

グラフの上のランダムな衝突モデル
A RANDOM COLLISION MODEL ON GRAPHS

統計数理研究所・総合研究大学院大学 伊藤 栄明

Yosiaki Itoh

The Institute of Statistical Mathematics
and The Graduate University for Advanced Studies

We investigate a random collision model on graphs which is an idealized situation for interacting types. The types may represent species, genotypes, brands, factions or other classifications.

Consider a graph G of a finite nonempty set $V = V(G)$ of p points together with a specified set $X(G)$ of q unordered pairs of distinct points. A pair $x = \{u, v\}$ of points in X is called a line of G and x is said to join u and v . The points u and v are adjacent, u and v are incident with each other, as are v and x .

$W(u_1, u_2, \dots, u_k)$ is the set of all pairs of distinct points $u_1, u_2, \dots, u_k$. We consider a population of particles of p types, $1, 2, \dots, p$ which consist of $N_1(t), N_2(t), \dots, N_p(t)$ respectively at time t . We write

$$\vec{N}(t) = (N_1(t), N_2(t), \dots, N_p(t)) \quad (1)$$

It is assumed that initially $\vec{N}(0) = \vec{\alpha}$, that in each unit of time a pair of particles are chosen at random from the n particles, and that all possible pairs are equiprobably chosen. There exist adjacency relations between the types as defined by the graph G , with points $V(G) = \{1, 2, \dots, p\}$ and adjacencies $X(G)$. Further it is assumed that a choice of a pair of particles of different types i and j , for which $(i, j) \in X(G)$, changes the pair of particles of the type i with probability $\frac{1}{2}$ and of the type j with probability $\frac{1}{2}$. A choice of a pair of particles of different types $(i, j) \in W(V) - X(G)$ does not make any effect, as well as a choice of a pair of the same type.

For each point $i \in V(G)$, consider variable x_i . $S_k(x_1, x_2, \dots, x_p)$ is k -th order elementary symmetric function. $W(u_1, u_2, \dots, u_k)$ is the set of all pairs of distinct points $u_1, u_2, \dots, u_k$. Let $|S|$ be the number of the elements of the set S . Consider

$$T_{k,l}(x_1, x_2, \dots, x(p)) = \sum_{|W(u_1, u_2, \dots, u_k) \cap X(G)|=l} x_{u_1} x_{u_2} \dots x_{u_k} \quad (2)$$

$\{N(t), t = 0, 1, 2, \dots\}$ is a Markov chain with transition probabilities defined by

$$Pr(\vec{N}(t+1) = \vec{n}_{ij} \mid \vec{N}(t) = \vec{n}) = \frac{n_i n_j}{n(n-1)} \quad (3)$$

for $(i, j) \in X(G)$, where $\vec{n} = (n_1, n_2, \dots, n_p)$, $n = \sum_{i=1}^p n_i$, $\vec{n}_{ij}$ is a vector with i -th component $n_i + 1$, j -th component $n_j - 1$, and all other components equal to those of $\vec{n}$. We have

$$Pr(N(t+1) = \vec{n} \mid \vec{N}(t) = \vec{n}) = \sum_{i=1}^p \frac{n_i^2}{n(n-1)} + \sum_{(i,j) \in W(V)-X(G)} \frac{2n_i n_j}{n(n-1)} \quad (4)$$

Theorem. Let $Q(t)$ be the number of existing types at time t .

$$Pr(Q(t) \geq k) \geq \frac{T_{k,0}(\vec{N}(0))}{M C_k \left(\frac{n}{M}\right)^k} \quad (5)$$

where $M = \max\{k \mid W(u_1, u_2, \dots, u_k) \cap X(G) \neq \emptyset\}$.

参考文献

- [1] Itoh, Y. (1973). On a ruin problem with interaction, *Ann. Inst. Statist. Math.*, 25, 635-641
- [2] Itoh, Y. (1979). Random collision models in oriented graphs, *J. Appl. Prob.*, 16, 36-44
- [3] Itoh, Y. (1982). Random collision models on graphs, *Pacific Area Statistical Conference*, 27-30
- [4] Harary, F. (1969). Graph theory, Addison-Wesley Publishing Company, Inc., Massachusetts.