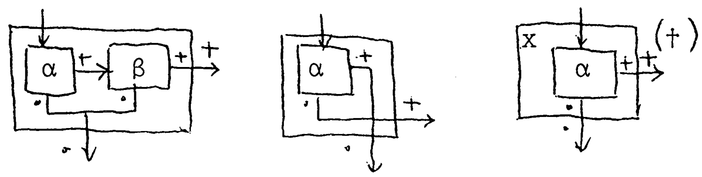
program 0 1 a p x $\alpha\beta$

flowchart



program $\alpha+\beta$ $\bar{\alpha}$ $\pi x\alpha$

flowchart



(†) Free variables x in α are regarded as equal to the whole program α.

B. TRANSLATION OF D CHARTS INTO PROGRAMS

Any D-chart[2] is translated in L as follows:

(1) $a \rightarrow a$,

(2) α then $\beta \rightarrow \alpha\beta$,

(3) if p then α else $\beta \rightarrow p\alpha + \beta$,

(4) while p do $\alpha \rightarrow \pi x(p\alpha x + 1)$.

Note that if α is of this type, then α contains at most one variable x and $|\alpha|_+ = \phi$. It is impossible to convert any program in L into D-chart. For example, $\pi x(pa + q + bx)$ has no flowchart equivalents.[6]